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Evolution of population-resource dynamics models

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I. Introduction

An economy has a potential to outgrow its supporting ecosystems, leading to a collapse.

In economics, there are two types of literature on resources and growth. The first type consists of models that assume that advances in technology are fast enough to overcome the increasing scarcity of renewable resources (*e.g.*, Solow 1999), or even nonrenewable resources (*e.g.*, Stamford da Silva, 2008; Cheviakov and Hartwick, 2009). The other type is characterized by models that accept the fluctuation of economic growth driven by resource dynamics. Brander and Taylor's (1998) so-called BT model, originally designed to replicate the population and resource dynamics of Easter Island (henceforth E.I.), belongs to this category. Since its initial appearance, the BT model has generated many descendants (Dalton and Coats, 2000; Erickson and Gowdy, 2000; Maxwell and Reuveny, 2000; Reuveny and Decker, 2000; Anderies, 2003; Pezzey and Anderies, 2003; Prskawetz et al., 2003; Basener and Ross, 2005; Dalton et al., 2005; Nagase and Mirza, 2006; Good and Reuveny, 2006; D'Alessandro, 2007; Basener et al., 2008; de la Croix and Dottori, 2008; Taylor, 2009).

This study examines existing BT-type models through the following set of attributes: (1) population growth, (2) substitutability, (3) innovation, (4) capital accumulation, (5) property rights and institutional designs, and (6) modelling approach. By integrating the existing models through a common set of attributes, this study aims to provide a better understanding of population and resource dynamics models in general, and the BT-type models in particular, that are suited to study the sustainability of certain types of economies, as revealed by the following sections.

Our comparative analysis of the models yields the following conclusions. An endogenous, rather than exogenous, population growth function allows a model to incorporate the effect of economic activities on population, through variables that reflect individuals' economic decisions. The issues of substitutability, innovation and capital accumulation are intertwined; a model that sheds light on the effect of an endogenous technological change on substitutability between natural and man-made capital or goods facilitates our investigation of sustainability issues. Allowing a model to internalize inter-generational externalities in resource use by incorporating property right changes and/or institutional designs is a useful exercise, but careful attention is needed for the consistency between such an arrangement and the mathematical representation of the depicted economy. Finally, the common criticism regarding the use of convenient mathematical assumptions applies to the existing BT-type models, but computer simulation allows for a wider array of functions that can better represent the intended relationships between the relevant variables.

The rest of the paper is organized as follows. Section II provides a brief introduction to the BT model. Section III compares and integrates the BT-type models through the above-mentioned six attributes. Section IV concludes our analysis.

II. Basic characteristics of a BT-type model

Figure 1 depicts the period-by-period material and cash flow and agents' behaviour in a typical BT-type model. A typical BT-type model has the following characteristics. It depicts a small, closed economy. It has a renewable resource (S) to be used to produce two types of good, a harvested good (H) and a manufactured good (M). The resource dynamics is hence given by the resource growth and harvesting activities. An additional input for each sector is labour (L_H, L_M), or population ($L \equiv L_H + L_M$), and population growth is endogenously driven by a fertility function. The economy is decentralized in the sense that the relative price of the goods and the wage are determined by market forces. Although people as consumers individually maximize utility in each period, the original BT model has one sector-level production function for each sector. In the original BT model, the aggregate production function for each sector is linear in labour, given the existing resource and population stock sizes. Therefore, a fully-decentralized (and possibly primitive) interpretation of production activities is possible, namely, each worker independently has her one-person production activity and receives the "wage" (w) that equals her marginal revenue product of one unit of labour, given the market prices of the two goods. Finally, in most of the BT-type models, individuals behave in a myopic manner; these agents do not maximize utility across multiple time periods and instead focus on the given period. Therefore, most of the BT-type models consist of a combination of agents' static optimization in each time period, taking the sizes of the resource stock and population as given, and transitional processes from one period to the next given by a set of dynamic equations for these stock variables. Figure 2 shows this mechanism using the static equilibrium values of the original BT model. A major appeal of the BT-type models is its ability to demonstrate potential volatility of an economic system. Also, its simplicity leaves room for incorporating variables that can address key issues of sustainability.

III-1. Population Growth

Although population growth has been treated as exogenous in many studies of economic growth and natural resources (*e.g.*, Dasgupta and Heal, 1974; Elíasson and Turnovsky, 2004; Economides and Philippopoulos, 2008), endogenous population dynamics is indispensable for models whose purpose is to address sustainability of an economic system. Empirical case studies support that there is a feedback mechanism between population and natural resource (*e.g.*, Diamond, 2004). In general, population dynamics models use ordinary differential equations in the form of:

$$\frac{dL/L}{dt} \equiv f(\text{weather, food, predators, etc.}),$$

where L denotes the population size. Population change per time period is typically defined as a summation of fertility at the individual level.

Since a feedback mechanism between population and natural resource is essential, it is better to discuss population dynamics along with resource dynamics. The most popular framework for modelling this type of predator-prey interactions has the following structure (Turchin, 2003):

dS/dt = "prey growth in the absence of predators" – "total killing rate by predators"

where S denotes the natural resource stock and

dL/dt = "predator growth (or decline) in the absence of prey" + "conversion of eaten prey into new predators."

The basic idea is that the right-hand side of each equation consists of two parts. The first part of each equation indicates the independence of one stock variable from the other, while the second part shows the interdependence between the two stock variables.

The original BT model uses Volterra's (1931) framework in which a natural resource grows logistically in the absence of the harvest (as cited in Turchin, 2003):

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{K}\right) - H,$$

where K denotes the carrying capacity for this resource, r denotes the intrinsic growth rate, and H denotes the predator L's harvest level (Figure 2).[1] The population growth

[1] One of the standard frameworks of population-resource dynamics in biology is the Lotka-Volterra

function in the BT model is given by:

$$\frac{dL}{dt} = \left(b - d + \phi \frac{H}{L} \right) L ,$$

where the amount of H in each static equilibrium depends on S (Figure 2). The BT model expresses Malthusian population dynamics in which population growth consists of two parts: the net birth rate ($b - d$) that is independent of the level of per-capita food consumption (H/L) and the fertility rate ϕ that affects the population growth only with nonzero level of H/L. Since $b - d$ is assumed to be negative, in the absence of harvest from the nature the population will be extinct.

This population growth function has two notable traits. First, the population growth rate is linear in H/L, which implies that the more they eat the more they produce offspring. This feature may contradict situations in some developed countries where there is a negative relationship between income level and population growth.[2] Second, the function assumes that consumption of the manufactured good (that could be regarded as a composite of, *e.g.*, medicine, fishing equipments, boats, and agricultural equipments) does not affect population growth. Brander and Taylor (1998) do not include such manufactured goods, because, as Reuveny and Decker (2000) point out, in equilibrium the per-capita manufactured good is always a constant: $M^*/L = (1 - \beta)$, where $1 - \beta$ is a parameter representing consumers' preferences for good M (Figure 2).

(L-V) model, a bilinear system that is the simplest possible version of this type of interaction. The original L-V model, however, is not very realistic, and there have been many descendants with other functional forms (Turchin, 2003).

[2] Galor and Weil (2000) develop a unified growth model that captures the transition from a Malthusian to a Post-Malthusian regime.

However, as we address later the effect of the consumption of manufactured goods on population growth matters when substitutability issues and the effects of capital accumulation are taken into account.

Descendants of the BT model fall into two groups in terms of population dynamics. The models in the first group use the population growth functions of the original BT model, either as it is or with slight modification. The models in the second group employ population growth functions that are very different from the one used in the original BT model.

Regarding the models in the first group, Dalton and Coats (2000), Pezzey and Anderies (2003), Dalton et al. (2005), Good and Reuveny (2006), and Taylor (2009) use the same equation of motion as that of the original BT model, whereas several others use variations. Erickson and Gowdy (2000) focus on the effect of manufactured capital (A) accumulated from the harvested good. Compared with the archaeological evidence of E.I., the population in the original BT model peaks about 200 years too early. To explain this gap and improve the fitness of the model (*i.e.*, to obtain the estimate of population dynamics that is more consistent with the archaeological evidence), the authors introduce the third equation of motion for A:

$$\frac{dA}{dt} = H - \delta A,$$

where parameter δ represents the capital depreciation rate. The accumulated capital contributes to the fertility rate, with the lag of 100 years (denoted as A_{100}):

$$\frac{dL}{dt} = L\left(b - d + \phi\frac{H}{L} + \phi\alpha\beta A_{100}\right),$$

where α and β are parameters representing the productivity of the H sector and consumer's preferences for *H*, respectively (Figure 2).

This approach invites us to contemplate its assumptions and formulation. First, this approach reflects the fact that individuals' well-being, including health and fertility, improves with the consumption of a capital good. The chosen lag period improves the fitness of the model for this specific case; as a general rule, theoretical reasoning and/or empirical evidence should guide such a choice. An alternative approach may be to let the effect of the capital good be felt immediately, with a coefficient that represents the marginal effect. Second, an interpretation of the supposed mechanism of capital formulation would be helpful to better understand the portrayed economy. In the above model, people consume the harvested good, while at the same time accumulating the same amount of the good as capital. That is, the harvested good in each period is used for both immediate consumption and capital accumulation. Whether capital should be accumulated from the harvested good or the manufactured good is another issue to consider. In another BT-type model by Anderies (2003), investments are made on the portion of the manufactured good that is set aside separately from immediate consumption purposes, to be accumulated for capital formation.

D'Alessandro (2007) provides a more general framework to account for the heterogeneity of environmental development paths. His model includes two types of natural resources: a renewable resource (forest) and an inexhaustible one (land). This model can explain the situation in which people may continue to exist as they exhaust the renewable resource stock, as it may have been the case with E.I. This is expressed as follows:

$$\frac{dL}{dt}=\left[\gamma\frac{C}{L}+\phi\frac{H}{L}+\left(b-d\right)\right]L,$$

where C denotes "corn" obtained from land, the harvested good H is obtained from forest, and γ and ϕ are the caloric units (or fertility rates) of consumption of C and H, respectively. Since land is assumed to be inexhaustible, people can survive even after depleting the forest.[3] An issue to consider here is the assumption of the perfect substitutability between the two types of goods, whose validity would depend on the characteristics of the specific cases.

Reuveny and Decker (2000) incorporate population management into the population dynamics. They replace the linear fertility coefficient ϕ in the original BT model with a function:

$$F = \phi\left(\frac{H}{L}\right)^x$$

that can be concave ($0 < x < 1$), linear ($x = 1$, the original BT case), or convex ($x > 1$). The characteristics of this fertility function depend on the value of x, a policy instrument. Although the authors' purpose for introducing x to the model is to examine the effect of population management, their population function can also address the criticism that, in the original BT model, fluctuation of the population size can be arbitrarily large when harvest is abundant (Basener and Ross, 2005). By employing $0 < x < 1$, growth can be tamed to a reasonable level. Also, nonlinearity of a fertility function in consumption of goods would be consistent with empirical evidence (the "Demographic Transition").

Maxwell and Reuveny (2000), followed by Prskawetz et al. (2003), relate

[3] In this model, good C replaces good M. C has a production function of labour input only, as the production function of M in the original BT model. C also contributes to the utility function in the same manner as M does in the original BT model. Therefore, another way to interpret this model is that the manufactured good contributes to fertility.

natural resource scarcity to emergence of conflicts. They assume that when per-capita resource level S/L is less than a given threshold level $\bar{V}$, conflicts emerge and increase the death rate, expressed as follows:

$$\frac{dL}{dt} = \left((b - \eta d) + \phi \frac{H}{L} \right) L,$$

where η represents the effect of conflicts. η is greater than 1 under conflicts and is equal to 1 otherwise. While the authors assume discontinuous changes in the dynamics once conflicts set in, Prskawetz et al. (2003) propose continuous changes by assuming that the death rate is a function of a threshold for conflict and natural resource scarcity, defined as follows:

$$\frac{dL}{dt} = \left[b - \eta\left(\bar{v}, \frac{S}{L} \right) d + \phi \frac{H}{L} \right] L\,; \;\; \eta\left(\bar{v}, \frac{S}{L} \right) = 1 + \frac{\eta_{max} \bar{v}^{p}}{\bar{v}^{p} + \left(\frac{S}{L} \right)^{p}}.$$

Here, η is a logistic function of S/L. $\eta_{\max}$ represents the maximum impact that a conflict may exert on the death rate. When the per-capita resource becomes very low, the death rate is at its maximum, *i.e.*, $\eta = 1 + \eta_{\max}$. Together with two more conflict-driven parameters that affect labour allocation and resource growth, both studies show that conflicts can serve as a stabilizing feedback mechanism as long as it becomes active early enough.

In contrast, models in the second group, proposed by Basener and Ross (2005) and Basener et al. (2008), abandon the framework used in the original BT model and adopt the logistic predation originally proposed by Leslie (1948), expressed as follows (Basener and Ross, 2005):

$$\frac{dL}{dt} = \left[a\left(1 - \frac{L}{S}\right)\right] L ,$$

where a is the intrinsic growth rate of population.[4] Although without the fertility component that represents the conversion of eaten prey into new predators, these models show better fitness to the archaeological data. Another advantage of this population function is that they can avoid the BT model's aforementioned problem of arbitrarily large population growth; with the logistic function, the population growth rate is capped by the nature's carrying capacity. Meanwhile, this population growth framework also has a disadvantage. The per-capita consumption (and hence production) level of the harvested good is constant by assumption, *i.e.*, scarcity does not affect individuals' economic decisions, contradicting neoclassical economic theory.[5]

Although the resource side of equations of motion lacks variations across the models (most BT-type models use the same logistic growth function minus harvest, as the equation of motion), we examine one variation given by D'Alessandro (2007) and Taylor (2009) who employ a critical depensation growth function:

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{K}\right)\left(\frac{S}{\underline{K}} - 1\right) - H .$$

$\underline{K}$ represents the "tipping point" of the resource stock level below which the regeneration rate becomes negative (Taylor, 2009). This arrangement allows their models to address the irreversibility problem. A tipping point becomes a determinant of

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4 Basener et al. (2008) propose the discrete version of the model.

5 Another study by de la Croix and Dottori (2008) takes a different approach. Instead of the BT-type predator-prey system, they incorporate competition between two tribes. It is an overlapping generations model in which each tribe chooses its fertility rate to maximize its tribal utility. Their approach is considerably different from those of the other studies, and we do not fully explore it here.

the stability of the interior steady states in D'Alessandro's (2007) model, and Taylor (2009) shows that it is one of the three preconditions for the system to reach an environmental crisis, including the collapse of the system.[6] Ecological studies support critical depensation growth functions, and these authors' results warrant further investigation of the use of this type of function.

In conclusion, there are two types of the population dynamics among the BT-type models, and each has different features. As Basener et al. (2008) suggest, unlike fundamental laws of physics there is no *right* single differential equation for the population dynamics. Hence we should choose one based on the purposes and the corresponding assumptions of the model. An advantage of the framework used in the original model is that it incorporates neoclassical economics considerations into the population growth in the sense that the harvested good is obtained from agent's optimization. There are pros and cons to the population growth functions explored in this section, and we further propose two directions for extending the original BT model to enhance its theoretical basis and empirical relevance in application. First, incorporating the manufactured good into the population growth function allows the model to capture the effect of broader economic activities on the population dynamics. Second, population growth as a function of the nature's resource capacity allows the population growth rate to be aligned with, or constrained by, the surrounding nature's carrying capacity.

III-2. Substitutability

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[6] Taylor (2009) defines an environmental crisis as "a dramatic, unexpected, and irreversible worsening of the environment leading to significant welfare losses."

Opinions on economic models that presume various degrees of substitutability between man-made and natural inputs are based on both theoretical and empirical arguments. Theoretically, for a constant elasticity of substitution (CES) function strong sustainability requires that the elasticity of substitution (henceforth denoted by σ) between man-made good (or input) and environmental amenities (or natural resources) must be less than one (Gerlagh and van der Zwaan, 2002; Lawn, 2003).[7] Using a nested CES production function and multinational data, a recent study by Markandya and Pedroso-Galinato (2007) provides two sets of estimates of σ: one set of values based on past studies (1971-1998), and another set freshly estimated by the authors, using more recent data. The first set consists of low values of σ between capital and energy (0.87), labour and energy (0.42), and labour-capital composite and energy (0.42 and 0.5). The second set consists of higher values, *e.g.*, between capital-human resource-energy composite and land (1.00), and capital-human resource-labour composite and energy (1.00). The only low figure from the second set is between capital and energy (0.37). Compared with the estimated value of 2.0 by Nordhaus and Tobin (1972), the estimates given by this more recent study suggest that the values of σ are lower than previously thought. Since we expect that the values of σ change as economies evolve, changes in estimated values as described above are not surprising, although a common view is that as economies develop the relationship between energy and capital tends to evolve from being complements to substitutes (Ayres, 1998). As for substitutability in consumption, although low degrees of substitutability have been observed in various surveys (Gelso and Peterson, 2005), we are not aware that empirical literature (in real, rather than

[7] We focus on what is known as Hicksian or full σ (Markandya and Pedroso-Galinato, 2007).

hypothetical, settings) on this issue is pointing to any particular direction.

Most of the BT-type models so far have not addressed substitutability issues. As for production functions, some of the BT-type models continue to employ variations of the original BT model's linear production functions in labour (L) for both good H and M: $H = \alpha SL_H$ and $M = L_M$, where α is an exogenously given productivity parameter and L_H and L_M ($L_H + L_M = L$) are the labour force allocated to the two sectors (Figure 2). Meanwhile, other BT-type models employ variations of Cobb-Douglas (C-D) functions. This latter approach allows the authors to introduce inputs in addition to labour. Among these models, Anderies' (2003) model is the most general in the sense that both H and M are functions of labour and manufactured capital (A):[8]

$$H = E_H S^{\alpha_S} L_H{}^{\alpha_H} A_H{}^{1-\alpha_H}$$

and

$$M = E_M L_M{}^{\alpha_M} A_M{}^{1-\alpha_M} ,$$

where E_H and E_M are efficiency factors (to be explained more in section III-3), α_S, α_M, and α_H are between 0 and 1, and $A_H + A_M = A$. As L and S, A is a stock variable and is given during each period. While introducing man-made capital is critical to address substitutability between natural and man-made inputs, C-D functions limit σ between inputs to be one. Nagase and Mirza (2006) employ a CES function $M = [\theta H_M{}^{\rho} + (1-\theta)L_M{}^{\rho}]^{1/\rho}$ where H_M denotes the amount of H used as an input. Their study provides

[8] Dalton et al. (2005) modify the production function for M to be a C-D function of L and land, a non-depletable but fixed amount of resource. De la Croix and Dottori (2008), due to their focus on the analysis of conflict and bargaining between clans, omit good M from the model and adopt for H a C-D production function of S and L.

sensitivity analyses with respect to various (exogenously given) values of σ.

A different approach by Prskawetz et al. (2003) adopts a production function $H(S, L) = eSL_H(fL_H + S)^{-1}$, where e and f are positive parameters (while keeping the linear production function for M). As a C-D or CES does, this function exhibits diminishing marginal returns and has a constant σ (= 0.5). Meanwhile, this function has a unique feature: for a given level of an input, the output is asymptotically bounded from above as the other input level goes to infinity.[9]

As for substitutability in consumption, most BT-type models maintain the C-D utility function adopted by the original model (Figure 2). In contrast, Nagase and Mirza (2006) employ a CES function for the utility function as well as the above-mentioned production function. Their simulation results show that reduced substitutability in both production and utility functions makes the population and resource dynamics more volatile. They also observe the fluctuation of agents' utility levels to address the issue of changes in agents' well-being over time.

Most of the existing BT-type models have population growth functions with endogenously determined per-capita consumption of the harvested good; therefore, by adopting utility functions that allow for varying degrees of substitutability these models can portray the effect of substitutability in consumption on harvesting activities, and hence on the population and resource dynamics. In addition, adoption of production functions with man-made capital and endogenous innovation will further allow these models to examine how consumers' preferences regarding substitutability affect the relative price of the two goods and drive the innovation path--these topics are addressed

[9] Prskawetz et al. (2003) classify this function as a Monod-type, although a classical Monod-type kinetics function does not have the predator stock (L) as part of the numerator.

in the next two subsections.

While the empirical investigation of σ still awaits further studies, analysing the existing BT-type models in terms of substitutability yields some points to consider. First, extending the existing models to allow for analyses of the effect of varying degrees of substitutability both in production and consumption on population and resource dynamics would serve one of the primary purposes of the BT-type models: studying sustainability. Second, allowing σ to evolve over time endogenously has both theoretical and empirical bases--this is to be addressed in **III-3**. Finally, while not explored by the authors, the production function of Prskawetz et al. (2003) provides another channel to address substitutability issues. As described above, this production function caps the output level, being consistent with the notion of strong sustainability. Since the amount of harvest is bounded by the existing stock size during any given time period and ultimately by the nature's carrying capacity, in addressing substitutability issues this function could be better suited for the manufactured good rather than the harvested good. Combined with the introduction of man-made capital as an input, this function allows us to examine the trade-off between man-made and natural capital under the strong sustainability criterion.

III-3. Innovation

Non-exogenous technological change can be distinguished into endogenous technological change (ETC) and induced technological change (ITC). A technological change caused by economic activities represented by endogenous variables in the model is an ETC, while a policy-induced additional change to ETC can be considered an ITC

(Edenhofer et al., 2006).[10] The economics literature on ETC and ITC is flourishing; for the purpose of this paper it suffices to say that there are strong supports for ETC and ITC in both theoretical and empirical literature. Economic theory dictates that economic agents respond to prices that reflect relative resource scarcity, and empirical studies on resource price and innovation support this implication (*e.g.*, Popp, 2002; Khatri et al., 1998; Thirtle et al., 1998).[11] Another critical issue surrounding ETC and ITC is the effect of learning curves. Studies show that in addition to the price of resources, existing knowledge base affects the innovation path (Gritsevskyi and Nakićenović, 2000; Köhler et al., 2007).

The original BT model has no innovation processes; it and some of the BT-type models provide comparative statics analyses of the steady state, with respect to parameters such as α (harvest productivity) , r (intrinsic growth rate), and K (carrying capacity). With the original BT model, the steady-state resource stock size S^* decreases with an improvement in the harvesting technology (an increase in α). The steady-state population size L^* increases with an innovation in biotechnology (an increase in r or K), while the effect of an increase in α on L^* depends on the steady-state resource growth level. Anderies (2003), as described earlier, adopts C-D production functions for H and M with efficiency factors E_H and E_M. This model also has parameter η, an indicator of negative impacts of agriculture on the natural resource base. The author provides the boundary combinations of the values of E_H and E_M for the existence of a steady state and a comparative static analysis on changes in η. His analysis shows that, with the

[10] Alternatively, the term ITC can be used to represent both ETC and ITC (Grubb et al., 2002).

[11] Popp (2002) uses U.S. patent data. Khatri et al. (1998) and Thirtle et al. (1998) provide case studies on innovation and agriculture in the UK and South Africa, respectively.

given set of parameter values, higher productivity in both sectors (larger E_H and E_M) increases the likelihood of population overshooting and collapse, and that reduced externalities (smaller η) do not prevent this scenario without systemic changes in the feedback loop between resource use and population. D'Alessandro (2007) examines the effect of an innovation introduced as a shock that increases the value of α. With the given set of parameter values, the author shows that a productivity shock reduces the resilience of the internal steady state and increases the risk of the collapse of S. As described in **III-1**, thanks to the non-depletable resource (land), even with the collapse of the renewable resource S this model yields a steady state with a positive population size. The author also examines the effect of technology shocks that yield increases in land productivity. Such changes can increase the steady-state population size but also reduces the resilience of the internal steady state.

Two of the BT-type models adopt time-dependent exogenous technological change and ITC. Reuveny and Decker (2000) employ time-dependent logarithmic and exponential growth functions for K, r, and α. Their simulation results show two intuitively sound results: innovation in harvesting technology, ceteris paribus, can cause population crash due to resource depletion, and higher resource growth rates, ceteris paribus, can sustain larger population sizes. One outcome that awaits an interpretation is that exponential growth of carrying capacity can trigger a feast-famine cycle. Contrary to all other studies, Dalton et al. (2005) incorporate ITC into their model. In this model, changes in L (embodiment of the existing knowledge and experience with technologies) affect the sizes of α and r, defined by the following difference equations: $\alpha_t = \alpha_{t-1}[1 +$

$\xi_\alpha\lambda(dL/L)]$ and $r_t = r_{t-1}[1 + \xi_r\lambda(dL/L)]$ for $dL > 0$ and $\alpha_t = \alpha_{t-1}$ and $r_t = r_{t-1}$ otherwise.[12] They find that, compared with the original BT model, making α and r endogenous following these rules worsen the feast-famine cycle. The negative effects (exacerbated feast-famine cycle with lower S^* and/or L^*) of increases in α or positive effects (more stable system with higher S^* and/or L^*) of increases in r, *ceteris paribus*, are qualitatively similar to those of above-mentioned studies. One puzzling aspect of their simulation result is that increases in r *alongside* increases in α affect the system negatively.

From these studies, we notice the general effects of certain types of technological changes, namely, stimulating harvesting technologies have negative effects whereas bio-technologies have positive effects (meaning of negative and positive as described in the previous paragraph). Meanwhile, the analyses conducted so far are mostly of innovation as one-time or time-dependent exogenous changes, not allowing us to study the effect of continuous innovation driven by scarcity and market prices on the stability of the system and its agents' well-being. One model by Dalton et al. (2005) employs ITC, and each innovation process is a function of the population L that represents the knowledge base of the economy. Such an innovation function could include other variables that allow the technology to evolve in response to changing relative scarcity of productive resources, including man-made or natural capital.

To conclude, since a major purpose of analysis using the BT-type models is to understand the interactions among population, resource use, and the stability of the

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[12] Parameter λ (> 0) represents the marginal effect of population changes. Parameters ξ_α and ξ_r represent the status of institutional arrangement of property rights, to be explained in **III-5**.

economy, letting this type of model depict the transitional adjustment process by incorporating both scarcity-driven ETC and policy-driven ITC that address additional needs for the depicted economy to reallocate resources is a beneficial direction for extending these models.

III-4. Capital Accumulation

Despite the remarks we have made so far on introducing capital accumulation into the models, the main obstacle for most of the existing BT-type models in motivating the agents to accumulate and maintain capital stocks is the agents' time preferences. Accumulation and maintenance of any form of capital (man-made or natural) takes place when agents in the modelled economy care about their future. Whereas in most of the BT-type models agents are myopic, except one by Good and Reuveny (2006) in which consumer's choice is modelled as a dynamic, multi-period optimization process. The resulting agent's optimal choice in this model takes account of the shadow prices of the two stock variables (population L and natural capital S). Such a model could potentially incorporate saving activities and man-made capital accumulation as typically done in Ramsey growth models.

Two models that include man-made capital are given by Erickson and Gowdy (2000) and Anderies (2003). In Erickson and Gowdy's (2000) model, accumulated man-made capital A affects the fertility function (see **III-1** for the description of the capital accumulation rule) but has no direct effects on other functions. As described in **III-2**, in Anderies' (2003) model production of both H and M are functions of A. The capital accumulation rule in this model is given by the difference between the investment and capital depreciation: $dA/dt = sw{\cdot}L/P_M - \delta A$, where s denotes the marginal propensity to save and δ the depreciation rate, both exogenously given. Total savings of the economy in each period ($sw{\cdot}L$) are used to purchase the manufactured good to form the man-made capital, as described in **III-1**.

The motivations behind the introduction of man-made capital are, in Erickson and Gowdy's (2000) case to hypothesize weak sustainability that seems to be indicated by the archaeological evidence of E.I., and in Anderies' (2003) case to analyse the effect of investment on demographic transition and the effect of innovation on the dynamics of the system through capital accumulation. To focus on these objectives, in both models agents' optimization processes are kept as static, and the question remains as to how to interpret the motivation behind the formation of capital in the portrayed economies. This question is to be more fully explored in **III-5** where we address the compatibility issue between agents' static views in the BT-type models and introducing into the models institutional designs or evolution of property rights.

The existing models shed light on possible directions to extend the models for the purpose of studying sustainability issues with the introduction of man-made capital. In addition to incorporating man-made capital into production functions, another dimension to consider is the consumption side. Having S and A in a model allows us to explore a variety of issues. For example, one can incorporate substitutability between the environmental amenity from the natural resource stock and the service flow from the man-made capital stock as social infrastructure. The effect of changes in the amount of such services, as a result of accumulation or depletion of these stocks, on individuals' well-being can be studied by observing changes in the utility levels. One could also introduce a threshold level of the natural resource stock size that maintains the minimum life-support system for individuals, or that keeps individual's consumption levels of goods and services above certain levels, with varying degrees of substitutability between the two stocks.

III-5. Property Rights and Institutional Designs

Two questions that help us address the treatment of property rights in the BT-type models are: what type of property rights historically existed in E.I., and what type of property rights are represented by the mathematical specifications of the models. For the purpose of replicating the population and resource dynamics of E.I., consistency between the two questions is critical. Meanwhile, if one's interest is to analyse property-right issues using the BT-type models as a tool, it is important, first, to understand what a chosen model framework represents and then extend the model accordingly. Dalton et al. (2005) point to the evidence that Polynesian-style communal ownership ruled by a chief or tribunal council, with tightly controlled access to the resource and strong focus on immediate consumption of resources, existed in E.I., and the design and operation of the BT-type models can be interpreted as such.[13]

When an economy is "compact", collective or common ownership can be effective (Demsetz, 2002; Libecap, 2009), whereas some of the authors of the BT-type models regard the potential of resource depletion that is inherent in these models as a market failure due to inter-generational externalities in resource use.[14] These authors

[13] We distinguish common or communal ownership, under which resources are subject to regulation and access to the resources is restricted, from open-access in which resources are up for grabs by all takers (Merrill, 2002). It is our understanding that, in order to focus on the property right issues surrounding the renewable resource, authors of the BT-type models presume implicitly that goods are traded as private goods, and that labour is traded as a privately owned input. When man-made capital is present in the production function, it is also presumed to be privately owned (Anderies, 2003, p.240).

[14] A "compact" economy is one in which economic interactions are biologically, geographically, and/or socially close so that cultural customs and feelings for others are influential (Demsetz, 2002).

introduce into their models measures that can reduce the risk of resource depletion and population overshooting, namely, institutional designs such as user charges (Pezzey and Anderies, 2003) and limitation on harvesting activities (Dalton and Coats, 2000; Pezzey and Anderies, 2003). These authors posit stronger (more private) resource ownership as the motivation behind the emergence of such instruments. Dalton and Coats (2000), for example, explain that when a resource is expected to become relatively scarce in the future, under stronger property rights people are more likely to assign a smaller labour force for harvesting today than they would under weaker property rights. Meanwhile, the mathematical specifications of these models represent the unchanging behavioural assumptions for agents across varying degrees of resource ownership. Therefore, an alternative interpretation for the emergence of these instruments is possible, and it is that they are institutional designs introduced by a chief or community leaders, while maintaining the existing common resource ownership.[15] Such an interpretation is consistent with the theoretical and empirical literature that suggests that privatization is

[15] Functional specification for an agent's optimization problem can be considered different under different property right regimes, reflecting agents' varying degrees of resource ownership and their alternative states of preferences (*e.g*., Caputo and Lueck, 2003). With private property rights, the production activity in this sector may be better defined at the individual agent's level as a function of her property rather than the sector-level stock size S. Dalton and Coats (2000), for example, introduce parameter χ that represents the varying degrees of property rights. This parameter appears in the reduced-form equilibrium labour allocation for the harvest sector L_H*. χ does not appear in agents' optimization process, suggesting that the behaviour of agents in this economy remains unchanged across the alternative states of property rights. Therefore, we could alternatively interpret these parameter changes as a representation of a chief's resource conservation policy under the existing property right regime.

not the only solution to resource overconsumption under common ownership and that historical examples suggest that long-lasting, self-governed management of common property resources is possible (Ostrom, 2002 and 1990).

In contrast to such institutional designs, Dalton et al. (2005) and Good and Reuveny (2006) choose innovation in harvesting technology and discount rates, respectively, as the areas affected by varying degrees of property rights. Dalton et al. (2005) examine the effect of strong or weak property rights on the direction of innovation, by incorporating two types of technological change: resource-conserving technology and resource-depleting technology, represented by parameters ξ_α and ξ_r as presented in **III-3**. In this model, stronger property rights promote resource conserving technology, whereas weaker property rights encourage resource-depleting technology.[16] Good and Reuveny (2006) examine the effect of various states of property rights on the level of harvesting through changes in consumers' discount rates for their multi-period utility maximization problem. In this model, with stronger (weaker) property rights people have low (high) discount rates.

The analytical results of these BT-type models are qualitatively consistent with the implications of comparative statics of the original BT model; changes in parameters (due to institutional designs or otherwise) that discourage harvesting activities tame boom-and-bust cycles of population and resource dynamics. These results are compatible with the motivations behind the introduction of property right changes into the models by these authors.

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[16] As described in **III-3**, the technology parameters ξ_α and ξ_r in Dalton et al. (2005) appear in the equations of motion for the innovation process but not in agents' optimization process; hence the same alternative interpretation as described in the previous footnote is applicable here as well.

Existing literature provides two prime candidates that explain the mechanism of evolution of property rights: interest-group theories and social-norm theories (Merrill, 2002). The former suggests that in in-egalitarian societies changes in property rights are imposed in a top-down manner by those who are more capable than others in an attempt to capture economic rent, whereas the latter suggests that in egalitarian, close-knit societies changes in property rights emerge in a bottom-up manner as a social norm or pressure, by societal members who shares strong common interests (*e.g.*, Barzel, 2000; Kaiser and Roumasset, 2007).[17] As indicated by Pezzey and Anderies' (2003) interpretation of institutional designs as "social norms, pressures, or taboos", or Dalton et al. (2005) and Good and Reuveny's (2006) representation of the effect of property rights on preferences and innovation paths, these models are consistent with social-norm theories, although in the BT-type models that incorporate property right issues, changes in property rights are exogenous.[18] Comparatively, as mentioned earlier these models could alternatively be interpreted as being consistent with Ostrom's (1990, 2002) finding that an introduction of institutional designs does not have to be interpreted as being driven by more *private* ownership but instead by "locally evolved institutions and norms", with the *existing* common ownership.

Meanwhile, there remains an issue of compatibility between most of the BT-type models being static optimization models and the long-term perspectives that

[17] Barzel (2000) analyses the trade-off for a dictator between alternative property right regimes, and Kaiser and Roumasset's (2007) case study of Hawaii is an example of the shift between the bottom-up and top-down evolution of property rights.

[18] While property rights are given exogenously in many studies, some studies incorporate property rights regimes as endogenous variables (*e.g.*, Birdyshaw and Ellis, 2007; Bhattacharya and Lueck, 2009).

motivate institutional designs or evolution of property rights (and, as discussed in **III-4**, capital accumulation). Although motivated by stronger property rights, with only static optimization provided in the models the aforementioned institutional designs are not based on long-term rationality. Evolution of institutional designs or property rights themselves requires long-term perspectives among individuals. In response to the question of how do norms for sustainable resource use evolve, Ostrom (2002) states that sustainable resource use is likely when long-term stewardship rights are given to compact groups of people who value fairness, trustworthiness, cooperation and reciprocation and who communicate with each other. With such groups of people in society, an introduction of institutional designs or property right regime changes that facilitate sustainable resource use tend to stem from concerns for future. Consequently, there seem to be two reasonable alternatives for the BT-type models to address these issues: (1) switch to multi-period optimization models (*e.g.*, Good and Reuveny, 2006) to introduce the forward-looking views of individuals in general, or (2) while maintaining most individuals' myopic views, assume that someone (*e.g.*, a social planner or a chief) has longer-term perspectives, and regard institutional designs and possibly property right regime changes as representative of such perspectives.

To conclude, both institutional designs and evolution of property rights are useful approaches, and which one (or possibly both) should be incorporated into a model depends on the circumstances to be analysed. For the purpose of using a BT-type model to analyse a compact economy, institutional designs that sustain common-resource ownership, along with the mathematical specification and the optimization process of the model that exemplify the chosen approach, are an option.

III-6. Modelling Approach

Dynamic modelling often faces the trade-off between mathematical representation of intended characteristics of economic activities and mathematical assumptions for the sake of convenience. In this section, we revisit the issues of substitutability and innovation and extend our analysis to consider alternative approaches to dynamic modelling based on various objectives of using these models.

Certain functions are popular for the easiness with which to obtain analytic solutions, but these functions may not necessarily represent the intended relationships between the relevant variables. As described earlier, most of the BT-type models so far employ linear or C-D production functions and C-D utility functions. These functions are easy to solve for equilibrium outcomes but restrict these models' scope to address substitutability issues (*cf.* **III-2**). One way to address this problem is to use a CES function and conduct sensitivity analyses with respect to the elasticity of substitution σ.

However, there still remains the issue of σ being exogenously given and constant across time. What is critical about substitutability is not its static value but the rate of change in this parameter over time (Beltratti, 1997). Introducing innovation into a model can help address the impact of technological progress on substitutability. For example, changes in relative scarcity of harvested versus man-made inputs, represented by the relative price of the two inputs, can drive the direction of innovation and affect the value of σ.

Convenient mathematical assumptions also arise when a model aims to provide a steady-state equilibrium. Neoclassical optimal growth models tend to employ linearly homogeneous functions so that steady-state growth rates can be expressed in per-capita terms.[19] While this is a generally accepted approach, whether it is desirable to require a steady state in a model of population and resource dynamics depends on the objective of the analysis. In reality an economy may never reach a steady state due to a continuous process of changes and disruptive forces that cause instability (Scrieciu, 2007; Barker, 2008). Most of the BT-type models can be classified as a combination of a static general equilibrium model and a simulation model whose transitional process is given by a set of differential equations.[20] By design a model of this type requires a static equilibrium for each period; however, such a model does not necessarily need to simplify functions to obtain an analytic solution; instead of solving by hand, computational tools are available to yield numeric solutions for simulation analyses.[21]

IV. Conclusion

This survey article provides a review of models that demonstrate the inter-dependency between population dynamics and natural resource dynamics. In particular, we focus on the BT framework and its descendents that are originally designed for a small, closed

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[19] Edenhofer et al. (2006) provide a general classification for models with innovation and resource issues: (1) *optimal growth models* (inter-temporal maximization of social welfare), (2) *energy system models* (cost minimization for the energy sector), (3) *simulation models* that start with a set of initial values for an economy and calculate the values for the following periods using a set of differential equations, and (4) *general equilibrium market models* that employ demand and supply analyses in multiple, inter-dependent sectors.

[20] Good and Reuveny (2006) present an optimal growth model. Basener and Ross (2005) and Basener et al. (2008) provide models that are defined outside the framework of neoclassical economic theory.

[21] Another possible direction is to employ a non-equilibrium approach in which we specify behavioural and interaction rules for agents and let the power of a computer reveal the dynamics of the model through repeated simulations (*the Economist*, 2009).

economy. These models are characterized by the feedback mechanisms between agents' individual, period-by-period optimization of how to allocate their labour endowment and consumption activities and the transitional processes from one period to next given by a set of laws of motion for the population and resource stocks. As a result, the consequences of individuals' static decisions are reflected in the population and resource dynamics. We believe that this branch of literature is of great interest for the study of sustainability issues. This literature prompts us to question our future prosperity, through our reflection on the demise of past civilizations, and also through the understanding of the modelling of population and resource dynamics in general. A little over a decade has passed since the initial appearance of the BT model, and with various extensions that have contributed to expand the literature in multiple directions as we demonstrated, it seems to be the right time to provide a summary of the literature.

This survey aims to integrate a group of models through a set of attributes that are commonly present across these models, namely, population growth functions, substitutability between man-made and natural goods or inputs, innovation, capital accumulation, property rights and institutional designs, and modelling approach through requirements on the types of solutions and corresponding functional choices. Through our analyses in this manner we aim to elicit how each attribute can be incorporated in various ways to address specific issues of one's interest. We hope that such a survey will facilitate a better understanding of this type of model and further application of the model framework to relevant modern circumstances. We regard the BT model as the skeleton of a general model of population and resource dynamics. As demonstrated by its descendents, the simplicity of the original model leaves room for incorporating variables that allow us to address various issues that are relevant in contemporary economies.[22]

Through our analyses we identify unexplored areas and suggest alternative approaches and interpretations as possible directions of extending the model framework. We are not proposing that a model should encompass every possible feature, but we hope that highlighting these features in relation to the existing models will stimulate further development of the literature.

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[22] Brander and Taylor (1998) and Maxwell and Reuveny (2000) provide examples of potential application.

Appendices

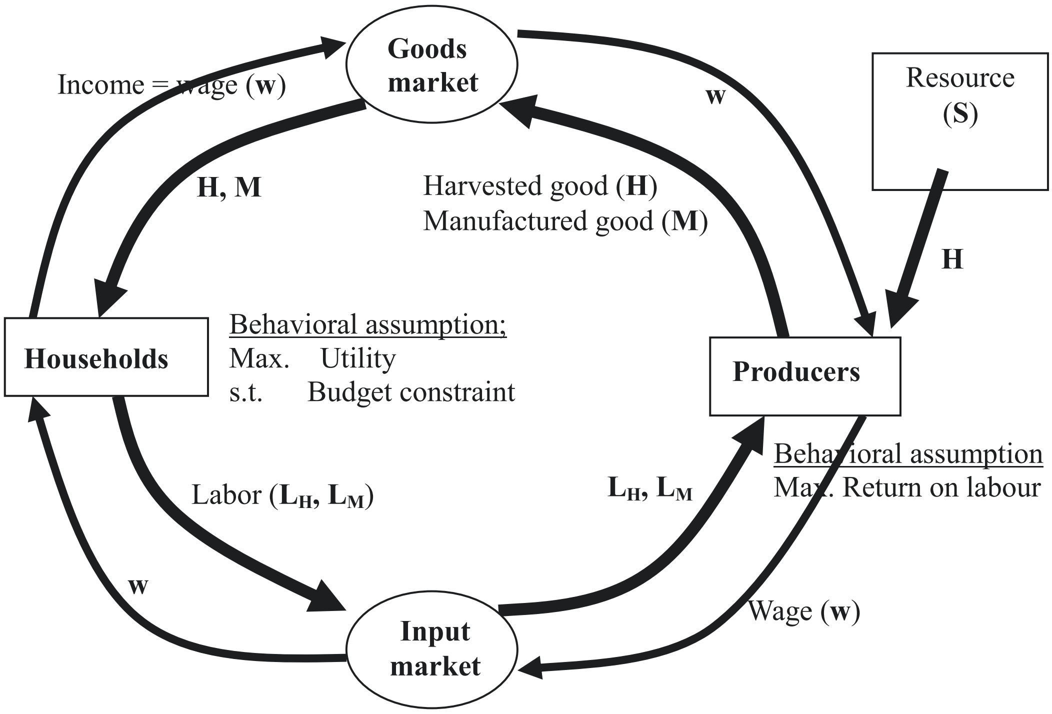

Figure 1. Period-by-period material and cash flow and agents' behaviour in a BT-type model.

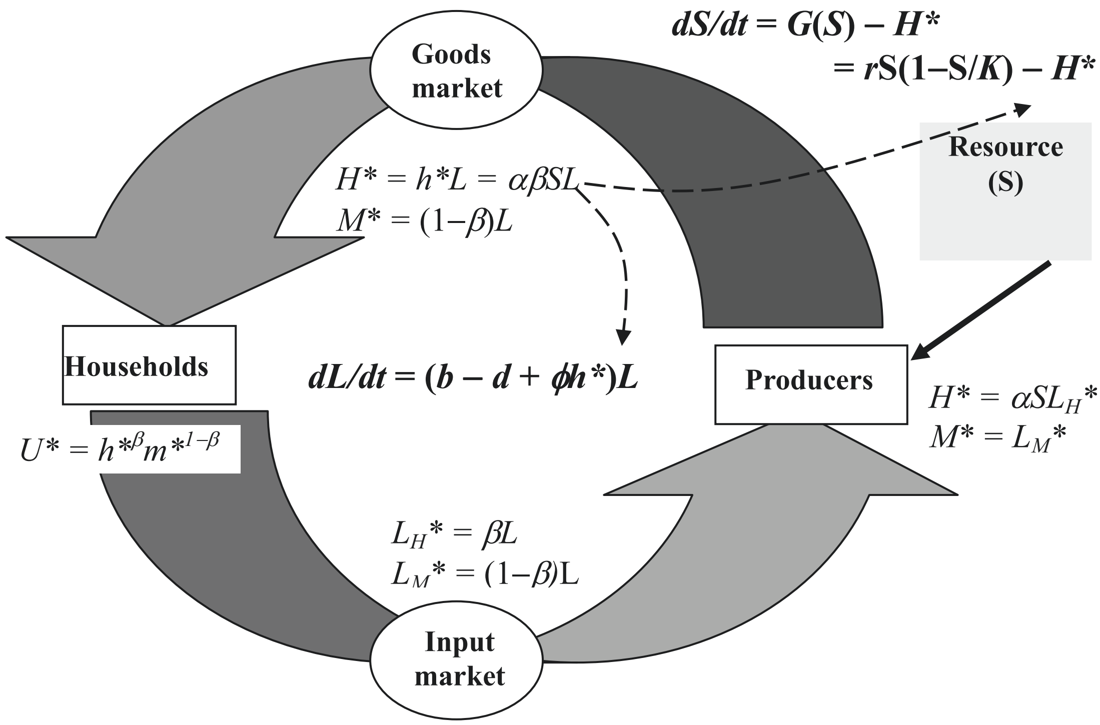

Figure 2. Static equilibrium outcomes and their feed-in mechanisms for the population and resource dynamics. (Equations are based on the original BT model; Asterisks indicate that these expressions are equilibrium values.)

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