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Parents, children and primary school mathematics: experiences, identity and activity

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Abstract

Parental involvement in children's learning plays a significant role in attainment in primary school. However, in the case of mathematics, a core subject in the primary school curriculum, research suggests that parents face a number of barriers to involvement.

Following an approach informed by the sociocultural theory, this project aimed to investigate parental involvement in children's school mathematical learning through a focus upon experiences, identity and activity.

Twenty-four parent-child pairs took part in the study. The children were all aged between 7 and 11 years old and attended primary schools in the UK. Parents took part in a semi-structured episodic interview and parent-child dyads were observed completing a 20-minute simulated school mathematical activity.

Data analysis consisted of four phases. Firstly, interview responses were subjected to a thematic analysis to examine parental experiences of: (1) school mathematics, (2) parent-child mathematical activity, and (3) home-school communication. Secondly, the interview transcripts were analysed using dialogical self theory to investigate mathematical identity. This concentrated on how parents constructed a mathematical 'self', to describe themselves, and a mathematical 'other', to describe their children. Thirdly, the observations of parent-child mathematical activity were analysed for mathematical goals, contingency and scaffolding. Finally, the results of the second and third phases were compared to study the relationship between identity and goals.

Analysis of parental experiences extended existing academic research in a number of areas. This included parental interaction strategies, particularly propinquity, and barriers to parental involvement, for instance divergent mathematical understandings.

Uniquely, in applying dialogical self theory to study mathematical identity, this research showed how the mathematical 'self' and 'other' shift spatially and chronologically through participation in sociocultural activity. Identity formation was also shown to be a reflexive process that embraced a range of diverse social influences.

Mathematical goals were seen to form and shift due to the activity structure, artefacts and conventions of the task, social interaction between the dyad, and the prior experience parents and children brought to the task. Analysing parent-child school mathematical interaction in this manner provides a distinctive contribution to understanding a widespread, but poorly understood social practice.

The final stage of analysis indicated that the mathematical identities parents assigned to children more closely match the goals in parent-child mathematical activity than the mathematical identities parents constructed for themselves.

The original and important findings generated by this project provide distinct implications for academics, educators and others working with parents and children.

Chapter 1

Introduction

1.1 Why study parents, children and primary school mathematics?

In order to appreciate why it is necessary to study the experiences, identities and activity of parents supporting children's primary school mathematics it is essential to understand the importance of mathematics as an educational subject and life-skill, and the significance of parental involvement in children's academic outcomes in primary school.

1.1.1 The importance of mathematics

Mathematics is a key subject in primary school. The *National Curriculum* (Department for Education & Employment, 1999a) sets out the statutory requirements for teaching and learning in UK primary schools. It describes mathematics thus:

> Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways. Mathematics is important in everyday life, many forms of employment, science and technology, medicine, the economy, the environment and development, and in public decision-making. Different cultures have contributed to the development and application of mathematics. Today, the subject transcends cultural boundaries and its importance is universally recognised. Mathematics is a creative discipline. It can stimulate moments of pleasure and wonder when a pupil solves a problem for the first time, discovers a more elegant solution to that problem, or suddenly sees hidden connections.
>
> Department for Education & Employment, 1999a, p.60

The importance of mathematics is mirrored in a raft of government publications produced since the adoption of the *National Curriculum* including the *National Numeracy Strategy* (Department for Education & Employment, 1999b) and *Primary National Strategy* (Department for Education & Skills, 2006). Indeed, the significance of mathematics in primary school is emphasized in the recently released draft *National Curriculum for Mathematics: Key Stages 1 and 2* (Department for Education, 2012), which states that:

> A high quality mathematics education provides a foundation for understanding the world, the ability to reason mathematically, and a sense of excitement and curiosity about the subject.
>
> Department for Education, 2012, p.1

A range of reports into mathematics education, including the recent *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools* (Williams, 2008), all highlight the significance of high-quality mathematics education in UK primary schools.

In their years at primary school, from the ages of five to eleven, children are expected to learn and develop conceptual understanding in a range of topics in mathematics. This forms the foundation for their learning of mathematics in secondary education (11-16 years old) and the numerical skills and capabilities that they will need in their future adult lives.

Children do not only learn mathematics at school. In this critical phase of their education, primary school pupils learn mathematics in two contexts: school and home. More research has looked at children's learning of mathematics at school than investigations into learning mathematics in the home. Understanding mathematics in the home is relevant because it is known that children bring knowledge, beliefs and orientations regarding mathematics from home into the classroom (Anderson & Gold, 2006). It is therefore logical to expect that these attitudes and understandings have been influenced by parents and carers.

1.1.2 The importance of parental involvement

Parental involvement has a major influence on children's achievement and attainment in primary school.

> Research consistently shows that what parents do with their children at home is far more important to their achievement than their [parents] social class or level of education
>
> Desforges & Abouchaar, 2003, p.87

Even after accounting for a number of socioeconomic variables the effect of parental involvement on academic achievement is still statistically significant (Desforges & Abouchaar, 2003). Parental involvement hugely shapes the early years of a child's development. This is highly influential in terms of attainment in

Mathematics, English and Science by the end of primary school (Duckworth, 2008). Parental involvement is not uniform. It is moulded by a range of social and cultural factors (Desforges & Abouchaar, 2003).

The importance of parental involvement on children's attainment has long been recognised and promoted in a UK government reports and policies from the 1960s to the current date (Department for Children, Schools & Families, 2007; 2008; Department for Education & Skills, 2007; Plowden, 1967; Rose, 2009, Taylor, 1977), these include key reports relating to mathematics education (Cockcroft, 1982; Williams, 2008).

> Parents can exercise, even if unknowingly, a considerable influence on their children's attitudes towards mathematics.
>
> Cockcroft, 1982, p.62

> Parents are a child's first and most enduring educators, and their influence cannot be overestimated. Parents should be at the centre of any plan to improve children's outcomes, starting with the early years and continuing right through schooling.
>
> Williams, 2008, p.69

1.2 What is parental involvement?

A comprehensive literature review on parental involvement, carried out for the UK government by Desforges and Abouchaar (2003), identified a number of elements that constituted parental involvement. This included: parenting skills in parent-child interaction, supporting schoolwork at home, and communication with school. These elements themselves were influenced by parental values and aspirations.

1.2.1 Parent-child interaction

In the Desforges and Abouchaar (2003) analysis, parent-child interaction appeared particularly significant. Indeed, research has shown that the quality of parent-child interaction in the early years of childhood is related to children's subsequent learning and development (Mattanah, Pratt, Cowen & Cowen, 2005; Morrison, Rimm-Kauffman & Pianta, 2003; Neitzel & Stright, 2003; Pratt, Green, MacVicar and Bountrogianni, 1992). However, interactions do not always influence attainment positively (Hyde, Else-Quest, Alibali, Knuth & Romberg, 2006). For example, negative or disproving behaviour is linked to poorer performance

(Gauvain, Fagot, Leve, & Kavanagh, 2002). Not all parents interact in the same manner with their children with individual differences commonly present. General differences in parent-child mathematical interaction have also been shown in terms of socioeconomic group (Hyde et al., 2006; Leseman & Sijsling, 1996; Saxe, Guberman & Gearhart, 1987; Vandermass-Peeler, Nelson, Bumpass & Sassine, 2009) and level of maternal education (Hyde et al., 2006; Neitzel & Stright, 2003).

Parents use a range of strategies when interacting with their children (Civil & Andrade, 2002; Hoover-Dempsey, Bassler & Burow, 1995; Solomon, Warin & Lewis, 2002). Interaction appears to be shaped by individual differences as well as social and cultural issues. It also typically reduces with age. As children grow older parents appear to grant them greater autonomy during school work interactions at home (Hoover-Dempsey et al., 1995).

Parents' mathematical interactions can be instructive, narrowly focusing on directing children towards solutions, or exploratory, using question to guide and support children, or observatory, watching and only intervening if absolutely necessary (Civil, Diez-Palomar, Menendez-Gomez & Acosta-Iriqui, 2008).

Solomon et al. (2002) interviewed parents and their teenage children regarding their experiences of completing homework together. They identified a number of parental support strategies from the interview responses. These are summarised in Table 1A below.

Table 1A Parenting styles

Form of parental support	**Definition**
No support	Parent doesn't offer homework support
Unconditional support	Parent praises and encourages child but doesn't appear to teach or explain activity
Promoting autonomy	Parent is present if required but leaves task for child to complete
Proactive involvement	Parent actively supports child
Monitoring	Parent exercise control over the task, directing the child's activity

Note: Contents from Solomon et al., 2002, p.608-609

Promoting autonomy was the preferred strategy of nearly half (48%) of the 58 families in the study. The other four forms of interaction were roughly equal across the sample. This highlights the wide variations in parent-child interaction, with the polar opposites of parental control (monitoring) or complete independence (no support) having a similar frequency. This suggests parents have very different views of what constitutes appropriate parental involvement in homework.

Hoover-Dempsey et al. (1995) conducted a similar study with 69 parents of elementary school children in the US. Their interviews showed parents engaged in promoting independent work, structuring homework tasks for their children, and supporting children through motivation and direct teaching. Again a variety of parental approaches to interaction were present with promoting independence and monitoring performance being the most popular.

A study carried out by Hyde et al. (2006) found wide variations in parent behaviour, emotion, confidence and enthusiasm during parent-child mathematical activity. Indeed, for many parents and children mathematical activity can be stressful and frustrating (Abreu & Cline, 2005; Else-Quest, Hyde & Hejmadi, 2008; Hughes et al., 2007; McMullen & Abreu, 2011; Lange & Meaney, 2011; Solomon et al., 2002). Else-Quest et al. (2008) explored mother-emotions during mathematics interactions. They found positive parental emotions linked to improved child performance and, adversely, negative emotions were linked to decreased performance. They suggest that reinforcement of particular emotions, for example confusion, during parent-child interaction can shape feelings and emotions about mathematics. In their examples of good parental practice they found joy, affection and pride, conversely in cases of poor practice they found tension, boredom and sadness.

Some parents feel anxious about their children's mathematical future (Solomon et al., 2002). A factor that may be exaggerated due to the aspirations many parents typically hold concerning their children. Negative emotion responses can also occur due to the conflict between involvement and time-pressures associated with modern family life (Hoover-Dempsey et al., 1995; Jackson & Remillard, 2005).

Understanding parent-child mathematical interaction is a key to understanding not just parental involvement but learning and development constructed by parents and children.

1.2.2 Supporting schoolwork at home

Parents and children participate in a range of mathematical activities at home (Baker, Street & Tomlin, 2006; Civil & Andrade, 2002; Jackson & Remillard, 2005). Parents' past experiences of mathematical activity, both as a child learning mathematics and as a parent interacting with their own child, influence how they perceive and participate in mathematical interaction (O'Toole & Abreu, 2005). Therefore it is important to understand parental experiences of involvement in children's primary school mathematics.

A survey involving over 5000 UK parents and carers, carried out by Peters, Seeds, Goldstein and Coleman (2008), showed that the vast majority of respondents felt it was extremely important to support their children's schoolwork at home. However, many felt a lack of confidence because of changing teaching methods and a consequent lack of understanding of children's work. As has already been discussed, confidence is implicated in the quality and character of parent-child interaction.

In the UK, school mathematical practices have changed markedly over the last decade and a half. UK government policy such as the *National Curriculum* (Department for Education & Employment, 1999a), *National Numeracy Strategy* (Department for Education & Employment, 1999b) and *Primary National Strategy* (Department for Education & Skills, 2006) advocate teaching practices and mathematical procedures that differ considerably from parents' own experiences of schooling.

A number of authors have researched the impact of these curricular changes (Abreu & Cline, 2005; Baker et al., 2006; McMullen & Abreu, 2011; Street, Baker & Tomlin, 2008). These findings are often similar to those of researchers in the US investigating reform of mathematics teaching (Civil & Bernier, 2006; Hoover-Dempsey et al., 1995; Jackson & Remillard, 2005; Remillard & Jackson, 2006). These investigations all appear to show how changing teaching practices have formed a barrier preventing parents from being able to support their children's school mathematical activity.

McMullen and Abreu (2011) showed how lack of knowledge and understanding of children's school mathematics frustrated some parents. Many parents saw their children's school mathematics as very different to their own knowledge and school experiences. This led to difficulties and feelings of exclusion during parent-child interaction. In a study of primary school parents, Abreu and Cline (2005) found the majority saw current mathematics teaching as different, particularly in terms of the methods and tools used in the classroom and the mathematical strategies children were taught. Difference in written algorithms for addition, subtraction and multiplication figured prominently in differences identified by parents. These and other studies show that parents often worry about confusing their child if they teach them their 'way' of mathematics.

Some parents tend to think of mathematics as formulaic and routine. This can be reinforced by the type of homework activities that children and parents are expected to complete (Civil et al., 2008; Warren & Young, 2002). In some cases when parents have different mathematical understandings to their children, it can lead them to prefer to use their own mathematics rather than their child's (Civil & Andrade, 2002; Civil et al., 2008). This can thus limit the involvement parents have in supporting children school mathematical development.

Issues around parents' knowledge and understanding of mathematics feed into processes of valorisation and identification (McMullen & Abreu, 2011). The availability of knowledge and consequent parental involvement is also linked to processes of home-school communication.

Homework is often located towards at the centre of the parent-child home-school relationship (Solomon et al., 2002). In Peters et al. (2008), 82% of parents reported supporting homework in Key Stage 1 (5-7 year old) and 73% in Key Stage 2 (7-11 years old). Research on the effectiveness of homework is contradictory (e.g. Hyde et al., 2006; Farrow, Tymms & Henderson, 1999; Levin et al., 1997). However, homework does involve parents in their child's education and creates a useful means of home-school communication (Sheldon & Epstein, 2005).

1.2.3 Home-school communication

It appears parental support of child's schoolwork is often bolstered or impeded by the ability of parents to communicate with school. Certainly, parent involvement programmes, such as Home-School Knowledge Exchange (Andrews & Yee, 2006; Feiler, Greenhough, Winter, Salway & Scanlan, 2006; Hughes & Greenhough, 2006; Hughes & Pollard, 2006) and Parents as Partners in Early Learning (Department for Children, Schools & Families, 2007) show how good communication between home and school can improve parental engagement and involvement with children's school learning. This is also the case in the US where parental involvement programmes in elementary school have been shown to be positively related to achievement (Jeynes, 2005). As might be anticipated, similar results occur in projects specifically focused upon mathematic learning and parental involvement, in both the UK (Merttens & Vass, 1993) and US (Sheldon & Epstein, 2005; Shumow, 1998). As might therefore be expected, parental involvement has been promoted in UK government policy (Department for Education & Skills, 2007).

Nevertheless many parents report problems with home-school communication, with research suggesting that the vast majority want more detailed and frequent communication with school (Peters et al., 2008). Parents feel that with more information from school they will be better able to support their children's mathematics (Remillard & Jackson, 2006). Home-school communication allows parents to feel that they are able to monitoring their children's progress in mathematics, which in turn influences their parental involvement (Jackson & Remillard, 2005).

Unfortunately, it is a one-way flow of information that typically characterises the home-school relationship in the UK (Abreu & Cline, 2005; Baker et al., 2008; Hughes & Greenhough, 2006). Many schools view communication from their own perspective as a means to progress their own ends (Edwards & Warin, 1999). In this sense the mathematical knowledge and experiences that parents and children can bring to schools from the home is ignored (Hughes et al., 2007).

Given the difficulties that parents face in supporting their children's school mathematics at home it is understandable that they would wish to have more information. As has been discussed more information can lead to more involvement. How these processes are reflected in parent-child mathematical activity is less well understood.

1.2.4 Values and aspirations

Pea and Martin (2010) found that parental values were important in guiding parents' involvement in children's mathematics. Here the values parents held regarding mathematics, for instance its importance or the importance of success at school, influenced levels of parental involvement. This ties into the idea of valorisations of social and cultural practices.

Abreu (1995, 1998, 2002) describes the importance of understanding how people assign social values, or social valorisations, to knowledge in practice. Social valorisations influence processes of social interaction and the mediation of cultural artefacts and tools. Abreu (1995) first raises this discussing how children experience mathematical activity differently in home and school contexts in Brazil. Later she showed how valorisation of mathematical practices could be linked to social identity (Abreu, 2002). In the UK, Abreu and Cline (2005) revealed how children produce valorisations based on the mathematical activity they undertake. Parents and children can have different views of what constitutes mathematics (Abreu, Cline & Shamsi, 2002; Civil & Andrade, 2002; McMullen & Abreu, 2011; Remillard & Jackson, 2006). For instance when Abreu et al. (2002) interviewed children they found that they tended to prefer or value school mathematics, what they had learnt in the classroom, over and above their parents' mathematics. This social valorisation led to children resisting their parents' views on mathematics.

McMullen and Abreu (2011) showed how parents' valorisations of mathematics differed depending on their knowledge of current school mathematics, linked to curricular differences discussed earlier. This valuing of mathematical strategies and approaches appeared to influence how parents interacted with their children, especially when a parent's valorisations conflicted with their child's. Differing valorisation led to conflict and difficulties during parent-child interaction.

Research on valorisation of mathematical practices, and links to knowledge and understanding, in the UK have tended to focus upon ethnic minority groups (Abreu & Cline, 2005; Abreu et al., 2003) or contrasting teachers and non-teachers (McMullen & Abreu, 2011). In these situations activity have been viewed through reconstructed experiences and social representations rather than observed directly.

Values can be related to aspirations in the sense that to aspire to something suggests that it has value. Saxe et al. (1987) showed mathematical aspirations of mothers were similar across class boundaries, though expectations were reduced in a lower socioeconomic class. Parental aspirations have been shown to have a positive relationship with achievement (Singh et al., 1995).

Parental involvement is a multi-faceted process that takes many separate forms and varies considerably between individuals. The act of parental involvement can be hampered or advanced by parent's own experiences of involvement in their children's primary school education. In these experiences it is possible to see the barriers and opportunities that influence the elements of involvement identified by Desforges and Abouchaar (2003).

1.3 Discussion

The findings presented above give an insight into the importance of parental involvement and parent-child mathematical activity in learning and development in mathematics. They also suggest how these processes are experienced by some parents and how such experiences could influence involvement and interaction. However, within these findings it is also possible to see gaps in contemporary knowledge.

School mathematical interaction has been infrequently studied in the home in a qualitative manner through observation, especially in the United Kingdom. An exception is the work of Street et al. (2008) who observed children in Reception, Year 1 and Year 2 doing school and non-school mathematics with their parents. In the US, insightful quantitative research centring on parent-child school mathematical interaction was conducted by Saxe et al. (1987) and Hyde et al. (2006). Qualitative research by Solomon et al. (2002) in the UK and Hoover-Dempsey et al. (1995) in the US looked at homework generally and not at mathematics specifically. This appears especially important given qualitative research on experience.

Research on parental experiences of parent-child school mathematical activity has tended to focus on minority groups, either ethnically or socially, or small samples of education workers and non-education workers. There is little research in the UK looking at predominantly white working- and middle-class parents and children. Furthermore there is even less understanding of how these kinds of experiences link to actual mathematical activity and knowledge construction. There is obviously a gap to study parental experiences, parent-child mathematical activity, and the influences of parental experiences on parent-child mathematical activity from a qualitative frame. Certainly when considering the emotions observed in mathematical interaction by Else-Quest et al. (2008) or the emotional experiences recounted by parents by Abreu and Cline (2005) then there appears scope to try to tie these together and understand the causes of negative experiences and how they play out in mathematical activity. Similarly how does knowledge and understanding of mathematics, highlighted so adeptly by McMullen and Abreu (2011), and patterns of communication, suggested by Hughes and Greenhough (2006), play out in a larger population of UK primary school parents? Can these facets be linked to subsequent parent-child mathematical activity?

In order to move beyond interpreting experiences and activity separately towards an understanding how experiences influence activity it is necessary to understand how values, aspirations and, perhaps crucially, identity arise through experiences. This is especially the case since, as O'Toole and Abreu (2005) showed, past experiences mediate present activity. How does this mediation work? Is it possible to create a better understanding of what constitutes a 'mathematical' identity? Certainly this is the topic of much contemporary debate in mathematics education (Sfard & Prusak, 2005).

1.4 Thesis structure

The next chapter sets out a detailed theoretical framework for this study. It orientates this thesis towards critiquing research in learning and conceptual development in mathematics that is relevant to parent-child interaction. It also assesses whether the lenses of sociocultural and activity theories are appropriate to study mathematical activity. It connects findings here linking parental experiences of support and involvement to a means to understand how such support can facilitate mathematical activity through the idea of mathematical goals. Furthermore it attempts to highlight how activity cannot be fully understood without an appreciation of identity, since both learning and identity formation can be seen to originate in activity.

Chapter three begins by producing a series of aims and research questions that are the product of both this introduction into the topic and parental involvement generally, and the specific conceptions arising of the theoretical framework. It presents a methodology for studying the experiences of parents, the mathematical identities parents construct for themselves and their children, and parent-child school-related mathematical activity. It also proposes a mechanism for linking together identity and activity.

Chapters four, five, six and seven present data and arguments linked to the areas of parental experiences, mathematical identity, parent-child school mathematical activity and the link between identity and activity. This shows evidence collected through observation and interviews of a sample of twenty-four parents and their primary school-aged children in the UK.

The final chapter in this thesis summarises the key findings of this project in terms of their theoretical and practical relevance to parents, children and primary school mathematics. It highlights the limitations of the study and presents areas for future academic research.

Chapter 2

Theoretical framework

2.1 Introduction

The theoretical framework draws upon a number of different approaches and constructions in order to locate this research endeavour in terms of the study of experiences, identity and activity associated with parent-child primary school-related mathematics. The previous chapter discussed the importance and relevance of parental involvement. This chapter moves further to produce a theoretical context into how elements of this activity can be better understood.

The first part of this chapter focuses upon the fundamental assumptions that underpin this project concerning learning and development. Here the emphasis is on discussing the ideas of Vygotsky (1978, 1981, 1986) and their links to sociocultural theory, particularly in understanding the social and cultural nature of cognition and learning.

Building upon this, the debate then turns towards a deeper understanding of the role of context and activity in mathematical learning and development. It highlights the work of Vygotsky's contemporary Leont'ev (1978, 1981) and shows how his activity theory allows the study of mathematical cognition through a focus on goals and goal-related activity. This is followed by a discussion on the key role of culture in mathematical understanding.

The work of Saxe (1991) is presented as a framework for bringing together the ideas of participation in a cultural practice, goals and socioculturally-situated cognition and learning. The debate then shifts towards supporting an analysis on emergent goals in parent-child mathematical activity with an understanding of two processes containing goal-related behaviour: contingency and scaffolding.

The chapter then moves into a deliberation concerning processes of identity formation and the relationship between goals, learning and identity. Here research on identity that comes from a sociocultural viewpoint is discussed before an approach based upon dialogical self theory is advanced.

Finally, at the end of this chapter, a summary sets out the key findings of this literature review and presents its implications for this research project.

2.2 Sociocultural theory

A sociocultural approach focuses upon the relationship between mental functioning and the social and cultural context in which the mental functioning arises (Wertsch et al., 1995). It sees social interaction and sociocultural contexts as essential in development (Ernest, 1995). It builds on the work of Vygotsky (1978, 1981, 1986), and can be seen in the ideas of apprenticeship (Rogoff, 1990), legitimate peripheral participation (Lave & Wenger, 1991) and guided intervention (Tharp & Gallimore, 1988). Its foundations lay primarily in the work of Vygotsky. Vygotsky believed that to understand the individual you must first understand the social environment in which the individual exists (Wertsch, 1985).

Lev Vygotsky was born in 1896 and died in 1934. His ideas and approaches were formed in a turbulent time in Russian history and have clear parallels with the theories of Karl Marx and Friedrich Engels (Bakhurst, 2007). It is the 1978 translation of Vygotsky's work *Mind in Society*, perhaps more than any other, which has been used to vindicate sociocultural theories of learning and development.

> The appeal of the sociohistorical theory offered by Vygotsky, Leont'ev, Luria, and others who have followed lies in the primacy it places on mind <u>in</u> society (Vygotsky, 1978) and the associated examination of cognitive development in socio-cultural activity.
>
> Rogoff, 1990, p.14

At the centre of Vygotsky's theory is the idea that:

> …intellectual development cannot be understood without reference to the social milieu in which the child is embedded. For Vygotsky, children's cognitive development must be understood not only as taking place with social support in interaction with others, but also as involving the development of skill with sociohistorically developed tools that mediate intellectual activity.
>
> Rogoff, 1990, p.35

In other words, Vygotsky rejected the idea that development was driven by any single factor, and so cannot be explained by any single corresponding principle (Wertsch, 1985). The fact that Vygotsky's work is still so highly influential is a testament to the strength of his theories (Wertsch, 1985). In particular four key areas of Vygotsky's work are relevant to the sociocultural approach taken in this thesis: the genetic method, internalization, mediation and the zone of proximal development.

2.2.1 The genetic method

A fundamental principle of a Vygotsky's theory is the 'genetic' approach to understanding development. This means you can understand development only if you understand the origins and transitions of development experienced by the individual. Vygotskian cultural-historical or sociocultural analysis focused on development relating to 'histories'. These were associated with the general history of humanity, the life history of the individual in society and the history of a particular psychological system. Scribner (1985, p.139) included the history of individual societies "as societies and cultural groups participate in world history at different tempos and in different ways". This approach has been used to study development across 'genetic' domains. Vygotsky himself was interested in development through social interaction, sociogenesis, as well as how the individual developed over time, ontogenesis. His ideas have also been used to study two other genetic domains: phylogenesis, the development of a group of individuals, utilising Engel's notion of the division of labour (Wertsch, 1995); and microgenesis, development over a short time span, often studying the characteristics of individual social and cultural interactions. Whilst Vygotsky never used the term microgenesis in his writing it is clear that he argued for a microgenetic analysis against much of psychological research which he saw as investigating and reporting 'fossilised' behaviour (Wertsch, 1981). This term relates to Vygotsky's likening of behavioural development to geological structures, or layers, which are built upon and superseded by new behaviour (Kozulin, 1986). Vygotsky's own research was primarily ontogenetic but he recognised that development does not occur in isolation in any one genetic domain, all are intertwined.

2.2.2 Internalization

Vygotsky (1978) supposed that language, and therefore social interaction, is the primary avenue through which learning occurs. This social interaction and transmission of culture allows the internalization of higher psychological functions, allowing the social to become psychological. Vygotsky (1978, p.56) writes that “We call the internal reconstruction of an external operation *internalization*.” This is not a simply mental function, as Leont’ev (1981, p.56-57) states:

> The process of internalization is not the transferral of an extended activity to a pre-existing, internal ‘plane of consciousness’: It is the process in which the plane is formed.

This internal phase is formed through cooperative social interaction and enabled by speech and language (Tharp & Gallimore, 1988). The process of internalization is critical in Vygotsky’s ‘general law of cultural development’, which states that:

> An interpersonal process is transformed into an intrapersonal one. Every function in the child’s cultural development occurs twice: first, on the social level, and later, on the individual level: first between people (interpsychological), and then inside the child (intrapsychological)… All the higher functions originate as actual relations between human individuals.
>
> Vygotsky, 1978, p.57

Wertsch (1985) describes internalization as ‘patterns of activity’ on the external plane that are internalized on the internal plane, as in the general law of cultural development, through socially and culturally mediated processes.

2.2.3 Mediation

Another theme in Vygotsky’s work is that an understanding of action on the social and individual level requires an understanding of ‘mediation’. This mediation occurs through cultural artefacts such as language, tools, signs and actions. Mediation implies social and cultural interaction. This has a range of sociological and anthropological implications, again highlighting the importance of cultural context in development. Cole (1996) suggests that the proposal that psychological processes develop through ‘culturally mediated’ activity is at the heart of Vygotskian theory.

> …everything that is cultural is social. Culture is the product of social life and human social activity. That way just by raising the question of cultural development of behaviour we are directly introducing the social plane of development
>
> Vygotsky, 1981, p.164

Wertsch (1985) suggests that the idea of mediation is the most important concept created by Vygotsky. It allows the connection of the internal and external in internalisation and facilitates development across genetic timeframes. If internalization is the plane of development, and mediation the catalyst, then the site can be located within Vygotsky's zone of proximal development.

2.2.4 The zone of proximal development

In order to ascertain learning and development Vygotsky (1978) developed the concept of the 'zone of proximal development' (ZPD). Wertsch (1985) argues that Vygotsky developed the idea of the ZPD to tackle the problem of assessing children's intellectual abilities and evaluating of instructional practices. The ZPD also responds to Vygotsky's views on fossilized behaviour. He believed post-testing of learning simply showed ingrained patterns of fossilized behaviour rather than what the child could learn, or how they could develop, through the use of culturally mediated tools. Vygotsky (1978, p.86) defines the ZPD as:

> …the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers.

Vygotsky (1978) uses a floral analogy suggesting the ZPD deals with buds and flowers rather than fruits, in other words processes which are advancing or maturing but have not yet been finalised or completed. The ZPD can be thought of as the difference between assisted and unassisted performance. It is a "fundamental nexus of development and learning" (Tharp & Gallimore, 1988, p.30). Vygotsky (1978) suggests that developmental processes lag behind learning processes. It is this lag that allows the formation of zones of proximal development.

The ZPD is not a physical entity or an internal psychological characteristic waiting to be activated or utilised, it is created through negotiation in a particular context by the child and the expert (McLane, 1987). The more capable peer or adult becomes an 'expert', compared to the child who is less able in that particular intellectual sphere, in that particular context. There is not a single ZPD for each individual (Tharp & Gallimore, 1988). Each experience or social and cultural interaction forms a unique ZPD since each experience or social and cultural interaction between two or more people is itself unique.

Culture and cognition are inseparable in the ZPD. Culture is fluid, created by people talking, working, living, interacting, using tools and other artefacts. Therefore the ZPD is no longer just a mechanism for assessing development, it is as Rogoff (1990) argues, a 'crucible' of development because interactions taking place within the ZPD allows participation in cultural activities that would be otherwise inaccessible to the non-expert.

The components of learning, development, language, social interaction, culture and context are all intertwined in a sociocultural approach. Several key authors have played an important role in the development of sociocultural theories in psychology and education building upon the work of Vygotsky. As might be expected, there is no single defining, or defined, sociocultural theory. Each of the key thinkers in the field has developed their own 'brand' of sociocultural understanding based on their interpretation of the field and the results of their own research (Cole, Engeström & Vasquez, 1997). The streams of sociocultural theory that are most relevant to this study are research concerning (a) the study of activity and (b) the role of context in learning and development.

2.3 The study of activity

Wertsch (1985) suggested that the appropriate unit for analysing consciousness is the theory of activity proposed by Leont'ev. Elements of activity appear across sociocultural research (Cole, 1995). When defining activity Wertsch (1985, p.212) states the following:

> …an activity can be thought of as a social institutionally defined setting. An activity or activity setting is grounded in a set of assumptions about appropriate roles, goals, and means used by the participants in that setting. In terms of the levels of analysis in the theory of activity, one could say that an activity setting guides the selection of actions and the operational composition of actions, and it determines the functional significance of these actions.

Note that this definition of activity does not stop at the idea of an individual carrying out an action; it also encompasses the idea of context.

2.3.1 Leont'ev's activity theory

The study of activity and activity theory (AT) originates in the work of Alexei Leont'ev (1978, 1981). Leont'ev was a follower and contemporary of Vygotsky.

Leont’ev built his activity theory on the foundations of Vygotsky’s work (Wertsch et al., 1995). In it he managed to blend approaches from anthropology and psychology (Nasir & Hand, 2006). Key elements within AT can be linked to Vygotsky’s writings. Indeed several authors have opined that Vygotsky himself was close to developing an activity theory before his premature death (Wertsch, 1985). Kozulin (1986) opposes this view believing that Vygotsky purposely steered away from developing a theory of activity, preferring to concentrate on social activity as a generator of consciousness and a mediator of activity.

Leont’ev worked in the Institute of Psychology at Moscow University with Vygotsky. He later moved to Khar’kov in the Ukraine where he formed his ‘activity approach’ or ‘psychological theory of activity’ (Zinchenko, 1995). Alongside contemporaries in Khar’kov, Leont’ev developed an activity theory based on Vygotsky’s ideas of internalization but with the central mediation being played by activity rather than language (Kozulin, 1986). AT was not developed in isolation by Leont’ev; it involved input from Khar’kov researchers such as Davydov, Zinchenko, and Gal’perin (Wertsch, 1985).

Within Leont’ev’s work there are strong features of Marxist ideas of labour and human activity (Wertsch, 1981, 1985; Engeström & Miettinen, 1999) and the philosophy of Hegel (Leont’ev, 1981). In their writing, Marx and Engels produced ideas around an active subject and how the subject interacts with the social environment. Their ideas around ‘labour’ and ‘tools’ can be seen in Leont’ev’s focus on the activity present in the division of labour between individual action and collective activity (Engeström & Miettinen, 1999).

Leont’ev’s approach uses Vygotsky’s ideas of internalization and tool mediation. He writes: “The tool mediates activity and thus connects humans not only with the world of objects but also with other people” (Leont’ev, 1981, p.56). He also adopts, and develops, Vygotsky’s ideas on higher mental functions, and intrapsychological and interpsychological functioning, together with his ideas on cultural-historical development, noting:

> "…higher psychological processes unique to humans can be acquired only through interaction with others, that is, through interpsychological processes that only later will begin to be carried out independently by the individual. When this happens, some of these processes lose their initial, external form and are converted into intrapsychological processes."
>
> Leont'ev, 1981, p.56

Leont'ev shares similar views to Vygotsky regarding consciousness writing, "…consciousness is not produced by nature: consciousness is a product of society, it is produced" (Leont'ev, 1981, p.56-57). So for Leont'ev, internalization, social interaction and activity are indelibly intertwined. Thus we can see Vygotsky (1978, 1981) as a basis for Leont'ev AT. Leont'ev (1981) himself credits Vygotsky with introducing the concepts of tools, goals and motive that Leont'ev developed and refined.

Within Leont'ev's AT there are several important ingredients – action, activity, goals, objects, operations and motives. These each require the existence of one another so are inseparably threaded together. This is the complexity at the heart at the heart of AT and provides both strength and weakness.

Activities are distinguished by their different objects, which in turn are connected to motives. There can be no activity without motive:

> …the main feature that distinguishes one activity from another is its object. After all, it is precisely an activity's object that gives it a specific direction. …an activity's object is its real motive. Of course, the motive can be either material or ideal. The main point is that some need always stands behind it.
>
> Leont'ev, 1981, p.59

Human activities are realised in reality through actions. Actions that are intended to achieve a conscious result are goals. "The actions that constitute activity are energized by its motive, but are directed towards a goal" (Leont'ev, 1981, p.60). Goals depend then on the motive of the activity. They also appear to be a response, or subjective answer, to range of variables present in social interaction. Leont'ev himself seemed to favour an evolution of goals through social interaction where goals are tested out through action or negotiation.

If actions are concerned with goals then operations are concerned with conditions. "The origin of an action is to be found in relationships among activities, whereas every operation is the result of the transformation of an action" (Leont'ev, 1981, p.64). Social interaction can be difficult to interpret, as motives, goals and actions respond to negotiation, evolution and transformations. These elements build a complex picture of activity, they are difficult to define and measure in interaction, making AT a difficult methodological proposition. Indeed, AT has been criticised as being too complex to use as an analytical tool (Axel, 1997).

So whilst Leont'ev (1978, 1981) provides us with a theoretical framework to study activity his approach is rarely implemented methodologically. Some researchers use elements of AT (e.g. Wertsch, 1985) whilst others have used it to develop more complex systems (e.g. Engeström, 1999). Much of the implementation of elements of AT has focused on goals and goal-directedness, since Leont'ev argued that the focus for studying social, cultural and cognitive processes should be goal-directed activity (Nasir, 2002).

For instance, Wertsch (1985) links the ideas of Vygotsky and Leont'ev into his research. He uses goals and goal-directed action, as well as theories of internalization and the ZPD. In his view goal-directed action is 'embedded' in objects. In his work, Wertsch (1985) draws upon a number of research projects in the 1970s and 1980s that used Vygotsky and AT and notes that they all appear to suggest the importance of goal-directed action. For Wertsch motive of activity guides action, goal formation and operation. Motive is socially-situated and therefore context-related. Kozulin (1986) notes how Wertsch sought to combine elements of Vygotskian theory and AT to understand the social interactions that are essential in the interpsychological phase of Vygotsky's model. By better understanding the social plane Kozulin (1986) suggests that Wertsch hoped to add to research on the relationship between cognitive development and cultural setting.

2.4 The role of context in learning and development

Culture and activity are interwoven. Context is inseparable from human activity (Rogoff, 1990). The activities we perform across contexts and cultures both mould and characterise learning (Nasir, Hand & Taylor, 2008). Here culture is defined as the particular context that envelops an individual participating in a particular practice. For instance the culture of the mathematics classroom, with its inherent norms (Yackel & Cobb, 1996), is different to the mathematical culture at home or in the workplace. A large number of studies have specifically studied learning and development and the surrounding culture to show how cognition is situated and resides within cultural practices. Brown, Collins and Duguid (1989) go as far as saying that learning is a process of enculturation. In mathematics this has been shown in a number of studies investigating out-of-school mathematics.

2.4.1 Out-of-school mathematics

Academic research into out-of-school mathematics, for example street vendors (Carraher, Carraher & Schliemann, 1985; Nunes, Schliemann & Carraher, 1993), candy sellers (Saxe, 1988a, 1988b, 1991), shoppers (Lave, 1988), farmers (Abreu, 1995), shopkeepers (Beach, 1995), basketball players (Nasir, 2000a) and domino players (Nasir, 2000b; 2002), shows how mathematical activity is a cultural phenomenon. Multiple analyses show not just that mathematical activity differs between school and non-school contexts but that mathematical activity is intertwined with the culture in which it takes place.

For example, Lave (1988) conducted a number of studies on non-school mathematics. In the Adult Math Project she studied how mathematics was used by people, who she terms JPFs – 'just plain folks', in their daily lives (e.g. shopping, dieting and money management). Her work showed how performance, strategies and attitude toward mathematics depended upon the context in which the mathematics was situated. She found that often mathematics was seen as a 'reified object' with little application outside institutional settings.

Similarly, research with children (Carraher et al., 1985; Nunes et al., 1993) showed how mathematical forms, reasoning and performance altered between classroom and non-classroom settings. This covered a range of written and oral forms of mathematics such as addition, subtraction, multiplication, division and proportionality. A 'form' is a cultural construction linked to participation in a specific social or cultural context (Saxe, 1991). For instance a number line or column subtraction is a mathematical form linked to participation in school mathematics.

2.4.2 Guided participation

Moving beyond the intrinsic nature of culture and cognition, Rogoff (1990) marries ideas within Vygotskian thought, particularly the notion of the zone of proximal development, and Leont'ev's activity theory to show how learning occurs through guided participation in cultural activity. For instance, through this approach we see participation as an active process whereby a parent structures a young child's participation in a practice, influencing goal-related activity. Indeed, Rogoff (1990) shows how goals, in part, help form active learners. It is this activity that is required to be successful in Rogoff's model. Within her plane of guided participation activity is directed towards implicit, explicit or emerging goals (Rogoff, 1995). She asserts that these actions involve motive, again linking to Leont'ev's ideas.

> The adult's structuring of the problem may be tainted to the child's level of skill. With a novice, the adult may take responsibility for managing the subgoals as well as making sure the overall goal is met. A more experienced child may assume responsibility for achieving the subgoals and eventually for managing the whole task.
>
> Rogoff, 1990, p.95

Embracing the themes of participation in cultural activity, goals and mathematical activity leads to a body of research that uses highly detailed analytical structures for viewing goals and goal-related activity. In this approach goals are seen as representative of culturally contextualized and mediated learning.

2.5 The study of goals, culture and cognition

Geoffrey Saxe has produced a number of papers, chapters and books on culture and mathematics (Saxe et al., 1987; Saxe, 1982, 1985, 1988a, 1988b, 1991, 1992, 1995, 1996, 1999, 2002, 2004; Saxe & Guberman, 1998; Saxe, Gearhart & Seltzer, 1999; Guberman & Saxe, 2000). His early work, as an undergraduate, was with Eskimo groups in sub-arctic Alaska. Here he began to study cognition in social and cultural contexts (Saxe, 1996). Later, he travelled to Papua New Guinea to study the Oksapmin people (Saxe, 1982, 1985). The tribe, which had had very little contact with the outside world, had a unique non base-10 number system. At the time Saxe visited the Oksapmin elements of the Western number system were being introduced through trade stores and mission schools. This presented an ideal situation in which to study social and cultural influences on mathematical activity and cognition. However, Saxe felt that this work did not contain enough analysis of how mathematical environments emerged or enough attention to analysis of social interaction (Saxe, 1996).

2.5.1 Goals and mathematical understanding

His later work in the US (Saxe et al., 1987), studied the relationship between numerical goals and social and cultural processes. Saxe et al. (1987) linked his approach to studying goals to Leont'ev's activity theory and the developmental approach of Vygotsky. The project utilised a sociocultural perspective to investigate children's numerical understandings, and how these were formed and negotiated in everyday activities. At the heart of this work was the idea that:

> …children's numerical understandings are their *goal-directed adaptations* to their numerical environments, therefore, the study of number development should entail coordinated investigations of children's emerging abilities to generate numerical goals and the shifting sociocultural organization of their numerical environments
>
> Saxe et al., 1987 p.4

Through this approach Saxe et al. (1987) studied the goal-related mathematical activity of parents and their 2½ and 4½ year old children. This showed that maternal instruction influenced children's goal-related activity. Linking goals and understandings, in a finding which supports the guided participation model of Rogoff (1990), they suggest that through maternal support children were able to achieve more complex goals than they would have been able to accomplish alone. Both child and maternal goals were seen to shift and evolve as a result of parent-child mathematical interaction. Mothers formed and pursued goals with their children that were related to the complexity of the task.

From this work Saxe developed his practice-based research framework that consists of three main elements: emergent goals, form-function shifts, and the interplay between cognitive developments across practices (Saxe, 1996). The first of these, emergent goals, reflected a Vygotskian focus on microgenesis. The later modes of analysis were more ontogenetic in character, focusing on development over time.

2.5.2 The emergent goal framework

Saxe moved on to studying situated cognition in Brazilian rural weavers and urban candy sellers (Saxe, 1988a, 1988b). His study of candy sellers utilised the emergent goal framework (Saxe 1991).

> Goals, then, are emergent phenomena, shifting and taking new form as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts
>
> Saxe, 1991, p.17

Goals are emergent because they form and evolve in response to a number of characteristics of the sociocultural environment. Saxe (1991) states that goals are shaped by four factors: activity structures, artefacts and conventions, social interaction and prior understandings. These are displayed in Saxe's four parameter model shown in Figure 2A.

Figure 2A The four-parameter model

Note: Adapted from Saxe, 1991, p.17, Fig 2.2

This emergent goal framework has also been used to study mathematical goals in a number of different contexts including pairs of children playing a mathematical classroom-based game (Saxe, 1992, 1995), children playing monopoly (Guberman, Rahm & Menk, 1998), middle- and high-school basketball players (Nasir, 2000a), domino players (Nasir, 2000b, 2002), mathematical activity in a high-school classroom (Saxe, 2002). To date it has not been used to study parent-child school-related mathematical activity in the home.

Activity structure

Saxe (1991) explains how the goal structure of an activity is related to practice-linked and task-linked motives. In other words the motives behind an activity, from counting coins to shopping or currency trading, shape the goals inherent in the activity. Using the example of candy sellers, Saxe (1991) shows how the four phases within the candy selling practice (purchase - prepare to sell - sell - prepare to purchase) structured the goals of candy selling. For instance the purchase phase had arithmetic involving currency for wholesale amounts of candy, whilst the prepare to sell phase involved calculations for converting these wholesale prices to individual unit prices plus a profit mark up. In the same way, Nasir (2000b) found that the activity structure of playing dominoes (choosing an opening play - making play choices - post-game wrap up) affected the choice of goals of people participating in the game.

Artefacts and conventions

Just as the currency system and pricing conventions influenced the goal-related activity of candy sellers (Saxe, 1991) so the rules and pieces in a game of dominoes influence the children and adults who play the game (Nasir, 2000b). The artefacts and conventions of a cultural activity influence the motives and hence goals pursued in an activity. Each cultural practice has its own set of social and cultural conventions and artefacts. These engender certain motives in order to participate in the practice. These motives in turn produce goal-related activity that is symptomatic of the practice.

Social interaction

Social interaction between individuals engaged in a cultural practice, primarily through language but also through other forms of communication such as gesture, influences the goals those individuals form in that practice. Saxe (1991) showed this in the social interactions candy sellers had with peers, customers and suppliers. For instance, bargaining and negotiation over prices influenced the mathematical goals constructed by candy sellers interacting with customers and suppliers. Likewise, pupil-pupil and pupil-teacher social interaction not only encourage and support goal formation, but also lead to reflection and negotiation of mathematical goals (Saxe, 2002). This was also shown, albeit at a simpler level, between parents and young children engaged in mathematical problem solving (Saxe et al., 1987).

Saxe (1992, 1995; Saxe & Guberman 1998; Guberman & Saxe 2000) designed a game to involve collective social practice, linking to Leont'ev's activity theory and ideas regarding the division of labour. From this Guberman and Saxe (2000) contend that in the division of labour between children working on a collaborative activity, different mathematical goals are formed. Saxe (2002) elaborates the way in which children take positions and justify their goals based on their interpretation of the information available to them. Through working with others these justifications get tested by others and amended as the child sees fit. This social negotiation allows the emergence and evolution of goals.

Prior understandings

The past experiences and prior understandings brought to a mathematical activity can logically be expected to influence the mathematical goals constructed within that activity. Saxe (1991) showed how sellers' goals could be seen to be related to their age and experience. He showed how sellers favoured different goals in the activity of setting the price of candy. Younger sellers (6-7 years old) relied heavily on support from others, older children (8-11 years old) still frequently used support but occasionally performed ratio calculations themselves, the oldest children (12-15 years) very rarely sought support and instead performed ratio calculations based upon addition and division to calculate pricing structures. Nasir (2000a, 2000b, 2002) similarly showed the complexity of mathematical goals in basketball players and domino players increased with age and that reliance on others, or goal support, reduced with age. She found the more people participated in practice, the more their mathematical goals shifted as learning occurred. In research with 3rd and 4th graders involved in collaborative activity, Saxe and Guberman (1998) similarly showed that the goals differed due to prior understandings, namely mathematical knowledge acquired in the classroom.

2.6 Other forms of activity linked to mathematical cognition

It can be seen that the emergent goal framework produced by Saxe (1991) provides a theoretical framework that can be used to study parent-child mathematical activity. However, how do the findings of others concerning parent-child interaction align with a focus on goal-related activity? Do we need to incorporate more than just a study of emergent goals if we are to thoroughly appreciate how parents and children participate in school-related mathematical activity?

Earlier, in section 1.2.1, a number of strategies were presented that appeared to typify parent-child interaction. These included instruction, direction, exploration, monitoring, motivation and promoting independence. It is unclear how these elements alone influence goals and cognition. Yet they can be seen in the two key elements that repeatedly standout in studies of adult-child interaction originating from a sociocultural perspective, including those with a focus on goal-related activity: contingency and scaffolding. Many studies investigating these topics have researched interaction using Vygotsky's (1978) ZPD or Rogoff's (1990) subsequent notion of guided participation. Because of this shared theoretical root their findings are directly relevant to research on goals and goal-related activity.

2.6.1 Contingency

Wood and Middleton (1975) studied pre-school children and parents completing a pyramid-building construction problem. They used the construct of a 'region of sensitivity to instruction' (RSI) to measure current ability and 'readiness' for new instruction. Although the RSI was developed independently, it has clear parallels with Vygotsky's (1978) notion of the ZPD. Wood and Middleton (1975) found that parents adapted their support and instruction based upon how the children responded to their assistance. This adapted instruction focused on the child's RSI. The more effective parents were at altering their instructions based upon the child's responses the more successful the child became in the construction task. They found that, in the majority of cases, the amount of help a parent gave was related the child's previous success. When the child was correct the next parental intervention provided less help. When incorrect the next intervention gave more help. This study showed that parent-child interaction constantly evolves as parents adapt their approach in the light of performance. This approach is referred to as contingent intervention.

Wood, Wood and Middleton (1978) followed this up and found that contingent teaching strategies, i.e. those more in tune with the child's ability, led to more effective performance by children than non-contingent approaches. In the Saxe et al. (1987) study of parent-child interaction they found contingency shifts in number tasks. They found that mothers adapted their practice based on their child's performance, targeting particular difficulties their children were experiencing. Leseman and Sijsling (1996) produced similar findings when studying mothers and 3 year olds undertaking a problem solving task. Similarly, in older children (10-11 year olds) completing long division mathematics homework, Pratt et al. (1992) showed parents enacting contingency approaches.

Contingent intervention appears to be a common feature of parent-child interaction but it is unclear in what manner mathematical goals and goal-related behaviour affect contingency and how the cultural practice of mathematical activity in the home supports this process.

2.6.2 Scaffolding

Wood, Bruner and Ross (1976) studied how tutors supported 3, 4 and 5 year olds in another pyramid construction task. Similar to Wood and Middleton (1975), the construction problem was outside the child's ability and so required the support of another. Whilst the interface was tutor-child rather than parent-child, the paper documented and explained a key interaction strategy termed 'scaffolding', which further developed the idea of contingency.

Wood, et al., (1976, p.90) described scaffolding as a "process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts". The scaffolder is the 'expert', in this case the tutor, and the 'novice' is the child. Here we see parallels with guided participation in the ZPD. The authors suggest that the process of scaffolding involves six functions: recruiting the interest of the novice in the task; diminish the degrees of freedom within the task to direct the novice towards the operation; maintain the novice's direction towards the action, goal or objective; highlighting the 'critical features' of the task operation for the novice; controlling the novice's frustration; and modelling or demonstrating solutions to the task.

Bruner (1985) shows how scaffolding is tied to internalisation. He suggested that the scaffold becomes a tool used by the child and converted into consciousness. Returning to Leont'ev's (1981) view of internalisation as a plane of consciousness, it is clear how the scaffold not only supports learning but provides the very foundations for the plane of learning. Here then, scaffolding is a fluid and dynamic process where both parent and child are active in the construction of knowledge through interaction (Stone, 1993).

Research on scaffolding

Many authors have investigated scaffolding in parent-child or teacher-child mathematical interactions (Hyde et al., 2006; Lindberg, Hyde & Hirsch, 2008; Neitzel & Stright, 2003; Mattanah et al., 2005; Pratt et al., 1992; Vandermass-Peeler et al., 2009) and others in parent-child or teacher-child interaction generally (Bliss, Askew, & MacRae, 1996; Connor & Cross, 2003; Kermani & Brenner, 2000; Laakso, 1995; Leseman & Sijsling, 1996; Pratt, Kerig, Cowan & Cowan, 1988; Rogoff, 1990; Tharp & Gallimore, 1988; van de Pol, Volman & Beishuizen, 2010, 2011).

As a process scaffolding appears to be more prevalent in the home than in school (Bliss et al., 1996). Indeed it appears to be infrequently used by teachers (van de Pol et al., 2011). This may be related to the amount of one-on-one interactions that occur in the home compared to the classroom.

Laakso (1995) studied how Finnish mothers and fathers completed a conveying and teaching task with their 8-10 year old children. She found evidence of scaffolding and, interestingly, noted that mothers seemed more sensitive in their scaffolding behaviour than fathers. However, both Pratt et al. (1988) and Mattanah et al. (2005) found no difference between US mother and father scaffolding effectiveness. In terms of child gender, Lindberg et al. (2008) showed no overall effect of gender on maternal scaffolding, but did find variation within groups, with more 'traditional' mothers, categorised through a scoring questionnaire, providing more scaffolding to girls than boys. Pratt et al. (1988) also found individual differences within paternal scaffolding. Kermani and Brenner (2000) compared Iranian and American mothers and found differences in scaffolding behaviour, even though both groups were equally sensitive to the children's level of development. Likewise, a number of authors have linked differences in maternal scaffolding to socioeconomic status (Hyde et al., 2006;

Leseman &Sijsling, 1996; Vandermass-Peeler et al., 2009). Findings such as these suggest both a social and cultural dimension to scaffolding.

In a longitudinal study of parent-child scaffolding, Connor and Cross (2003) found that generally over time parents reduced the amount of support they gave as children grew older, however the support they did give was more efficient and better targeted to the child's RSI. So overtime it appears parents become more adept at scaffolding.

Neitzel and Stright (2003) worked with a sample of 68 mothers-child dyads (pre-schoolers) and found that maternal scaffolding behaviour helped predict elements of academic competence after a year of schooling. Pratt et al. (1992) and Mattanah et al. (2005) also found parental scaffolding behaviour predicted academic performance.

Hyde et al. (2006) studied mother-child (10-11 year olds) interaction around mathematics homework. They found a wide variation in the quality of scaffolding by mothers. They suggested that mathematical self-confidence was an important variable in scaffolding quality. Their analysis showed that maternal education levels correlated with scaffolding quality, in other words more educated mothers were better at aligning their efforts to the child's ZPD. Neitzel and Stright (2003) also found that maternal education influenced scaffolding behaviour in terms of both the metacognitive information provided, and the amount of emotional support and responsibility transferred.

Scaffolding and goal-directed action are linked within the ZPD (Tharp & Gallimore, 1988). Rogoff (1990) discusses how scaffolding helps children to determine goals and sub goals in adult-child interaction, linked to the social mediation of guided participation. However she does not show how the individual components of scaffolding interact with goal-related activity. This is an area of goal-related research for which few findings exist.

2.7 Activity and identity

Litowitz (1993) argues that development in the ZPD cannot be fully understood without appreciating both activity and identity. From her reading of Vygotsky, she argues that language is a route for internalisation of both cognitive development and personality. She writes that:

> We may say that, as our inner speech is internalized speech of others, our self is constituted by the internalized others who speak.
>
> Litowitz, 1993, p.189

Learning and identity are interwoven in the internalisation of the social into the intrapsychological. Both must be appreciated if a single one is to be understood. Similarly, Stone (1993) argues that understanding scaffolding requires an appreciation of how such dyadic interactions involve complicated social dynamics and relationships.

Key factors in parental involvement, such as values, aspirations and assumptions, experiences and beliefs, can all be seen in the notion of identity. Research shows the connection between identity and activity. Furthermore, it is theoretically possible to show the interweaving of goals, identity and learning, allowing the tying together of the many strands of this theoretical framework.

2.7.1 Goals, identity and learning

Nasir (2002) studied the microgenetic emergence of mathematical goals but also attempted to align this with an ontogenetic study of identity. In her study of emergent goals in adults and children playing dominoes and basketball she focused on the link between identity and goals. She suggested that there was a 'multifaceted, bidirectional' association between goals, identity and learning, as shown in Figure 2B.

Figure 2B The relationship between goals, identity and learning

Note: Adapted from Nasir, 2002, p.239, Fig 4

Within Nasir's (2002) approach it is proposed that two-way relationships exist for each of the three elements. For instance, between goals and identity, where she writes:

> New goals are often structured in line with emerging identities in practice. As participants take on new identities vis-à-vis others in the practice and in relation to the activity at hand, they begin to construct more sophisticated practice-linked goals (increasingly aligned with those of experts) …new goals lead to new identities. Here, participants (through the alignment of their goals with experts) begin to see themselves as more expert, hence changing their identity in relation to the practice
>
> Nasir, 2002, p.240

Nasir (2002) views identity as being constructed by individuals through their active participation in cultural activities, this is very much a sociocultural definition. She draws on Wenger (1998) and Holland, Lachicotte, Skinner and Cain (1998) to see identity as:

> ...a fluid construct, one that both shapes and is shaped by the social context. Indeed, identity is not purely an individual's property, nor can it be completely attributed to social settings. From Wenger's perspective, identity develops both through individual agency and through social practice
>
> Nasir, 2002, p.219

Sociocultural theory generally views identity as constructed in and through social and cultural situations. Rather than an internal construction, as cognitive-biological theories attest, sociocultural theorists see identity, in line with Vygotskian theory, as forming as the result of interaction on the interpsychological plane.

Lave and Wenger (1991) saw identity as linked to participation in a community of practice. For them, ideas around identity formation and development are crucial to understanding the concept of legitimate peripheral participation in communities of practice. Wenger (1998) later built upon these ideas placing identity, alongside practice, as a key element in his theory of learning. Wenger's ideas assume identity is community-based rather than an individual representation, and moves away from the psychological study of the individual and their experiences, towards the study of communities they are part of and how these explain the experiences, belonging and 'identity' of the individual.

Holland et al. (1998) also perceived identity as part of social practices, and social and cultural forms. For them, identity is discovered through the study of cultured or figured worlds. These are encounters which are socially organized and produced through everyday activity. In their view people have multiple identities depending upon the practice and activity in which they are engaged.

2.7.2 Mathematical identity

Academic research has looked at processes of identification and mathematical identities in various groups and contexts. It follows that if identity arises out of participation in social and cultural activity, then activities in different communities of practice give rise to different identities. We may have distinct mathematical identities at home, at work and in the supermarket.

The idea of mathematical identity is neatly defined by Martin (2007, p.150) when he writes that:

> Mathematics identity refers to the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person's self-understandings and how they are seen by others in the context of doing mathematics

A number of researchers studying identity in mathematics from a sociocultural perspective follow a similar definition to Martin (2007). These generally incorporate Harre and van Langenhove's (1991) notions of positioning and positionality. This defines positions as a discursive mechanism through which "people locate themselves and others within an essentially moral space" (Harre & van Langenhove, 1991, p.396). So through dialogue and participation in sociocultural activity people form positions from themselves and others regarding mathematics.

Boaler and Greeno (2000) and Esmonde (2006; 2009) draw on positioning to study identity in mathematics classrooms. Boaler and Greeno (2000) found that the identities that are brought to, and constructed in the classroom influence how pupils engage in mathematics learning. Likewise, Esmonde (2006; 2009) found identity processes involved in learning mathematics in group-work situations.

McMullen and Abreu (2011) produced four themes that appeared to be part of how mothers constructed their mathematical identities. They suggested that perceptions of mathematical ability, memories of learning mathematics, valorisations attached to mathematics in the parent's sociocultural context, and experiences of work experience all influenced mathematical identity. This shows elements of self-positioning.

Self-identification, or self-positioning, was the subject of research with children and parents conducted by Esmonde et al., (2011). They found evidence of self-identification occurring around mathematics. Abreu and Cline (2003) linked identities and mathematical practices to not just self-identification but also the social processes of 'being identified' by others. This can also be seen in work on pupil and teacher identities by both Crafter and Abreu (2010) and Gorgorió and Prat (2011). Abreu (2002) showed how parents project identities on to their children through encouraging them, or not, to take part in certain types and forms of mathematics. Here parents position their children mathematically.

Whilst these results are highly useful and insightful to understanding mathematical identity their approaches are not necessarily as comprehensive as other recent theoretical advances in the field. A systematic approach to understanding identity through positioning of the 'self' and 'other' via participation in sociocultural activity is presented in dialogical self theory. This approach can be seen to have theoretical similarities to the above arguments but produce findings at a deeper level of understanding.

2.7.3 Dialogical self theory

Dialogical self theory (DST) originated in the 1990s through the work of Hubert Hermans and his various collaborators (Hermans, 1996; Hermans & Kempen, 1995; Hermans, Kempen & van Loon, 1992). The notion of a dialogical self has evolved and expanded over the past twenty years, finding application in many areas of psychology, particularly in clinical and cultural divisions (Raggatt, 2007), but more recently in educational contexts (Akkerman & Meijer, 2011; Ligorio, 2010, 2011). In its latest versions DST presents a detailed and comprehensive theoretical model for understanding identity and notions of self (Hermans & Gieser, 2011; Hermans & Hermans-Konopka, 2010).

The origins of DST lie within the work of the American philosopher and psychologist William James and the Russian literary critic Mikhail Bakhtin (Hermans et al., 1992).

William James and the 'self'

In his work *Principles of Psychology: Volume 1*, James (1890) sets out an early vision of psychology as a discipline, but one that still has relevance now, particularly in the case of DST. When discussing his conception of the self, James distinguished between the self as object and the self as subject (Hermans et al., 1992). The self as object is characterised as 'Me' and the self as subject as 'I'. 'I' is the self as knower, a sense of personal identity, whilst 'Me' is the self as known (Hermans, 2001). James also promoted the notion of 'Mine', to which end we can see elements of our mental 'belongings' (e.g. my daughter) help to shape the self (Hermans & Hermans-Konopka, 2010).

By forwarding the principle of 'Me' and 'Mine', James suggested that the self was not restricted to the internal but incorporated the external, what could be thought of as the sociocultural environment. Notions of 'I', 'Me' and 'Mine' suggest consistency and unity, as elements of the self perpetuate over time. But they also suppose change, as new experiences and 'belongings' are incorporated in our 'self'. For James, the self appears to be made up of different, often conflicting, multiple selves (Hermans, 2001). Salgado and Hermans (2005) suggest that James's division of the self into 'I' and 'Me' led him to embrace the idea of a range of selves, which are determined by different social circumstance. In his ideas about self, James appears to draw together arguments of unity and multiplicity as well as a sense of internal and external relations or negotiation of the self.

Mikhail Bakhtin and dialogue

The second theoretical pillar of DST originates in the literary criticism of Mikhail Bakhtin. Through his reading and analysis of Dostoevsky's works, Bakhtin developed the notion of the polyphonic novel (O'Sullivan-Lago & Abreu, 2010). In *Problems of Dostoevsky's Poetics*, Bakhtin (1984) shows how the Russian author Dostoevsky uses multiple voices in his stories. Bakhtin argued that the characters in Dostoevsky's work have separate, distinct voices, what Bakhtin terms a polyphony of voices, through which the novel is told, rather than a single voice of the author (Hermans et al., 1992). These voices, which are involved in complex dialogical relationships, take alternative points-of-view and show ideological independence (Hermans, 1996).

Taking the notion of polyphonic voices and dialogue a step further, Bakhtin proposed that 'dialogue' can be applied from literature to the concept of personality (Hermans et al., 1992). Bakhtin suggested that throughout our lives we are involved in communication and dialogic processes (Salgado & Hermans, 2005). Indeed, he suggested that self-narrative forms where different voices interact (O'Sullivan-Lago & Abreu, 2010). Bakhtin believed that dialogical processes involved the interaction, or juxtaposing, of different voices (Hermans, 1996). He proposed that the narratives constructed by these voices could be explained diachronically, related to time, and synchronically, in terms of space (Raggatt, 2007).

Positioning - the spatial and temporal nature of the dialogical self

Hermans and colleagues applied the ideas of James and Bakhtin to develop the notion of the dialogical self. In this James's 'I' and Bakhtin's polyphony of voices across space and time are embraced to envisage a multiplicity of self. Hermans and Hermans-Konopka (2010, p.120) suggest that "the self and identity can only be properly understood when their spatial and temporal nature is fully acknowledged".

The dialogical self is narrative, and hence temporal, and so evolves and changes over time. It is evident in the stories we tell about ourselves and the ways in which the past shapes the present 'I'. It is also spatial in the sense that the 'I' shifts depending on the context we find ourselves in.

DST builds on Harre and van Langenhove's (1991) notion of position and positionality. Positioning is the key to the temporal and spatial nature of DST (Raggatt, 2011). According to Raggatt (2007) dynamic positioning emerges through our conversations, relationships, narratives and stories, as well as the social and political order that surrounds us, what the author terms the 'micro encounters of daily life'. Through these encounters we may take 'internal' positions (e.g. I as optimist), 'external' positions (e.g. an imagined voice of another), and ‘outside’ positions (e.g. interlocutors) (Raggatt, 2011). Both Holland et al. (1998) and Wenger (1998) use positioning as part of their theorisations of identity. However, their models are not as detailed or comprehensive for the study for positioning as DST.

I-positions

The self is not an internal physical construct, rather it is located in the positions we take in ‘real or imagined space’ (Hermans & Kempen, 1995). The ‘I’ can and does move from position to position.

Through an understanding of ‘I’ as positional in time and space, the dialogical self imagines the self to be made up of a number of different I-positions.

> …we conceptualize the self in terms of a dynamic multiplicity of relatively autonomous *I* positions in an imaginal landscape. In its most concise form this conception can be formulated as follows. The *I* has the possibility to move, as in a space, from one position to another in accordance with changes in situation and time. The *I* fluctuates among different and even opposed positions. The *I* has the capacity to imaginatively endow each position with a voice so that dialogical relations between positions can be established.
>
> Hermans et al., 1992, p. 28

The self is decentralised though a polyphony of I-positions. In this conception there is not a central, overarching ‘I’ or ‘Me’. Harking to its origins in Bakhtin, Hermans (2001) likens I-positions to characters in a story, each of which has a separate background that shapes its voice, producing a narrative, storied self.

As Hermans and Hermans-Konopka (2010, p.139) write:

> The theoretical advantage of the notion of I-position is that it brings unity and continuity in the self, while preserving its multiplicity. The I is continuous over time: in the process of appropriation and rejection, it is one and the same I who is doing this. At the same time, the I, located in time and space and intrinsically involved in the process of positioning, is confronted with a wide variety of new positions and possible positions. As a reaction, the I appropriates some of them and rejects others. Those that are appropriated are experienced as ‘mine’ and as ‘belonging to myself’ and, as a consequence, they add to the unity and continuity of in the self.

A single person can occupy a variety of I-positions that can be in agreement or disagreement with one another. I-positions emerge, fluctuate, evolve and dissipate as a result of interaction with the social and cultural environment and each other (Hermans et al., 1992). Furthermore, through sociocultural synergy, I-positions evolve in on-going processes of positioning, repositioning and counter-positioning (Hermans & Gieser, 2011).

Properties and characteristics of I-positions

I-positions can be thought of as internal or external (Hermans, 2001). The diagram below shows Herman's (2001) model of spatial positions in a multi-voiced self, where I-positions are located within one of two concentric circles.

> *Internal* positions, depicted by dots within the inner circle, are felt as part of myself (e.g. I as a mother, I as an ambitious worker, I as an enjoyer of life), whereas *external* positions, depicted by dots within the outer circle, are felt as part of the environment (e.g. my colleague Peter becomes important to me because I have an ambitious project in mind). In reverse, internal positions receive their relevance from their relation with one or more external positions (e.g. I feel like a mother because I have children). In other words, internal and external positions receive their significance as emerging from their mutual transactions over time.
>
> Hermans, 2001, p.252

Figure 2C Positions in a multi-voiced self

Note: From Herman, H. J. M. (2001) The dialogical self: towards a theory of personal and cultural positioning. *Culture and Psychology, 7* (3), p.253, Fig 1

The size of the dots reflects the closeness of the I-position to the current dialogue that is occurring. It is through dialogue that positioning becomes an active, recursive process where internal and external I-positions are constantly in motion as a result of changing social and cultural circumstance (Hermans, 2001).

The dialogical self, sociocultural circumstance and the 'society of mind'

DST relies on the link between the individual and society; it is culturally embedded (Hermans & Hermans-Konopka, 2010). The notion of I-positions necessitates dialogue and mediation. In this way DST is linked to the sociocultural philosophies expounded by Vygotsky, Leont'ev and Wertsch (Hermans & Kempen, 1995). It sees the self co-exisiting on two planes - interpsychological, through heterodialogue, and intrapsychological, via autodialogue (Valsiner, 2002). The self is constituted in society, and becomes a society itself - a society of mind (Hermans & Gieser, 2011). In the society of mind the polyphony of voices forms a multitude of positions through dialogical action.

Dialogical interactions in sociocultural contexts, such as that between parents and children, assume power relationships and role construction. In these situations a more dominant individual, the parent, may impose or foster an I-position in the less dominant individual, the child. This power relationship can be likened to sociocultural processes of learning in the zone of proximal development, where knowledge is created in the interpsychological space between the novice and the expert prior to its internalisation by the novice. Therefore clear linkages exist between DST and sociocultural theory (for further discussion see Ligorio, 2010).

Other forms of positioning in the dialogical self

I-positions are not the only forms of positioning in the dialogical self. Raggatt (2011) identifies twenty-one different forms of positioning influencing the self. Some of these, such as I-position, internal position, external position and outside position, have already been discussed. Others, such as meta-positions and social and reflexive positions are relevant in the construction of I-positions in sociocultural interaction.

A meta-position is a superordinate position which forms as the product of two or more positions (Raggatt, 2011). They arise as a result of self-reflection, allowing the self to take an overview or over-arching assessment of other positions (Hermans & Gieser, 2011). For instance we might see 'I as a parent' as a meta-position which encompasses a myriad of other positions around images of what a parent is and does.

A social position is one, such as gender, socio-economic status or level of education, which acts as an outside position influencing our internal and external I-positions (Raggatt, 2011). Reflexive positioning occurs when we reflect upon the label or positions placed upon us from outside. As such reflexive positions are mediated, for instance by a parent labelling us as 'good' or 'bad' (Raggatt, 2011). I-positions have been shown to alter and shift depending on the sociocultural context (Aveling & Gillespie, 2008) in a manner consistent with Raggatt's (2011) definitions of social and reflexive positioning. Aveling and Gillespie (2008) showed how second-generation immigrants in the UK held divergent I-positions that reflected the 'voices' present in their sociocultural context. These voices originated from others (e.g. family members and the community) but also through cultural tools (e.g. language). Akkerman and Meijer (2011) similarly showed how teacher identity could be seen as both individual and social.

Dialogical self theory and goals

Finally, it is possible to see that the theoretical link between goals and identity, shown in Nasir's (2002) model, can be studied by focussing attention on emergent goal construction and identity construction via dialogical self theory. Logically following this path offers 'doing' mathematics as creating goals but also creating positions and therefore notions of the mathematical 'self'. It is already evident that goals are linked to learning and learning influences goals. Similarly, learning influences internal and external positioning, and how we position ourselves influences our activity and thence learning.

2.8 Summary and key implications of the theoretical framework

The theoretical framework outlined in this chapter has shown how parent-child mathematical activity cannot be separated from the culture in which it occurs. Children's learning of school mathematics at home occurs through participating in the practice of school mathematics. By using the sociocultural theories of Vygotsky, Leont'ev and Rogoff, together with the emergent goal framework of Saxe, it is theoretically possible to gain an insight into parental involvement in children's school mathematical activity. Not only could this provide a window onto mathematical cognition but a better understanding of the ‘activity’ of primary school-related mathematics in the home. This is currently conspicuous by its absence in the debate on parental involvement in children’s school mathematical development.

Moreover by also including the dialogical construction of mathematical identity we gain a greater understanding of not just mathematical identity but the way in which identity influences activity and vice versa. This could present a deeper understanding of activity and issues around parental involvement beyond interaction. This is especially relevant given the findings introduced in the first chapter of this thesis. Like the two-way relationship between identity and goals, understanding parental experiences should allow an understanding of activity and identity, which in turn produces a better understanding of one of the mechanisms through which parents make sense of experiences.

Certainly, to produce a holistic exploration of parental involvement in school-related mathematical activity it is necessary to employ a framework that considers experiences, identity and activity.

Chapter 3

Methodology

3.1 Introduction

A number of elements came together to create the methodological conceptions and procedures utilised in this study. This chapter attempts to give an account of the philosophies and methods that were used and, importantly, a justification for their appropriateness. Whilst this chapter reads as a linear progression building from ideas towards data gathering approaches and then analysis the actual process was more cyclical due to periods of reflection. Nevertheless, this chapter reports in sequential steps: (1) a conceptual framework, (2) a discussion of the aims and objectives of the research, (3) the characteristics of the sample of parents and children who participated in the research, (4) the data collection methods that were employed, (5) a successful pilot study, and (6) the analytical approaches applied to the data set.

3.2 Conceptual framework

This section sets out the philosophical positions and theoretical-methodological conceptions behind this research study. It begins with the elemental facets of biography, ontology and epistemology and shows how these personal beliefs influenced the chosen research paradigm, which can consequently be seen to inform the aims and objectives of the study, the choice of participants, and the methods and approaches of enquiry that are all outlined in this chapter.

3.2.1 The researcher

Denzin and Lincoln (2003) suggest that qualitative research is a process defined by ontology, epistemology and methodology, which in turn are informed by the researcher’s biography. It seems appropriate therefore to reflect upon my biography, ontology and epistemology so that the reader can see how these have clearly influenced the methodological decisions taken in this study and the qualitative stance adopted.

Biography

It is important to recognise how my ideas and beliefs, shaped by a lifetime of experiences, have influenced this research project. As Gouldner (1962) asserts, research is not a value-free enterprise. As a researcher the decisions I make are a result of my value-laden judgements, judgements that rely not just upon the books and articles I have read but also upon the life I have lived. My interest in learning and conceptual development in mathematics, identity, and experiences of parents and children can be connected to my career as a primary school teacher. Working with pupils and parents stimulated an interest in how parents supported children's school mathematical learning. Over time this led eventually to a PhD and transformed into the research study outlined within this thesis. These experiences influenced what I wanted to research (experiences, identity and mathematical activity), who I wanted to research it with (parents and primary school children), where I wanted to research these phenomena (in individual homes), and why I wanted to research them (to understand mathematical learning).

Ontology

Ontology is what we believe to be the nature of reality (Denzin & Lincoln, 2003). Different authors give different categories of ontological assumptions. For instance in terms of our social reality, Bryman (2008) distinguishes between objectivist and constructivist whilst Cohen, Manion and Morrison (2007) differentiate between objective realism and subjective normalism. I prefer to think of a division between realist and relativist. Realism concerns a measurable reality existing externally and independently to the individual. It is a belief in an objective reality. Relativism suggests there is no concrete reality just individual subjective experience. In this later belief reality is relative to different individuals.

My ontological assumption of a relativist reality corresponds with constructivist theory. I see a social and cultural construction of reality, where reality is expressed through thought and language.

Epistemology

Ontology and epistemology are closely linked. If ontology is the nature of reality then epistemology is how we gain knowledge of reality (Wellington, 1996). It can be thought of as beliefs concerning the theory or nature of knowledge. Epistemological assumptions concern what can be known and understood. Bryman (2008) distinguishes between two types of epistemological assumption prevalent in social research: positivism and interpretivism, what Cohen et al., (2007) term positivism and anti-positivism. Positivism is historically associated with the natural sciences and relies upon the tenets of objectivity, inductivism, deductivism and realism (Bryman, 2008). Interpretivism is associated with subjectivity, interpretation and relativism and is prevalent in qualitative research.

My ontological assumptions lead me towards an epistemological position of interpretivism where there is no objective, concrete knowledge which exists external to the individual. Instead knowledge and understanding of human experience is a subjective interpretation. This places my philosophy close to Denzin and Lincoln's (2003) constructivist-interpretivist paradigm and Schwandt's (2000) social constructivism. When I align my epistemology with sociocultural theory, particularly the work of Vygotsky (1978, 1981, 1986), Leont'ev (1978, 1981), Rogoff (1990) and Saxe (1991), I conceptualise knowledge as subjective and individually-constructed through participation in social and cultural practice. Knowledge and learning are situated activities. The beliefs I hold as a researcher conspire to place me within the qualitative paradigm.

3.2.2 The qualitative research paradigm

Kuhn (1962/1996) coined the term paradigm to refer to collective theoretical understandings and beliefs that mould research. These beliefs are collective in the sense that they are shared by researchers who engage in a discourse defined by common understandings. Qualitative research is one such research paradigm. Debates concerning different paradigms and the growth of the qualitative movement have been the subject of much academic discussion amongst philosophers and social scientists (for a detailed discussion see Denzin & Lincoln, 2003). However, this is a lengthy discussion that it is not prudent to replicate in the space available within this thesis.

Two key features of qualitative research are reflexivity and interpretation. Qualitative research relies upon reflexivity on the part of the researcher during all phases of research, from formulation to reporting (Flick, 2009). Reflexivity involves reflecting critically on the self as a researcher (Lincoln & Guba, 2003). This process is central to, and pervades throughout this project. At every stage it is a reflexive enterprise, not just reflecting and evolving as a researcher but in constantly reflecting upon the research itself. This creates a fluid research entity that shifts and responds to the process of research rather than just the philosophy and theory of research.

Qualitative research is an interpretive enterprise that relies upon the researcher's subjectivity to interpret phenomenon. Denzin and Lincoln (2003, p.4) write that:

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible

Interpretative enquiry has the meanings, representations and perceptions of human beings as its primary data source (Mason, 2002). In this sense the qualitative approach is reflected in the heart of this project, namely the understandings of parents and children. To investigate these understandings a number of research questions were formulated (see 3.3). Then in order to access these human experiences a number of parents and children were recruited to the project (see 3.4). Next, understandings were collected qualitatively through interviews and observations (see 3.5) before being interpretatively analysed (see 3.7).

3.3 Aims and objectives

The overall aim of the project was to investigate parent-child mathematical activity, mathematical identity and parents' experiences of mathematics. It sought to build upon much of the work discussed in the previous chapter and, importantly, to produce a unique contribution to knowledge in the fields of psychology and education through well-constructed, reflexive qualitative enquiry.

3.3.1 Research questions

Mason (2002) likens research to an 'intellectual puzzle', the aims of which are expressed by research questions. Indeed, the appropriate formulation of research questions is critical to the success of any research endeavour (Flick, 2009). Research questions should organise the project, suggest the data required to answer them, and give a structure for reporting findings (Punch, 2005). They should also be clear, focused and reflected upon throughout the research process (Flick, 2009). The eight research questions listed below are a product of these intentions. They were manufactured in line with the aims, objectives and conceptual framework of the project and in concert with the literature review reported earlier. Throughout the research cycle the questions have driven the research but also been flexible enough to allow the study to grow and evolve.

Three questions are preoccupied with studying parents' 'experiences'. Here experiences are defined as the narratives, or opinions shaped by narratives, that parents recount which are concerned with (a) supporting children's development of conceptual understanding of primary school mathematics (b) potential issues and difficulties parents face, specifically during parent-child school mathematical interaction, and (c) home-school communication.

RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?

RQ2: What barriers do parents face in supporting their children's mathematical development?

RQ3: How are parental teaching practices shaped by, and through, communication with schools?

The next two questions focus on advancing understanding of the role parental mathematical identity plays in supporting children's learning of mathematics.

RQ4: How do parents dialogically construct identities for themselves and for their children?

RQ5: How does mathematical identity influence parent-child school-related mathematical activity?

The final three questions are geared towards studying parent-child mathematical activity, particularly how parents and children interact and co-construct mathematical learning.

RQ6: How do parents and children form, negotiate and operate mathematical goals?

RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?

RQ8: To what extent do parents ‘scaffold’ learning and conceptual development in mathematics?

Table 3A below shows how the three main themes of this thesis: experiences, identity and activity, connect to the eight research questions presented above.

Table 3A Linking research questions and the themes of experiences, identity and activity

Research question	**Experiences**	**Identity**	**Activity**
RQ1: What techniques, strategies and mechanisms do parents use to support their children’s mathematical development?	✓		
RQ2: What barriers do parents face in supporting their children’s mathematical development?	✓		
RQ3: How are parental teaching practices shaped by, and through, communication with schools?	✓		
RQ4: How do parents dialogically construct identities for themselves and for their children?		✓	
RQ5: How does mathematical identity influence parent-child school-related mathematical activity?		✓	
RQ6: How do parents and children form, negotiate and operate mathematical goals?			✓
RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?			✓
RQ8: To what extent do parents ‘scaffold’ learning and conceptual development in mathematics?			✓

3.4 Participants

In order to answer the research questions and investigate experiences, identity and activity associated with mathematics the study recruited a number of parents, both mothers and fathers, and their primary school-aged children. These participants displayed a range of ages, abilities and attitudes towards mathematics.

3.4.1 Sample selection

It was the original intention of this research to focus solely upon mathematical experiences, identities and interactions in parents and 7-9 year old children. However recruitment issues led to the sample being expanded to parents and 7-11 year old children.

Given the findings of a number of studies investigating parental experiences of mathematics and supporting children's mathematical learning, which found difficulties arising from curricular changes, the ideal age group for this study appeared to be 7-9 year olds. At this point in their primary school education children are exposed to a range of mental and written forms for calculation. This variety of strategies lessens as children grow older and use quicker, more mechanistic forms such as column algorithms. It was envisaged that Year 3 and 4 children would produce different forms allowing comprehension of their different conceptual understandings. It was also supposed that parents would have different experiences and understandings to their children. This could therefore provide a rich data set.

Problems in recruiting sufficient number of 7-9 year olds led to the study parameters being expanded to include 10-11 year old children. As expected these children did use a small number of forms but their parents still reported similar experiences to the parents of 7-9 year olds.

Whilst a great deal of research has studied parent-child mathematical interaction only one study has specifically investigated parent-child mathematical goals. This study looked at 2 and 4 year olds (Saxe et al., 1987), so by concentrating on an older sample this project sought new unique findings. Similarly, no research on mathematical goals, using Saxe's (1991) approach has investigated parent-child mathematical interaction or been located in the UK, giving the potential for a unique contribution to knowledge.

3.4.2 Ethical considerations

The process of undertaking academic research requires the researcher to constantly evaluate their decisions and judgements ethically (Sikes, 2004). Ethics is of central importance in the entire research process (Wellington & Szczerbinski, 2007), especially where the research involves young children (Hitchcock & Hughes, 1995).

In order to operate in an ethically-sound manner, researchers need to be conscientious in constructing their study (Bryman, 2008), in data collection through obtaining participant consent, maintaining confidentially and anonymity, and ensuring recruitment does not involve coercion (Robson, 2002) and in data analysis and presentation (Flick, 2009). Therefore the researcher reflected on the importance of ethics throughout the research process. At every stage of formulating aims, objectives and methodology, of collecting and analysing data, and of presenting my findings I have endeavoured to act in an ethically-responsible manner.

The study design and operation were built upon other professionally- and ethically-sound research studies reported earlier together with an awareness and desire to meet the ethical requirements of not just Oxford Brookes University, which was essential before the study could progress, but the guidelines set out by relevant professional bodies, namely the British Psychological Society and the British Educational Research Association.

Informed consent, which requires participants to voluntarily agree to take part in the study whilst understanding its background and potential impacts upon themselves, was required for parents and children. Consent, from both parents and children, was given in writing and could be withdrawn at any moment should the participant wish to withdraw from the study. In this case any data the participant provided would have been destroyed. Copies of the information sheet and participant consent form are included in Appendix A and B.

The identities of individual parents and children are known only to the researcher. All data related to participant identity was maintained securely. The names of the parents and children who took part in the research have been changed in this thesis in order to maintain anonymity and confidentiality.

Of crucial importance is that the research analysed and reported in this project has been presented in an honest and fair manner. The analysis undertaken and the quotes selected represent what I believe to be the 'truth' for these participants, the stories they told and the activities they undertook. Examples in this document, of which there are a great many in chapters 4, 5, 6 and 7, were representative of the participant and of the analysis. Evidence used out of context and yet presented or attributed to participants would be a gross betrayal of their trust.

3.4.3 Recruitment

Qualitative research generally utilises small samples of participants obtained through purposive sampling (Miles & Huberman, 1994). In other words samples are purposely selected because of their specific characteristics. In this case the sample was defined by the need to recruit parents and their primary school-aged children.

Participants were recruited to the study using a variety of mechanisms over the course of eighteen months. Initially schools were approached to act as a channel for contacting parents. Many schools were not interested in taking part but some did agree to act as a conduit of information. In this manner several hundred leaflets advertising the study were sent out to parents alongside large posters to be displayed in schools. This garnered a very limited response, with only one participant being recruited in this way. Because of this the recruitment strategy shifted away from targeting schools towards convenience and snowball sampling of parent-child pairs.

In convenience sampling participants are recruited in part because they are easily accessible to the researcher. It results in a sample which only represents itself (Cohen et al., 2007). In this case this strategy is acceptable since the research does not seek to generalise experiences, identity or activity for all parents. Through convenience sampling individuals known to the researcher were approached and given information about the study. They then decided whether or not to take part in the study.

Snowball sampling was also used to recruit participants. In snowball sampling participants are identified by other people who believe they will present an ‘information-rich’ case (Miles & Huberman, 1994). This typically involved a previous participant or other individual informing the researcher that their friend or family member was interested in taking part. These acquaintances were then approached as with convenience sampling above.

Many qualitative projects rely on the notion of theoretical- or data-saturation to determine the conclusion of recruitment and data collection. This is the point at which no new patterns, themes or theories appear to be emerging from the data. Its origins lie in grounded theory (Bryman, 2008). Mason (2010) argues that many PhD studies that claim to use saturation do not in fact use the concept as the guiding principle in sample size. Issues such as time, resources and the need to specify sample sizes for ethical approval influence eventual sample size. Mason (2010) surveyed 560 qualitative PhD’s and found a mean sample size of 31 for single data stream projects (e.g. interview only). Guest, Bunce and Johnson (2006) interviewed 60 participants in a qualitative study of health issues in two African nations. They found that the vast majority of patterns and themes were present in a much smaller sample. They argue that when the sample, as in this case, is a rather homogenous group of individuals then just 12 interviews are required. In their own research 88% of all their patterns and themes, and 97% of their key findings, appeared in their first 12 interviews.

In this study sampling was driven by the availability of participants and also the knowledge that qualitative research typically requires small samples to reach saturation. Eventually it was possible to recruit 24 parent-child pairs. This is when data collection ceased. All the themes discussed in the next chapter were later compared to their initial occurrence in the parental interviews. This showed that 95% of all the themes had arisen by the eighth interview and 100% by the twelfth interview. This appears to support the assertion of Guest et al. (2006) regarding sample sizes. It also suggests that whilst data saturation was not actively sought it was achieved.

3.4.4 Participant characteristics

Twenty-four parents and twenty-four children took part in this research. The following sub-sections below discuss the characteristics of these participants.

Parents

Table 3B describes the characteristics of the parent sample used in this study. All the parents were between the ages of 20-49 and had at least two children. Sixteen parents (71%) were mothers and eight (29%) were fathers. The parents reported a range of qualifications in mathematics from none to A-level. The majority stated that they used mathematics regularly in their work or daily life. The parent-child dyads that took part in the mathematical task are listed in Table 3C.

Table 3B Characteristics of parent participants

Parent	**Gender**	**Age Group**	**Highest qualification in mathematics***	**Use of mathematics in daily/work life**	**Number of children**
Abigail	Female	30-39	GCSE	Yes	2
Beth	Female	30-39	GCSE	No	3
Carl	Male	40-49	GCSE	Yes	2
Charlotte	Female	40-49	GCSE	Yes	3
Chris	Male	30-39	None	Yes	2
David	Male	40-49	A-Level	No	2
Deborah	Female	30-39	GCSE	Yes	2
Gary	Male	40-49	None	Yes	2
Gemma	Female	30-39	GCSE	Yes	3
Ian	Male	30-39	GCSE	Yes	2
Imogen	Female	40-49	GCSE	No	2
Jayne	Female	40-49	A-Level	Yes	2
Jennifer	Female	30-39	GCSE	No	2
Julia	Female	30-39	None	Yes	3
Lindsay	Female	30-39	None	No	3
Natalie	Female	30-39	GCSE	Yes	3
Neil	Male	30-39	GCSE	Yes	2
Niamh	Female	40-49	GCSE	No	2
Peter	Male	40-49	GCSE	Yes	2
Rebecca	Female	30-39	GCSE	No	2
Robert	Male	40-49	GCSE	Yes	2
Ruth	Female	30-39	GCSE	Yes	3
Suzy	Female	40-49	GCSE	Yes	2
Vicky	Female	40-49	GCSE	Yes	3

*GCSE = GCSE equivalent qualification typically achieved at 16; A-level = A-level equivalent qualification typically achieved at 18

Table 3C The parent-child dyads

Parent	Child	Parent	Child
Abigail	Zach	Jennifer	Jacob
Beth	Scott	Julia	Declan
Carl	Karen	Lindsay	Ben
Charlotte	Callum	Natalie	Daisy
Chris	Lizzie	Neil	Daniel
David	Grace	Niamh	Connor
Deborah	Caitlin	Peter	Jessica
Gary	Shaun	Rebecca	Zoë
Gemma	Kitty	Robert	Alex
Ian	Megan	Ruth	Michael
Imogen	Owen	Suzy	Matthew
Jayne	Oliver	Vicky	Sam

Children

The children in the study were all aged between 7-11 (Key Stage 2) and attended fourteen different state primary (5-11) or junior (7-11) schools across five local education authorities in the UK. Table 3D shows that the most popular age group was children attending Year 4, 14 children or 58% of the overall sample. This was due to the original parameters of the sample selection. Fifteen boys (63%) took part in the study compared to nine girls. The Year 4 group was the only one to have an equal number of girls and boys.

Table 3D Age, school year and gender of child participants

School year	Age (in years)	Number of children in sample	Number of male children in the sample	Number of female children in the sample
Year 3	7-8	4	4	0
Year 4	8-9	14	7	7
Year 5	9-10	3	1	2
Year 6	10-11	3	3	0
Total		24	15	9

Each of the four age groups shows a different pattern of gender and ability. Table 3E breaks down the participants into school year groups and shows each child's estimated attainment in mathematics. This description is a reflection of the researcher's professional judgement as a teacher, conversations with the parent (e.g. Rebecca - "They're not at the top, they are sort of middle. They're alright I think.") or child (e.g. Kitty - "I like maths because I'm really good at it and it's really fun...") and observation of the child completing the mathematical task (see 3.5.2). Table 3E suggests that overall the children were more able than might be expected in a randomly selected sample of children. It may be that the confidence and ability of the child was a deciding factor in whether a parent or their child wanted to take part in the study.

Table 3E Characteristics of child participants

Child participant	**Gender**	**Year group**	**Child's estimated attainment in mathematics**
Ben	Male	Year 3	Average
Declan	Male	Year 3	Below average
Owen	Male	Year 3	Below average
Sam	Male	Year 3	Below average
Alex	Male	Year 4	Average
Caitlin	Female	Year 4	Above average
Callum	Male	Year 4	Average
Connor	Male	Year 4	Average
Daniel	Male	Year 4	Average
Grace	Female	Year 4	Above average
Jacob	Male	Year 4	Average
Karen	Female	Year 4	Above average
Kitty	Female	Year 4	Above average
Lizzie	Female	Year 4	Above average
Megan	Female	Year 4	Below average
Scott	Male	Year 4	Average
Zach	Male	Year 4	Average
Zoë	Female	Year 4	Average
Daisy	Female	Year 5	Average
Jessica	Female	Year 5	Above average
Michael	Male	Year 5	Above average
Matthew	Male	Year 6	Average
Oliver	Male	Year 6	Above average
Shaun	Male	Year 6	Below average

3.5 Data Collection

In this project data collection occurred through two methods: interview and observation. Parents took part in episodic interviews with the researcher, generating data relevant to experiences and understandings of mathematics, mathematical activity and mathematical identity. Observations of parents and children working together on mathematical word problems provided data on mathematical activity and mathematical identity.

These two data collection streams provided the necessary means to generate information that could be analysed in order to address the project's eight research questions. Table 3F below shows how the parental interviews and the parent-child mathematical task were each designed to support a number of research questions.

Table 3F Linking research questions and data collection techniques

Research question	**Data collection technique**	
	Parental interview	**Parent-child mathematical task**
RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?	✓	
RQ2: What barriers do parents face in supporting their children's mathematical development?	✓	
RQ3: How are parental teaching practices shaped by, and through, communication with schools?	✓	
RQ4: How do parents dialogically construct identities for themselves and for their children?	✓	
RQ5: How does mathematical identity influence parent-child school-related mathematical activity?	✓	✓
RQ6: How do parents and children form, negotiate and operate mathematical goals?		✓
RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?		✓
RQ8: To what extent do parents 'scaffold' learning and conceptual development in mathematics?		✓

Validity and reliability

Quantitative research relies upon validity and reliability, which some authors (e.g. Cohen et al., 2007) argue applies to qualitative research as well. Certainly procedural reliability (Flick, 2009), replicating the observation instrument, was evident in this study. Validity is a more loaded term. Rather than validity and reliability, Lincoln and Guba (1985) argue for a focus on confirmability, credibility, dependability, transferability and trustworthiness. This appears far more apt for a qualitative enquiry and the ethical standpoint outlined earlier. Indeed Flick (2009) shows how Lincoln and Guba's (1985) criteria support 'valid' and 'reliable' qualitative research. This methodology chapter hopefully shows the reader how the project meets these five criteria, for instance in the credibility of its instruments and approach, the trustworthiness of the focus, researcher and participants, or the dependability of a reflexive and recursive approach to research design, data collection and analysis.

3.5.1 Parental interviews

The aim of the parental interview was to allow the researcher access to parents' experiences of (a) mathematics, (b) parent-child mathematical interaction and (c) home-school communication. They also provided evidence of the parents' dialogical constructions of the 'self' and 'other'.

Interview theory

Interviews are widely used by qualitative researchers in both education (Anderson, 1998: Drever, 2003) and psychology (Willig, 2008). They provide an adaptable and flexible method (Robson, 2002) that is able to generate insights and understandings into human experience (May, 2001). Interviews also allow researchers to investigate deep and complex issues (Cohen et al., 2007) and study perceptions, meanings, and individual constructions of reality (Punch, 2005). Furthermore, they provide a tool to study subjective viewpoints (Flick, 2009). As such, interviews are well-suited to the conceptual framework and research questions outlined earlier.

Interviews fall into three general categories: structured, unstructured and semi-structured. The category of interview chosen should ideally complement a study's research questions (Fontana & Frey, 2005). Structured interviews typically contain direct, closed questions that create exploratory inflexibility (Opie, 2004). They lack the depth of both unstructured and semi-structured interviews. Their repeatability, due to standardisation, does give higher levels of reliability and validity (May, 2001). Nevertheless closed questioning restricts the investigation of different experiences that characterise individuals, making them inappropriate for this particular project.

Unstructured interviews are the direct opposite to structured interviews. They are very flexible and allow the interviewer to respond to the interviewee in a highly reflexive manner. However the lack of structure means the data generated can be unpredictable (Wellington, 1996). Unstructured interviews suit open-ended, exploratory research projects, not necessarily those more tightly focused on particular areas and theories. Therefore they seem of limited use in the context of this project.

Semi-structured-style interviews contain elements of both structured and unstructured approaches and are a popular technique in research (Flick, 2009; Robson, 2002). They have some structure in terms of a number of predetermined questions relevant to the object of the study but they also give the researcher the opportunity to ask follow-up questions and probe responses (Hitchcock & Hughes, 1995). This flexibility appears well-suited to understanding and investigating parents' subjective experiences.

Interviews have several limitations that must be considered by the researcher. They can be highly subjective (Flick, 2009) and suffer from a lack of standardisation and repeatability (Bogdan & Biklen, 2007). This can be minimised by using a pre-determined instrument with all participants. Researchers also have to be aware of leading questions (Kvale, 1996). Whilst it is clearly the aim to ask questions on certain topics it is not appropriate to create contexts in which the participant is channelled towards saying what the researcher 'wants to hear' at the expense of what the interviewee 'wants to say'. In this project these dangers were avoided through reflexivity, i.e. constantly checking the actions and interpretations of the researcher.

A large number of different types of semi-structured interviews exist which are operated in a range of fields. When selecting an approach it is essential that the chosen type of semi-structured interview fits with the conceptual framework and object of study. Because of this a form of semi-structured interviewing termed 'episodic interviewing' was selected for this project. This approach, pioneered by Uwe Flick (1997, 2000, 2009), is based upon assumptions around episodic and semantic memory and asks interviewees to provide narrative episodes and opinions linked to pre-selected questions.

Episodic interviewing relies upon theories in psychology that assert that narrative is a mechanism by which we understand and make sense of our experiences (Flick, 2000). This matches the debate on identity in the previous chapter. Episodic interviewing also relies upon episodic and semantic theories of knowledge.

> ...episodic knowledge comprises knowledge which is linked to concrete circumstances (time, space, persons, events, situations), whereas semantic knowledge is more abstract, generalised and decontextualized from specific situations and events.
>
> Flick, 1997, p. 4

The reason why a particular experience is remembered and therefore has meaning is crucial. Why such an episode is significant to the participant requires an understanding of the event and its sociocultural context. For instance a mathematical experience may be significant to an individual because of a combination of success or failure and the context in which it occurred. Because of this the details are remembered and readily recalled. Another instance where actors or events differed may not be remembered because it was not significant to the individual.

Semantic knowledge is founded on assumptions and relations (Flick, 2009). Here research focuses upon the relationship between concepts and the subsequent experiences that arise from these. An example would be a respondent's opinion or definition of mathematical activity and how this relates to their mathematical experiences.

By combining these two forms of knowledge, episodic interviews allow insights into experiences, views and narrative understandings. Episodic interviews have been used previously by researchers investigating parents' representations of mathematics (O'Toole & Abreu, 2005; McMullen & Abreu, 2011), processes of identity formation in mathematics (Crafter & Abreu, 2010) and dialogical self theory (O'Sullivan-Lago & Abreu, 2010).

Episodic interview procedure

Flick (2000) presents a clear process for designing, conducting and analysing episodic interviews. This approach provided a template for the construction and operation of episodic interviews in this project.

Firstly, an interview schedule, shown in Appendix C, was constructed containing questions relevant to the topic. In this case twenty-three questions were formulated to cover parents' experiences of mathematics, mathematical interaction and home-school communication. These questions also allowed insights into positioning of the 'self' and 'other' with regard to mathematics.

The interview began by familiarising the interviewee with the style of the subsequent episodic questions and the emphasis on recounting concrete situations. Following a ‘warm-up’ question that sought to relax the interviewee, the first set of questions on the interview schedule covered the participant’s subjective understandings of the topic, for example:

What do you associate with the word mathematics? How does this word make you feel? Can you think of a time when you felt this way?

As with all the episodic questions probes were used to follow-up on interesting or insightful dialogue generated by the respondent. Next, biographical-style questions were asked concerning experiences of mathematics and the parent’s opinion of their child’s views on mathematics. This was followed by highlighting everyday experiences, therefore regular involvement in mathematics and mathematical interaction, through questions such as:

Does your child often talk about what he/she has being learning in class? Can you recall the last time your child spoke to you about their mathematics work in school?

Thereafter a number of questions were focused on parent-child mathematical interaction. Lastly, the final episodic questions were dedicated towards home-school communication, for example:

Can you think of a situation where you have discussed your child’s maths with their teacher/school?

At the end of the interview any follow-up questions or clarifications arising from points made earlier were addressed. As suggested by Flick (1997) the interview ended with the participant being asked to evaluate the interview experience.

In order to provide a context for the episodes and opinions that were collected in the interview, details about the situation and the participant were taken at the end of the session. In this case participants were asked a series of questions, included in Appendix C, the details of which were used to help reflect upon the data and also describe the sample.

Episodic interview operation

The interviews lasted from 20-43 minutes depending on the participant and took place in either the evening after school, weekends or during school holidays. Each interview occurred in the participant's home and was digitally recorded.

The order of task and interview was varied to check whether the positioning of the task and interview influenced responses. For instance whether parents referred to the task in the interview or whether the interview appeared to cue them act in a certain manner in the task. Parents who undertook the task first did occasionally mention the task in their interview but not to any consistent level or apparent relevance to their reported experiences or identities. No visible influence was seen in parental activity for parents who undertook the interview first.

3.5.2 Parent-child mathematical task

The second data collection strand in the research project was a simulated school mathematics task involving parents and children. The aim of the task was to replicate the work parents and children regularly complete as homework, and children complete at school.

Observation theory

Observation has a history central to qualitative research (Flick, 2009). Its many different types can be thought of as existing along a 'spectrum of observation' (Wellington & Szczerbinski, 2007). At one end is systematic non-participant observation, typically using set, standardised instruments. Here the researcher tries to be outside the activity and limit interaction with the participants. The opposite end of the scale is participant observation where the researcher becomes immersed in the situation. Here the boundary between observer and actor becomes blurred due to the inevitable interaction between the researcher and the participant.

Researchers also need to be aware that the process of observation typically influences the behaviour of participants (Flick, 2009). The behaviour of the parent-child dyad cannot be expected to be a complete replication of usual activity since the researcher is observing. This does not mean observation is inappropriate but rather that the researcher's presence needs to be reflected upon and appreciated during data analysis.

Previous research on mathematical goals has often involved observation of mathematical activity (Guberman et al., 1998; Guberman & Saxe, 1998; Nasir, 2000a, 2000b; Saxe, 1988b, 1991, 1992, 2002; Saxe et al., 1987). For instance Saxe (1991) observed the practice of candy sellers and the transactions that they made. His observation instrument was structured in order to differentiate the different mathematical goals constructed between sellers and customers, sellers and peers, and as sellers converted wholesale cost to retail unit prices. When Guberman et al., (1998) observed the mathematical goals of children playing monopoly they noted each mathematical problem a child faced and the solutions they adopted. Therefore it is appropriate to use a similar approach to observe the mathematical goals constructed by parents and children during a simulated school mathematical activity. Before the task could be designed two factors had to be considered, the mathematical focus and format of the task.

Focus: subtraction

A huge amount of research has been carried out into children's learning of mathematics, primarily at school but also at home. The focus of the task was not selected in order to further mathematical understanding but rather as a vehicle to study parents' and children's conceptual understanding through goal-related interaction. The topic of subtraction was chosen for three main reasons. Firstly, as outlined earlier, the teaching of subtraction has changed a great deal over the last 15 years following the introduction of the *National Curriculum* (Department for Education & Employment, 1999a), *National Numeracy Strategy* (Department for Education & Employment, 1999b) and *Primary National Strategy* (Department for Education & Skills, 2006). Therefore parents could be expected to have different experiences and use different mathematical forms and strategies for subtraction than their children. This would hopefully generate opportunities for dialogue and dyadic interaction. Secondly, subtraction is often conceptually problematic for children (Barmby, Bilsborough, Harries & Higgins, 2009), so parents would hopefully need to support and work with children, again generating dialogue and access to understandings. Finally in UK primary schools, particularly Years 3 and 4, children are taught and use a variety of different techniques for subtraction, which rely on different conceptual understandings of mathematics. Thus the choice of subtraction may give rise to children undertaking different mathematical goals for which different mathematical forms or strategies could be observed.

Format: word problems

In school mathematics, word problems act as a link between abstract concepts and 'real-life' situations, modelling reality and providing a context through which children can negotiate mathematical meaning (Hiebert, 1984). Word problems act in a similar way to other models and images in the classroom, such a diagrams, pictures and concrete materials, in that they are all an intermediate phase between the language of mathematics, its symbols, rules and procedures, and abstract ideas and concepts. They are used in classrooms and homework situations, presenting an appropriate context to study parent-child school mathematical interaction.

Task design

When you encounter a word problem you have to decode the semantic meaning of the sentence and connect it to a mathematical procedure or operation. Then you have to decide upon a strategy to achieve the goal of answering the question. These four steps (decode the text>select the operation>perform the computation>answer the problem) have been shown to be used in studies on children's answering of word problems (Greer, 1997). Not all children will decode the semantic structure the same, nor use the same approach to solve a certain problem (De Corte & Verschaffel, 1991). A child may well use an armoury of different calculation strategies flexibly when solving semantically different word problems. Whilst children may not all choose the same approach, the semantic structure of a word problem can influence the mathematical form or strategy children use to solve them (Anghileri, 2006; Carpenter, Hiebert & Moser, 1983; Carpenter & Moser, 1982; De Corte & Verschaffel, 1987, 1991; Fuson, 1992; Riley, Greeno & Heller, 1983).

A great deal of research has taken place into word problems in the early years of primary education (for a summary see Fuson, 1992). This has resulted in the evolution of a categorisation of word problems, produced by Fuson (1992), which is still broadly accepted in contemporary research. This categorisation contains four main types of word problem: (1) change - subtraction or addition to a quantity; (2) compare - comparison of two quantities; (3) combine - adding two quantities; and (4) equalize - finding the difference between two quantities. Word problems can also be categorised by their semantic component structure, namely: the number of items (unary or binary), the implied procedure (active or static), word order (e.g. missing start, change or end) and suggested procedure (e.g. take from or add to).

A range of different types of word problem were chosen for the task to vary difficulty and prevent repetition of questions. Ten question types, shown in Table 3G, were selected. This was seen as appropriate since the task was set at 20 minutes to mimic the typical length of homework in primary school.

In terms of difficulty, the problems were differentiated for the four different age-groups who undertook the tasks. The problems were designed with reference to expected levels of attainment for each age group, taken from the *National Curriculum* (Department for Education & Employment, 1999a) and the guidelines set out in the *Primary National Strategy: Pitch and Expectations* documents for Years 3, 4, 5 and 6 (Department for Children, Schools & Families, 2010a, 2010b, 2010c, 2010d).

Within each year group it was expected that variations in ability would be found, especially given recent research (Department for Education, 2011). Therefore each child was first given five problems aimed at average expectations of the year group. The performance of the child over these five questions then dictated the final five questions they would be asked to answer. Different sets of final questions were produced to cater for children struggling or excelling at the task. This design had the advantage of minimising frustration for children and allowing 20 minutes of observation to occur for virtually all participants. The list of different word problems used in the four year groups is included in Appendix D.

Table 3G Types of word problems used in parent-child task

Word problem type	Example
Compare take from (difference unknown)	Katie is 135cm tall. Josh is 109cm. How much shorter is Josh than Katie?
Equalize take from (difference sentence cues solution)	Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table?
Change take from (missing change)	Katie had 44 stickers. She lost some of them. She now has 9 stickers. How many did she lose?
Equalize add to (difference sentence cues opposite solution procedure)	Katie has 59 marbles. If Josh buys 16 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?
Change take from (missing end) – multi-step problem	Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?
Combine physically (missing part)	Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have?
Equalize take from (difference unknown)	Josh has 143 Lego pieces. Katie has 59 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Compare add to (difference sentence cues opposite solution procedure)	Josh has 261 counters. He has 134 more counters than Katie. How many counters does Katie have?
Compare take from (difference sentence cues solution)	Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have?
Change add to (missing start)	Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Each of the problems was presented on a single sheet of A4 paper. The word problem was displayed at the top and underneath was a large blank space for any written workings. This format allowed the collection of written mathematical forms.

Task procedure

Parents and children were all observed by the researcher completing the task. The tasks all took place in the family home and were introduced by the researcher who explained that the dyad should 'do what they normally do' when completing mathematics homework. It was hoped that this would help relax the participants and encourage them to act in a manner typical of their usual interactions. The researcher then observed the participants and only intervened to clarify any relevant instructions. The verbal dialogue of the participants was recorded. Field notes were taken during the observation of mathematical activity. These concentrated on capturing behaviours not detected by the audio recorder.

Children and parents were able to skip questions or return to them later. On three occasions the researcher felt it necessary to intervene to encourage the dyad to switch questions because of a participant's frustration or difficulty.

At the end of the task participants were asked to explain any interesting cases and asked for their evaluation of the activity. Children were also asked a series of simple questions concerning their views and opinions of mathematics to help gauge their performance in the task and whether they felt the task was representative in terms of their usual mathematics school work and homework. Afterwards the researcher wrote up initial impressions of the task and any notes and interpretations that appeared relevant to the situational context.

3.6 Pilot study

A small-scale study was carried out to pilot the data collection strategies previously described. Three participants took part in the pilot study. Two of the participants were known to the researcher and expressed an interest in taking part and the third responded to a locally-placed advertisement.

The first dyad took part in the mathematical task. They attempted to answer a range of word problems giving the researcher a greater understanding of the suitability of the task and practice at running this element of data collection (e.g. field notes, timing etc.). The transcript from the dyad also allowed data analysis procedures to be trialled. The analysis of the goals constructed by this parent-child pair was later published as part of proceedings of the Seventh Congress of European Researchers in Mathematics Education (Newton & Abreu, 2011).

The other two parent-child pairs took part in both interview and observation. This allowed the interview schedule to be trialled and refined and gave the researcher practice of carrying out and analysing episodic interviews. Only slight changes were made in question wording and order. The mathematical task data gave further evidence of the suitability of the task and practice of collecting and analysing this data.

For all three participants in the pilot stage the mathematical task was video recorded. This seemed to cause some distraction for both parent and child. Therefore the main study switched to digital audio recording as this was seen as less invasive and more attractive to potential participants.

3.7 Data analysis

Four different methods of data analysis were selected in order to answer the research questions central to this project. Table 3H shows how each analytical strategy enabled certain research questions to be approached. It shows how interview data was used to support research questions 1-4, task data was utilised in answering research questions 6-8, and both forms of data were used to investigate research question 5.

Data from the interviews was first subjected to a thematic analysis studying parental experiences. The same data set was then used to study dialogical constructions of the ‘self’ and ‘other’. Data from the parent-child task was examined through an analysis of mathematical goals and goal-related activity. In order to consider links between identity and goals the results of the dialogical and goal analyses were compared.

Table 3H Linking research questions and analytical strategies

Research question	**Analytical strategy**			
	Thematic analysis - interview data	**Dialogical self analysis - interview data**	**Goal analysis - task data**	**Linking identity and goals - interview & task data**
RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?	✓			
RQ2: What barriers do parents face in supporting their children's mathematical development?	✓			
RQ3: How are parental teaching practices shaped by, and through, communication with schools?	✓			
RQ4: How do parents dialogically construct identities for themselves and for their children?		✓		
RQ5: How does mathematical identity influence parent-child school-related mathematical activity?				✓
RQ6: How do parents and children form, negotiate and operate mathematical goals?			✓	
RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?			✓	
RQ8: To what extent do parents 'scaffold' learning and conceptual development in mathematics?			✓	

3.7.1 Thematic analysis

All parental interviews were first transcribed then checked against the original recordings to ensure consistency. The transcripts were then subjected to a thematic analysis.

Methodologies can be divided into those that are theory-laden or require a particular 'theoretical' framework, for instance conversation analysis or interpretative phenomenological analysis, and those that are theory-free, such as thematic analysis (Braun & Clarke, 2006). This gives thematic analysis the flexibility to be applied in a range of situations but also demands that any results must be supported by theoretical and conceptual framework, such as the one presented in 3.2.

Within research literature thematic analysis does not appear to be clearly demarcated or applied in a universally agreed manner. Whilst its different proponents agree on many of its essential criteria they do present contrasting approaches to implementing a thematic analysis. Still other approaches have large similarities to thematic analysis, such as Miles and Huberman's (1994) pattern coding, a meta-level of abstraction above initial coding. Three popular methods of thematic analysis were first compared, authored by Boyatzis (1998), Flick (2009) and Braun and Clarke (2006), before a single approach was selected and applied.

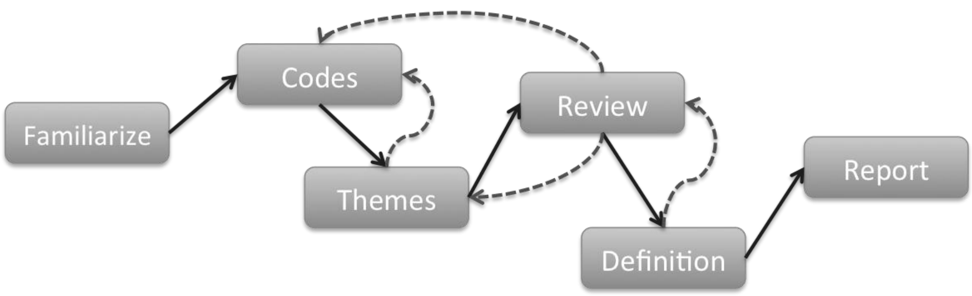
Boyatzis (1998) produces three types of thematic analysis based upon whether the study is theory-driven, prior research-driven or data-driven. All three approaches involve coding segments of data which are then checked and compared. His systematic approach places a heavy importance on sampling, reliability and validity, meaning that codes can also be scored and statistically analysed. This is arguably the most positivist of the three approaches and as such its reliance on sampling and its apparent desire to minimise interpretation in favour of consistency means it follows a path inapt for the conceptual framework and sampling strategies used in this study.

Flick's (2009) thematic coding originally developed out of grounded theory, but differs in its basis on a priori assumptions, and has been used to support episodic interviewing. In his version thematic analysis is a multi-staged approach. Firstly, a written description of each case is produced describing the participant and the external social context. Secondly, a deeper analysis studies the way the participant deals with or interacts with the object of study. This occurs through open and then selective coding, mirroring procedures in grounded theory. Selective coding generates thematic domains, which are cross-checked and amended as the study grows and more cases are analysed.

Braun and Clarke (2006) sought to draw together many of the issues and divergences in the different forms of thematic analysis and present a clearer approach and philosophical justification. They demonstrate a recursive technique that is becoming increasingly popular in qualitative research. Their model has a great many similarities with the approach of Flick (2009), in terms of coding and creating thematic domains, however it proffers a more structured and systematic analytical technique. It also draws attention to the importance of thematic maps, what Attride-Stirling (2001) calls thematic networks, to illustrate the coding structure, and its subsequent evolution and compression or expansion. As a method it has also been used by authors conducting similar research with parents (e.g. McMullen & Abreu, 2011). Because of these points Braun and Clarke's (2006) model of thematic analysis was chosen to investigate parents' experiences.

Braun and Clarke (2006) provide a six-stage approach to thematic analysis, shown in Figure 3A. Here the solid arrows show how the stages follow each other whilst the dotted arrows hint at the constant reflexivity underpinning the technique.

Figure 3A Braun and Clarke's (2006) process of thematic analysis



a) Familiarization with the data

In this first stage the twenty-four transcripts were read numerous times and initial ideas and thoughts were noted in the margins of each page. The purpose of this immersion is for the researcher to develop a deep understanding and knowledge of the data. This meant that before the first code was formed the researcher had begun to develop a much stronger awareness of the data set as a whole.

b) Generating initial codes

Theoretical coding, in line with the research questions under investigation, was then carried out. This focused attention on the aforementioned areas of understanding parents' perceptions and experiences of primary school mathematics, their experiences of completing school work at home with their children, and home-school communication. At this stage all individual transcripts were coded by hand by the researcher, producing codes that were closely linked to the data.

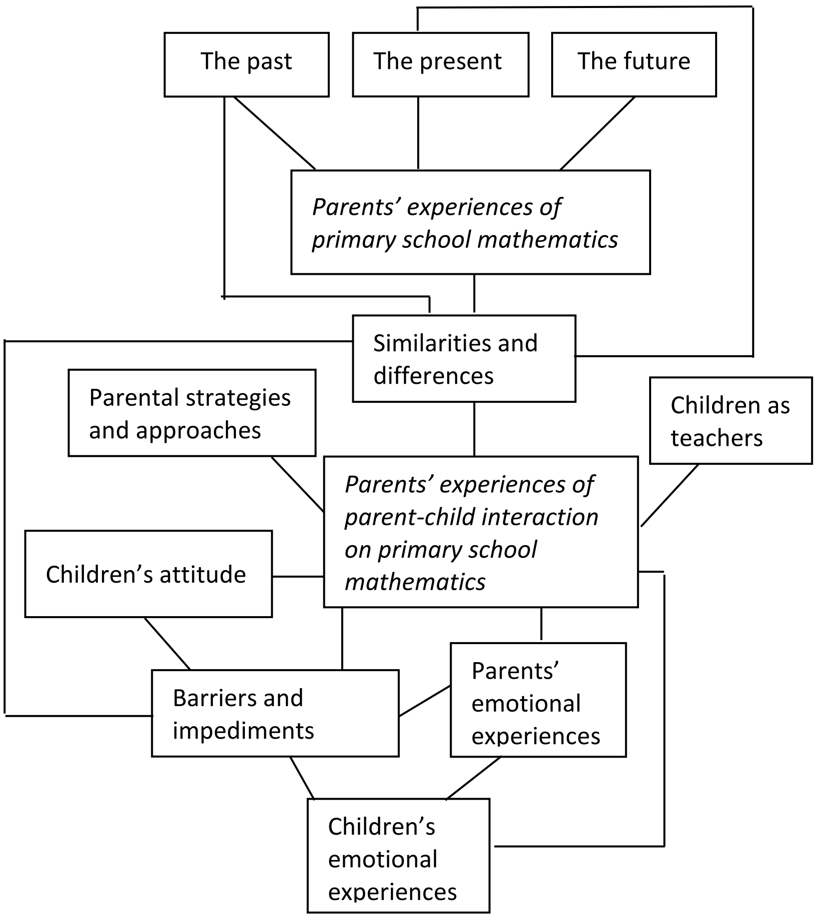
This theoretical approach did not mean that codes were pre-determined prior to data analysis. On the contrary all the codes, and subsequent themes, emerged from the data but were the result of attention being focused upon data relevant to the research questions underpinning this thesis.

c) Searching for themes

A list consisting of all the codes present in the transcripts was produced. These codes were compared between and across participants to build a series of themes. Thematic maps showing potential themes, connections and relationships were produced as an alternative way of viewing the data. An example of an early thematic map exploring the interconnectedness between several themes is shown in Figure 3B.

In this highly reflexive stage a great number of themes were created and discarded, and codes amended and reconceived. Codes and themes were then inputted into the qualitative research computer software package NVivo 8.

Figure 3B Exploring the connections between themes



d) Reviewing themes

When the themes were complete and conceptualised they were then reviewed and compared to each other. If there was too much overlapping data or the themes appeared too diverse or similar then themes and codes were reflected upon again leading to potential change and evolution. This process was supported by NVivo 8 which provided a quick and simple way to compare themes and codes across participants. The programme was only used as an aid to organising and sorting data. Interpretation and analysis still resided with the researcher.

e) Defining and naming themes

When the process of theme building and checking was complete each theme was then studied to identify its main central point. Themes were compared to initial extracts and coded sections of data to check consistency. They were then defined with a clear explanation and criteria for inclusion and exclusion, leading to further refinement and development. All the coded extracts in each theme were compared to ensure consistency. This led to alterations in theme definitions and to the merging and separation of some themes. Themes were structured in terms of a superordinate or overarching theme, in this case related to the focus upon parents' experiences, then themes, sub-themes and in some cases second-order sub-themes. The linking of levels of theme was visualised in a series of thematic maps and hierarchical diagrams, the purpose of which was another was to view and check themes and linkages. As such hierarchical diagrams are presented throughout chapter 4.

f) Producing the thematic analysis report

The result of the thematic analysis is presented in chapter 4 of this thesis. Here each superordinate theme and level below is defined, discussed and supported using a relevant example taken from a parental interview transcript. Occasionally two examples are used where only short segments of text existed, or a second example was thought to highly-strengthen the argument. Examples were selected to ensure, where possible, that the different voices of as many participants as possible could be heard.

Several themes that emerged out of the data were not included due to (a) relevance to the study research questions, (b) limited support or occurrence in the data set, or (c) limited space to report the theme within the chapter. As with the other stages these decisions were taken by the researcher based upon the data, factors discussed within the conceptual framework and the aims and objectives of the study. When the thematic analysis was completed it was observed that many of the themes appeared to be interconnected. Therefore a structural analysis of narrative episodes was carried out.

Structural analysis

A number of different techniques exist to interpret narratives. Structural analysis is a core form of narrative analysis (Riessman, 1993). In purely narrative-centred research it is often supplemented by further more complex analysis, for example thematic modelling or performative or interactional analyses (Esin, 2011). Structural analysis, pioneered by Labov (Cortazzi, 1993), gives an insight into how stories are constructed. It also "helps us understand how people give shape to events, how they make a point, their reaction to events and how they portray them" (Gibbs, 2007, p69-70). However, it does not directly tackle the interviewee-interviewer relationship (Riessman, 1993) or the sociocultural context (Esin, 2011). Therefore in this case the conceptual framework covered earlier guided its implementation.

Structural analysis shows how people represent past events and experiences through a series of chronological stages: (1) abstract, (2) orientation, (3) complication, (4) evaluation, (5) result and (6) coda (Cortazzi, 1993). The abstract summarises the story that follows giving brief details that suggest why it is important. Orientation covers the time, place, situation and participants within the narrative (Riessman, 1993). The complication is a series of events that link to the elements within the orientation. In the evaluation the respondent represents why these complicating events are relevant. In other words its significance and meaning to the narrator (Riessman, 1993). The result element of the narrative covers what finally happened (Cortazzi, 1993). The final stage, coda, is optional and occurs if the respondent chooses to complete the narrative by returning to the present time (Riessman, 1993).

The first step of the structural analysis was to collect all the parental narrative episodes contained within each transcript. Each narrative was then divided into the above five sections (abstract, orientation, complication, evaluation and result). The example in Table 3I shows the segmenting of a narrative episode produced by Gary.

Table 3I A narrative episode prepared for structural analysis

Narrative elements	**Dialogue**
Abstract	There was one we could get wasn't there. There was one that we had to leave altogether. There was an algebra question, yeah there was an algebra question
Orientation	and I always thought that I was pretty good at it
Complicating Action	and, and the problem that we'd got is that I had forgotten a vital part of how it operated. Erm and Shaun didn't know it either so neither of us was sure.
Evaluation	And I think we, we did, we answered it the best as we could with what we assumed was the solution but
Resolution	to be honest with you neither of us ever got to the bottom of it.

Unfortunately, the majority of episodes were not developed to a sufficient length to identify all the structural stages. A total of nineteen episodes, from twelve different parents, could be divided and analysed. These episodes were then studied by comparing them to the coded sections of data from the thematic analysis. Studying the narratives in this manner proved a useful tool in displaying the interweaving of themes within narratives, as well as showing the links between certain themes and consequent positive and negative emotional experiences.

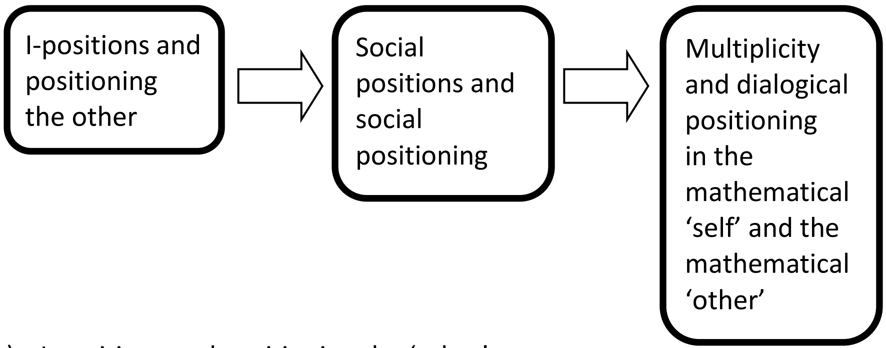
3.7.2 Dialogical self analysis

Bruner (1990) argues that one of the ways that people understand the world is through narratives. As an approach, narrative elements have been widely used to study elements of mathematical identities and positioning (e.g. Boaler & Greeno, 2000; Crafter & Abreu, 2010; Esmonde, 2009; Esmonde et al., 2010; Gorgorió & Prat, 2011; Sfard & Prusak, 2005). Similar methods have also been used in research into dialogical self theory. Since the approach and practice initially grew out of clinical psychology, several prominent authors have used rating scales and semi-structured or narrative interviews to identify I-positions (Hermans, 1996; Raggatt, 2000). Yet the theory is also interested in the realisation of self through dialogue. Consequently researchers have also used a variety of different methods to investigate the dialogical self. Analysis of biographic texts (Raggatt, 2007), autobiographical constructs (Josephs, 2002), blogs and classroom talk, both child-child and teacher-child (Ligorio, 2010), interviews, focus groups and observations (Aveling & Gillespie, 2008), and, methodologically relevant to this study, semi-structured episodic interviews (O'Sullivan-Lago & Abreu, 2010) have all been used in the past.

In this project the episodic interviews, discussed in 3.5.1, were analysed with regard to the narrative episodes co-constructed by the parent and researcher. At this point it is prudent to note the term 'co-constructed'. Whilst the interviews were carried out with the minimum of interference or interruption the parents' episodes were still co-constructed with the researcher. The identity that a participant creates is a result of that person and the surrounding social world (McAdams, 1993). A parent is producing a story for their audience, the researcher. The story is shaped and created for the ear of the researcher and therefore can be thought of as co-constructed between interviewee and interviewer.

The dialogical self analysis followed three stages, shown in Figure 3C, each investigating a different form of positioning. The basis for this approach was drawn from a number of different academic studies explained below. Parental interviews had already been transcribed and checked as part of the first stage of thematic analysis reported above and so were already prepared for analysis.

Figure 3C Three stages of dialogic analysis of the mathematical 'self' and mathematical 'other'



a) I-positions and positioning the 'other'

O'Sullivan-Lago and Abreu (2010) studied I-positions in their work on identity in cultural contact zones. In their study, I-positions were identified through the coding of episodic interview transcripts. In this case codes were applied to segments of text where the interviewee was positioning themselves whilst discussing elements relevant to the study. Their approach allowed the classification of a number of different I-positions in the text and produced more in-depth knowledge of dialogical processes in cultural identities. This is one of the few approaches to use both episodic interviews and a straightforward, systematic coding system.

Using an approach similar to O'Sullivan-Lago and Abreu (2010), the interview transcripts were investigated for the mathematical positioning of the 'self' and of the 'other'. The focus of analysis was on situations where parents spoke about themselves or their children with reference to mathematics or mathematical contexts. In this sense the inquiry focused on categorising, studying and comparing the mathematical positions that parents assign to themselves and to their children.

Sections of text involving mathematical positioning were firstly open coded. This long list of codes was then studied for patterns and commonalities resulting in the combining of some open codes into larger pattern or thematic codes.

In both the first and second stages of dialogical self analysis coded segments for the 'self' generally, but not exclusively relied upon 'I' in the first person (I, we, me, us), for instance "*I like numbers. Alright, I like numbers, I like adding things up*" was coded as 'I as enjoying mathematics', whilst positioning the 'other' tended to be in the third person (he, she, it , they), for example "*she always responds positively so I don't have any, I don't have any doubts that she enjoys maths*" was coded as 'My child as enjoying mathematics'.

As with the earlier thematic analysis, transcripts were first coded by hand before codes were placed into NVivo 8 in order to more quickly and easily check and compare codes and ensure consistency across the whole sample. When the long list of 'self' and 'other' codes was complete it was apparent they fell into three identifiable themes, associated with behaviours, competencies and emotions. Codes were divided in these categories and then checked again for appropriateness and consistency.

b) Social positions and social positioning

The second stage of the dialogical analysis centred on the role of the sociocultural environment on mathematical positioning. This studied the 'voices' or social positions that can be seen in parental narratives. Again positions were investigated with reference to positioning of the 'self' and of the 'other'.

This approach followed the model of Aveling and Gillespie (2008). Their study looked at I-positions in second generation Turks living in the UK. In particular they focused on the relationship between I-positions and the sociocultural environment. After identifying I-positions, their analysis sought to code the 'voices' present in narratives. This coding of verbal data was used to discover the social origin of I-positions. It focused on reported speech and what they termed 'echoes', which are "utterances that are not attributed to others, but that nonetheless seem to have a distinct social origin beyond the speaker" (Aveling & Gillespie, 2008, p.6).

Following this approach, this stage of analysis began by open coding segments of data and then combining and refining codes through a recursive cycle and the use of NVivo 8. In this case codes could be seen to reflect actual specific voices and more ephemeral generalised individuals. Codes could also be often connected to positions of the 'self' and 'other'. For instance 'I as good at mathematics', a mathematical I-position, could also be 'I as more successful at mathematics than others', a general voice, or if it involved more specific voices 'I as more mathematically successful compared to my friends'.

c) Multiplicity and dialogical positioning in the mathematical 'self' and the mathematical 'other'

Any approach which separates and studies elements of data, in this case related to coding dialogical identity, naturally fragments individual narratives in order to study patterns and support general theoretical constructs. However, a key factor in dialogical self theory is the notion of multiplicity of positions, a constant evolution in space and time, and the idea that people construct a highly complex 'self' in and through dialogue. These were the subject of the third and final stage of analysis.

Firstly, multiplicity was assessed by comparing the different positioning of the 'self' and 'other' within individuals. This showed not only the number of positions but the variety of similar and different 'self' and 'other' identifications. Next, the positions within each individual were studied in terms of chronology to see whether positions shifted or remained constant over time. This was possible since parents often produced mathematical narratives concerning different events and different stages of their lives. Finally, positions were compared spatially by looking at the context in which they occurred, for instance school, work and home.

The three stages of dialogic analysis of the mathematical 'self' and mathematical 'other' are reported in chapter 5 of this thesis. Findings are supported with examples taken from parental interviews. As with the thematic analysis these quotes are taken from a wider number of participants as practical. Some miscellaneous 'self' and 'other' positions that were not relevant to the study aims and research questions were discarded.

3.7.3 Goal analysis of parent-child task

Goal formation and operation is an active process. Mathematical goals emerge from social and cultural constructions which are active and constantly evolving. This flux is represented in Saxe's (1991) model. As noted in the previous chapter, the emergent goals framework produced by Saxe (1991) provides an approach that can be used to study learning and development in social and cultural processes at the microgenetic scale. This approach has been used by a number of authors to study observations of emergent goal behaviour (e.g. Guberman et al., 1998; Nasir, 2000a, 2000b, 2002; Saxe, 1991, 1992, 1995, 2002).

The digital recordings of the parent-child tasks were first transcribed. This included notes of pauses and pause lengths. The field notes were then compared to the recording and text to add detail and depth to the transcript, for example linking an observed attitude, process or emotion to the transcribed dialogue. Description of written work completed by the dyad was also added to the transcript to add further depth. Within each transcript the text was divided into segments relating to the particular word problem being tackled.

Natural breaks in speech, for instance completing a calculation silently, and changes in speaker, typically parent, child or researcher, were used to split the text into turns. Each turn was sequentially numbered and time coded. The use of timing for turns and pauses helped interpretation of activity by suggesting mathematical thinking-time and allowing analysis of chains of turns or utterances (i.e. how parents and children responded to goal-related dialogue). Once the transcripts had been formatted in this fashion they were subjected to goal analysis.

Firstly, the task and completed transcripts were read and re-read several times, with sociocultural theory and research in mathematical learning and development in mind, to interpret the ingredients present in each of Saxe's (1991) four parameters. Once these had been identified then goals and goal-related behaviour was sought in each parameter. A consequence of this was an overlap of goals where intentions and actions could be ascribed to more than one parameter.

The coding of what was or was not a mathematical goal, where a certain goal began or ended, whether identical goals in different parts of the transcript were the same goal or different goals, was a labyrinthine undertaking. Indeed, Saxe (1991, p.16-17) himself notes that:

> Specifying the goals individuals form in cultural practices is an analytical endeavour of some complexity. Not only do individuals shape and reshape their goals as practices take form in everyday life, but they also construct goals that vary in character as a function of the knowledge that they bring to practices. Like Suchman's recent characterization of running rapids in a canoe (1987), while one may have a general plan of approach - goals and sub-goals to accomplish in the run - when one hits the white water, goals and means of accomplishing them emerge and shift with the exigencies of the situation and one's expertise.

At no time do any of the studies discussed previous clearly set out how they code a manuscript for goals and goal-related behaviour or what does in fact counts as a 'goal'. They all present a method of analysis where it appears that the four parameters (activity structures; artefacts and conventions; social interaction; and prior understandings) are separated and goals are found in each. To minimise the complexity in identifying mathematical goals this analysis concentrated on a definition of a mathematical goal as 'a conscious action carried out as a result of a particular mathematically-related motive or intention'. This narrowed the focus of attention to elements of the transcripts where specific mathematical actions could be identified in association with a certain parameter. Even so intentions of parents and children could only be inferred from their words and their observable actions. The coding of goals was still a highly interpretative exercise that relied heavily upon the theoretical knowledge and understanding of the researcher. Coding of goals within and between transcripts was compared almost constantly during analysis to maximise consistency and repeatability. This recursive approach, like the previous analyses on interview data, meant that coded segments were reduced, expanded, abandoned or retained until a point in the analytical cycle where only minimal change or doubt existed.

This approach was first tested on three transcripts (Rebecca, Robert and Vicky), the results of which are reported in Newton and Abreu (2012). The method was then extended to the other transcripts, with the three analysed transcripts being re-embraced into the reflective cycle. Upon completion of this analysis, goals in each of the parameters could be compared within and between individual transcripts allowing a range of factors, such as number of goals, the originators and completers of goals, age of child and ability of child to also be investigated. The final analysis of goal-related activity is presented in chapter 6. Here excerpts from task transcripts are used extensively to support findings.

The previous chapter highlighted the importance of processes of contingency and scaffolding in parent-child interaction. This led to the formation of two research questions to tackle these twin areas. The parent-child observation data was first analysed for contingency shifts and then for scaffolding.

Contingency shifts

Wood and Middleton (1975) studied contingency through coding interactions for five separate levels of assistance, varying from general verbal instruction to demonstration, and then comparing these across a pyramid building task. Wood et al. (1978) followed a similar approach to study four different types of teaching strategy, including a specific contingency method. Again this focused on the five levels of intervention used in Wood and Middleton (1975). Both papers used statistical analysis to study the relationship between the different levels and showed a resulting contingency 'shift' as parents gave more or less support depending on the success or failure of the previous activity. In this study it was thought to be overly complex to code every parental intervention on the five levels then undertake statistical comparison. Therefore the dialogue and notes for each completed word problems was compared with the next to see if the child's previous performance appeared to influence the parent's level of support. Whilst this is clearly not as rigorous as other studies it does give an indication of presence or absence of contingency processes in the sample.

Scaffolding

Scaffolding, as first outlined by Wood et al. (1976), has been the subject of much academic research over the last 30 years. Different authors have used alternative definitions and approaches to analyse scaffolding in a variety of contexts. This debate was covered extensively in van der Pol et al. (2010). They reviewed 66 academic articles in order to eventually propose a framework for the analysis of scaffolding that builds on the original ideas of Wood et al. (1976) and subsequent work in the field. Their framework contained five types of scaffolding 'intentions' (direction maintenance, cognitive structuring, reduction of degrees of freedom, recruitment, and contingency management) and six types of scaffolding 'means' (feedback, hints, instruction, explaining, modelling, and questioning). Elements of this approach were then used to study teacher-pupil interaction (van der Pol et al., 2011).

Because of its logical approach and strong theoretical foundation, the van der Pol et al. (2010) framework was used to analyse scaffolding in this this study. The goal-related dialogue between parents and children was studied for the various scaffolding means and intentions in line with the definitions and examples included in Appendix E. The aim of the analysis was to observe the extent to which scaffolding was present in parent-child interaction. Therefore in this case the presence or absence of the elements were noted not their frequency, length or juxtaposition to other means or intentions.

3.7.4 Linking identity and goals

The final analysis carried out in this project attempted to connect findings from the dialogical analysis of 'self' and 'other' to analysis of the parent-child interaction in order to investigate the connections between goals and identity suggested in research literature (e.g. Nasir, 2002). In this case goal-related activity of the parent and child was compared to the 'self' and 'other' positions that had been identified for each parent participant. This was relatively straightforward given that the goal and identity analyses had each already been separately completed.

Social positions and social positioning could not be scrutinised directly since they were not accessible in parent-child dialogue during the interaction. However social positions could be interpreted from some forms of behaviour. For instance, supportive behaviour in the task might reflect I-position of 'I as supporting my child's mathematical development', which could have originated in a social position of 'I as more supportive than other parents'.

Because of the number of examples required to support dialogical positioning, and the number of excerpts needed to link to goal-related behaviour, a single parent-child pair (Abigail and Zach) is used to illustrate the links between mathematical identity and mathematical goals in chapter 7. Here examples from other parent-child dyads are used sparingly and only when they add greatly to the argument.

3.8 Conclusion

In this chapter the core conceptual and theoretical assumptions underpinning the research project have been explained and justified as an interpretative qualitative endeavour. The study design, including the eight research question that guided the project, was clarified. The mothers, fathers and children that took part in the study, whose experiences and activity form the later analytical and discussion chapters, were described in-depth. The two methods of data collection and four analytical approaches expounded and defended. This sets the scene for the four analytical chapters that now follow.

Chapter 4

Parents' experiences of mathematics, mathematical interaction and communication with teachers and school

4.1 Introduction

This analysis contributes to the investigation of three research questions concerning parental involvement in children's mathematical development:

RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?

RQ2: What barriers do parents face in supporting their children's mathematical development?

RQ3: How are parental teaching practices shaped by, and through, communication with schools?

Certainly, the research reported earlier suggested that parent use a variety of approaches during parent-child interaction (Civil & Andrade, 2002; Civil et al., 2008; Hoover-Dempsey et al., 1995; Hyde et al., 2005; Solomon et al., 2002), that they face a number of impediments to supporting children's school mathematical development (Abreu & Cline, 2005; Civil & Bernier, 2006; McMullen & Abreu, 2011; Street et al., 2008), and that typical patterns of home-school communication exist that appear to influence parental involvement (Abreu & Cline, 2005; Hughes et al., 2007; Street et al., 2008). Underlying all these are processes of valorisations highlighted by Abreu (1995, 1998, 2002).

What is not clear is how these factors operate in non-minority groups in the UK or necessarily how experiences feed into valorisations, which then influence parent-child primary school-related mathematical activity.

The thematic analysis of twenty-four parental interviews reported here resulted in the emergence of three superordinate themes associated with experiences and valorisations of:

- School mathematics
- Parent-child school mathematical interaction

- Communication with school shaping parental mathematical practices

The analysis resulted in four levels of themes emerging from the participant responses. These are ranked as: superordinate theme>themes>sub-themes>second-order sub-themes.

In this chapter each of the superordinate themes is reported in turn with particular attention to its component themes and sub-themes. Each theme and sub-theme is defined and illustrated with an example taken from interview transcripts. Thematic hierarchies, together with frequencies of themes and sub-themes across the sample, are reported at the beginning of each of the three superordinate theme sections.

Many of the themes and sub-themes produced in this analysis overlap, sharing characteristics, origins and consequences. The analysis, being highly focused on individual patterns, does not address the interconnected nature of many of the themes. In order to provide such scrutiny, a narrative structural analysis was also conducted on the interview data. The results of this approach are also reported here. The chapter concludes with a discussion of the meaning and relevance of these results in conjunction with the contemporary academic debate.

4.2. Parents’ experiences of mathematics

The first superordinate theme arising from the analysis was parents’ experiences of mathematics. Four main themes concerning parents’ experiences were identified within the data set: similarities and differences between parents’ and children’s mathematics, experiences of learning mathematics, valorisations of mathematics, and perceptions of the role of mathematics in children’s futures. The last three themes concern respondents’ past, present and imagined future.

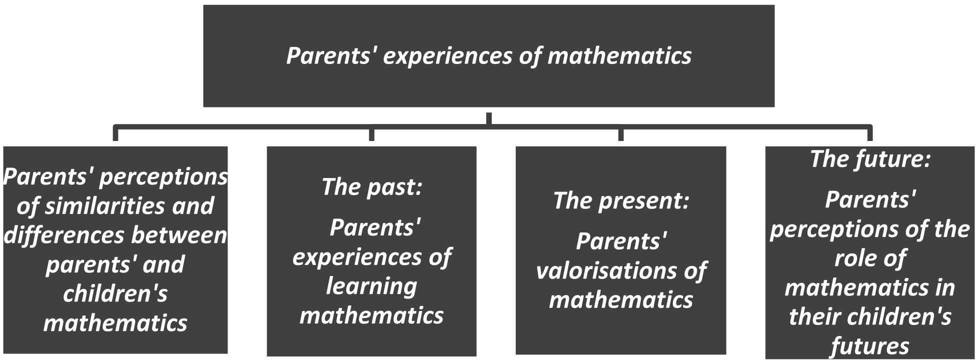


Figure 4A Thematic hierarchy for parents' experiences of mathematics

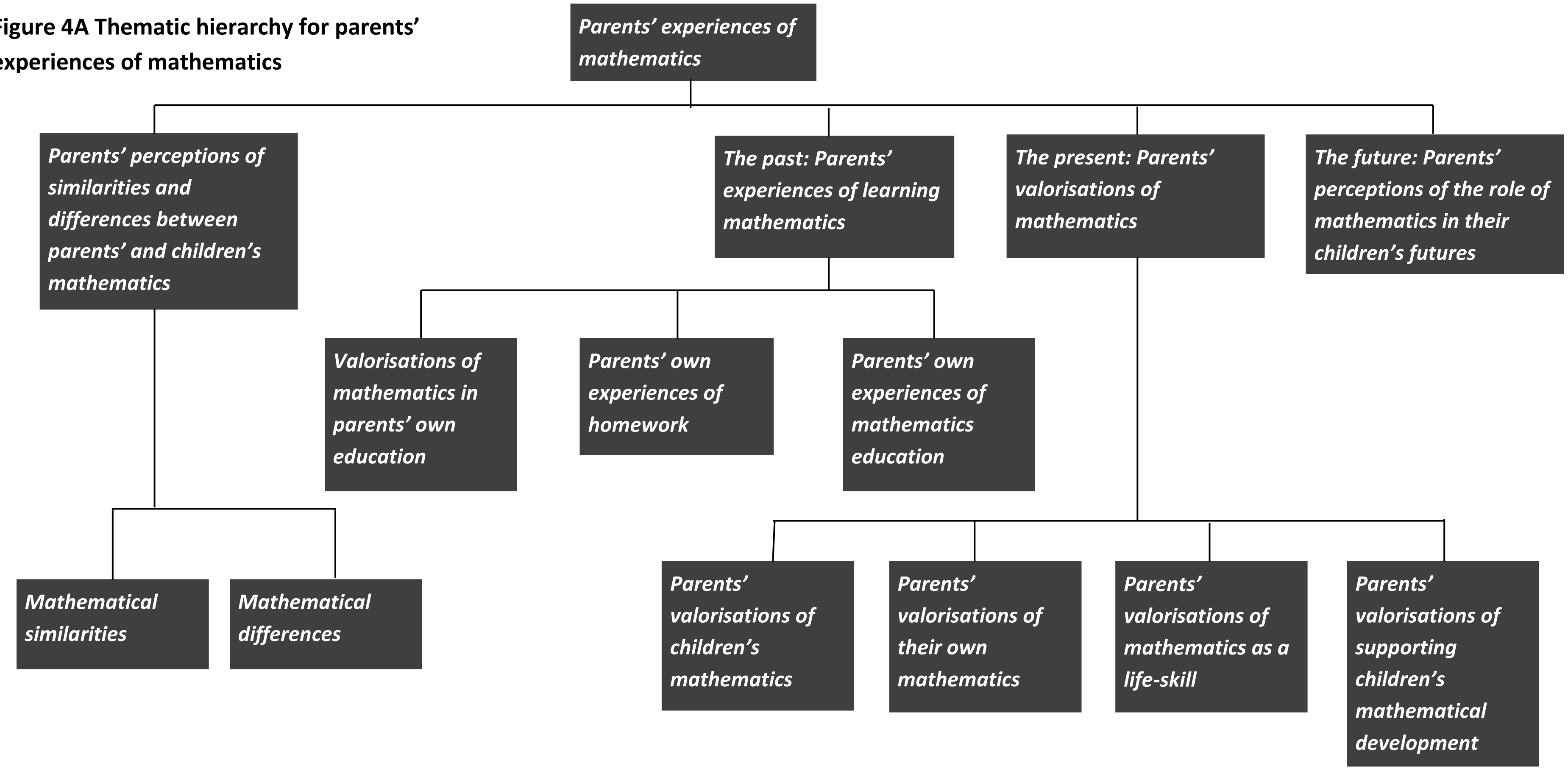


Table 4A Parents' perceptions of similarities and differences between parents' and children's mathematics and parents' experiences of learning mathematics – thematic frequency table

	Themes	***Parents' perceptions of similarities and differences between parents' and children's mathematics***		***The past: Parents' experiences of learning mathematics***		
	Sub-themes	***Mathematical similarities***	***Mathematical differences***	***Valorisations of mathematics in parents' own education***	***Parents' own experiences of homework***	***Parents' own experiences of mathematics education***
Year 3	Imogen			✓	✓	✓
	Julia		✓	✓	✓	✓
	Lindsay		✓	✓	✓	✓
	Vicky			✓	✓	✓
Year 4	Abigail		✓	✓		✓
	Beth	✓	✓	✓	✓	✓
	Carl	✓	✓	✓	✓	✓
	Charlotte					✓
	Chris		✓	✓	✓	✓
	David		✓		✓	✓
	Deborah		✓	✓	✓	✓
	Gemma		✓	✓	✓	✓
	Ian	✓		✓	✓	✓
	Jennifer		✓	✓		✓
	Neil		✓	✓	✓	✓
	Niamh	✓	✓	✓	✓	✓
	Rebecca		✓	✓		✓
	Robert		✓		✓	✓
Year 5	Natalie		✓	✓	✓	✓
	Peter		✓	✓	✓	✓
	Ruth	✓		✓	✓	✓
Year 6	Gary		✓	✓	✓	✓
	Jayne		✓		✓	✓
	Suzy		✓		✓	✓
	Total	**5**	**19**	**19**	**20**	**24**

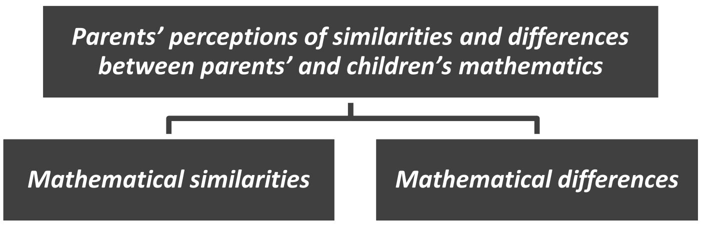
Table 4B Parents' valorisations of mathematics and perceptions of the role of mathematics in their children's futures – thematic frequency table

	Themes	***The present: Parents' valorisations of mathematics***				***The future: Parents' perceptions of the role of mathematics in their children's futures***
	Sub-themes	***Parents' valorisations of children's mathematics***	***Parents' valorisations of their own mathematics***	***Parents' valorisations of mathematics as a life-skill***	***Parents' valorisations of supporting children's mathematical development***	
Year 3	Imogen			✓	✓	✓
	Julia			✓	✓	
	Lindsay		✓	✓	✓	
	Vicky			✓	✓	
Year 4	Abigail	✓		✓	✓	
	Beth		✓	✓	✓	✓
	Carl	✓	✓	✓	✓	✓
	Charlotte		✓	✓	✓	✓
	Chris	✓	✓	✓	✓	✓
	David	✓		✓		✓
	Deborah	✓	✓	✓	✓	
	Gemma	✓		✓	✓	✓
	Ian	✓		✓		
	Jennifer	✓			✓	
	Neil	✓	✓	✓	✓	
	Niamh	✓	✓	✓	✓	✓
	Rebecca	✓				✓
	Robert	✓			✓	✓
Year 5	Natalie	✓		✓	✓	
	Peter					✓
	Ruth			✓	✓	
Year 6	Gary			✓	✓	✓
	Jayne		✓	✓	✓	✓
	Suzy	✓	✓	✓	✓	
	Total	**14**	**10**	**20**	**20**	**13**

Figure 4A shows a diagrammatical representation of all the themes and sub-themes within the superordinate theme of parental experiences of mathematics. A total of four themes and nine sub-themes emerged from the data. The frequencies of each sub-theme within the sample are shown in Tables 4A and 4B. Here frequency refers to the presence or absence of the theme in each individual interview. For instance, Table 4A shows that the sub-theme 'Parents' own experiences of homework' was present in 20 of the 24 interviews. The two tables split the participants sequentially into four groups relating to the age of their child: Year 3 (aged 7-8), Year 4 (8-9), Year 5 (9-10) and Year 6 (10-11). This allows the comparison of themes across age groups.

4.2.1 Parents' perceptions of similarities and differences between parents' and children's mathematics

A striking feature of the interview data is the way in which parents' perceive the mathematics that their children are learning in primary school as similar or different from the mathematics that parents covered in their own schooling. Parents also saw differences between their children's school mathematics and the mathematics parents used in their working lives. Similarities appeared to help parents identify with, and feel more comfortable supporting their children. Perceiving mathematics as different was seen on occasion to lead to more problems and difficulties for parents when helping their children.



Mathematical similarities

Some parents spoke about how their children's mathematics appeared similar to their own. By the end of Key Stage Two (Years 5 and 6) children's mathematical strategies and approaches, particularly for addition and subtraction, begin to converge with the methods parents were taught and use in everyday life.

Beth highlighted this point when she noted that the mathematics in Standard Assessment Tests (SATs) that children take at the end of Year 6 appeared to be the same as the mathematical strategies she was taught at school.

Parent participant: Beth
Theme: Parents' perceptions of similarities and differences between parents' and children's mathematics – Mathematical similarities

Yeah, yeah and looking, to be honest looking in the SATs, because I've bought quite a few err SATs revision books for William up to SATs and looking in there it seems to be the way we did it in school yeah.

Mathematical differences

Nineteen of the parents perceived the mathematics that their children were learning in primary school as distinctly different from their own schooling or knowledge of mathematics. It is clear from analysing the parental narratives that an inability to support or identify with their children's mathematics appeared to prevent some parents from supporting their children's mathematical learning. Parents discussed differences in strategies for addition, subtraction, multiplication and division, as well as the teaching of shape, and the use of calculators, computers and interactive software.

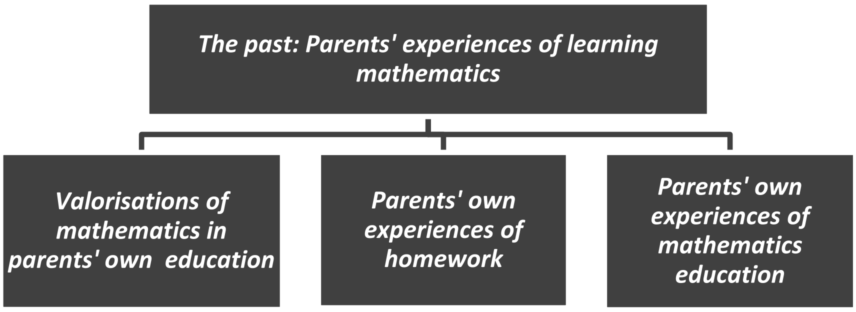
Lindsay found the current teaching of mathematics difficult to comprehend, here comparing it to a foreign language. The knock-on effect for this on her mathematical interactions with her children was plain to see in the narratives she produced.

Parent participant: Lindsay
Theme: Parents' perceptions of similarities and differences between parents' and children's mathematics – Mathematical differences

Well they do it a lot different now, which is confusing as a parent, you know we do things one way and they do it a completely different way, it's like foreign [language] to us, do you know what I mean, and sometimes that can be a hard to understand.

4.2.2 The past: Parents' experiences of learning mathematics

Within the interviews parents were asked questions concerning their own mathematical histories. The analysis of these narratives produced insights into the value or importance they placed on mathematics during their schooling, including the social significance of homework. Furthermore, thematic analysis of these narratives displayed parents' experiences of learning mathematics at school and undertaking homework with their own parents.



<u>*Valorisations of mathematics in parents' own education*</u>

When parents discussed the value they placed on mathematics during their own education and schooling it was clear that the majority felt that it was important to learn mathematics. They often felt expectation or even pressure from parents or school to work hard and succeed at exams. None of the parents perceived mathematics as unimportant during their childhood, though several suggested that it had a limited value or importance.

Several parents talked about the homework practices that characterised their childhood. From these it was possible to infer the values attached to specific activities. These valorisations were connected to the time their own parents dedicated to the activities, the rules that monitored homework and the expectations that were thrust upon them.

Neil presented an example of the predominant view amongst the respondents in this study. He presented his childhood valorisations of mathematics as linked to both his parents' views and his desire to 'do well in life', which for him involved securing employment and a home.

Parent participant: Neil
Theme: The past: Parents' experiences of learning mathematics – Valorisations of mathematics in parents' own education

Yeah and that comes from my parents. Erm I suppose maths, English and science were the key subjects in school. Science wasn't as big a thing with my parents because it wasn't a big school subject when they went to school. But maths and English was, "If you don't do maths and English well you'll never get a job". And I always wanted to do well in life so I always thought if I don't do well in maths and English I'm never going to live anywhere.

Parents' own experiences of homework

When parents talked about their own mathematical experiences of homework they tended to discuss general attitudes towards support or an absence of support. Where parents commented on the approach to the support they received as a child they often presented their own practice as replicating that of their parent. Where a parent identified their own support as more extensive than that which they received as a child, they sometimes reasoned that this was precisely because of the lack of support they had received from their own parents. In this sense we see parents perceiving their own practice as radically different to that of their parent.

Deborah presented an example of a parent who perceived her practice as replicating her own upbringing. She brought forth a view of the guidance and support she received as a child from her mother, who valued and prized academic achievement and supporting her children, and likens this to how she works with her daughter Caitlin.

Parent participant: Deborah
Theme: The past: Parents' experiences of learning mathematics – Parents' own experiences of homework

The same kind of thing like we have here [referring to her helping Caitlin]. I think you carry on what your parents have done with you. So you come home, you've got your homework, "Mum I don't get it", mum would help you and guide you through it and do it in her way (laughs). "We never did it like that in my day" (laughs). She would do it her way, you would do it how you had been taught, and if you got the same answer great and if you didn't she'd explain how she'd got to the answer and help you through it so that you could get to the right one.

Parents' own experiences of mathematics education

Respondents often spoke in great detail about their experiences of learning mathematics at school. They recalled both positive and negative episodes, oftentimes with strong emotional reminiscences. These experiences provided parents with feelings that they were 'good' or 'bad' at mathematics as a child. Whether or not parents felt comfortable, confident or enjoyed mathematics appears, in part, related to a particular teacher or dynamics of a childhood classroom.

Vicky presented a narrative that included both positive and negative aspects. Her early education in mathematics was shaped by a teacher who she saw as cruel and bullying. These deep negative experiences were easily recalled in detail. Her transition to secondary school changed her perception of mathematics into a more positive practice. By the end of secondary school she had developed a positive, confident outlook towards mathematics and secured employment in a bank, a sector of employment she has remained in for over twenty years.

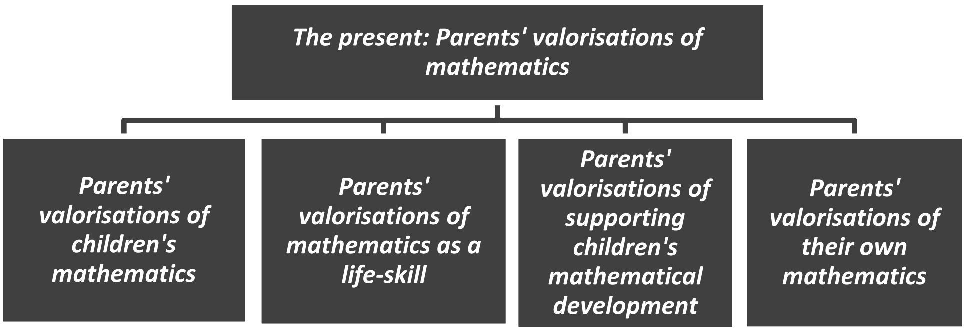
Parent participant: Vicky
Theme: The past: Parents' experiences learning mathematics – Parents' own experiences of mathematics education

For me it depended which teacher because in junior school I had a teacher who was horrible and used to pick on me and bully me, well that's what it felt anyway, and I used to struggle with my times tables. And she used to make me scared and nervous, I used to feel sick and my stomach used to churn but then when I got into comprehensive I enjoyed it because it was more... it wasn't just your adding and taking away it was better and we used to have a laugh with the teacher... so it made it better.

4.2.3 The present: Parents' valorisations of mathematics

In their discussions parents showed the relative value they placed on the mathematics their children learnt at school and reproduced at home, and the mathematics that parents learnt, understood or used regularly. These value judgements varied between participants. Indeed, there appeared to be clear linkages between the value a parent placed on their own mathematics and the valorisations they produced concerning both their child's mathematics and primary school mathematics in general. In simple terms the stronger the parent valued their own mathematics the more difficult they found it to accept their child’s mathematics.

Respondents also presented valorisations of mathematics as an everyday life-skill, where the value they placed on mathematics varied but was clearly evident in several accounts. Likewise, parents spoke about the value they attached to supporting their child’s mathematics at home, a process most advocated.



Parents’ valorisations of children’s mathematics

Parents produced valorisations concerning different aspects of their children’s mathematics. Firstly, as we have already seen the vast majority of participants saw their children’s mathematics as different to their own mathematics. It is therefore perhaps unsurprising that most of the parents in the sample valued the mathematics of their children differently to their own. They tended to view children’s mathematics being long-winded, overly complex and time-consuming. In other words producing a lower valorisation of children’s mathematics than they tend to give to their own approaches.

However, the majority of participants, even those who attached a lower value to their children’s mathematical strategies, clearly valued their children’s mathematical learning and accomplishments in school. Particular focus was paid to proficiency in number operations and recall of times-tables. Parents also felt it important to support their children with their school learning in mathematics, even if in private they denigrated the strategies and approaches their children used.

Robert saw his son Alex’s mathematics as different to his own and his inclination was to tutor his son in the mathematics that he learnt as a child. He did not follow this path because he did not want to handicap his child's learning of mathematics at school. Robert also shows value attached to the judgement of the school, and therefore the mathematics they are teaching. This valorisation of school knowledge similarly causes him to support his son by attempting to adopt modern mathematical strategies, shown in Alex's use of number lines in the later parent-child structured task.

Parent participant: Robert
Theme: The present: Parents' valorisations of mathematics - Parents' valorisations of children's mathematics

My temptation was, 'No you do it like this', but I don't want to destroy what the school's doing and so I go along with it even if sometimes I think, 'Well, I can't see how this is going to develop to later...'. But I accept that often they know best.

Parents' valorisations of their own mathematics

As mentioned in the previous sub-theme parents often valued their own mathematics above that of their children, but still tended to support their children's mathematical learning at school. Most parents supported children by using, or attempting to use, modern school mathematics. Nevertheless, this could prove frustrating when parents attached markedly different values to the two practices.

Deborah, who highly valued her own mathematics due to her previous academic success in the subject and her regular use of mathematics in her occupation in finance, talked about the frustration of supporting her daughter's use of number lines. She clearly valued this approach below her own mathematical technique for subtraction, a vertical algorithm, a conflict which led to annoyance.

Parent participant: Deborah
Theme: The present: Parents' valorisations of mathematics - Parents' valorisations of their own mathematics

She'll have like, she'll have had to write the number lines in the book and you have to stop yourself because I just don't see the point! You've got to write out the number line and do the bouncy thing [jumps] and bouncy thing back and, why don't you just write it down two minus one equals one. And I just patiently let her get on with it while gritting my teeth (laughs).

Parents' valorisations of mathematics as a life-skill

Respondents perceived the extent to which they saw mathematics as a real-life skill. This sometimes consisted of the degree to which they saw elements of mathematics as either concrete or abstract entities. Mathematical knowledge was seen as important for their children's future academic success, employment prospects and day-to-day lives. It was evident that the parents who placed a high value on mathematical success for academic or employment prospects were those who also valued practices of parental support and involvement.

David, who himself had experienced academic success in mathematics, saw the subject as highly important in real-life. He saw it as a necessity to achieve a level consistent with a formal secondary qualification in mathematics in order to properly function as an adult.

Parent participant: David
Theme: The present: Parents' valorisations of mathematics - Parents' valorisations of mathematics as a life-skill

Maths is obviously an important and integral part of learning, whatever subject it is that you are going to learn and however you are going to get on in life. At least having a basic, you know, GCSE level maths is vital. You can't really get on without that sort of level.

Parents' valorisations of supporting children's mathematical development

Generally participants placed a high value on supporting their children's mathematical development both at home and at school. The majority of parents dedicated time and effort to supporting their children and attempting to communicate to their children that homework was an important activity. Support was also judged to be important in order to: aid parents' understanding of current teaching practices, feel informed about what children were learning at school, help children through any difficulties or misconceptions, encourage children to complete homework, foster responsibility and discipline through regularly completing tasks, promote attainment, and to provide additional support above that children experience through schooling.

Carl felt that supporting his daughter Karen both helped and encouraged her to succeed in mathematics. He suggested that the activity of completing work regularly at home generated a sense of discipline. Carl was keen for his daughter to be successful at school and placed a high-level of importance on potential academic achievement.

Parent participant: Carl
Theme: The present: Parents' valorisations of mathematics – Parents' valorisations of supporting children's mathematical development

Err yeah absolutely yeah I think it just encourages them to, well it gives them a bit of, a bit of discipline that says you've got to do something out of school as well to supplement what you are learning really.

4.2.4 The future: Parents' perceptions of the role of mathematics in their children's futures

Just as parents value mathematics in the present, and produce narratives which disclose the significance they assign to mathematical experiences, they also imagine mathematical futures for themselves and their children. These depictions draw upon parents' present valorisations of mathematics, particularly how parents value children's mathematics, their own mathematics and mathematics as a life-skill. Future aspirations centre around two points. The first of these is formal educational achievement and qualifications in secondary and higher education. The next is a degree of confidence, familiarity and day-to-day ability to deal with numbers and figures. In this latter category these aspirations were sometimes linked to parents' own perceived shortcomings or past difficulties with mathematics. Some discussed their apprehension about the possibility that they would struggle to support of assist their children's learning in the future due to their own level of mathematical understanding.

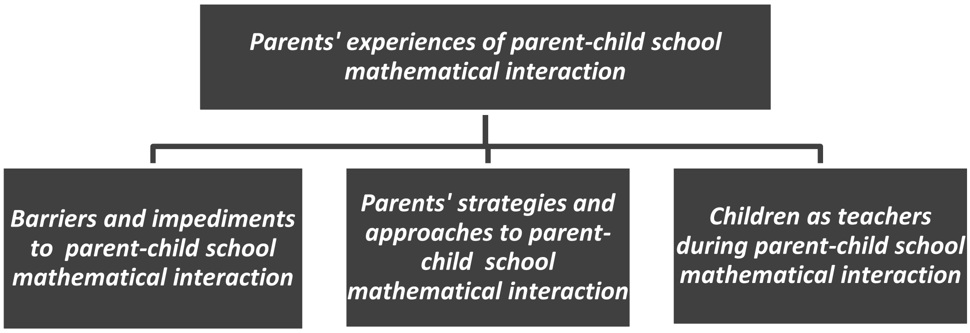
Charlotte talked about aspirations not in terms of academic achievements in mathematics, which she herself obtained, but in the sense of confidence in mathematics. Like many parents she did not want her children to be panicked by using number in everyday life.

Parent participant: Charlotte
Theme: The future: Parents' perceptions of the role of mathematics in their children's futures

Yeah, yeah definitely. I want them just to be able to do things easily without having to think about it, and not struggle or panic, just be able to do it.

4.3 Parents' experiences of parent-child school mathematical interaction

Three key themes were associated with parents' experiences of parent-child school mathematical interaction.



Firstly, the barriers and impediments parents faced when trying to support their children's mathematical development at home. Secondly, the strategies and approaches employed by parents during parent-child school mathematical interaction. Thirdly, parents seeing children as teachers during school-related mathematical interaction.

Figure 4B shows the thematic hierarchy for themes and sub-themes that arose during data analysis. Table 4C and 4D display the frequencies for each sub-theme. From this it is evident that the strongest sub-themes were parents' knowledge and understanding, propinquity and children's rejection of parents' mathematics.

Figure 4B Thematic hierarchy for parents' experiences of parent-child school mathematical interaction

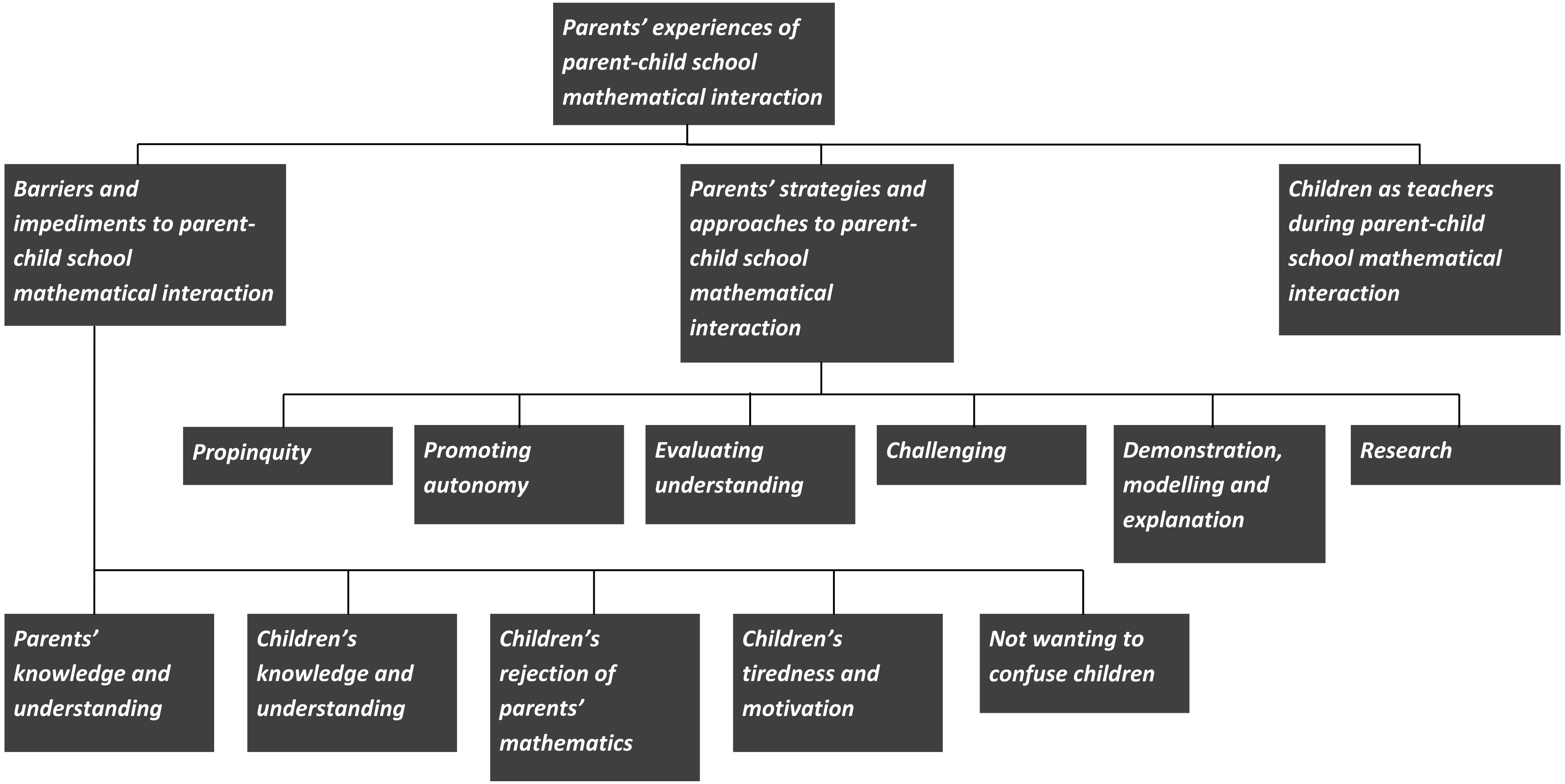


Table 4C Barriers and impediments and parental strategies concerning parent-child school mathematical interaction - thematic frequency table

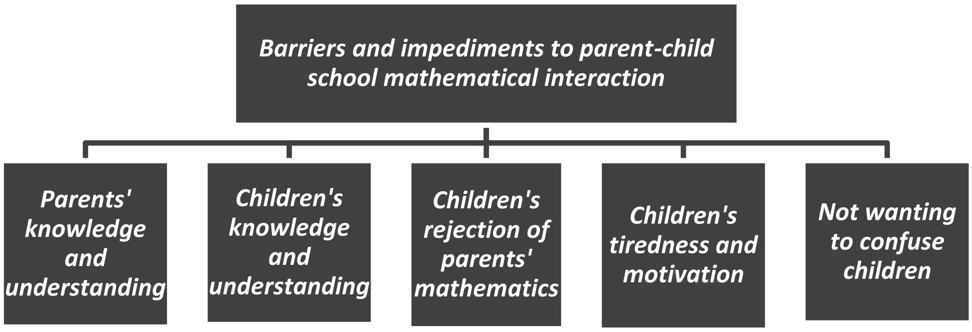
	Themes	***Barriers and impediments to parent-child school mathematical interaction***					***Parental strategies and approaches to parent-child school mathematical interaction***					
	Sub-themes	***Parents' knowledge and understanding***	***Children's knowledge and understanding***	***Children's rejection of parents' mathematics***	***Children's tiredness and motivation***	***Not wanting to confuse***	***Propinquity***	***Promoting autonomy***	***Evaluating understanding***	***Challenging***	***Demonstration, modelling and explanation***	***Research***
Year 3	Imogen	✓				✓					✓	
	Julia	✓			✓						✓	
	Lindsay	✓		✓	✓		✓					
	Vicky		✓	✓			✓		✓		✓	✓
Year 4	Abigail	✓	✓	✓	✓	✓	✓	✓				
	Beth	✓		✓	✓		✓		✓	✓	✓	✓
	Carl						✓					
	Charlotte		✓	✓			✓		✓	✓	✓	
	Chris	✓		✓		✓	✓			✓	✓	
	David	✓				✓	✓	✓	✓	✓	✓	
	Deborah	✓	✓	✓		✓	✓	✓		✓	✓	✓
	Gemma		✓				✓	✓	✓	✓		
	Ian	✓			✓						✓	✓
	Jennifer	✓	✓	✓			✓	✓	✓			
	Neil	✓	✓	✓	✓		✓				✓	✓
	Niamh	✓		✓	✓		✓	✓				
	Rebecca	✓										
	Robert			✓			✓			✓		
Year 5	Natalie	✓						✓				
	Peter	✓		✓				✓	✓			
	Ruth							✓		✓		
Year 6	Gary	✓	✓		✓		✓			✓	✓	
	Jayne	✓		✓			✓	✓	✓		✓	✓
	Suzy	✓		✓	✓		✓			✓		
	Total	**18**	**8**	**14**	**9**	**5**	**17**	**10**	**8**	**10**	**12**	**6**

Table 4D Children as teachers during parent-child school mathematical interaction - thematic frequency table

	Themes / ***Sub-themes***	***Children as teachers during parent-child school mathematical interaction***
Year 3	Imogen	
	Julia	
	Lindsay	✓
	Vicky	
Year 4	Abigail	✓
	Beth	✓
	Carl	
	Charlotte	
	Chris	
	David	
	Deborah	
	Gemma	
	Ian	
	Jennifer	✓
	Neil	
	Niamh	
	Rebecca	✓
	Robert	
Year 5	Natalie	✓
	Peter	
	Ruth	✓
Year 6	Gary	
	Jayne	✓
	Suzy	
	Total	**8**

4.3.1 Barriers and impediments to parent-child school mathematical interaction

Twenty-two of the twenty-four respondents in this sample of parents discussed the impediments that they faced when trying to support their children's primary school mathematics at home. Many of the obstacles that parents experienced appeared to have negative emotional consequences, which in turn influenced parent-child interaction.



Not all the parent accounts were completely negatively disposed to mathematical interaction. Several parents had positive experiences supporting their children. Indeed, many of the facilitators that parents presented as supporting mathematical interaction with their children are discussed later (see section 4.4). However, nearly all the participants produced at least one episode where they felt hindered during parent-child school mathematical interaction.

<u>*Parents' knowledge and understanding*</u>

In the majority of interviews, parents' narratives of mathematical interaction with their children suggested that a common obstacle to successful co-operation was parents' limited knowledge and understanding of primary school mathematics. This is understandable given the changes in the primary school mathematics curriculum over the past 10-15 years. This experience was particularly strong in the parents of Year 3 and Year 4 children. Towards the end of primary school the teaching of mathematics and the strategies and approaches children use at home aligns more closely with parents' own understandings. In parents of Year 6 children parents' knowledge and understanding appeared as a more minor obstruction because of convergence of understanding. Here children and parents were using the same strategies for mathematical calculations.

An example of parents' knowledge and understanding as a barrier is the use of number lines by children in Years 3 and 4. Many parents felt hindered and handicapped by their lack of knowledge of this approach. On occasion these understandings caused argument, conflict and negative emotional experiences in children and parents.

Abigail recounted the experience of trying to help her son Zach with homework involving number lines. She had limited understanding of the strategy and so was restricted in the help she could offer Zach. This resulted in the homework taking up three whole afternoons during a school holiday. This long, frustrating experience, which resulted in great upset to her son, lead to her formally complaining to the school about the homework and the lack of support offered to parents.

Parent participant: Abigail
Theme: Barriers and impediments to parent-child school mathematical interaction - Parents' knowledge and understanding

I remember once instance last year which it was absolutely horrendous with the homework. We had 20 questions, which sounds great you know, and at the bottom of the sheet you had to put how much you enjoyed it and how long it took. Erm so I put down, "This took three afternoons". It was erm as I say a list of twenty questions and it was all to do with number lines, but the problem is they'd not explained to us what a number line was!

Children's knowledge and understanding

Another topic which frequently arose from the data was parents' perceptions of their children's knowledge and understanding of school mathematics as an impediment to parent-child co-operation. Parents often spoke about their children not communicating what they had done at school or not being able to remember instructions to homework that they were given (also see section 4.4.1). Parents appeared impeded by not knowing what their children could do, by the homework task being too difficult for their children, or being unable to build upon their children's existing learning. When children's and parents' knowledge of mathematics differed, then clear divergent understandings were encountered.

The episode below presented by Vicky was an unfortunately common occurrence. Sam struggled with mathematics at school and the work he brought home was frequently too difficult for him to complete without intensive support from Vicky. This example shows the upset and frustration lack of knowledge and understanding caused Sam.

Parent participant: Vicky
Theme: Barriers and impediments to parent-child school mathematical interaction - Children's knowledge and understanding

I was asking him to read the question and he'd read it out but then he'd just shout numbers out for the answer because it was obvious he didn't know what he was doing, and he'd just be shouting random numbers in the hope that he got the right one. And I [said] 'no, no no no no' so he says 'I can't do it. I'm rubbish, I'm rubbish. I can't do it' and storms off.

Children's rejection of parents' mathematics

Another barrier that was commonly experienced was when children rejected their parent's mathematics. In the context of doing homework children clearly valued the mathematical approaches they were taught in school more highly than their parent's mathematics. Even when the two were the same many appeared to value the mathematical knowledge of their teacher higher than the mathematical knowledge of their parent. These valorisations had the potential to cause friction between parent and child.

Suzy described the value her son attached to the mathematical knowledge of his teacher. In this example she discussed how he rejected her view because it conflicted with the view of his teacher as 'right'. The disagreement lead to a negative emotional response in Matthew, her son, and frustration for Suzy that her help was rebuffed.

Parent participant: Suzy
Theme: Barriers and impediments to parent-child school mathematical interaction - Children's rejection of parents' mathematics

And we had some homework, and everybody makes mistakes, but we had some homework and it was like a Sudoku puzzle, his teacher had done the first bit wrong. We had a big tantrum about that because 'He couldn't have done it wrong' and I said 'But he has done it wrong and it won't fit'. That was a nightmare, and I said 'Look', and he wouldn't let me explain the workings out because his teacher was right and I was like, 'No he can't be because you've got to put this in here'.

Children's tiredness and motivation

Many participants perceived that an obstacle to parent-child interaction was children's tiredness and motivation. Homework was typically completed in an evening after school. Here parents mentioned that children's tiredness after a full of day of education presented an impediment to successful co-operation. Similarly, parents also spoke about children's lack of motivation, sometimes allied with tiredness, as a problem when trying to get their children to do homework. The distractions of television, computer games and social activities were often blamed for children lacking motivation.

Beth talked about her son's lack of motivation to complete his homework. She suggested that he saw time spent on his computer console as more attractive. These competing interests caused conflict as Beth had to persuade her son to complete his homework.

Parent participant: Beth
Theme: Barriers and impediments to parent-child school mathematical interaction - Children's tiredness and motivation

Yeah, yeah, like we do have problems sometimes because he's got an X-box and he sits up there, especially in winter times when they come home from school and its dark nights, but sort of if he's sort of talking to some of his friends on the X-box, I have to say come on we've got to go, to get this done. He'll have a bit of an argument with me and then he'll come. I'll say to him, "The quicker we get it done, the quicker you can go back to what you are doing".

Not wanting to confuse children

The topic of parents' not wanting to confuse children during homework sessions was linked to parents' perceptions of primary school mathematics as different, and parents' and children's knowledge and understanding. A lack of knowledge, divergent understandings, personal experiences of schooling, and previous negative episodes of collaboration all appeared to influence certain parents in their interactions.

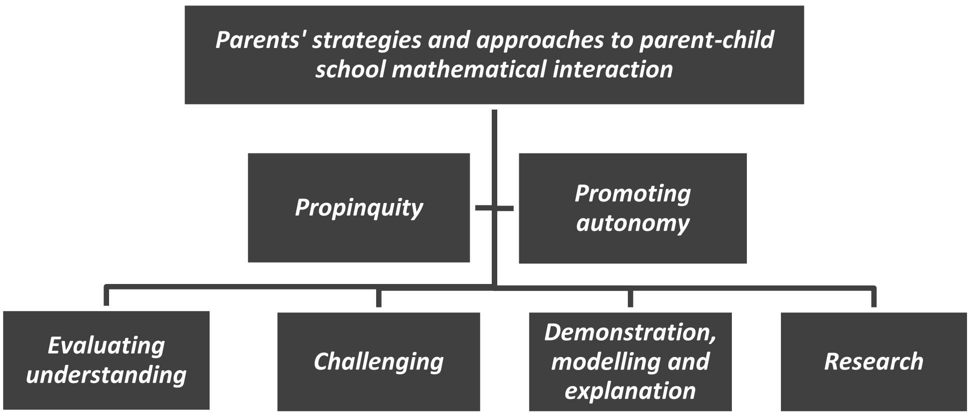
For instance, Abigail discussed her concern that she might be confusing her son or showing him forms of mathematics different to those he was being taught at school. This appeared to be linked to a feeling that she did not fully understand her son's school mathematics.

Parent participant: Abigail
Theme: Barriers and impediments to parent-child school mathematical interaction - Not wanting to confuse children

Yeah, yeah, erm well yeah because you need to know, you need to know whether you're steering them in the right direction. I mean at the moment, I mean we did one the other week, it wasn't maths it was (pause) I can't remember what it was [but] it was really ridiculous. Erm and I'm thinking am I steering him the right way with this?

4.3.2 Parents' strategies and approaches to parent-child school mathematical interaction

In order to support their children's learning and, perhaps, cope with the difficulties outlined in the preceding theme, parents operated a number of strategies and approaches to support parent-child interaction. These could be defined as: propinquity; promoting autonomy; evaluating understanding; challenging; demonstration, modelling and explanation; and research.



<u>Propinquity</u>

The strongest sub-theme within parents' strategies was that of propinquity, which refers to nearness in place, relation or time. So in the context of parent-child interaction it concerns a parents' desire to be near or close to their child whilst they are completing homework. In this manner they are showing that the task is important in terms of the time they are giving and their availability of support. Yet during propinquity parents do not want to take over responsibility for the homework from the child or directly supervise every step. Propinquity was not the same for all parents. Rather it was an approach or ideal that was seen as desirable by almost all the parents in the sample, where parents attended to spatial proximity, time and attention in a manner which reflected their experience and relative valorisations.

Charlotte neatly summed up propinquity when she described the typical manner in which she supported her son Callum. She showed the value she placed on proximity and access to her son's homework, the monitoring she undertook and the importance she placed on being able to assist where required.

Parent participant: Charlotte
Theme: Parents' strategies and approaches to parent-child school mathematical interaction - Propinquity

I probably wouldn't always be sat next to him whilst he does his homework, I might be pottering around the kitchen, but he'll be here doing it. He'll be here doing it in the kitchen, the same room as me, erm and I'll just keep looking. I'll leave him to it, if I know he's doing alright I'll leave him to it… If he's struggling I'll come and sit next to him and we'll try and work through it together.

Promoting autonomy

Allied to, and frequently occurring beside, the sub-themes of propinquity and challenging was promoting mathematical independence during interaction. This approach was clearly perceived as important by various participants in the sample. The strategy was often operated with the proviso that their children knew they could come to them if they needed assistance.

Niamh provided a typical example of the way in which she encouraged autonomy in her child.

Parent participant: Niamh
Theme: Parents' strategies and approaches to parent-child school mathematical interaction - Promoting autonomy

I, he brings a piece home every week, more or less, and I, I try and let him do it and then if he gets stuck I always, I'll always sit down and help him.

Evaluating understanding

Evaluating their children's conceptual understanding during parent-child mathematical interaction was an approach utilised by eight parents. This strategy was used to check work during or at the end of a homework session. It was also present when questioning children directly to establish reasoning and understanding. The level of sophistication of this strategy varied across the parents but it was frequently seen as an approach that parents took to doing schoolwork at home with their children.

Gemma, a teacher, perceived her practice as rigorously analysing her daughter's understanding during a homework session, then using this information to adapt her support accordingly. Throughout her interview Gemma presented herself and her co-operative activities over homework as proactive and highly sophisticated.

Parent participant: Gemma
Theme: Parents' strategies and approaches to parent-child school mathematical interaction - Evaluating understanding

And I will always read it with Kitty, I will check she understands what she's got to do and before she even picks up her pencil I will say to her, "Right, tell me what you think they are asking you to do and how are you going to do it?" And if she can give me that feedback I'm quite happy or if she can't I'll address what she's got to do.

Challenging

Several parents, particularly those who placed a high value on mathematical success, spoke about how they wanted to challenge their children to succeed mathematically. This was enacted through encouraging children to try harder questions at school or at home or provide more than just essential examples and answers during homework. Parents also spoke about creating school mathematical-style activities to supplement homework.

Challenging his son at home and at school was a clear aim of Robert. He talked about his desire to 'push' his son Alex when completing homework, often by adding extra activities or providing additional examples to those requested. Robert also encouraged Alex to attempt harder, more complex mathematical tasks at school.

Parent participant: Robert
Theme: Parents' strategies and approaches to parent-child school mathematical interaction - Challenging

I like to push him. I don't want to give him something that 10 minutes and he can do and think 'Oh that were alright'. They do like extension work at school, so I do encourage him, 'Oh I do Level 2 dad'. 'So next week you have a go at Level 3. If you can't do it drop down to 2 but have a go at 3 first'.

Demonstration, modelling and explanation

From recounted episodes of parent-child interaction it was possible to determine instances of demonstration, modelling and explanation. Parents would demonstrate mathematical techniques and solutions. They described modelling approaches and operations, frequently addition and subtraction. They also reported explaining concepts and strategies.

Ian provided an example of modelling when he mentioned techniques he would use to help his daughter to understand and calculate two-digit additions.

Parent participant: Ian
Theme: Parents' strategies and approaches to parent-child school mathematical interaction - Demonstration, modelling and explanation

There has been a few times where you, probably not so much lately but sort of when she was a bit younger, when you're doing the sort of, what I, what I used to do, what we both used to do is if it's like 12 plus 16, or things like that, gets things out and count them.

Research

When faced with a perceived inability to support their children with homework parents often spoke about utilizing a social network of family, friends or other parents, or alternatively using the Internet to research a certain mathematical method. In this manner the strategy of research was used to overcome parents' and children's lack of knowledge and understanding of primary school mathematics. It also highlighted the wider support structure that some parents required in order to assist with school mathematics.

Both Internet research and contacting other family members was an approach taken by Jayne to help support her son in the exemplar produced below. Here an activity calculating the area of a triangle was solved by ‘Googling’, also frequently portrayed as a research strategy by respondents, and by contacting a knowledgeable other, in this case the child’s grandfather who was seen as highly skilled in geometry.

Parent participant: Jayne
Theme: Parents’ strategies and approaches to parent-child school mathematical interaction - Research

And sometimes we’ve not known exactly what the sum was so we’ve just Googled it, half base times height and things like that. So yeah and some of the angles and things his granddad’s an ex-pattern maker so he’s quite up on that so we keep ringing him up and saying, “What’s this?” So we do a bit a research. If we don’t understand it we don’t leave it we look round it and find, find an answer.

4.3.3 Children as teachers during parent-child school mathematical interaction

The final theme that emerged from parents’ experiences of parent-child mathematical interaction was the perception of children playing the role of teachers, informing and educating parents on school mathematics. This role reversal, as the child becomes the expert and the parent the pupil, provides an interesting insight into co-operative activity. Here the parent relinquishes their role as expert when the child’s knowledge and understanding is greater. The strategy was understandably more common in older children.

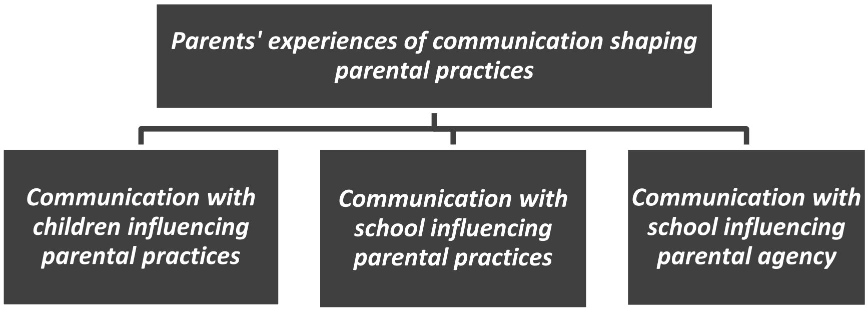
Jayne presented an example of a place value activity involving decimals in which her lack of understanding was overcome with support from her son Oliver. She was unable to comprehend the activity until she was ‘taught’ by her son.

Parent participant: Jayne
Theme: Children as teachers during parent-child school mathematical interaction

And I haven’t got a clue even though I’m like a scientist I’d just not got a clue, I’m “Oh my god!” And Oliver taught me. He went, “Oh it’s that”. And I went, “Are you sure”. So I like I had to have a really long, good think about it and I said, “Yeah you’re right it is”, and I could see it then.

4.4 Parents' experiences of communication shaping parental practices

This section concerns how parents teaching practices are shaped through communication with schools.



It was evident that communication can be seen to directly influence behaviour and the conditions for behaviour. Support for this is contained within the twin themes of influencing parental practices and agency. Children appeared to act as a conduit for information from school regarding mathematical procedures. Studying this phenomenon also gives an insight into how communication with children shapes parental practices.

Figure 4C shows the hierarchy of themes and sub-themes that emerged during the analysis of interview data. Here three levels of theme are present. This ranking contains six second order sub-themes. These were generated due to the wealth and richness of the data concerning home-school communication.

Figure 4C Thematic hierarchy for parents' experiences of communication shaping parental practices

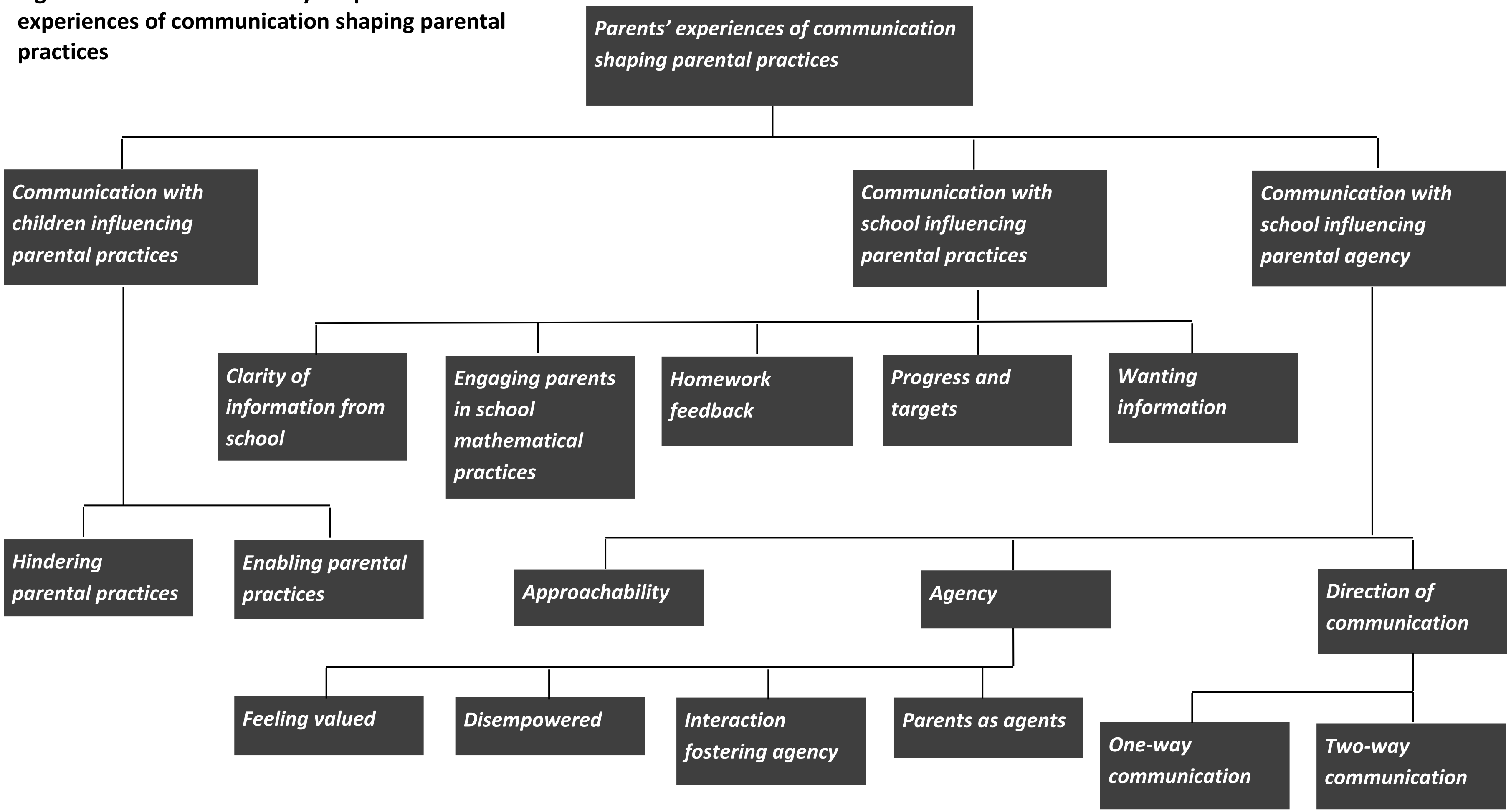


Table 4E Communication with children and school shaping parental practices – thematic frequency table

	Themes	***Communication with children influencing parental practices***		***Communication with school influencing parental practices***				
	Sub-themes *Second-order sub-themes*	***Enabling***	***Hindering***	***Clarity of information***	***Engaging parents in school mathematical practices***	***Homework feedback***	***Progress and targets***	***Wanting information***
Year 3	Imogen		✓				✓	✓
	Julia					✓	✓	✓
	Lindsay		✓	✓		✓	✓	✓
	Vicky		✓			✓	✓	✓
Year 4	Abigail	✓	✓	✓	✓	✓	✓	✓
	Beth	✓	✓	✓		✓	✓	
	Carl	✓	✓			✓	✓	
	Charlotte		✓			✓	✓	
	Chris		✓	✓		✓	✓	✓
	David		✓					✓
	Deborah	✓		✓	✓	✓	✓	
	Gemma	✓				✓	✓	
	Ian	✓				✓	✓	
	Jennifer	✓	✓		✓	✓	✓	
	Neil	✓	✓		✓		✓	✓
	Niamh	✓	✓			✓	✓	✓
	Rebecca	✓	✓		✓	✓	✓	✓
	Robert	✓				✓		✓
Year 5	Natalie		✓		✓	✓	✓	
	Peter	✓	✓	✓	✓	✓	✓	
	Ruth	✓	✓			✓	✓	✓
Year 6	Gary		✓		✓	✓		
	Jayne	✓			✓	✓		
	Suzy		✓	✓		✓	✓	✓
	Total	**14**	**18**	**7**	**9**	**21**	**20**	**13**

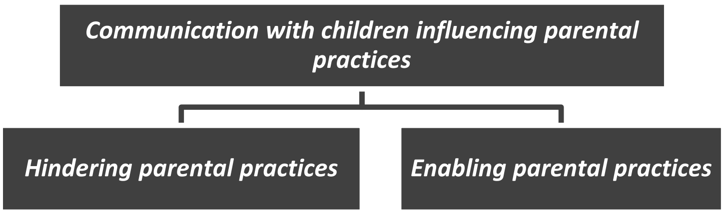
Table 4F Communication with school shaping parental agency – thematic frequency table

	Themes	***Communication with school influencing parental agency***						
			Agency				***Direction***	
	Sub-themes / *Second-order sub-themes*	***Approachability***	*Feeling valued*	*Disempowered*	*Interaction fostering agency*	*Parents as agents*	*One-way*	*Two-way*
Year 3	Imogen	✓	✓		✓		✓	✓
	Julia	✓		✓			✓	
	Lindsay	✓	✓				✓	✓
	Vicky	✓	✓	✓			✓	
Year 4	Abigail	✓	✓	✓	✓	✓	✓	
	Beth	✓	✓			✓	✓	
	Carl	✓		✓			✓	
	Charlotte	✓	✓		✓		✓	✓
	Chris	✓	✓	✓			✓	
	David	✓		✓			✓	
	Deborah	✓	✓			✓	✓	
	Gemma	✓	✓			✓	✓	
	Ian	✓	✓				✓	✓
	Jennifer	✓	✓		✓	✓	✓	✓
	Neil	✓	✓	✓	✓	✓	✓	✓
	Niamh	✓	✓				✓	
	Rebecca	✓		✓			✓	
	Robert	✓	✓		✓	✓	✓	✓
Year 5	Natalie	✓	✓				✓	
	Peter	✓	✓	✓	✓	✓	✓	✓
	Ruth	✓		✓			✓	
Year 6	Gary	✓	✓		✓		✓	✓
	Jayne	✓	✓			✓	✓	
	Suzy	✓		✓			✓	
	Total	**24**	**18**	**11**	**8**	**9**	**24**	**9**

Tables 4E and 4F display the frequencies for each sub-theme and second-order sub-theme. Here we see the strongest sub-themes are approachability and homework feedback. The strongest second-order sub-themes are one-way communication and feeling valued.

4.4.1 Communication with children influencing parental practices

Thematic analysis of participant narratives showed a number of ways in which communication with children enables or hinders parental activity.



Hindering parental practice

The majority of participants discussed a lack of communication from children regarding their mathematical learning at school. For many of these parents the only way to discuss school mathematics with their children was to initiate the conversation, even then the information gleaned was minimal, or of a nature which did not aid parent-child mathematical interaction. Many of the children in the sample appeared to operate as a poor channel of information on school mathematical practices.

Neil presented communication with his child as ‘like getting blood out of a stone’. The difficulty of getting his son to talk about mathematics and explain homework, to Neil’s satisfaction, caused Neil to wonder sometimes whether his son actually understood certain concepts. The technique that this parent used to get around the hindering action of limited child-parent communication was to encourage his son to demonstrate strategies.

Parent participant: Neil
Theme: Communication with children influencing parental practices - Hindering

It’s like getting blood out of a stone. He’ll say stuff and things and occasionally he’ll give specifics but you have to really sort of crowbar it out of him. He tell you when you’re doing homework, in a long-winded sort of way, how he’s done it at school erm but the easiest way is to get him to show it to you because if he describes it, it takes a long, long time to get any information out of him, which is why I believe he doesn’t understand what they are actually teaching him sometimes.

Enabling parental practices

There were instances where participants spoke about the strength of communication with their children and how it enabled them to support homework sessions. Generally, communication regarding homework was perceived as being better than communication regarding children's school learning.

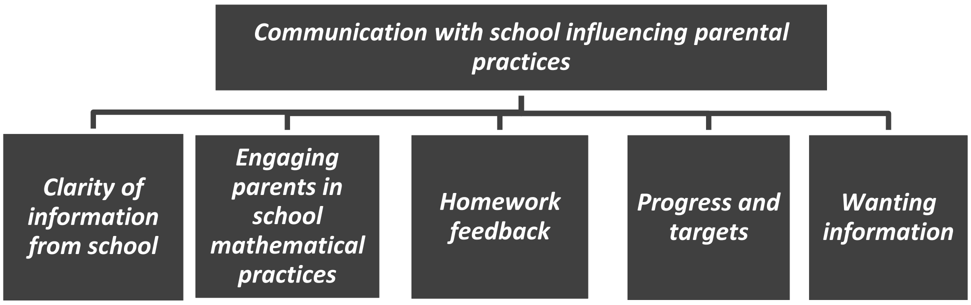
Gemma spoke about communication with her daughter. Kitty was a mathematically able and socially adept child who was able to explain the aims and objectives of a piece of homework to her mother. This channel of communication offered Gemma a source of information on her daughter's mathematical understanding.

Parent participant: Gemma
Theme: Communication with children influencing parental practices - Enabling

And very often when it's the maths she'll say "I know exactly what we've got to do because we've been doing," I don't know, "symmetry or data in class and what we've got to do is..." And she can tell me without having to read it word for word. So I clearly know that she has understood in class because the homework is always and extension now of the class [work].

4.4.2 Communication with school influencing parental practices

A complex topic emerging from the data was how communication with school influenced parental teaching practices. Within this theme the importance placed on information was highly visible, particularly the desire for a range of information which could be potentially utilised to support mathematical interactions. These processes of informing can be divided into parents wanting clearer information from school, schools attempting to engage parents in school mathematical practices, parents and children receiving feedback on homework, parents' desire to understand children academic progression, and parents' wish to have more understanding of school mathematical techniques and approaches.



<u>Clarity of information from school</u>

When considering how information from school appeared to influence parental practices it was clear that alongside quality and quantity, the clarity of information that parents received influenced their understanding of their children's mathematical development and attainment as well as their knowledge of mathematics teaching. This appeared to effect their ability to support their children's mathematics at home. Several parents discussed how poor instructions presented a barrier to assisting their children, whilst others spoke about the difficulty in understanding how well their children were progressing mathematically compared to age-related expectations.

Chris spoke about the clarity of homework instructions. Confusion arose when instructions were not immediately apparent. This influenced how difficult Chris felt it was to support his daughter.

> **Parent participant: Chris**
> **Theme: Communication with school influencing parental practices - Clarity of information from school**
>
> I've had to sit, I've had to sit a couple of times and read through the instructions of what they were trying to [do]. You can glance at most of it and know what they are trying to achieve. There's a couple of times when you think well it's not very clear this. Are they wanting us to do it this way or that way? It can be as silly as the photocopier missing the bottom bit off and stuff like that.

<u>Engaging parents in school mathematical practices</u>

For a minority of parents in the study, communication with teachers and school had engaged them in school mathematical practices. Several of the parents spoke about attending mathematics workshops run by their child's school. At these events parents were given information about mathematical strategies. These experiences were seen as beneficial by all the participants in terms of their understanding of their children's learning, and by a majority in terms of being able to better support their children. Some parents spoke about how teachers had directed them towards resources on the Internet or activities that they could do at home to support school mathematics.

Rebecca spoke several times during her interview about a mathematics workshop she and her partner had attended. She clearly valued the experience as highly useful and believed it enabled her to better support her child. The product of the workshop, a piece of paper with various strategies and teaching forms, was kept in the kitchen close to where homework took place in case of difficulties.

Parent participant: Rebecca
Theme: Communication with school influencing parental practices – Engaging parents in school mathematical practices

It's just in a classroom. It's Zoë's teacher. He just got one of them wipeboard things and shows you how to... Did all this stuff [shows paper from earlier] on the computer. Just exactly the same as that telling us how to help them at home with this, basically. So we've kept it so as and when they do get some we can refer to it.

Homework feedback

Feedback and feedforward on formative assessment and learning tasks is generally given to judge performance and influence future behaviour. Therefore feedback on homework to parents and children could be a mechanism for influencing parental teaching strategies. When questioned, half of the repondents spoke about seeing no feedback on work completed at home. Many of these parents also did not know whether their children received feedback on their homework during classes at school. Parents who did see feedback varied in their opinions of its usefulness. The feedback itself also varied in quality from ticks and crosses for correctness to written comments and suggestions. For some parents the desire for either feedback, or clearer feedback, was a source of frustration and a cause of conflict with school.

Abigail, who spoke at length about the communication difficulties she experienced with her son's school, reflected on homework feedback. She contrasted her child's previous teacher, and the good experiences of communication and feedback on homework, with the present status quo. Her frustration about a lack of feedback was compounded by the failure of her efforts to get the school to alter their practice.

Parent participant: Abigail
Theme: Communication with school influencing parental practices – Homework feedback

Yeah, yeah, we get absolutely nothing now. It is really, really bad. Erm and I've asked about it but I've not got anywhere.

Progress and targets

Nearly all the parents discussed the information they received from school regarding their child's progress in mathematics. The level of detail provided by school differed widely from general comments to *National Curriculum* sub-levels for mathematical attainment. Information on progress and targets was given to parents through parents' evenings and end-of-year reports. Again, parents differed in their contentment with information on progress and targets. A small group felt that the school did not provide them with enough information and were not satisfied with the facts they did receive. A slightly larger group felt informed about progress but still wanted more information. In both groups individuals generally wanted more specific details of activities that could be done at home to support children and clear targets for them to achieve. A final group, consisting of over half the parents in the sample, felt content with the communication from school regarding their child's mathematical learning and did not appear to seek extra information.

Imogen discussed the format of a communications book she received concerning her son's activities and progress. Her satisfaction over her son's attainment is informed by communication with school. She explains how her son made above average progress over recent times to assume a level commensurate with that expected of a child of his age.

Parent participant: Imogen
Theme: Communication with school influencing parental practices – Progress and targets

You see they have got actually they have got things in maths, core learning in maths, but obviously they don't tell you how to do it. So, but they have got levels and he has moved up, from infants to junior school, he has moved up more levels than he should to get actually onto his age level, because he was behind.

Wanting information

A strong theme emerging from the data was the fact that many parents wanted more information about mathematics teaching from schools in order to support their own parental practices in the home.

Niamh presented an example of this when she talked about the difficulty of supporting her child with multiplication homework. In this instance her son, Connor, was struggling to complete homework using the grid method for multiplication. This is a strategy over which Niamh had a limited knowledge. She did not wish to confuse her son by showing him a different method however she felt compelled to do this because of the lack of information from school. Like many parents, Niamh saw the provision of more information as a potential mechanism to enable her to better support her son's mathematical development.

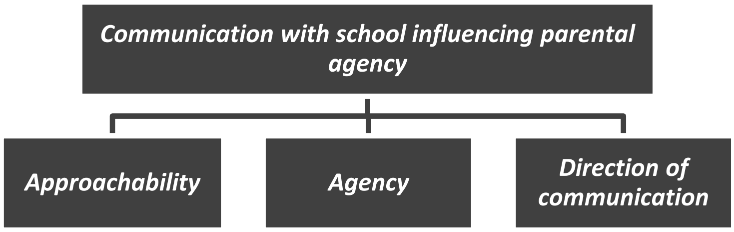
Parent participant: Niamh
Theme: Communication with school influencing parental practices – Wanting information

The thing that I don't like is that I don't know how he's doing them. So unless he tells me, they don't send anything home that says this is how we do multiplication and this is how we want Connor to do it. Because if I come to, if he brings something home and it's not, and he's struggling and I don't know the way that he's doing it at school I'll try and teach him the way I know, which obviously won't help him if he's not doing that way at school. So it would be helpful if they sent things home to say this is how we do this kind of thing, you know.

Of the thirteen parents who wanted more information on teaching practices in order to support their children at home only three also spoke about feeling that their child's school had engaged them in school mathematical practices. This suggests that parental workshops and even simple written information can help shape parental practices.

4.4.3 Communication with school influencing parental agency

Another way to view how teachers and schools support or inhibit parental teaching practices is to analyse how access to information from schools influences parental agency. In this sense agency is defined simply as the ability to take action and the feeling that one has the ability to take action. So, in other words, how access and communication with school influences parental perceptions of their ability to take action concerning their children's mathematics.



Approachability

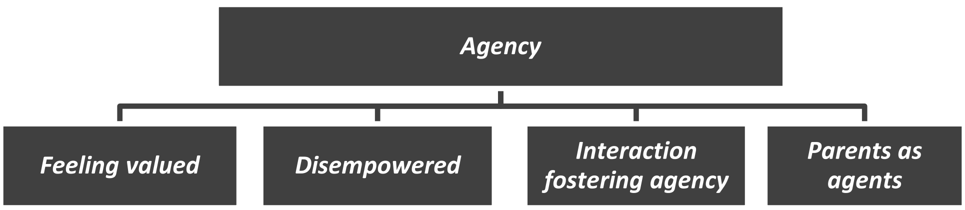
All the parents spoke about the physical access they have to teachers and school. In the most part schools and teachers were painted as approachable and welcoming both for parents and children. Parents spoke about the ease with which they felt they could approach teachers with their concerns or queries about mathematics or homework. Most parents who had spoken about conflicts with teachers, lack of information, and feelings of inability in being able to support their children mathematically still perceived school as a welcoming place and teachers as approachable. This suggests that the approachability or otherwise of a school is not a key determinant in enabling parents to support their children's mathematical learning. Rather, it may be the case that more emphasis needs to be placed on schools being proactive in engaging parents as opposed to just reactive in responding to their queries.

For instance, Carl spoke about the ease to which he felt he could access school with any concerns he had, whereas Suzy felt constricted by the arrangement at her son's school which prevented her from contacting teachers outside pre-arranged appointments.

Parent participant: Carl **Theme: Communication with school influencing parental agency -** **Approachability** It's an approachable school so if we had any concerns I wouldn't hesitate I would just go and chat to her class tutor.
Parent participant: Suzy **Theme: Communication with school influencing parental agency - Approachability** Whenever you go in, you've got like the woman on the main reception. It's very hard to get in to see the teachers because they're always in the class or whatever, so you literally walk through the front door and if you'd got any money to hand in or anything you'd see the receptionist.

Agency

When studying parents' narratives it was possible to see how participants feel valued or disempowered when communicating with school. Furthermore, narrative episodes often produced glimpses of how interaction with school fostered parental agency, enabling parents to support children's mathematics at home. It was also possible to observe how access to information, coupled with feeling valued, was present when parents proactively acted as agents approaching schools with mathematical queries.



Feeling valued

Two-thirds of respondents discussed how interaction with school made them feel valued. They noted how their views and opinions on their children's education carried weight with the school. They reported the results of phone calls, letters, meetings and visits to school that concerned issues communicated by parents to schools and vice versa. These cases all showed how parents thought that their opinions and actions were positively valued.

Lindsay described part of her relationship with school. She felt valued because the school kept her informed and enquired whether she had any concerns. Furthermore, her agency was enhanced by a feeling that the school took her views and opinions into account.

> **Parent participant: Lindsay**
> **Theme: Communication with school influencing parental agency – Agency – Feeling valued**
>
> They sort of tell me stuff, do you know what I mean, as to what's happening, as to where they are, and to how they feel they're doing. They ask me if I've got any concerns obviously. So they tell me how they feel that he's doing and if I've got any concerns, which obviously I'll share with them and they seem to take that into account.

Disempowered

Just under half of parents described instances of communication with school that were not resolved to their satisfaction. In these episodes it was apparent that a lack of communication or ability to influence problems concerning mathematical learning left parents feeling disempowered. These difficulties included issues around homework, schools responding to parent queries, contrasting views and opinions of education, and worries concerning potential developmental disorders.

Rebecca experienced disempowerment over her communication with school regarding homework. She valued homework highly as an activity but felt that school did not share this valorisation, creating conflict. This was further compounded by the school's lack of response to her worries, highlighting in her mind that they did not value her views and opinions.

> **Parent participant: Rebecca**
> **Theme: Communication with school influencing parental agency – Agency – Disempowered**
>
> I've even asked him on open night. So I don't know what happens to homework. I've never seen it be marked so... I feel like in Y4 it like it doesn't matter now. We've got it but it doesn't matter what you think.

Interaction fostering agency

Eight participants presented episodes of communication with school in which interaction fostered agency in parents. In these situations information was provided to parents that allowed them to take action, or feel able to take action, and play a larger role in their children's mathematical education. This has clear links with the earlier sub-theme of engaging parents in school mathematical practices, but on this occasion it focuses upon agency and empowerment to support mathematical strategies, rather than knowledge of mathematical structures per se.

In a number of cases parents were asked to support children's learning, for instance multiplication tables, and be more involved in mathematics at home. This was valued by the school and appeared to foster agency in the parents concerned. Gary recalled a conversation with his child's teacher which gave him a channel of communication and hence empowered him to support his son's learning.

Parent participant: Gary
Theme: Communication with school influencing parental agency – Agency – Interaction fostering agency

... it is made clear by the school that if at any time we want to contact them we are completely welcome. And we know that the teachers respond to that and that they do seem to like it if you do so. They are more than willing to work with you, which is why I said we got a phone call about a relatively minor incident in class because they were just keeping us in the picture.

Parents as agents

Nine parents, all of whom had been identified as 'feeling valued', took the initiative in contacting schools about concerns around their child's mathematical development. Here parents mentioned how they were able to influence their children's mathematical education at school. In other cases access to teachers led parents to alter their support of children during mathematical work at home. These instances of parents acting as agents in interacting with school covered issues such as children not being challenged enough at school, contacting school regularly concerning homework queries, and the school's approach towards homework.

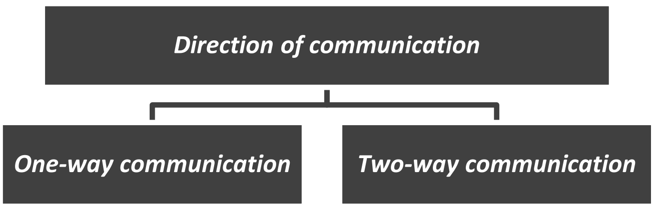
In the next example Jayne can be seen to take the initiative, and act with agency, in communication with her son's teacher. Jayne clearly believed that her child would not have made as much progress if she had not intervened.

Parent participant: Jayne
Theme: Communication with school influencing parental agency – Agency – Parents as agents

So I saw his teacher and said, because he'd get this maths work and sit and do it watching the telly and I taught that's, that's not pushing him at all. So I went in and said, "It's not pushing him enough", and I said, "and he's not interested because he can do it too easy". So she said, "Well I'll put him on the next book but if it gets to hard..." So she put him on the next book and in a term, whereas they're lucky if they move up one level, Oliver moved up 2 levels in maths because he was getting pushed more.

<u>Direction of communication</u>

The final theme which emerged from the analysis of communication with school influencing parental agency is that of the direction of communication. As shown in the earlier review of literature, communication can be unidirectional, or one-way between home-school or school-home, or bidirectional where information flows in channels of communication backwards and forwards between home and school. Research suggests that the bulk of communication between school and parents is unidirectional. It also presents the notion that two-way communication is richer and promotes greater agency in parents. This agency should allow parents to better support their children's mathematics.



One-way communication

The vast majority of instance of communication between parents and schools discussed in the interviews were unidirectional. They concerned parents being informed about progress at parents' evenings, attainment in end-of-year reports, instructions and examples on homework, and correspondence from schools. In the opposing direction were uninformed parents seeking information from teachers and school.

An instance of one-way communication is described in this extract. Chris discusses the end of year report he received concerning his daughter's progress at school. The report communicates what the school wants the parent to know. It does not necessarily communicate what the parent actually wants.

Parent participant: Chris
Theme: Communication with school influencing parental agency – Direction – One-way

The written, written report that's just, yeah I'm mean there's the where she is and points and stuff but (pause) I, I suppose in some ways it's the same with some of my businesses that run really smoothly and stuff you write very little and, you know, your plans for the year are so simple because, you know, it's great because it does this and it does that and we get that back from Lizzie, you know, she's great, lovely, polite, she's right up here so there's no need to worry about anything but, yeah but no-one is perfect at everything, where does she need help? That's what you want, I would want is more, "Yeah she's doing great but you really could do with focussing just around this bit because it's not her strongest bit of maths."

Two-way communication

Several participants presented episodes of two-way communication. These often were directed towards particular problems children were experiencing or utilised formal channels of communication such as planners, diaries or homework books. These allowed parents and teachers to inform each other of problems and issues. In these cases the quality of communication appeared to give parents agency, potentially influencing their interactions with their children.

Because of recent problems concerning his son's progress Neil had regular two-way communication with school. They would inform him of Daniel's activities and progression at school and Neil would inform them of mathematics activities they were doing at home. In this manner they hoped to work together. Later in his interview Neil recalled how his son became interested in a game involving mathematics called 'Warhammer'. He spoke about this with his son's teachers as a mechanism for working together to motivate and encourage Daniel.

Parent participant: Neil
Theme: Communication with school influencing parental agency – Direction – Two-way

So yeah they, they wanted to get feedback from us. We talked to them weekly, they give us their feedback weekly, erm like a mini parents evening, erm but that was the teacher more than the school. Because she said, "Just off the record can you just come and tell me what you're doing because we get somewhere we need to really go for it. Because it is obviously, we're hitting a brick wall and he's not a, he's a stubborn kid (laughs) so we need help, erm and we can't do it alone and you are the parents".

4.5 Structural analysis of episodes of parent-child mathematical interaction

Due to the connections between themes, which have already been touched upon throughout this chapter of analysis, a study of the structure of parental narrative episodes was undertaken. Thematic analysis separates and groups data. However, this process can detach experiences from their context, including circumstances, contributory factors and emotional consequences. This is the case with the analysis reported so far in this chapter.

The approach to narrative structural analysis outlined in Chapter 3 was used to study the linkages between themes reported earlier in this chapter. It also allowed the analysis of themes and effects, particularly emotional consequences. The episodes which are described below can be divided into those which appeared to have a negative emotional outcome, for either the child or the parent, and those that had a positive emotional result. A total of ten negative episodes, from ten parents, were appropriate for structural analysis. Nine positive episodes, from five parents, were also studied. Positive incidents tended to be more fragmented and less structured as fully formed narratives, thereby making it more difficult to find episodes to analyse. Table 4G shows the number of episodes selected from individual respondents.

Table 4G Positive and negative episodes structurally analysed

Positive episodes		**Negative episodes**	
Parent	**Number of episodes analysed**	**Parent**	**Number of episodes analysed**
Beth	1	Abigail	1
Jayne	3	Beth	1
Neil	2	Charlotte	1
Robert	2	Chris	1
Suzy	1	Deborah	1
		Gary	1
		Jennifer	1
		Lindsay	1
		Suzy	1
		Vicky	1
Total positive episodes	**9**	**Total negative episodes**	**10**

4.5.1 Positive episodes

Positive incidents tended to be centred on successful interactions where children's knowledge and understanding was high, or where a homework problem was solved by either individuals or cooperation between parent and child.

The following was a narrative produced by Jayne in response to a question about whether she ever struggled to help her child, Oliver, with his school mathematics.

Parent participant: Jayne

No I don't think... Yeah they do a lot where they do, it'll be say 0.0185 and it'll say you've got to name which are the hundredths, which are the thousandths and which are the tenths. And I haven't got a clue even though I'm like a scientist I'd just not got a clue, I'm "Oh my god!" And Oliver taught me. He went, "Oh it's that". And I went, "Are you sure". So I like I had to have a really long, good think about it and I said, "Yeah you're right it is", and I could see it then. But he was quite capable of doing that and thank god that that was the bit that I struggled on. But no, not really, I mean they do a lot of trigonometry and I can never remember doing that at junior school level. And sometimes we've not known exactly what the sum was so we've just googled it, half base times height and things like that. So yeah and some of the angles and things his granddad's an ex-pattern maker so he's quite up on that so we keep ringing him up and saying, "What's this?" So we do a bit a research. If we don't understand it we don't leave it we look round it and find, find an answer.

Structural analysis breaks Jayne's story into two separate narratives. The first one below shows how the parent's lack of knowledge and understanding leads to confusion and a negative emotional response. Through communication with her child, and the activity of Oliver in a role of mathematical expert, Jayne was able to improve her understanding leading to a positive emotional solution.

The first part of the narrative opens with an abstract which hints at the volume of potentially problematic homework. Next it introduces the context, in this case the mathematics of decimal place value. The problem, or complicating action, is the parent's lack of understanding of place value. This leads to a negative emotional evaluation of the situation. The problem is solved through cooperative work with the child. When evaluating the experience the parent responds with relief that her child was able to assist her. This narrative shows the linkages between barriers and impediments and children as teachers and enabling styles of communication with children.

<table>
<tr><th>Narrative elements</th><th>Dialogue</th><th>Themes</th><th>Emotional response</th></tr>
<tr><td>Abstract</td><td>No I don't think... Yeah they do a lot where they do,</td><td rowspan="6">Parents' knowledge and understanding

Children as teachers

Enabling parental practices</td><td rowspan="6">Negative emotion - parent

Positive emotion - parent</td></tr>
<tr><td>Orientation</td><td>it'll be say 0.0185 and it'll say you've got to name which are the hundredths, which are the thousandths and which are the tenths.</td></tr>
<tr><td>Complicating Action</td><td>And I haven't got a clue even though I'm like a scientist I'd just not got a clue,</td></tr>
<tr><td>Evaluation</td><td>I'm "Oh my god!"</td></tr>
<tr><td>Resolution</td><td>And Oliver taught me. He went, "Oh it's that". And I went, "Are you sure". So I like I had to have a really long, good think about it and I said, "Yeah you're right it is", and I could see it then.</td></tr>
<tr><td>Evaluation</td><td>But he was quite capable of doing that and thank god that that was the bit that I struggled on.</td></tr>
</table>

The second part of Jayne's narrative showed that the problem of insufficient knowledge and understanding, this time of both parent and child, was resolved by research using the Internet and social networks. No emotions are observed in this section of narrative. The abstract outlines the issue, in this example trigonometry. The context is the parent's knowledge of this topic. In this narrative the complicating action is the parent and child's lack of understanding of how to calculate the area of a triangle. The solution to this problem is to consult the Internet for information or contact a family member. The child's grandfather is chosen because of his subject knowledge, which is valued by the parent. Finally the parent evaluates the episode by confirming that the research strategy is utilised when parent and child lack understanding of particular mathematical concept. This narrative shows the linkages between barriers and impediments and parental strategies and approaches.

Narrative elements	**Dialogue**	**Themes**	**Emotional response**
Abstract	But no, not really, I mean they do a lot of trigonometry	Parents' knowledge and understanding Children's knowledge and understanding Research	
Orientation	and I can never remember doing that at junior school level.		
Complicating Action	And sometimes we've not known exactly what the sum was		
Resolution	so we've just googled it, half base times height and things like that. So yeah and some of the angles and things his granddad's an ex-pattern maker so he's quite up on that so we keep ringing him up and saying, "What's this?" So we do a bit a research.		
Evaluation	If we don't understand it we don't leave it we look round it and find, find an answer.		

4.5.2 Negative episodes

The main cause of negative episodes was a lack of mathematical understanding, displayed by either parent or child. Poor home-school communication and rejection of parents' mathematics also lead to adverse emotional responses.

The narrative analysed below is taken from an episode recalled by Charlotte when asked to describe the last time she did any mathematics with her son Callum. A negative emotional response is felt by Callum as a result of his knowledge and understanding and rejection of Charlotte's mathematics. The parent begins by summarizing the nature of the mathematical activity, a homework investigation where Callum had to calculate the perimeter of his bedroom. Initially she encourages Callum's mathematical autonomy and asks him to try to solve the problem, with the proviso that she will check his solution at the end. The complicating action in this narrative arises when the parent assesses the child's solution as incorrect and so decides to work more closely with her son. As they work together it becomes apparent that Callum does not have a sufficient understanding of metric measurement to tackle the task. However, when Charlotte attempts to demonstrate and explain measuring skills Callum rejects her knowledge as incorrect. As she evaluates the situation, the parent discusses the frustration her son felt at being unable to complete the problem. This emotional response leads Charlotte to postpone the activity until later in the day, at which point the mathematical problem was resolved. This narrative shows the linkages between parental strategies and approaches and barriers and impediments.

Narrative elements	**Dialogue**	**Themes**	**Emotional response**
Abstract	He wanted to measure his room to find out the perimeter and the area		
Orientation	so we got the tape measure out and he went up to measure it and he came down. I just said, "You go and measure it", and I left him to it because I was doing tea and other things. [She says] "You come down and you just write the measurements for each side and we'll have a look".	Promoting independence Evaluating understanding	
Complicating Action	And he came down and he'd got some obscure measurements, 69cm or something. I went, "What that's just like two rulers." He's going, "No it's not". So we had to go back upstairs and get the tape measure and he's going, "No you're reading it wrong". And I said, "I'm not". [He says] "But that's centimetres". [She replies], "I know its centimetres".	Evaluating understanding Child's lack of understanding Demonstration, modelling and explanation Rejection of parents mathematics	
Evaluation	But he just wasn't listening. I think he'd got himself all frustrated and worked up and he started crying.		Negative emotion - child

Complicating Action	So I said, "Well you just stop there and when you decide you want me to help, and you're going to listen to me, then come down and tell me and I'll come back up". So he then had his tea and I then went, "Are you ready now?" And he went, "Yes".		
Resolution	So we went back up and did it again.		

Another negative episode of parent-child mathematical interaction comes from Abigail and concerns a piece of homework on number lines completed with her son Zach. In her opening she labels the experience very negatively as a terrible event. Abigail then orientates the listener by outlining the homework activity and the amount of time it took to complete. The crux of the narrative is that her knowledge and understanding of number lines did not allow her to support her son adequately. Initially Zach was able to support his parent but when the calculations became more difficult he could not complete the subtractions using a number line strategy. To tackle the problem Abigail showed Zach a different method for subtraction, but because he had no knowledge of his parent's approach and the task took a long time he became upset. In evaluating her response Abigail talks about needing to give herself time and distance suggesting a negative emotional outcome. When concluding the narrative she speaks about how the experience lead to her contacting school and becoming more involved in its operation. In the last sentence she alludes to her role as an agent in facilitating better home-school communication and becoming more aware of modern school mathematical practices. This narrative shows the linkages between multiple barriers and children as teachers.

Narrative elements	Dialogue	Themes	Emotional response
Abstract	I remember once instance last year which it was absolutely horrendous with the homework.		Negative emotion - parent
Orientation	We had 20 questions, which sounds great you know, and at the bottom of the sheet you had to put how much you enjoyed it and how long it took. Erm so I put down, "This took three afternoons". It was erm as I say a list of twenty questions and it was all to do with number lines,		
Complicating Action	but the problem is they'd not explained to us what a number line was! So you got then a 7 year old explaining to their mum and dad. We kind of figured number lines but then you got to one where, I don't know, 315 minus 175 so it wasn't an easy calculation to do, you know err like a subtraction. We couldn't figure out how to do it on a number line so we ended up showing him our way and as I say because obviously that was a complete new way for him it took us that long that we had tears, we had tantrums, and this is from me (laughs) you know.	Parent's lack of understanding Children as teachers Parent's lack of understanding Child's lack of understanding	Negative emotion - parent Negative emotion - child Negative emotion - parent
Evaluation	I had to walk away erm that is one, that is one that sticks in my mind as absolutely...		
Resolution	and on the back of that there was a few changes up at school		
Coda	and a few things that happened that which we'll probably go on to later.	Parents as agents	

4.5.3 Patterns of themes in the narrative analysis

Examining the structure and frequency of themes in individual narratives presents a number of findings in terms of interconnectedness of themes. It allows us to state that parental knowledge and understanding, and hence mathematical activity during parent-child interaction, was often connected to communication with school. When communication with school was limited parents felt unable to support their children, which often had a negative emotional consequence. These parents, such as Jayne, often relied on communication with children or social networks.

The most connected theme was parents' views of mathematics as different, which could be seen alongside knowledge and understanding, rejection of parents' mathematics, not wanting to confuse children, as well as the selection, operation and success of parental teaching practices such as evaluating understanding and demonstration, modelling and explanation.

Valorisations of mathematics and mathematical activity often also overlapped. Past experiences of learning mathematics fed into present valorisations and co-operative practices, again connecting to the nexus theme of different forms of mathematical knowledge held by children and parents. The narratives of past experience could also visibly be seen to influence the strategies they adopted in their support of their children's mathematical learning.

The web of relationships which characterise the results of this thematic analysis are perhaps unsurprising given the complex and multifaceted nature of the subject. As a number of participants stated in their valorisations of the subject, 'mathematics is everywhere'. Daily participation in mathematical practices builds and refines valorisations. The narratives and opinions presented by parents are a result of a lifetime of mathematical practice and several years of school mathematical interaction with their children. These experiences can be broken down and compartmentalised individually but during interaction they are drawn upon in conjunction.

4.6 Discussion

The wealth of interview data provided by the respondents in this study supports a series of findings into how parents support children's development of conceptual understanding of primary school mathematics. In particular this analysis buttresses the results of a number of studies of parental involvement undertaken over the past twenty years. It also builds knowledge in key several areas. Findings are discussed with reference to the three research questions which drove the thematic analysis.

4.6.1 RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?

In terms of parents' own experiences of mathematical interaction, this analysis presents six main strategies and techniques used to support children's mathematical development:

- Propinquity
- Promoting autonomy
- Evaluating understanding
- Challenging
- Demonstration, modelling and explanation
- Research

This supports research that suggests that not all parents interact with their children in the same manner and that parents use a range of strategies when supporting children (Civil & Andrade, 2002; Hoover-Dempsey et al., 1995; Solomon et al., 2002).

The findings of Hoover-Dempsey et al. (1995), which indicated parents promoted independent work, supported children though motivation and instruction, and structured homework tasks, are mirrored in this thematic analysis. For instance, demonstration, modelling and explanation allows the structuring of homework whilst propinquity means parents are available to support children. The data from the four age groups, where the theme of promoting autonomy appears more prevalent with age, also appears to support the suggestions of Hoover-Dempsey et al. (1995) that parents appear to grant children greater independence during school work interactions at home as children age. Promoting autonomy was also a popular approach uncovered in parents and teenage children in Solomon et al. (2002). Their finding that 48% of families promoted autonomy is only slightly higher than in this study (41%) and would be expected given their older children.

Solomon et al. (2002) found four other strategies of interaction: no support, unconditional support, proactive involvement and monitoring. Civil et al. (2008) defined strategies of instruction, direction, exploration and observation in their work. These techniques were present in the sample discussed in this chapter and are present in some of the above mechanisms. Rather than looking at specific approaches, e.g. instruction, the thematic analysis presented broader experiences or perceptions of strategies. For instance the spectrum-like approach of propinquity, which is driven by valorisations, expectations and experiences, is far more multi-faceted than a single mechanism such as instruction. Certainly this work supports the findings of Civil et al. (2008) and Solomon et al. (2002), particularly the idea of parents having different views of appropriate parental involvement, but presents them in a different, unique conceptualisation.

Without doubt the themes of propinquity and research help build deeper knowledge on parental approaches to supporting parent-child mathematical interaction. The enlargement of the Internet in terms of school-related content and increasing connectivity of households to the World Wide Web means that parents have a means of accessing information to support their children that they did not have a decade ago. They use this because they want to be involved and support their children but also because information from schools is typically poor.

Supporting research elsewhere (e.g. Abreu et al., 2002; Civil & Andrade, 2002; McMullen & Abreu, 2011; Remillard & Jackson, 2006) these findings show that parents and children appear to have different views of what constitutes mathematics. This was visible in the accounts relayed by many parents.

Parents' past experiences mediate their current practice (O'Toole & Abreu, 2005) and values guide parental involvement (Pea & Martin, 2010). Therefore it would be expected that parents' strategies, approaches and mechanisms of support would be reflected in experiences, perceptions and valorisations of mathematics. Indeed, this appears to be the case. The themes presented here involving parents' valorisations of mathematics in the past, present and future demonstrate this. For instance, the manner in which parents value their own mathematics and the mathematics of their children clearly shapes mathematical activity. Without doubt this supports the arguments put forward in McMullen and Abreu (2011) that parents' valorisations of mathematics differed depending on their understanding of contemporary school mathematics. As in their work, the conflicts arising from different valorisations are also clear. Generally, the stronger the parent valued their own mathematics the more difficult they found it to accept their child's mathematics. Interestingly, this factor of competing valorisations led to situations where some parents felt they must support and provide assistance because they highly valued homework even though they did not value the mathematics within the homework. Here it is clear that valorisations led, as Civil and Andrade (2002) and Civil et al. (2008) found, to some parents preferring their own calculation methods rather that their child's, influencing parental involvement.

Experiences in childhood, particularly parents' valorisations of their own mathematical learning and the support they received as a child, influenced their own parental involvement. Many parents spoke about the support they received as children. This was reflected in how they valued mathematical practices and parental involvement. For instance, this is evident in the theme of parents' experiences of learning mathematics (4.2.2) and through the sub-themes of (i) valorisations of mathematics in parents' own education, including the example given by Neil; and (ii) parents' own experiences of homework, supported by the example from Deborah. Indeed, as shown in Table 4A, all the parents spoke about their own experiences of mathematics education, and the vast majority also spoke about homework and their valorisations of these practices. Many of these parents, shown in Table 4B, also presented examples, in the theme of parents' valorisation of mathematics, of the values they attached to their children's and to their own mathematics.

Those who praised their own parental support appeared to replicate it, as was seen in the example of Deborah. Those who did not value their own parental support often sought to 'do things differently' with their children and become more involved, similar to the findings of O'Toole and Abreu (2005). Whilst examples were not included in the main body of this chapter for reasons of space, Carl, David, Gary, Peter and Rebecca all spoke about wanting their children to experience a different level or quality of support to what they received as children. These ideas were also clearly seen in how parents' perceived homework and the mechanisms of parental support such as propinquity, promoting independence and challenging. It was also evident in the values still placed on their knowledge that some parents still contacted their own mothers or fathers when they faced mathematical problems.

Valorisations of mathematical futures showed that parents had broadly similar aspirations for their children. This reflected valorisations of mathematics as a real-life practice. Generally, all parents aspired to mathematical proficiency and confidence for their children. Several had worries about their ability to support their children, matching the findings in parents and teenagers produced by Solomon et al. (2002).

In summary it is clear that parents engage in parental mathematical involvement through a variety of mechanisms. These mechanisms are influenced by experiences and valorisations. Valorisations can influence involvement positively and negatively.

4.6.2 RQ2: What barriers do parents face in supporting their children's mathematical development?

Parents face a number of barriers in supporting children's mathematical development. The following impediments emerged from the parent interviews:

- Parents' knowledge and understanding
- Children's knowledge and understanding
- Children's rejection of parents' mathematics
- Children's tiredness and motivation
- Parents' not wanting to confuse children

The first of these, parents' knowledge and understanding, is perhaps unsurprising considering the fact that parents in the sample all attended primary school prior to changes brought about by the advent of the *National Curriculum* (Department for Education and Employment, 1999a), *National Numeracy Strategy* (Department for Education and Employment, 1999b), and the subsequent *Primary National Strategy* (Department for Education and Skills, 2006). Certainly, this phenomenon of divergent understandings has been found elsewhere by a number of researchers investigating parental involvement in mathematics (Abreu & Cline, 2005; Baker et al., 2006; McMullen & Abreu, 2011; Street et al., 2008). The barrier of knowledge and understanding, and its emotional consequences, was shown vividly in several parental accounts and through the structural analysis. This gains extra credence when we consider the findings relating emotions and mathematical learning (e.g. Hyde et al., 2006; Else-Quest et al., 2008).

These results, in a predominantly white working- and middle-class sample, replicate the findings of others looking at minority groups or more specific cohorts (Abreu & Cline, 2005; McMullen & Abreu, 2011, Street et al., 2008). It was clear that parents who understood contemporary mathematics through information with school experienced fewer barriers to involvement. These barriers could be easily addressed by schools, especially since a wealth of UK government policies and reports seek to further involvement as a form of increasing academic attainment.

Children's knowledge, particularly their ability to remember instructions or strategies, could also be a hindrance especially when it coincided with a lack of understanding by the parent. In some instances the homework was clearly too difficult for children, the examples recounted by Vicky and Imogen attest to this. These presented difficult emotional situations for both parents and children. Likewise, the theme of not wanting to confuse children, and the worry and confusion this causes, again replicating findings elsewhere (Abreu & Cline, 2005; McMullen & Abreu, 2010), links to both parents' and children's knowledge and understanding of mathematics.

A frequently mentioned impediment to parental involvement was children's rejection of parents' mathematics. This process is linked to the valorisations mentioned earlier. As in other studies (e.g. Abreu et al., 2002), it was clear that for some parents their children valued school mathematics more than the parents' mathematics. This valorisation formed a barrier that led to children resisting parental support. This again is linked to a divergence in parents' and children's mathematical knowledge and understanding.

Children's tiredness and motivation formed a barrier limiting the occasions where parents felt able to become involved in their children's school mathematics. For children that did not enjoy or value homework practices, certainly if they valued them less than other activities, it was parents to engage them in school mathematics. Whilst this was not predominant in the sample it still represents and interesting finding that is worthy of deeper investigation, particularly connecting to children's valorisations.

Another interesting finding relates to how parents perceive children to be 'teachers' of school mathematics. This theme, only present in a third of interviews, appeared to act as both a mechanism for involvement, for example for Jayne, and a barrier, for instance for Abigail. This appeared to be related to the flexibility of the role parents formed in parent-child interaction. Within this study the psychological processes involved in this are unclear but bear further exploration, certainly given the importance of children as a conduit of information given divergent understandings.

In summary barriers appear widespread and often interconnected, shown clearly in the narrative structural analysis, with many impediments having negative emotional consequences. These need to be tackled and can, as the next section shows, be mitigated by the schools and parents.

4.6.3 RQ3: How are parental teaching practices shaped by, and through, communication with schools?

Communication with school shapes practices in a number of ways. Firstly, access to and interaction with teachers fostered a sense of agency in some parents, making them feel they were aided and empowered to support their children. Secondly, and most frequently attested to in the results of the thematic analysis, is that parental behaviours and support strategies were influenced, either inhibited to supported, by:

- The clarity of information from school regarding homework, mathematical strategies and children's work in school
- Engaging parents in school mathematical practices
- Receiving feedback on the correctness or otherwise of homework
- Awareness of children's mathematical progress and how parents can support children's learning at home

This research shows that linked to these four points is the strong desire and need respondents felt for information to enable them to better support their children, and conversely that a lack of such information or poor clarity of material inhibits parental teaching practices.

These findings support research on parental involvement that shows the importance of good communication between parents and schools (e.g. Hughes & Greenhough, 2006). They also explore in greater depth issues raised by large-scale surveys, such as Peters et al. (2008) who also showed that parents typically wanted more information. By delving deeper it is possible to see the reasons and impacts of this lack of communication. Most interestingly it is also possible to see what happens when parents have more information and how this changes not just their perception of school mathematics but also the quality and features of parent-child interactions. These parents, like those in several US studies (Jackson & Remillard, 2005; Remillard & Jackson, 2006) feel that with more information they are better able to support and monitor their children's school mathematics.

As with other research in the UK (Abreu & Cline, 2005; Baker et al., 2008; Hughes & Greenhough, 2006) it is possible to see one-way flows of information between home and school. Because of this some parents were particularly critical about aspects of their relationship with school. Parents who felt engaged and able to communicate with school in a bi-direction manner appeared better able to support their children. Though such two-way communication was experienced by less than half of the sample.

It certainly appears that through relatively simple, low-cost and minimally time-intensive strategies school can better inform and involve parents in their children's school mathematics. This partnership not only leads to a greater ability to support mathematics at home but also potential changes in valorisations of mathematical practices. This was unquestionably evident for some parents and helped give them more confidence in parent-child mathematical interactions.

The next chapter of this thesis returns to the question of how mathematical identities are formed. These can be seen to influence parental support children's development of conceptual understanding of primary school mathematics. Through a study of identity it is possible to observe how many of the themes, experiences and valorisations contained within these chapter present insights into constructions of the mathematical 'self' and 'other'.

Chapter 5

The mathematical 'self' and the mathematical 'other'

5.1 Introduction

Mathematical identity is the subject of this chapter, specifically an investigation into:

RQ4: How do parents dialogically construct identities for themselves and for their children?

Dialogical self theory is founded upon the premise that the self is made up of a multiplicity of positions (Hermans et al., 1992). These positions evolve through interaction with the sociocultural environment and through constant dialogue between positions. Through positioning, complex and dynamic representations of the self are constructed (Hermans et al., 1992). These positions, which are held as part of a dialogical self, are spatially and chronologically dependent. In other words they change and evolve over time and also depending on the location of the self in terms of sociocultural context.

At the heart of dialogical self theory, and a focus for its study, is the notion of the I-position. These can be internal (e.g. I as a student) or external (e.g. the voice of an imagined other) (Hermans, 2001) and provide both multiplicity in the self but also a mechanism for unity and change over space and time (Hermans & Hermans-Konopka, 2010). An I-position is constructed through dialogue that establishes a relationship to an object, be it a feeling, experience, idea or concrete entity.

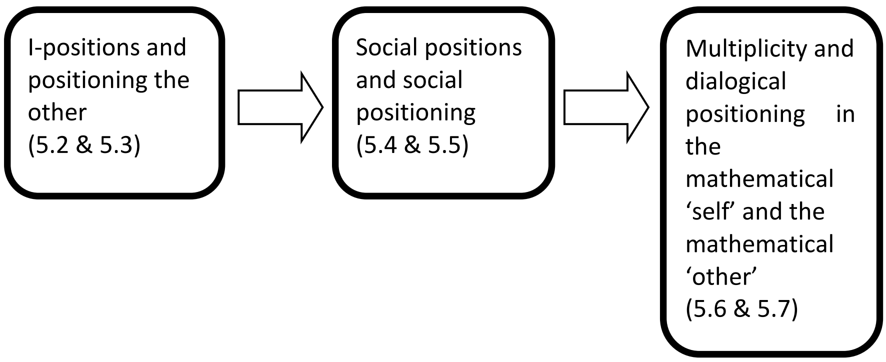
The investigation of mathematical identity can be extended beyond the notion of 'I' positions to look into how the social and cultural environment influences the mathematical self. This is achieved through looking at external 'voices' in participant narratives (Akkerman & Meijer, 2010; Aveling & Gillespie, 2008). In this context, the term 'voices' does not purely refer to utterances or reported speech but also imagined voices and perceived expectations or norms associated with another. These are social positions.

Research has also shown how individuals extend identities to others through positioning (Abreu, 2002; Abreu & Cline, 2003; Crafter & Abreu, 2010; Gorgorió &

Prat, 2011) Therefore using a dialogical approach it should be possible to better understand how parents position children mathematically. This ‘other’ positioning is analogous the process of outside positioning proposed by Hermans (2001) and Raggatt (2011).

As noted within the methodology section of this thesis, the study of mathematical identity was undertaken through a three-tiered sequence of analytical lenses, as shown in Figure 5A. The results of each of these levels are presented within this chapter.

Figure 5A Three levels of dialogic analysis of the mathematical ‘self’ and mathematical ‘other’



The first level of analysis concentrates on investigating the mathematical positioning of the ‘self’ and of the ‘other’. In this sense the inquiry focuses on categorising and studying the mathematical positions that parents assign to themselves and to their children. The second level of analysis centres on the role of the sociocultural environment on mathematical positioning. Here social positions are investigated with reference to positioning of the ‘self’ and of the ‘other’. The linkages between external social positioning and internalization of reflexive positioning are considered. In the third level of analysis, multiplicity and the individual mathematical ‘self’ and individual ‘other’ are studied to show how complex mathematical identities are constructed and evolve chronologically and spatially.

The chapter concludes by discussing the findings of the various analyses undertaken and drawing attention to its relevance in both addressing the research question posed at the beginning of this chapter and also contemporary debate on mathematical identity.

5.2 Parental I-positions: co-constructing identities of the self

In the initial stages of applying a dialogical self theory (DST) to the study of mathematical identity, specifically the mathematical identity co-constructed through dialogue with the researcher, it is appropriate to begin by studying one of the fundamental building blocks of DST, namely dynamic I-positions.

This was achieved by coding I-positions in parental narratives, as set out in 3.7.2. This first level of analysis produced a total of fifty-two different mathematical I-positions across the entire sample. Several of these were unique to individual parents. Forty-one mathematical I-positions could be seen in at least two separate parental narratives. When co-constructing a mathematical self it appeared that I-positions could be largely classified as falling into one of three categories. The mathematical 'I' appeared to emerge in dialogic conjunction with (1) perceived mathematical behaviours, (2) individual competencies, abilities or aptitudes concerning mathematics, or (3) emotions and feelings associated with mathematics. In the following sub-sections each of these three categories is discussed in turn. It is unfeasible to discuss all fifty-two positions within this chapter. Instead the most common mathematical I-positions in each of the three categories will be considered. In the examples included in this section mathematical I-positions are underlined and preceded by a number to specify their position in the text.

5.2.1 Behaviourally-related I-positions

Twenty-two mathematical I-positions were linked to perceived behaviours enacted by parents. These are shown in Table 5A. Here we see examples of how the self was identified as comprising of specific behaviours associated with mathematics. Many of these have similarities, such as the fine differences between behaving in a proactive or supportive manner, and alternatively identifying the self as being a tutor, a teacher or an instructor. Here, as elsewhere in this chapter when describing positions, a balance has been sought between differentiating between unique, often personalised I-positions, and enabling a comparison of I-positions across of group of diverse individuals.

Only six of these behavioural positions were unique to certain individuals, suggesting common and diverse self-identifications in the parent sample.

Mathematical I-positions such as ‘I as an infrequent user of math’ or ‘I as lacking motivation regarding mathematical activity’ were self-identifications used in only single cases.

Table 5A Mathematical I-positions linked to perceived behaviours

Mathematical I-position	**Number of respondents co-constructing I-position**
I as supporting my child’s mathematical development	24
I as playing a proactive role in my child’s mathematical development	21
I as organised regarding mathematical activity	9
I as encouraging my child’s mathematical development	9
I as challenging my child mathematically	8
I as flexible during mathematical interaction	8
I as instructional during mathematical activity	6
I as involved in my child’s mathematical development	6
I as replicating my own upbringing	6
I as co-operative during mathematical interaction	5
I as agreeable during mathematical interaction	3
I as practical regarding mathematical activity	3
I as promoting independence	3
I as a tutor regarding mathematical activity	3
I as assertive regarding my child’s mathematical development	2
I as a teacher regarding mathematical activity	2
I as lacking motivation regarding mathematical activity	1
I as monitoring my child’s mathematical development	1
I as motivated regarding mathematical activity	1
I as non-instructional during mathematical activity	1
I as not wanting to pressurise my child	1
I as supporting a work ethic	1

All the parents in the sample assumed an I-position of ‘I as supporting my child’s mathematical development’ when describing themselves and their mathematical interactions with their child. Part of their mathematical ‘self’ was the notion that they supported their children’s learning in some manner. That is not to say that all the parents were equally supportive of their children, just that they all identified themselves as supportive, some with a couple of references classified as ‘I as supporting my child’s mathematical development’ and others with over a dozen. In the example below Beth built an I-position as ‘I as supporting my child’s mathematical development’. She discussed how she supports her son’s homework by giving time and attention to his needs and maintaining spatial proximity.

Beth (parent) - Interview	
Dialogue	Mathematical I-position
(1) I tend to be in the kitchen when he's sat here doing his homework and like in literacy or words if he's stuck on it I'll like explain it.	(1) I as supporting my child's mathematical development

A majority of the parents co-constructed a mathematical I-position in which they identified themselves as proactive. In this sense they saw themselves as often taking the lead in providing out-of-school opportunities for mathematical learning and initiating contact with teachers over concerns and queries. Some respondents identified themselves as proactively seeking to improve their knowledge of mathematics in order to better support their children. Ruth, who also produced multiple references to 'I as supporting my child's mathematical development', positioned herself as proactively involved with her son's education. Here she recalled a meeting with his teacher where, not content that Michael was excelling mathematically, she approached the teacher with her concerns that he was not being challenged enough.

Ruth (parent) - Interview	
Dialogue	Mathematical I-position
I think it was probably the last parents' evening we had, and he's in the top set and he's at the top end of the top set (1) and my question was actually, "Michael's doing really well but do you think he's being pushed to stretch him because my concern is that he'll get bored if he's not stretched."	(1) I as playing a proactive role in my child's mathematical development

5.2.2 Competency-related I-positions

Parents identified their self as comprising of a range of attributes, abilities and aptitudes in relation to mathematics. Table 5B shows all twelve mathematical I-positions in which the labelling of the self appeared to be related to perceived competencies.

Table 5B Mathematical I-positions linked to perceived competencies

Mathematical I-position	**Number of respondents co-constructing I-position**
I as confused by mathematics	13
I as a competent user of mathematics	12
I as not good at mathematics	9
I as good at mathematics	8
I as a novice and learning mathematically from my child	8
I as finding mathematics difficult	4
I as improving mathematically since I left school	3
I as understanding mathematics	3
I as mathematically successful	2
I as struggling with mathematics	2
I as an expert in mathematics	1
I as held back by my lack of knowledge	1

Again fine but noticeable differences existed between positions such as finding mathematics difficult, confusing or perceiving the self as 'not good' at mathematics.

Identifying that they conceptualised mathematics as confusing, and therefore that they were confused during many forms of mathematical activity, was a common I-position assumed by parent participants. Confusion commonly stemmed from a perceived lack of knowledge or understanding of their child's mathematical activity. In the example below it is possible to interpret Deborah's comment about the difficulties she faced understanding her daughter's grid multiplication and subtraction as 'I as confused by mathematics'.

Deborah (parent) - Interview	
Dialogue	Mathematical I-position
<u>(1) I think it's the younger stuff that confuses me because I think it's the way that they're teaching them how to learn maths with the grids and the counting back and all that kind of stuff.</u>	<u>(1) I as confused by mathematics</u>

When discussing their competencies associated with mathematics, or their ability to support their children, many parents presented a self that was competent at mathematics. In this sense they did not see themselves as either good or bad at mathematics, but rather suggested that they were 'good enough' for the mathematical activities in which they were involved. Chris positioned himself as a competent user of mathematics, as shown below, when he described his daughter's homework as "relatively simple" and himself as "not the brightest but I'm not stupid".

Chris (parent) - Interview	
Dialogue	Mathematical I-position
I'm looking at some of the stuff she's doing with division and stuff, you know yeah <u>(1) it's relatively simple now but, you know, I'm not the brightest but I'm not stupid</u>, you know in two or three years I'm going to think...	<u>(1) I as a competent user of mathematics</u>

Generally, parents varied in the consistency with which they held a singular I-position regarding mathematical ability. As we would expect, and as is discussed later, parents produced mathematical I-positions which varied across space and time.

5.2.3 Emotionally-related I-positions

A range of emotions and feelings were incorporated into mathematical I-positions by parents. Within interviews parents often took both positive and negative positions depending on the experience or opinion they were presenting. The different mathematical I-positions associated with emotions are shown in Table 5C. It shows positive positions, for example invoking a confident self or a self which associated joy with mathematics, and negative positions, where the 'I' and mathematics were synonymous with emotions such as panic, fear, apprehension, worry and pressure.

Table 5C Mathematical I-positions linked to emotions

Mathematical I-position	**Number of respondents co-constructing I-position**
Positive emotions	
I as enjoying mathematics	15
I as feeling supported by my parents	8
I as mathematically aspirational	8
I as confident with mathematics	7
I as feeling vindicated through mathematical activity	1
I as unconcerned about my child's mathematics	1
Negative emotions	
I as not enjoying mathematics	11
I as feeling unsupported by my parents	7
I as apprehensive of mathematics	6
I as not interested in mathematics	6
I as regretful of mathematical activity	6
I as scared of mathematics	6
I as frustrated by mathematics	4
I as negative towards mathematics	3
I as nervous of mathematics	3
I as pressured by mathematics	3
I as worried by mathematics	3
I as panicked by mathematics	1

Fifteen parents portrayed themselves as enjoying mathematics. They spoke about the positive mathematical self that existed at different times and during different activities, for instance in the course of their schooling, their working life or at home with their children. Jayne discussed her interest in mathematics and her liking of the subject. She particularly enjoyed the problem-solving and reasoning aspects of the discipline.

Jayne (parent) - Interview	
Dialogue	Mathematical I-position
<u>(1) I like maths and I don't mind doing it and I see it, it is like doing puzzles really, it's not, it's not a chore it's like quite interesting.</u>	<u>(1) I as enjoying mathematics</u>

Conversely to the last I-position, just under half the parents in the sample co-constructed a self that did not enjoy mathematics. The position 'I as not enjoying mathematics' is shown in the next example. Here Gary, who frequently recounted negative experiences and attitudes associated with mathematics, described his uncomfortable feelings towards the subject. Even though Gary worked in a numerate occupation he did not enjoy using mathematics.

Gary (parent) - Interview	
Dialogue	Mathematical I-position
(1) I'm more uncomfortable with it, I mean I work in accounts incredibly, I left school without any qualifications whatsoever yet I do work in accounts.	(1) I as not enjoying mathematics

This first element within the primary level of analysis has concentrated on the mathematical 'I' and described I-positions co-constructed by parents in their dialogue with the researcher. These positions can be seen to be evident in relation to mathematical behaviour, competencies and emotions.

5.3 Parents positioning of children: co-constructing identities of the other

The second element within the first level of analysis follows a similar approach to the previous section, but rather than concentrating on positioning the 'self' it concentrates on positioning the 'other', in this case how parents create mathematical positions and identities for their children through 'other' positioning. So rather than positioning the self by way of 'I as...', our focus turns to positioning the other via 'My child as...'.

The parental interviews contained a total of thirty-four relevant dialogical positions. Twelve of these 'other' positions were unique to individual children. As in the previous section parents positioning of children could be seen to fall into three categories. Here again positioning was connected to (1) mathematical behaviours, (2) mathematical competencies, and (3) emotions and feelings connected to mathematics and mathematical activity. As before, these categorisations are discussed in turn with supporting examples provided from parental interview transcripts. The 'other' positions displayed in this analysis are double underlined and prefixed by a capital letter to denote order.

5.3.1 Behaviourally-related other positions

Parents identified their children as having a number of different labels regarding mathematical behaviour. Several were common across the sample, others like 'My child as behaving flexibly during mathematical activity' or 'My child as stubborn during mathematical activity' were positions restricted to individual children. The thirteen positions defined as behavioural are listed in Table 5D.

Table 5D Positioning the other through perceived behaviours

Other positioning	**Number of respondents co-constructing position**
My child as not communicating mathematical knowledge and activity	13
My child as lacking motivation regarding mathematics	11
My child as motivated regarding mathematics	11
My child as communicating mathematical knowledge and activity	10
My child as a mathematical tutor	7
My child as diligent regarding mathematical activity	6
My child as not always listening during mathematical activity	2
My child as struggling to maintain concentration during mathematical activity	2
My child as argumentative about mathematics	1
My child as behaving flexibly during mathematical activity	1
My child as behaving inflexibly during mathematical activity	1
My child as defensive about mathematics	1
My child as stubborn during mathematical activity	1

As has been established earlier in this thesis (see 4.4.1) parents represented communication with children as influencing parental mathematical practices through hindering or enabling activity. In this sample thirteen participants dialogically positioned their children as 'My child as not communicating mathematical knowledge and activity'. In her interview, Imogen positioned her son as uncommunicative and hinted at the problems that this caused in effective parent-child mathematical interaction.

Imogen (parent) - Interview	
Dialogue	Other positioning
I don't, we don't struggle doing it because we can do it, but it's just whether knowing if we are doing it the correct way because <u>(A) Owen is not a very open child so he won't turn round and say, "No you don't do it like that we do it like this mum"</u>	<u>(A) My child as not communicating mathematical knowledge and activity</u>

Several parents positioned their children as lacking motivation regarding mathematics. These labels originated in both narratives of mathematical episodes and in descriptions of general attitudes and behaviour. In the following example Niamh commented on the difficulty of getting her son Connor to do his mathematics homework. She clearly positioned him as lacking motivation regarding mathematics, someone who will "put it off until, you know, tomorrow and tomorrow never comes".

Niamh (parent) - Interview	
Dialogue	Other positioning
But getting him to do it is always not easy. <u>(A) You know he'll put it off until, you know, tomorrow and tomorrow never comes.</u>	<u>(A) My child as lacking motivation regarding mathematics</u>

5.3.2 Competency-related other positions

Generally parents positioned their children using positive competencies and aptitudes regarding mathematics. In fact, the respondents tended to use more positive labels for their children than they used to describe their own mathematical 'I'. The full list of other positioning linked to parentally designated mathematical competencies is shown in Table 5E.

Table 5E Positioning the other through perceived competencies

Other positioning	**Number of respondents co-constructing position**
My child as good at mathematics	16
My child as a competent user of mathematics	10
My child as not needing my help during mathematical activity	10
My child as gaining mathematical knowledge	6
My child as mathematically intelligent	4
My child as challenged by mathematics	3
My child as struggling with mathematics	3
My child as confused by mathematics	1
My child as doing well at school mathematically	1
My child as growing in mathematical self-confidence	1
My child as slow at mathematical activity	1

Children were positioned as 'good' at mathematics in a variety of situations and contexts. Often the speed and accuracy by which they completed mathematical tasks was referenced when a parent created an identity position for their child. Neil used a remembered conversation with his son Daniel to position him as good at mathematics, shown in the excerpt below. Neil remembered a time when Daniel used elements of algebraic reasoning, which he interpreted as a sign that his child was mathematically able.

Neil (parent) - Interview	
Dialogue	Other positioning
I mean it was, we were coming home from school one day and he was talking about something that he did at school and I said, "Oh brilliant that's fantastic". And I said, "It won't be long before you are doing algebra". And he said, "What's algebra?" And I said, "Oh that's like when A equals 1 and B equals 2 and C is A plus B". <u>(A) And he said, "So C is 3?" So I was like, "What! Spot on!" He was beaming in his face and I thought wow this kid is getting it straight away</u> and just instantly, "So B [C] is 3 then".	<u>(A) My child as good at mathematics</u>

Parents who often identified their children as motivated, diligent, enjoying mathematical activity or being good at mathematics also positioned their children as not requiring assistance during mathematical activity. They identified their children as having sufficient confidence and mathematical capability to only necessitate a minimum of parental supervision. David positioned his daughter as not needing support because she found homework easy and because he was not often called upon to assist her.

David (parent) - Interview	
Dialogue	Other positioning
I think she gets homework once a week for maths. <u>(A) Homework that she doesn't find that difficult so she can sit and do the whole thing and I will usually... She will just say, 'Daddy is that right?'</u>	<u>(A) My child as not needing my help during mathematical activity</u>

Unlike parents, children were not positioned as 'bad' at mathematics. They were described more favourably as good, competent or struggling.

5.3.3 Emotionally-related other positions

Table 5F shows the ten positions used in the sample to describe children's mathematical self, in a way that linked the position to a perceived emotion. The most notable of these positions are those that provide a generalised character label for the child, for example 'My child as not enjoying mathematics' or 'My child as confident with mathematics'.

Table 5F Positioning the other through perceived emotions

Other positioning	**Number of respondents co-constructing position**
Positive emotions	
My child as enjoying mathematics	17
My child as confident with mathematics	5
My child as ambivalent towards mathematics	2
Negative emotions	
My child as frustrated by mathematics	7
My child as not enjoying mathematics	6
My child as lacking self confidence in mathematics	3
My child as panicked by mathematics	2
My child as feeling conflicted between home and school mathematics	1
My child as scared by mathematics	1
My child as self-conscious of his mathematical aptitude	1

The most common feeling or emotion assigned to children was of liking and enjoying mathematical activity. Respondents spoke about their children enjoying mathematics at school, at home in the form of homework, and in a range of informal non-school context. The excerpt below is typical of respondents in the sample. Here Carl positioned his daughter as enjoying school mathematics, a view he based on previous conversations.

Carl (parent) - Interview	
Dialogue	Other positioning
I think generally when we ask her about her subjects she, she says, yeah she always responds positively so I don't have any, <u>(A) I don't have any doubt that she enjoys maths and that she enjoys school, which is really encouraging. So, so yeah I think she likes maths yeah.</u>	<u>(A) My child as enjoying mathematics</u>

In opposition to the positive feeling of mathematics as enjoyable are the identifications of mathematics as frustrating and mathematics as not enjoyable. Whilst these two more negative outlooks are less frequent they are present in a number of parental narratives. An example of this is the way in which Suzy positioned her son Matthew. She suggested he quickly became frustrated if he did not completely understand a mathematical activity.

Suzy (parent) - Interview	
Dialogue	Other positioning
(A) If he doesn't, if he's not 100% with it he gets frustrated very, very quickly.	(A) My child as frustrated by mathematics

The analysis above shows the broad range of positioning that is present within the data set. It shows how some 'other' positions, like mathematical I-positions, can be seen to arise in conjunction with certain perceived behaviours, competencies or feelings.

5.4 Parents social positioning: sociocultural voices in the self

In the second level of dialogical analysis of identity interview data was coded to connect, where possible, mathematical I-positions with any social positions. Through this approach to the coding of interview transcripts it is possible to link social positions to two distinct types of external 'voices' in parental narratives. The first is a generalised voice, representing societal influences and imagined others in the cultural environment. The second are specific voices of parents, teachers, friends and acquaintances whose labels and positions are, to a greater or lesser degree, rejected or absorbed into the mathematical self through a process of reflection. In the extracts below social positions are displayed in italics, preceded by a lower-case Roman numeral in brackets to indicate its order and position within the text. As in the previous section I-positions are shown by underlining the relevant segment of text, preceded by a number.

5.4.1 General voices

Looking at how parents position themselves with regard to various actors and societal and cultural expectations, gives an insight into the formation of the dialogic mathematical self. The external voice of another, perhaps an interaction with a teacher during a remembered exam, may produce social positions such as 'I as successful at mathematics as defined by teachers and school'. The position carries weight and may be absorbed and internalised through reflection as 'I as successful at mathematics'. Similarly, social and cultural experiences involving mathematics invite us to reflect and make comparisons with others, leading to positions such as 'I as successful at mathematics compared to my brother', or 'I as unsuccessful at mathematics compared to my friends'. These again may influence the 'I as successful at mathematics'. In this manner 'I as successful at mathematics' becomes not just a mathematical I-position but also a reflective position. Reflexive positions are formed when we reflect upon the label or positions placed upon us from outside.

Within the respondent interview data are imagined voices from non-specific sources, for instance other imagined parents, or more ephemeral voices representing perceived sociocultural norms. This is shown in the range of social positions displayed by the interviewees in Table 5G.

Table 5G Social positions and general voices

Social position	**Number of respondents co-constructing position**
I as reflecting the influences of my social environment	11
I as more supportive than other parents	7
I as more successful at mathematics than others	5
I as more able to support my child than other parents	4
I as different for not reflecting a perceived social view of mathematics	2
I as reflecting the influences of my school environment	2
I as competent at mathematics as determined by others	1
I as less mathematically able than others	1
I as more organised than other parents	1
I as more proactive than other parents	1
I as replicating successful social behaviour	1

As was shown earlier, when describing their mathematical behaviour all the parents in the sample exhibited the position 'I as supporting my child's mathematical development'. For several parents in the sample this I-position was clearly linked to seeing themselves as different to others in their sociocultural sphere. This comparison of their 'self' to non-specific others resulted in the position 'I as more supportive than other parents'. Gary identified himself as a supportive parent who had regular, high-quality communication with his son's school. He reflected on the imagined voice of a teacher to justify his opinion that the level of support he gives his son places him in a minority.

Gary (parent) - Interview		
Dialogue	Mathematical I-position	Social position
(1) (*i) You get the feeling that we might be in the minority, I hope we're not, but it seems, it almost seems like: "Wow parents who are interested. What a refreshing change. Come on in, sit down and let's, you know, talk about it."*	(1) I as supportive	*(i) I as more supportive than other parents*

Similarly some parents saw themselves as more organised or more proactive in supporting their children than other parents. Several parents used their mathematical I-position concerning their aptitude in mathematics to create a position of 'I as more able to support my child than other parents'. Jayne reflected on her confidence and enjoyment of mathematics and used this to justify a position in which she saw herself as more able to support her son Oliver than other imagined less-able parents.

Jayne (parent) - Interview		
Dialogue	Mathematical I-position	Social position
I mean *(1) (i) I'm fine because I'm alright with maths but I do think that some parents might struggle because it is a lot further forward than where we were at that point in our lives and erm how many people remember to do basic maths and fractions anyway?*	(1) I as a competent user of mathematics	*(i) I as more able to support my child than other parents*

As well as comparisons with other parents, judgements were made to differentiate the mathematical self from other imagined social actors. This occurred through social positions such as 'I as competent at mathematics as determined by others', 'I as less mathematically able than others' and 'I as more successful at mathematics than others'. In this next except Chris, who was discussing his commonly used mathematical skills, suggested he is often surprised when people cannot do the mathematics that he can easily accomplish.

Chris (parent) - Interview		
Dialogue	Mathematical I-position	Social position
(1) *(i) ...the number of people that I come across, we're talking about margins and percentages and they can't work a percentage out in their head. It, it, I'm just like, well how do you [cope]?*	(1) I as a competent user of mathematics	*(i) I as more successful at mathematics than others*

It is possible to interpret how parents construct other positions based on how they themselves are positioned by imagined voices of society, social expectations. In a number of accounts we can see societal factors in positions like 'I as replicating successful social behaviour' and 'I as reflecting the influences of my social environment'. Chris reflected on the social and cultural environment in which he grew up. He rejected academic expectations but reflected a more general societal expectancy regarding the value of mathematics.

Chris (parent) - Interview	
Dialogue	Social position
(i) Yeah the time when I went to school it was all about you got to this and then get to university and stuff, which I didn't, or even bother doing A-Levels either. But it was all about maths and English. You must be good at maths and English.	*(i) I as reflecting the influences of my social environment*

5.4.2 Specific voices

A number of specific voices are seen incorporated in parental social positions. These social positions allow I-positions to form by reflexively comparing the self to others. Table 5H shows a list of the different social positions, with specific voices, dialogically co-constructed during parental interviews. In this case the positions have been grouped to show the origin of specific voices.

Table 5H Social positions and specific voices

Social position	Number of respondents co-constructing position
Partners (wife, husband, boyfriend, girlfriend etc.)	
I as less mathematically able than my partner	4
I as less able to support my child than my partner	2
I as more mathematically able than my partner	1
I as similar to my partner	1
Parents	
I as reflecting the influence of my parents	7
I as replicating the mathematical practices of my parent	2
I as mathematically successful as determined by my parents	1
I as more supportive than my parents	1
I as not as mathematically able as my parent	1
I as similar to my parent	1
Siblings	
I as competent at mathematics compared to my sibling	1
I as mathematically unsuccessful compared to my sibling	1
I as more mathematically able than my sibling	1
Children	
I as similar to my child	9
I as different to my child	2
Friends	
I as mathematically successful compared to my friends	1
I as supportive like my friend	1
Peers	
I as competent at mathematics compared to my peers	2
I as mathematically successful compared to my peers	1
I as working harder at mathematics than my peers	1
Teachers	
I as mathematically successful as determined by my teacher	2
I as supportive as determined by child's teacher	2
Colleagues	
I as less competent at mathematics than my colleagues	1

The majority of these different voices are concerned with social positions formed around mathematical competencies or aptitudes. Some respondents recalled the voice of a teacher or parent and how this positioned them as good or successful at mathematics. In this case the voices of others are used to position the 'self' in relation to others. In the example below, Lindsay compared herself to her husband Tony. She used his response to her mathematical activity to create a social position where she saw herself as less mathematically able than Tony.

Lindsay (parent) - Interview		
Dialogue	Mathematical I-position	Social position
<u>(1) I just never enjoyed it at school.</u> *(i) Err and even now my husband he laughs at me, you know, if we're playing darts, because we've got a dart board up there, and he's worked it out and I'm still like, you know, on the fingers trying to (laughs).* <u>(2) I'm getting better at it the more we play darts but err... I can do it but it just takes me a while to get my head round it sort of thing, do you know what I mean</u>. <u>(3) So I'm not a lover of maths.</u>	<u>(1) I as not enjoying mathematics</u> <u>(2) I as a competent user of mathematics</u> <u>(3) I as not enjoying mathematics</u>	*(i) I as less mathematically able than my partner*

Similar patterns exist where parents either directly compare themselves with others or recall interaction with another. This arose in social positions like 'I as more supportive than my parents' and 'I as supportive as defined by child's teacher'.

Another specific voice that can be seen to influence parents' mathematical I-positioning is 'I as reflecting the influence of my parents'. This includes both uttered expectations and implicit evaluations. In the following example Jayne, who highly valued mathematics and saw herself as mathematically successful, discussed her view of her father and desire to reflect or 'be like' him.

Jayne (parent) - Interview	
Dialogue	Social position
Yeah my dad he was always good at maths. He used to sit and do little things with us and *(i) so you were always trying to emulate people you admire and I always thought it was brilliant.*	*(i) I as reflecting the influence of my parents*

Similarities between parents and children were used reflexively to both position the parent and for the parent to position the child. Nine respondents co-constructed a position of being similar to their child. Two participants communicated a position of being different to their child. Both are visible in the next excerpt. Here Lindsay compared herself to both her son and daughter. She contrasted herself with her eldest son, who she saw as good at mathematics, and positioned herself as similar to her daughter, who struggled with mathematics.

Lindsay (parent) - Interview		
Dialogue	Mathematical I-position	Social position
<u>(1) I used to enjoy it</u> I just it, you know, *(i) it just doesn't come naturally to me. I think for some people like my elder son he's... it just seems to come to him really easily, (ii) whereas my daughter is a bit like me. She struggles with it so she's not interested in it, do you know what I mean, so <u>(2) I wasn't as interested in maths</u>* as I probably was, you know, in English because I used to like to write and make stories up and things so...	<u>(1) I as enjoying mathematics</u> <u>(2) I as not interested in mathematics</u>	*(i) I as different to my child* *(ii) I as similar to my child*

Thirty-four social positions we found in the twenty-four respondent interviews. When looking for the origin of these positions it was possible to see both general and specific voices. It is also possible to infer the process of reflexivity catalysing the transformation of external social positions into internal mathematical I-positions.

5.5 Parents social positioning of children: sociocultural voices on the other

Following on from social positions and mathematical 'I' positions, a similar approach can be utilised to investigate the manner in which parents use social positioning when co-constructing mathematical identities for their children. It is possible to see how mathematically positioning a child as 'successful' or 'unsuccessful', or 'good' or 'struggling' at mathematics is replicated with reference to the sociocultural environment in social positions like 'My child as good at mathematics as compared to his/her peers' or 'My child as good at mathematics as determined by his/her teacher'. Such social positions can conceivably be absorbed through reflective activity into an 'other' position of 'My child as good at mathematics'.

As with the previous analysis, two types of voices are apparent in social positioning of the other: general and specific. This duo is considered in turn. Social positioning of children is highlighted in bold and preceded by upper-case Roman numerals in brackets, to indicate location in the extract. As previously 'other' positioning is double underlined and preceded by a capital letter.

5.5.1 General voices

General voices in the social positioning of children appear to come from social influences or expectations of children. These include the expectations contained with national targets or objectives but also perceptions of the general characteristics of 'typical' children. Table 5I shows four types of social position contained in interview transcripts that seem to link to general voices.

Table 5I Social positioning and general voices

Social positioning	**Number of respondents co-constructing position**
My child as compared to national standards in mathematics	7
My child as reflecting his/her gender	5
My child as typical	3
My child as more communicative than other children	1

General gender stereotypes exist in the sample as parents compare their children through the voices they perceive in society. For example, Niamh commented on her son's lack of motivation with regard to mathematics ('My child as lacking motivation regarding mathematics') and lack of communication concerning homework ('My child as not communicating mathematical knowledge and activity'). In justifying these positions she used not only personal experiences of working with Connor, but also a social perception that his behaviour is archetypal of his gender. In other words Connor was a boy therefore he was not interested in, and does not talk about, school mathematics.

Niamh (parent) - Interview		
Dialogue	Other positioning	Social positioning
(A) **(I) He's, to me he's a typical boy in that he's not interested in school work** (B) **and trying to get information out of Connor is quite difficult.**	(A) My child as lacking motivation regarding mathematics (B) My child as not communicating mathematical knowledge and activity	**(I) My child as reflecting his/her gender**

Other comparisons of children's mathematical behaviour or competencies to 'typical' children, drawing on social and cultural influences, occurred in the sample. For instance, again on the subject of communication, Jennifer compared her son's lack of communication not with his gender but with all 'typical' children. This enabled her to support a position of 'My child as not communicating mathematical knowledge and activity'.

Jennifer (parent) - Interview		
Dialogue	Other positioning	Social positioning
(I) He's a bit erm (pause) you know typical child, (A) **you ask him, "What have you done at school today?" And we get, "Can't remember".**	(A) My child as not communicating mathematical knowledge and activity	**(I) My child as typical**

Some sociocultural voices are highly valued by parents and play a visible role in social positioning and identity construction of children. In particular *National Curriculum* standards and levels of achievement, which parents often profess not to understand, are used to position children as 'good', 'competent' or 'struggling' at mathematics. Beth talked about her son's progress in mathematics. She appeared satisfied and not worried about his progress because he was "on his levels", a reference to a discussion with her son's teacher connected to age-related expectations in the form of *National Curriculum* levels.

Beth (parent) - Interview		
Dialogue	Other positioning	Social positioning
Err I mean we had parents' evenings, we had one, when was the last one, March I think not so long ago, and obviously (A) **(I) he's on his levels so we're not too concerned about him so yeah.**	(A) My child as a competent user of mathematics	**(I) My child as compared to national standards**

5.5.2 Specific voices

When studying social positioning of children three specific voices appear in parental accounts, namely siblings, teachers and peers. The sibling is a clear physical entity with a distinct voice. The teacher or teachers represent concrete beings but their voices often merge or are inferred. Finally children's peers make up a larger group of potential known and unknown individuals whose voices and identities are often perceived and imagined. Table 5J below shows social positions connected to these three specific voices.

Table 5J Social positioning and specific voices

Social positioning	**Number of respondents co-constructing position**
Siblings	
My child as different to his/her sibling	10
My child as similar his/her sibling	1
Teachers	
My child as progressing well in mathematics as defined by teachers	9
My child as not progressing well in mathematics as defined by teachers	3
My child as progressing appropriately in mathematics as defined by teachers	3
My child as having potential as defined by teachers	1
Children's peers	
My child as good at mathematics as compared to his/her peers	8
My child as comparing himself/herself to his/her peers	3
My child as judging his/her performance via feedback from his/her peers	1

Parents often identified and positioned their children not by how similar they were to each other but by how different they were. In terms of mathematics comparisons of behaviours, competencies and emotional responses are visible. An example of this is shown below when Julia positioned her son Declan, in terms of competency, between his siblings Ursula and Paul. This social positioning is a multi-way relationship between numerous other positions. Each child is positioned in relation to each other, but also with regard to social and cultural influences and expectations. This wider social positioning is evident in "you know like dyslexia but with math" and "absolutely brilliant at it, like top group at maths", where socially supposed voices or labels are used to position Ursula and Paul.

Julia (parent) - Interview		
Dialogue	Other positioning	Social positioning
No because **(I) they're all totally different. Like Ursula [her Y9 daughter] is really... she finds maths really difficult and they think she's got this problem where... you know like dyslexia but with maths, they think she's got that. Then Paul's like absolutely brilliant at it, like top group at maths or whatever, and then** <u>(A)</u> **<u>Declan he's ok but you can tell he gets fed up easily.</u>**	<u>(A) My child as lacking motivation regarding mathematics</u>	**(I) My child as different to his/her sibling**

The voices of teachers also play a role in how respondents position their children in terms of mathematical competencies. In the data are instances of parents positioning children with regard to their progress, inferring acceptance of teacher judgement or specified levels of attainment, for instance the *National Curriculum* levels discussed above. Teachers' voices are visible as parents position children as progressing appropriately, not progressing well, or progressing well in mathematics. The next excerpt shows a sample of this. Niamh referred to a conversation with her son's teacher, producing the social position 'My child as progressing well in mathematics as defined by teachers', which in turn influenced a position of 'My child as a competent user of mathematics'.

Niamh (parent) - Interview		
Dialogue	Other positioning	Social positioning
(A) So as far as I know I think he's ok with it. **(I) And the last parents' evening I was told that they'd done assessments and he was slightly above average. So as far as I'm concerned, you know, as far as I know he's alright.**	(A) My child as a competent user of mathematics	**(I) My child as progressing well in mathematics as defined by teachers**

The last set of specific voices in the interview data arose from children's peers. This shows parents positioning children based on comparisons to their children's peers, and based on the children's own comparisons to their peers. Here again processes of reflexivity guide positioning. The example below contains the social position 'My child as good at mathematics as compared to his/her peers'. David recounted a conversation with his daughter's teacher. He used the teacher's comments about his daughter's performance in a test to label her as "more than alright at maths", producing a position of 'My child as good at mathematics'.

David (parent) - Interview		
Dialogue	Other positioning	Social positioning
And in fact the teacher was very proud because she said, 'Actually she's doing really brilliantly because tomorrow I'm going to tell her that she's beaten Jonah'. **(I) He's the brainy kid of the class and she'd beaten him by half a point or something in the maths test. And the teacher was almost as proud of it as we were, to be quite serious. She said, 'Oh don't tell her tonight because I want to be able to tell her tomorrow. I want to see the look on her face when she sees her mark and realises that she's...'** (A) **At that point you think, well she's more than alright at maths.**	(A) My child as good at mathematics	**(I) My child as good at mathematics as compared to his/her peers**

A variety of social positions, used by parents to position children mathematically, are evident in parental narratives. These are generated by both general and specific voices that are heard by parents and incorporated in their own social positioning of their children through a mechanism of reflexivity.

5.6 Multiplicity in the mathematical ‘self’: The polyphony of the mathematical ‘I’

The third level of analysis in this chapter focuses on characteristics of individual dialogical identities. This section is addressed towards the mathematical ‘self’, whilst the next is concerned with positioning the ‘other’.

Multiplicity is a fundamental principle of dialogical self theory. In order to study multiplicity and its features, the mathematical I-positions and social positions discussed previously were compared and contrasted. Taking this analytical approach resulted in findings related to multiplicity, conflict, stability and instability.

5.6.1 Multiplicity

As suggested previously, a wide-range of mathematical I-positions and social positions exist within the data set. When looking in depth at these voices within individual parent-cases a multiplicity of positions is apparent.

Table 5K shows the number of different mathematically-related positions concerning the self that were co-constructed by each parent. It is clear that even regarding a subject as specific as mathematics, individuals hold a number of different positions. Between six and twenty-six positions were used to position the mathematical self in this sample. The mean number of positions per participant was 15 and the median 14.5.

Table 5K Number of different positions held by parent participants

Participant	Number of different mathematical I-positions	Number of different social positions	Total number of positions used during interview
Abigail	16	8	24
Beth	16	5	21
Carl	15	1	16
Charlotte	12	2	14
Chris	16	2	18
David	14	6	20
Deborah	16	2	18
Gary	22	4	26
Gemma	14	2	16
Ian	12	2	14
Imogen	13	4	17
Jayne	11	7	18
Jennifer	11	1	12
Julia	8	0	8
Lindsay	8	4	12
Natalie	5	1	6
Neil	13	7	8
Niamh	10	4	14
Peter	12	1	13
Rebecca	10	5	15
Robert	13	5	18
Ruth	5	2	7
Suzy	10	2	12
Vicky	8	4	12

It is possible to illustrate multiplicity of positioning by looking at two 'average' parent-cases. Both Charlotte and Rebecca sit in the middle of the sample with regard to the total number of positions held, fourteen and fifteen respectively. Table 5L shows all the mathematically-related I-positions and social positions held by both parents. It presents twenty-seven different positions, of which only two, 'I as supporting my child's mathematical development' and 'I as playing a proactive role in my child's mathematical development', are held by both participants. Even then it should be remembered that these were highly-popular positions held by twenty-four and twenty-one parents respectively.

Table 5L Individual mathematically-related positions held by Charlotte and Rebecca

Mathematically-related positions	**Charlotte**	**Rebecca**
Mathematical I-positions		
I as involved in my child's mathematical development	✓	
I as playing a proactive role in my child's mathematical development	✓	✓
I as replicating my own upbringing	✓	
I as supporting my child's mathematical development	✓	✓
I as a tutor regarding mathematical activity	✓	
I as a competent user of mathematics	✓	
I as confused by mathematics		✓
I as good at mathematics	✓	
I as improving mathematically since I left school		✓
I as not good at mathematics		✓
I as a novice and learning mathematically from my child		✓
I as understanding mathematics	✓	
I as apprehensive of mathematics		✓
I as confident with mathematics	✓	
I as enjoying mathematics	✓	
I as feeling supported by my parents	✓	
I as mathematically aspirational	✓	
I as not enjoying mathematics		✓
I as not interested in mathematics		✓
I as worried by mathematics		✓
Social positions		
I as more successful at mathematics than others		✓
I as more supportive than other parents		✓
I as less able to support my child than my partner		✓
I as less mathematically able than my partner		✓
I as similar to my parent	✓	
I as similar to my child		✓
I as mathematically successful as determined by my teacher	✓	

It is evident that whilst Charlotte and Rebecca hold a similar number of positions they hold very different positions. This multiplicity and diversity is a characteristic of both this sample of parents and the highly-individual nature of the dialogical self. By looking more closely still at a single case, that of Rebecca, as she had the mean number of mathematically-related positions, a complex mathematical 'I', build-up from a plethora of voices, is evidently visible.

Multiplicity and diversity: the mathematical self of Rebecca

Rebecca was a mother with two daughters Zoë and Carly, who were both in Year 4. Generally, she lacked confidence in school mathematics and, in the following example, presented several negative mathematical I-positions.

Rebecca (parent) - Interview	
Dialogue	Mathematical I-position
(1) It makes me think urgh (laughs). (2) I just was never any good at maths at school… I knew I wasn't… And I knew I couldn't do it I didn't... want to do it. (3) So it didn't interest me. (4) I sort of got better at it, I suppose maybe, once I started at court because I had to do a lot of... workings out with legal aid and things like that. So I probably got better out of school (laughs) with figures than when I was there. (5) But you say maths and I just... it just takes me back to the day when I couldn't do.	(1) I as not enjoying mathematics (2) I as not good at mathematics (3) I as not interested in mathematics (4) I as improving mathematically since I left school (5) I as not good at mathematics

Against this negative view of the mathematical self she wanted to be supportive of her children's education, though she did not participate greatly in mathematics in the home, often leaving homework support to her partner, who she judged to be more mathematically competent.

Rebecca (parent) - Interview		
Dialogue	Mathematical I-position	Social position
(1) Just a feeling of maths that I just get now yeah, and I struggle to think yeah it's going to get worse for the kids and can we... are we going to be able to help them because... (2) it's something that I'm not good at. *(i) My other half's not... he's a bit better, he takes it all in*, he was the one that was writing it all down and stuff [Rebecca is referring to a list of the methods their children have been using at school. *(ii) So if they do come home with a bit of maths, its like 'go to your dad' (laughs).* (3) You know but no I just can't get my head round...	(1) I as worried by mathematics (2) I as not good at mathematics (3) I as confused by mathematics	*(i) I as less mathematically able than my partner* *(ii) I as less able to support my child than my partner*

When describing the level of support she offered her daughters she drew upon a 'voice' from the sociocultural sphere involving social expectations of parental support. In the following example, when discussing mathematics workshops run at her daughters' school, she compared herself to other parents who she perceived as less supportive.

Rebecca (parent) - Interview		
Dialogue	Mathematical I-position	Social position
(i) There weren't many parents there to be honest. I thought there might have been quite a lot at these... <u>(1) Because I do think that you need to try to help them if you can.</u>	<u>(1) I as supporting my child's mathematical development</u>	*(i) I as more supportive than other parents*

This polyphony of voices acts to construct a mathematical self for Rebecca. This is a unique mathematical self that is used to make sense of past experiences and present activity.

By studying multiplicity of positioning in individual cases, then comparing those patterns across all the respondents, three dialogical phenomena come to prominence. It is possible to witness conflict occurring as a result of the interaction of opposing positions, and also to see examples of stability and instability in the mathematical self.

5.6.2 Conflict

Conflict occurring from interaction between positions was found within eighteen of the twenty-four parent interviews. Commonly this interaction was associated with an emotional response. The two points of dialogical conflict evident in the data are shown in Table 5M.

Table 5M Conflict between mathematical I-positions

Interacting mathematical I-positions	**Number of respondents exhibiting conflict**
I as supporting my child's mathematical development + I as confused by mathematics	16
I as supporting my child's mathematical development + I as a novice and learning mathematically from my child	2

All the parents in the sample constructed positions that saw themselves as supportive of their children's mathematical development. By far the most common conflict between positions occurred when 'I as supporting my child's mathematical development' met 'I as confused by mathematics'. Here emotion and conflict was clearly evident in the narratives constructed by parents. These two positions interacted, but in a manner where a resolution between them was difficult to achieve. Many parents clearly did not want to relinquish or weaken their desire to be supportive or to see themselves as unsupportive. Therefore this supportive position was inflexible when it came up against a view of the self as unable to understand a mathematical task or tasks. This inability to support and feel that one could support generated rich remembered experiences. This interpretation is reinforced in the two examples below. Imogen discussed the problems she faced helping her son with his mathematics homework. Here conflict between the positions led to frustration.

Imogen (parent) - Interview	
Dialogue	Mathematical I-position
(1) It's, it's the working out and how they come to get it worked out that it the problem. Because I think well it's alright me doing things and then it being the totally wrong way round of doing it then it's confusing him even more, you know, which is not a good thing. (2) We're trying to support him not...	(1) I as confused by mathematics (2) I as supporting my child's mathematical development

In Jennifer's account it was possible to see the stress and upset caused as two positions interact. This was further reinforced by her desire for her son to enjoy mathematics.

Jennifer (parent) - Interview	
Dialogue	Mathematical I-position
I guess, well I guess that you feel a little bit stressed (1) because you can't help them and because they're upset, you know. (2) Erm (pause) that's why we chose to do it how he was confident doing it rather than trying to push him into doing a way that he didn't understand because it would have, it would have made it so awful for him to not understand what he was doing. And because he is confident in maths you don't want to knock that off as well and make it into something uncomfortable.	(1) I as confused by mathematics (2) I as supporting my child's mathematical development

In a few cases it was possible to see when conflict between positions led to a new position or an alteration in behaviour or action. Natalie discussed an instance of supporting her daughter Daisy. After assuming a position of ‘I as confused by mathematics’ Natalie appeared to suggest that a subtle change occurred in her supportive I-positioning. Possibly because of the feeling that Daisy was mathematically competent (“And she had to explain the way”) Natalie repositioned herself (“Just do it and if you need me I’ll help”).

Natalie (parent) - Interview	
Dialogue	Mathematical I-position
I can’t really remember what it was (pause) I can’t remember but <u>(1) I remember saying, “You do it so different and I don’t know how you are doing it”.</u> And she had to explain the way and I went, “Well I don’t think that’s as easy as how I did it”. But obviously she found it easier because that was the way she was taught. <u>(2) So I just said, “Right just get on with it”. And I think that was when it was, “Just do it and if you need me I’ll help”.</u>	<u>(1) I as confused by mathematics</u> <u>(2) I as supporting my child’s mathematical development</u>

5.6.3 Stability

Generally there were elements of mild contradiction and variation in most of the parental interviews. Mathematical positioning of the self was not wholly consistent or stable. As would be expected from an understanding of dialogical self theory, changes in time and space produced visible changes in positioning. However, what was also clear from analysing the data was that some parents have a more stable mathematical ‘I’ than others. In these respondents there appears limited change in their mathematical self either spatially or chronologically.

Stability: the mathematical self of Gary

An example of a stable mathematical self is evident in the positions co-constructed by Gary in his interview with the researcher. Gary was in his forties and had a son Shaun, in Year 6, and a daughter Lily, in Year 1. In the interview Gary produced more mathematical I-positions than any other parent in the sample. Nevertheless, this diversity and polyphony did not lead to instability and broad changes in positioning across time and space. The positions Gary assumed, shown in Table 5N, back a view of Gary as supportive of his children’s learning, but also holding an identity which is negatively disposed towards mathematics.

Through positions such as 'I as confused by mathematics', 'I as not good at mathematics' and 'I as nervous of mathematics', Gary produced a mathematical self which showed consistency across time and space.

Table 5N Gary's mathematically-related positions

Mathematically-related positions
<u>Mathematical I-positions</u>
I as challenging my child mathematically
I as co-operative during mathematical interaction
I as encouraging my child's mathematical development
I as flexible during mathematical interaction
I as playing a proactive role in my child's mathematical development
I as supporting a work ethic
I as supporting my child's mathematical development
I as a competent user of mathematics
I as confused by mathematics
I as held back by my lack of knowledge
I as improving mathematically since I left school
I as not good at mathematics
I as feeling unsupported by my parents
I as frustrated by mathematics
I as negative towards mathematics
I as nervous of mathematics
I as not enjoying mathematics
I as not interested in mathematics
I as panicked by mathematics
I as pressured by mathematics
I as regretful of mathematical activity
I as scared of mathematics
<u>Social positions</u>
I as more supportive than other parents
I as similar to my child
I as similar to my partner
I as supportive as determined by child's teacher

In the extract shown below a number of these positions are presented by Gary. He associated a range of negative experiences and emotions with mathematics, primarily connected to his schooling. These appeared to persist through time suggesting stability in Gary's positioning, for instance in the quotes "the natural association and that goes right back to the experiences I had with it at school" and "that is something that will make me panic to this day".

Gary (parent) - Interview	
Dialogue	Mathematical I-position
Researcher: What do you associate with the word mathematics? What does it make you think? What does it make you feel?	
Gary: (1) Panic	(1) I as panicked by mathematics
Researcher: What makes you say panic and what makes you think of that emotional response?	
Gary: Because that's the one that I have always associated with it from... that's the natural association and that goes right back to the experiences I had with it at school. (2) I'm more uncomfortable with it, I mean I work in accounts incredibly, I left school without any qualifications what so ever yet I do work in accounts. And I am fine, I am, I'm not great at mental arithmetic at all (3) but that is something that will make me panic to this day. (4) If you sit me down and ask me a simple subtraction or an addition I'll, because I'm under the spotlight and I'm being expected to come up with a result like that I will probably just block. I can get round that by sort of, calming, calming myself down and what have you but... (5) I suppose that the panic is, like I said earlier on, going back to my classroom experiences being... (6) That I just completely didn't understand a lot of it and was (7) nervous, too nervous to stick my hand up and say I don't understand. So it is probably rooted back there.	(2) I as not enjoying mathematics (3) I as panicked by mathematics (4) I as pressured by mathematics (5) I as panicked by mathematics (6) I as not good at mathematics (7) I as nervous of mathematics

There was evidence of shifts in Gary's positioning over time, as in the example above when he states "And I am fine, I am, I'm not great at mental arithmetic at all". However Gary's dialogical positioning was generally very stable. For example, even though Gary worked in a highly numerate discipline, as an accountant, his perceptions of his own ability and experiences of past mathematical activity influenced his current mathematical positioning. He compared himself to his son, suggesting that they both panic around mathematics, repeating Gary's earlier comments about his schooling.

Gary (parent) - Interview		
Dialogue	Mathematical I-position	Social position
And I think it is the, *(i) Shaun is like me and (ii) Paula in that* (1) *he will easily panic with maths* *and the confidence can easily be tipped.*	(1) I as panicked by mathematics	*(i) I as similar to my child* *(ii) I as similar to my partner*

Overall it is possible to see chronological and contextual stability in Gary's positioning by studying his remembered school experiences, his present working life and descriptions of his current mathematical interactions at home.

5.6.4 Instability

Whilst most parents in the sample showed variation in their mathematical self, some parents displayed a far greater degree of instability. In this sense the mathematical self was very clearly seen to shift in response to changes in time and sociocultural context.

Instability: the mathematical self of Ian

A prime example of this is the case of Ian. He was the father of two daughters, Megan, a child in Year 4, and Louisa, who attended Reception. Ian's mathematically-related positions are shown in Table 5O.

Table 5O Ian's mathematically-related positions

Mathematically-related positions
Mathematical I-positions
I as replicating my own upbringing
I as supporting a work ethic
I as a competent user of mathematics
I as confused by mathematics
I as good at mathematics
I as not good at mathematics
I as a novice and learning mathematically from my child
I as apprehensive of mathematics
I as enjoying mathematics
I as feeling supported by my parents
I as not interested in mathematics
I as regretful of mathematical activity
Social positions
I as similar to my child
I as reflecting the influences of my social environment

When studying these positions it is evident that many appear contradictory. For instance when describing his mathematical competencies, Ian positioned himself as ‘I as a competent user of mathematics’, ‘I as confused by mathematics’, ‘I as good at mathematics’ and ‘I as not good at mathematics’. As we see in the following excerpts, Ian’s mathematical position changed depending on context and across time. In the first coded example Ian was talking about helping his daughter Megan with her homework. In this context he positioned himself as a competent user of mathematics, able to support Megan, even though he acknowledged that he “wasn’t brilliant at it at school”. In the second coded sentence, again when talking about helping with homework, Ian saw himself as mathematically competent and did not attach any negative emotions to this type of mathematical activity.

Ian (parent) - Interview	
Dialogue	Mathematical I-position
<u>(1) Something, it keeps, I wasn’t brilliant at it at school but I’ve got a pretty good grasp of anything like that.</u> <u>(2) On the whole it’s not that bad.</u>	<u>(1) I as a competent user of mathematics</u> <u>(2) I as a competent user of mathematics</u>

When asked directly what he thought of mathematics Ian evoked his schooling. He saw mathematics as a school activity, an activity he believed he was not good at. In this position, and in this time and context, Ian saw mathematics with apprehension and dread.

Ian (parent) - Interview	
Dialogue	Mathematical I-position
Researcher: What do you associate with the word mathematics? Ian: Probably Numbers, that’s probably the first thing that comes into your brain, numbers. <u>(1) I don’t know because I was rubbish at maths at school (laugh).</u> <u>(2) You probably think, “Oh god not maths again”. Numbers, dread, homework really.</u>	<u>(1) I as not good at mathematics</u> <u>(2) I as apprehensive of mathematics</u>

In a different context, this time a more informal mathematics situation involving playing darts with his father, Ian positioned himself as 'I as good at mathematics'. This stems from an ability to perform mental multiplication quickly and accurately. In his working life, in the retail sector, Ian saw himself as able to perform mental calculations involving money quickly to a high level. A change in context and image of mathematics brought a re-positioning towards mathematics.

Ian (parent) – Interview	
Dialogue	Mathematical I-position
I'm quite quick with things like that and she's asked me before, "How do you know that so quick?" And I always say the same thing to her, because growing up when I was 9, 10, 11, 12 me and my dad used to play darts all the time. <u>(3) And I learnt all my numbers from playing darts, growing up with my dad. So anything that's multiplied into 16, treble this, treble that, double that, I got it straight away.</u>	<u>(3) I as good at mathematics</u>

In the next example we can see a change in positioning over time. Ian positioned himself first as enjoying mathematics as a teenager at secondary school. This is opposite to the position he reconstructed of his primary-level education. He likened his attitude during primary school to that of his daughter's current feelings and approach.

Ian (parent) - Interview		
Dialogue	Mathematical I-position	Social position
Err yeah, well (pause) <u>(1) probably more from secondary school really because I probably started getting it at secondary school</u> whereas sort of *(i) when I was sort of Megan's age I was probably exactly like she is.* <u>*(2) As can't be bothered; don't matter.*</u>	<u>(1) I as enjoying mathematics</u> <u>(2) I as not interested in mathematics</u>	*(i) I as similar to my child*

When comparing himself to his child again Ian appeared to re-position himself, now seeing himself as either good at school mathematics "I'd be good at doing it" or competent "I'd just wing it and get through it".

Ian (parent) - Interview		
Dialogue	Mathematical I-position	Social position
I wouldn't say that it's her strongest subject really at school. *(i) She, she's very, very like I used to be at school I think.* <u>(1) *If I could be bothered to do it I'd do it and I'd be good at doing it.* (2) *If I couldn't be bothered to do it, I'd just wing it and get through it and be like that.*</u>	<u>(1) I as good at mathematics</u> <u>(2) I as a competent user of mathematics</u>	*(i) I as similar to my child*

In a case such as that of Ian it is possible to see chronological instability over time as remembered events and experiences lead to the creation of different mathematical I-positions. Similarly there is spatial instability as different activities result in different I-positions.

The analysis of dialogical positions is complex exactly because of the multiplicity of voices and inherent consistency and inconsistency in accounts. There is a degree of difficulty in separating temporal and spatial factors and interpreting the strength of positioning. However in this section it has been possible to see several elements that make up the multiplicity of the mathematical 'I', elements that clearly support the tenets of dialogical self theory.

5.7 The multiple positions of the other: Children's perceived mathematical identities

Just as it is possible to see multiplicity in parents' own mathematical positioning it is similarly evident that the same polyphony exists in parents positioning of their children. This section follows the same analytical approach applied to the study of multiplicity in the mathematical self, but now focusing on positioning the other rather than positioning the self. In line with dialogical self theory it would be expected that time and space influence positioning of the other. Following the premise that the other is dialogically positioned spatially and temporally, 'other' positions and social positioning of the other were studied in individual respondents and patterns compared across the sample as a whole.

As expected multiple voices could be heard in the mathematical identities that parents formed for their children. A discussion of this is presented below. Further generalisations and commonalities with regard to positioning the other were more difficult to find. For instance no appreciable evidence was found to support conflict arising from interaction between 'other' positions. Stability and instability did seem to be present but with much less substantiation than in the preceding section. A reason for this 'fuzzy' picture is the much lower number of positioning the other (34) compared to mathematical I-positions (52). Similarly social positions constructed concerning the self (34) were much more numerous than social positions formed in relation to children (13).

5.7.1 Multiplicity

As shown previously in sections 5.3 and 5.5, respondents co-constructed a number of dialogical positions when describing positioning their children.

Table 5P Number of different positions of the other used by parent participants

Participant	Number of different 'other' positions	Number of different social positions of the other	Total number of positions of the other used during interview
Abigail	10	2	12
Beth	10	6	16
Carl	6	0	6
Charlotte	8	1	9
Chris	9	3	12
David	6	4	10
Deborah	5	2	7
Gary	12	3	15
Gemma	6	3	9
Ian	6	2	8
Imogen	9	4	13
Jayne	9	3	12
Jennifer	7	1	8
Julia	2	2	4
Lindsay	3	2	5
Natalie	7	1	8
Neil	7	2	9
Niamh	5	4	9
Peter	7	2	9
Rebecca	3	2	5
Robert	8	3	11
Ruth	7	2	9
Suzy	9	1	10
Vicky	7	2	9

The number of different mathematically-related 'other' and social positions held by each parent is shown in Table 5P. The number of positions ranged from four to sixteen. The mean number of positions was nine and the median value for the sample was also nine.

By looking at two typical parents Ruth and Vicky, both of whom co-constructed nine positions, it is possible to see multiplicity and variety in positioning. Table 5Q shows the different positions created by the two parents and their two overlapping codes: 'My child as enjoying mathematics' and 'My child as not communicating mathematical knowledge and activity'. Both parents use an array of positions to describe their children mathematically. They constructed very different identities for their children, replicating the individualised character of the dialogical self. The dialogical other is seen to mimic the dialogical self in its multiplicity of positioning.

Table 5Q Individual voices used by Ruth and Vicky to position their children

Positioning children	**Ruth**	**Vicky**
Other positioning		
My child as competent at mathematics	✓	
My child as good at mathematics	✓	
My child as not needing my help during mathematical activity	✓	
My child as struggling with mathematics		✓
My child as communicating mathematical knowledge and activity	✓	
My child as not always listening during mathematical activity		✓
My child as behaving flexibly during mathematical activity	✓	
My child as lacking motivation regarding mathematics		✓
My child as not communicating mathematical knowledge and activity	✓	✓
My child as enjoying mathematics	✓	✓
My child as frustrated by mathematics		✓
My child as lacking self confidence in mathematics		✓
Social positioning		
My child as not progressing well in mathematics as defined by teachers		✓
My child as different to his/her sibling		✓
My child as good at mathematics as compared to his/her peers	✓	
My child as progressing well in mathematics as defined by teachers	✓	

Magnifying our attention to focus upon one of these parents, Vicky, allows a more in-depth examination of the multiplicity of voices that are utilised to create a mathematical 'other'.

Multiplicity and diversity: the mathematical other as formed by Vicky

Vicky had three children: Stephen (Year 7), Sam (Year 3) and Jenny (Year 2). She worked in finance and held strong, confident I-positions towards mathematics and the importance of supporting her children's mathematical development. When defining a mathematical identity for Sam she labelled him as someone who struggles with mathematics. This is apparent in the following narrative example.

Vicky (parent) - Interview	
Dialogue	Other positioning
I can't remember what task it was exactly but we were sat trying to do his homework in here, and I was asking him to read the question and <u>(A) he'd read it out but then he'd just shout numbers out for the answer because it was obvious he didn't know what he was doing, and he'd just be shouting random numbers in the hope that he got the right one.</u> And I 'no, no no no no' [as in calm down] so <u>(B) he says 'I can't do it. I'm rubbish, I'm rubbish. I can't do it.' and storms off.</u> So I went 'come on, come back and let's get it finished'. <u>(C) 'No no no, I can't do it, I can't do it. I'm rubbish'.</u> I went 'no come on'. I got him back down, broke it down and showed him what he needed to do and then he did it.	<u>(A) My child as struggling with mathematics</u> <u>(B) My child as lacking self confidence in mathematics</u> <u>(C) My child as lacking self confidence in mathematics</u>

Vicky used information from teachers at school to support her positioning of Sam. In the next example she drew on a social positioning of 'My child as not progressing well in mathematics as defined by teachers' to reflect and support a position of 'My child as struggling with mathematics'.

Vicky (parent) - Interview		
Dialogue	Other positioning	Social positioning
Researcher: Do they discuss maths much at parents' evenings? Vicky: It's only sort of discussed like with Sam that (A)**(I) he is behind and he needs to do more.** That sort of thing. But it's just mentioned in general yeah they're good at this, and that sort of things.	(A) My child as struggling with mathematics	**(I) My child as not progressing well in mathematics as defined by teachers**

She constructed distinct mathematical identities for her children. In Sam's case it was an identity constructed through comparison to his siblings and social positioning.

Vicky (parent) - Interview		
Dialogue	Other positioning	Social positioning
(I) Because Stephen was always really good, and Jenny is really good, (A) **and Sam's not bless him** **(II) Our Stephen enjoys it, he likes it.** (B) **Sam likes if he knows what he's doing,** (C) **but if he gets a little bit stuck he doesn't like it and doesn't want to do it. He only wants to do the easy stuff, whereas Jenny will sit down and will try anything and she will run it by you first 'Is that right?' 'Do you think that's right?' first, whereas** (D) **with Sam if he can't do it straight away then that's it. He gets the face on [upset] and wanders off so yeah.**	(A) My child as struggling with mathematics (B) My child as enjoying mathematics (C) My child as lacking motivation regarding mathematics (D) My child as lacking motivation regarding mathematics	**(I) My child as different to his/her sibling** **(II) My child as different to his/her sibling**

In the interview Vicky used seven 'other' positions and two social positions of the other to construct a perceived mathematical identity for Sam. This is comparable to other parents in the sample where all but one used social positioning to describe their child, and all used multiple positions. In this sense it is apparent that parents construct distinct mathematical identities for their children to make sense of their experiences of activity at home and interpretation of information gained from others.

5.7.2 Stability

The majority of parents produced stable identities for their children that did not show a great deal of change over time or context. In a sense this is to be expected given a number of factors. Firstly, the lower number of positions used to describe the other. A reduced number of positions led to less visible variation. Secondly, the shorter amounts of chronological time children have had to exhibit change. Parents have thirty or forty years of lived experience to call upon when positioning the self chronologically. A parent can only call upon a maximum of ten or eleven years of experience when positioning their chid. Thirdly, parents presented a limited number of different contexts under with their children completed mathematical activity. Typically experiences were restricted to homework activities and parents' understandings of children's school activity, as communicated by teachers. In a few cases there were instances of positioning through non-school mathematical activity.

An example of a stable case of positioning the other is the identity constructed by Gemma for her daughter Kitty.

Stability: the mathematical other as formed by Gemma

Gemma was a mathematically confident parent who formed a very supportive identity. She worked as a primary school teacher and thus had understanding of school-related mathematics. Gemma had three daughters. Her middle daughter Kitty attended Year 3 at a local primary school. As is apparent from the positions listed below in Table 5R; Gemma positioned Kitty as both able and confident with regard to mathematics.

Table 5R Gemma's positioning of Kitty

Positioning the other
Other positioning My child as communicating mathematical knowledge and activity My child as motivated regarding mathematics My child as a competent user of mathematics My child as good at mathematics My child as confident with mathematics My child as enjoying mathematics Social positioning My child as comparing himself/herself to his/her peers My child as good at mathematics as compared to his/her peers My child as judging his/her performance via feedback from his/her peers

The first extract shows three examples of Kitty being positioned as mathematically confident. They also show Gemma's perception that Kitty felt able and confident in mathematics because she compared herself to her peers, a common occurrence in children in the sample.

Gemma (parent) - Interview		
Dialogue	Other positioning	Social positioning
(A) She's very positive about maths (B) ...but the point is that she is confident to try to have a go because that first question [in the parent-child mathematical task] she said, "Oh I think I'll try and do it like this". But, well to hear Kitty talk about maths (C) she feels confident about maths **(I) because, she would say, she's on quite a high table.** But in fairness from me viewing Kitty's work, (D) I feel that Kitty's confident at maths because even if she can't get the right answer (E) she can always think of a method that she can use confidently.	(A) My child as confident with mathematics (B) My child as confident with mathematics (C) My child as confident with mathematics (D) My child as confident with mathematics (E) My child as a competent user of mathematics	**(I) My child as comparing himself/herself to his/her peers**

Gemma positioned her daughter as a competent user of mathematics who was able to work through any problems she encountered. Kitty was also seen as able to communicate mathematical knowledge and activity proficiently.

Gemma (parent) - Interview	
Dialogue	Other positioning
Often with her homework she will come home and I'll, I'll always say, you know, "Can you remember your homework?", rather than her just read it out. (A) (B) I'll say, "What have they said you've got to do?" And very often when it's the maths she'll say "I know exactly what we've got to do because we've been doing..." (C) So I clearly know that she's understood what they've been doing for two or three days because she even gets the homework book out in black and white she can tell me what she's got to do.	(A) My child as communicating mathematical knowledge and activity (B) My child as a competent user of mathematics (C) My child as a competent user of mathematics

In the final example Gemma positioned her daughter as communicative and suggested that Kitty judged her performance positively based on interaction with her peers. Again in the context of school mathematics Kitty is seen as adept.

Gemma (parent) - Interview		
Dialogue	Other positioning	Social positioning
(A) Kitty obviously came home yesterday and she said, **(I) "Somebody had said to me that I'd been really clever using that column mum, that we sat together".**	(A) My child as communicating mathematical knowledge and activity	**(I) My child as judging his/her performance via feedback from his/her peers**

In Gemma's case study limited positioning appeared linked to temporal experiences. Spatial positioning appeared coherent and highly stable.

5.7.3 Instability

There was limited evidence of instability in the brief mathematical identities created for children. Cases of instability were seen in only four parent interviews. The clearest example was shown in the case of Neil's positioning of his son Daniel.

Instability: the mathematical other as formed by Neil

Neil was a mathematically confident parent who constructed a supportive and proactive role for himself in terms of Daniel's education. Table 5S shows a number of conflicting voices that Neil drew upon to position Daniel mathematically. For instance opposing positions such as 'My child as enjoying mathematics' and 'My child as not enjoying mathematics', and 'My child as good at mathematics' and 'My child as confused by mathematics' were utilised at different points of the interview.

Table 5S Neil's positioning of Daniel

Positioning the other
Other positioning My child as lacking motivation regarding mathematics My child as not communicating mathematical knowledge and activity My child as stubborn during mathematical activity My child as confused by mathematics My child as good at mathematics My child as enjoying mathematics My child as not enjoying mathematics Social positioning My child as compared to national standards in mathematics My child as not progressing well in mathematics as defined by teachers

Evidence of chronological shifts in positioning was shown when Neil discussed Daniel's progress in school mathematics over the previous year. In the example below Neil's positioning of Daniel changed from 'My child as good at mathematics' to 'My child as struggling with mathematics' as a result of a conversation held with his son's teacher concerning national age-related expectations.

Neil (parent) - Interview		
Dialogue	Other positioning	Social positioning
(I) At the beginning of the year he was erm one point off a Grade 4 which is where he needs to be at Year 6, and he's only in Year 4. (A) **So he was quite advanced for his age. He dropped back to a three point... a 3a I think it is which is right at the beginning of 3.** (B) **(II) And so he, he obviously was struggling erm and not showing any kind of progression or stable work.**	(A) My child as good at mathematics (B) My child as struggling with mathematics	**(I) My child as compared to national standards** **(II) My child as not progressing well in mathematics as defined by teachers**

Contextual changes also appear to be taken into account when constructing an identity for Daniel. In the next example Neil positioned Daniel's mathematical self as highly context dependent. Here the type of activity, and Daniel's interest in it, helped to define whether he was seen as 'My child as good at mathematics' or 'My child as confused by mathematics'.

Neil (parent) - Interview	
Dialogue	Other positioning
So it's just a case of (A) he doesn't like it because (B) he doesn't understand it and he doesn't want to feel stupid. But (C) when he actually understands it he loves it and he does it all the time. (D) He excels in it once he understands it but then obviously (E) he just refuses to do it if he doesn't.	(A) My child as not enjoying mathematics (B) My child as confused by mathematics (C) My child as enjoying mathematics (D) My child as good at mathematics (E) My child as lacking motivation regarding mathematics

The final excerpt again shows the role of context in positioning the other. Neil discussed a game he played with his son. The game necessitated regular addition and subtraction but in an informal non-school manner. The context differs, game at home as opposed to mathematics at school, and Neil's positioning of Daniel also changed.

Neil (parent) - Interview	
Dialogue	Other positioning
(A) He literally started excelling straightaway. (B) It's just reminded me that so I think he loves it when he's got his head into it, (C) but getting him away from away from a DS or something is your biggest challenge.	(A) My child as good at mathematics (B) My child as enjoying mathematics (C) My child as lacking motivation regarding mathematics

Whilst the analysis with regard to positioning the other is not as clear as the evidence stemming from positioning the self, it is possible to draw broad conclusion on multiplicity, stability and instability that support dialogical self theory. Within the expositions in this section there is support for the dialogical construction of identity, polyphony and chronological and spatial influences in positioning.

5.8 Discussion

Using three separate analytically levels based upon the tenets of dialogical self theory, this chapter set out to investigate mathematical identity. Its findings throw light onto processes of identity formation in terms of how parents dialogically construct a mathematical 'self', as well as how they produce a mathematical 'other' for their children.

5.8.1 RQ4: How do parents dialogically construct identities for themselves and for their children?

The analysis shows that the mathematical 'self' is characterised by a number of features and component positions. A vast array of internal positions makes up the mathematical self. These mathematical I-positions can be classified as being related to dialogue concerning behaviours, competencies and emotions connected to past mathematical experiences and present mathematical activity.

This process of positioning the self reflects the findings of others in the area of mathematical identity (e.g. Abreu & Cline, 2003; Esmonde et al., 2011). In the work of Mullen and Abreu (2011), mathematical identity was seen to comprise of: perceptions of mathematical ability, memories of learning mathematics, work-based experiences and valorisations of mathematics. Certainly the first three of these elements are seen in the mathematical I-positions formed by parents. Perceptions of mathematical ability connect to competency-related positions. Memories of learning mathematics and work-based experiences are present in behavioural, competency and emotionally-related positions.

The three classifications of positions (behavioural, competency and emotional) can also be seen in the manner in which parents position their children mathematically, although a smaller variety of these 'other' positions are used across the sample. The process of parents positioning children mathematically was also highlighted by Abreu (2002).

External or social positions can be seen in parents' conceptualisations of the 'self' and 'other'. Here influences from the broader sociocultural environment are visible. Elements akin to social positioning have previously been seen in parents, teachers and children (e.g. Abreu & Cline, 2003; Crafter & Abreu, 2010; Gorgorió

& Prat, 2011) but not from a dialogical perspective. Using a dialogical approach it was possible to detect specific and general 'voices' in the social positions. These voices form social positions and influence internal positions, as suggested by Raggatt (2011). Such social positions in the 'self' and 'other' have been seen before in other arenas but not in terms of mathematical identity (e.g. Akkerman & Meijer, 2010; Aveling & Gillespie, 2008).

The social positions observed in parents are clearly reflexive and mediated by information and activity obtained within the sociocultural sphere. The actual or imagined voice of a parent or teacher influences how parents position themselves and their children. This appears to occur in a manner consistent with Raggatt's (2011) definitions of social and reflexive positioning. It is logical to assume that processes of valorisation are embedded within the mediation of reflexivity. Indeed the voices present in the narratives tend to be 'significant' in the sense that they originate from teachers, friends or loved ones, people from whom a view or opinion would be valued.

As would be expected from a dialogical perspective (Herman, 2001), analysis showed that a multiplicity of positions was used to form the mathematical 'self' and 'other'. Indeed on average 15 positions were used in constructing a mathematical self. Given that this represents a specific topic (mathematics), and responses to limited number of questions, it hints at the abundance of I-positions at the disposal of the individual.

These positions are defined spatially and chronologically. The narratives here show this dynamic nature of positioning. This connects to the notion of Martin (2007) that mathematical identity is related to mathematical context. A mathematical 'self' in one context may not be replicated in a different space or time.

Dynamic positioning is shown clearly in the constancy of 'self' and 'other' positions within parental narratives. Here it is clear that individual dialogically-constructed mathematical identities exhibit degrees of stability and instability that can be traced to chronological and spatial factors. As asserted by Hermans et al. (1992) we see I-positions in agreement and disagreement with each other. It is also possible to see a wide variety of positions across the sample, suggesting highly unique mathematical identities.

As shown in the narratives of Gary, Ian, Gemma and Neil, identity fluctuation is related to the sociocultural environment, particularly changes in time and space. Parents' differ in the consistency of their positioning of the 'self' and the 'other'. Children may be labelled with different mathematical identities during their lives. These may shift at home and at school. Similarly a parent may position themselves differently as they recall their own schooling, home- and work-life. A 'successful' mathematical identity at work may not be replicated when a parent reminisces their own schooling. The level of stability appeared to reflect the strength of feelings, those strongly positive or negative about mathematics, due to mostly to such experiences, had more consistency that those who reported a mixture of positive or negative experiences and a range of valorisations.

In summary, this analysis shows the mathematical 'self' and 'other' as fluid conceptualisations that shift and change in relation to participation in sociocultural practices across time and space. As such it supports and adds further validations to work on dialogical self theory (Hermans, 1996; Hermans & Gieser, 2011; Hermans & Hermans-Konopka, 2010; Hermans & Kempen, 1995; Hermans et al., 1992). Its crucial contribution is in the area of mathematical identity, previously unaddressed using an approach based upon dialogical self theory. The results in this chapter not only present a method for studying mathematical identity but also illustrate its topography. These features are shown to be indelibly linked to social and cultural experiences.

The debate now moves forward with a discussion of parent-child mathematical activity, in chapter 6, before bringing together identity and goals in order to test the relationship between these elements.

Chapter 6

Goals in parent-child mathematical activity

6.1 Introduction

In this chapter the focus shifts from individual experiences and mathematical identity to analysing joint activity. Twenty-four parent-child dyads took part in a twenty-minute simulated-school mathematical task that involved attempting to solve ten word problems centred on concepts in subtraction. The aim of the task was to investigate the following research questions:

RQ6: How do parents and children form, negotiate and operate mathematical goals?

RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?

RQ8: To what extent do parents 'scaffold' learning and conceptual development in mathematics?

Saxe (1991) showed how a child's numerical understandings were related to their mathematical goals. Furthermore, he revealed how children's goals in microgenetic activity can be seen to develop as a response to the (a) structure of the activity, (b) the artefacts and conventions surrounding and inherent to the activity, (c) social interaction during the activity, and (d) the prior understandings participants bring to an activity. This, as discussed previously, is termed the four parameter model of emergent goal formation. As an approach it has been used to study mathematical goals in a variety of contexts (e.g. Guberman et al., 1998; Nasir, 2000a, 2000b, 2002; Saxe, 1991, 1992, 1995, 2002; Saxe & Guberman, 1998) but not parent-child school mathematical interaction.

Therefore, following Saxe's (1991) model, this chapter uses data from the twenty-four parent-child dyads to study each of these four parameters in turn in order to investigate how parents and children form, negotiate and operate mathematical goals. Here a goal is defined as a conscious action carried out as the result of a mathematically-related motive or intention.

A number of previous studies have also shown learning and development is influenced by processes of contingency and scaffolding.

Contingency (Wood & Middleton, 1975; Wood et al., 1978) refers to parents adapting their level of support based upon children's performance in a task. It has been shown to occur in parent-child mathematical activity (Pratt et al., 1992; Saxe et al., 1987). Contingent approaches enacted by parents lead to more effective performance by children (Wood et al., 1978). However, it is unclear how contingency is executed in parent-child school mathematical interaction at home and its relationship to goals. Because of this data from the task participants is studied for instances of contingency.

Scaffolding (Wood et al., 1976) is a process with parallels to guided participation in the zone of proximal development. In scaffolding the 'expert' enables the 'novice' to complete an activity that would be otherwise impossible for them to accomplish. Scaffolding includes a number of different components. As a concept it has been shown to occur in parent-child mathematical interaction (Hyde et al., 2006; Lindberg et al., 2008; Neitzel & Stright, 2003; Mattanah et al., 2005; Pratt et al., 1992). For that reason scaffolding is also studied in this chapter, complementing the analyses of emergent goals and contingency.

The chapter concludes by discussing the relevance of the findings with regard to the research questions listed above and reaffirming how microgenetic goal analysis provides a mechanism to understand learning and development through participation in cultural activity.

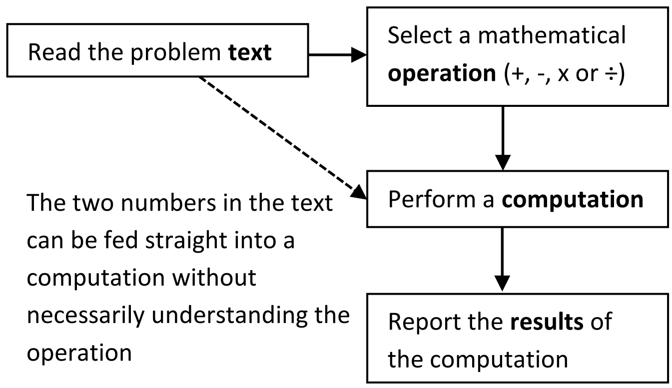
6.2 Activity Structures

The first element in the emergent goal framework is that of activity structures. Here the analysis focuses upon how the fundamental structure of the cultural practice itself influences goal construction. In the case of the parent-child mathematical task, the activity presents the participants with a series of mathematical word problems, such as the one shown below.

Katie is 135cm tall. Josh is 109cm tall. How much shorter is Josh than Katie?

The aim of the participants is to try to solve the problem. In this sense the structure of the activity necessitates that certain goals are applied to the situation. As shown earlier, in 3.5.2, research involving word problems has suggested a common approach can be observed in such activity. Therefore the problem solving model shown in Figure 6A was taken as a starting point for the analysis of activity structure. This supposed that the activity of solving word problems would necessitate goal formation around four separate elements: text, operation, computation and results.

Figure 6A Common model of problem solving



Note: Adapted from Greer, 1997, p.295, Fig 1

From this perspective the first step in order to solve a problem is to read the question. This is followed by a goal around the identification of the operation, or alternatively feeding the numbers from the word problem straight into a calculation. The third and fourth stages result in goals emerging in order to perform a computation and then report the results of the computation.

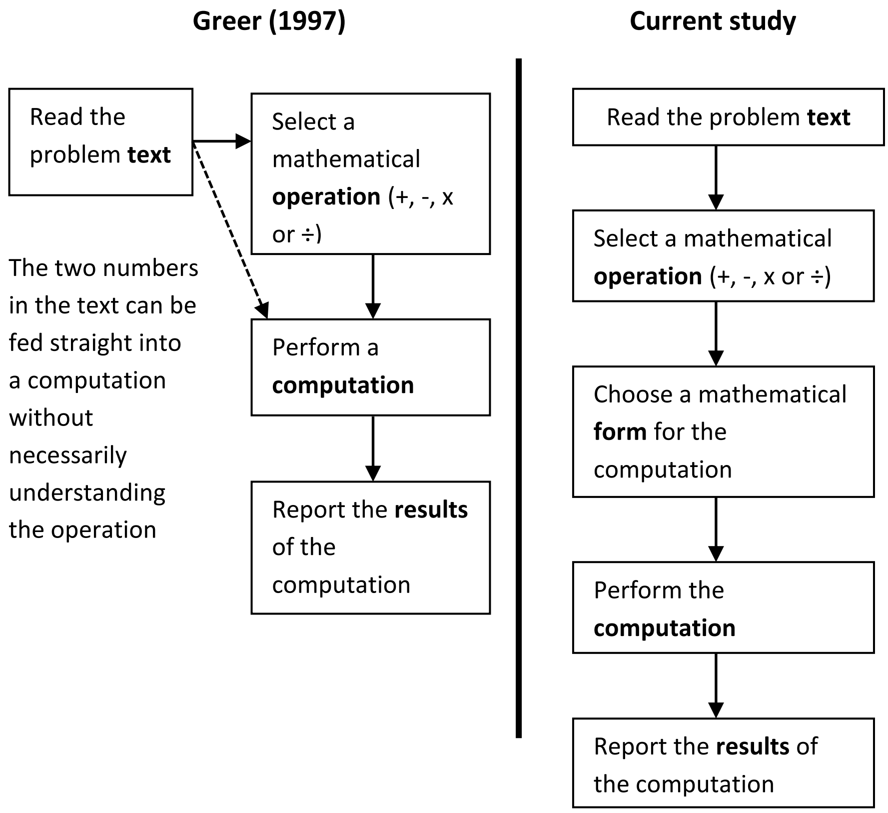
6.2.1 Analysis of activity structure goals

To ascertain whether the parent-child dyads replicated such a typically school-based-research model of goal behaviour, data from the tasks was studied and goals pertaining to the activity structure were initially coded for the four stages shown in Figure 6A.

Analysis showed that parents and children did indeed construct goals in line with a problem-solving approach similar to the one above but in a slightly different manner. Figure 6B compares Greer's (1997) approach with the model that emerged from the analysis of activity structure goals. This differs from the prior model in the inclusion of a separate stage in the problem solving process in which goals emerge directed towards selecting and defining mathematical forms through which a computation can be carried out. Mathematical forms are cultural-related constructions, for example a long division algorithm, that are related to participation in certain social and cultural contexts. In school mathematics they often occur as mental and written strategies for addition, subtraction, multiplication and division.

Figure 6B also differs in the fact that it removes the option to proceed from **text** straight to **computation** as the choice of a form and subsequent computation relies upon the selection of an operation, even if that operation has been chosen at random.

Figure 6B Comparison of models of problem solving



Activity structure goals could be seen in the dialogue contained within the parent-child transcripts or be logically inferred by the activity of the participants in the task. From studying the role of activity structure on goal formation a number of patterns were evident. On many occasions it was possible to see the originator of the goal, how the goal developed through social interaction (see 6.4) and who completed the goal. Progression through each of the five stages in the current model in Figure 6B could cause difficultly and involve potential misconceptions.

For instance, a question may be misunderstood, an incorrect operation might be applied, a child may not be able to conceptualise a suitable form or use it to perform a computation, or an answer could be reported inaccurately or incompletely.

Table 6A Activity structure-linked goals

Parent	**Child**	**Number of activity structure-linked goals**	**Number of activity structure-linked goals formed by parent**	**Number of activity structure-linked goals formed by child**	**Number of activity structure-linked goals completed by parent**	**Number of activity structure-linked goals completed by child**
Abigail	Zach	40	2	38	1	39
Beth	Scott	50	6	44	0	50
Carl	Karen	50	6	44	0	50
Charlotte	Callum	50	7	43	0	50
Chris	Lizzie	50	0	50	0	50
David	Grace	30	1	29	0	30
Deborah	Caitlin	40	9	31	5	35
Gary	Shaun	20	0	20	0	20
Gemma	Kitty	35	5	30	0	35
Ian	Megan	30	17	13	13	17
Imogen	Owen	20	13	7	4	16
Jayne	Oliver	50	1	49	0	50
Jennifer	Jacob	39	1	38	1	38
Julia	Declan	30	19	11	16	14
Lindsay	Ben	35	27	8	18	17
Natalie	Daisy	50	1	49	1	49
Neil	Daniel	40	22	18	11	29
Niamh	Connor	50	7	43	2	48
Peter	Jessica	50	0	50	1	49
Rebecca	Zoë	30	6	24	8	22
Robert	Alex	50	12	38	3	47
Ruth	Michael	50	2	48	0	50
Suzy	Matthew	40	4	36	0	40
Vicky	Sam	40	9	31	9	31
Total		969	177	792	93	876
Percentage			18%	82%	10%	90%

In total the twenty-four dyads attempted 194 word problems. All but one of these problems was completed following the five stages, shown in Figure 6B. 969 activity structure-linked goals were observed or inferred from the transcripts. The origin and completion of each of these goals was assigned to either parent or child. The breakdown of participants and activity structure-linked goals is shown in Table 6A. This shows that the vast majority of activity structure-linked goals were initiated and completed by children, suggesting an awareness and familiarity with the school mathematical-style activity. Through their engagement in prior instances of this sociocultural activity most children knew the stages they need to complete in order to solve the problems and set their goals accordingly. This validates the simulated task as representative of the cultural practice of school-related mathematical activity.

In many cases children accomplished these goals following support from their parent. In a minority of cases the parent completed the goal for the child, either inadvertently or because it appeared the child could not fulfil the goal. Table 6A shows that individual interactions contained contrasting amounts of parent and child completed goals, linked to the how difficult the child found the word problems. In ten dyads children completed all of the goals, sometimes with support. In two dyads children completed less than half the goals, again even including support.

Looking in greater depth at activity structure and goals, further analysis showed that 124 goals (13% of all goals) appeared to require dialogue or support between parent and child. However this support occurred in 92 out of 194 word problems (47%). This suggests support was directed towards individual goals in each problem rather than all the goals. Indeed, parental intervention was predominantly focused around performing a computation (68 out of 124 goals) or identifying a required operation (26 out of 124 goals). Table 6A shows that parents completed 93 of the 969 activity structure-related goals. Most of these were identifying the operation for children, who could then go on to choose a mathematical form or strategy and perform a calculation. Parents completed just 3% of computations.

From this it appears that the practice of school-style mathematical activity in the home is based around promoting independence in children, supporting them to understand the question and ascertain the operation required, and helping them to complete a calculation. In this practice it is acceptable for a parent to inform a child of an operation or ‘tell them what to do’ but it is not seen as appropriate to do it for them. In these ways it appears that this practice mirrors activity structures that might be expected to occur in schools.

In order to support these assertions three contrasting examples are now presented that illustrate the ways in which activity structure influences goal construction. The first example shows the predominant pattern in the data, children setting and achieving activity structure-linked goals. The second example demonstrates the kind of goal support that occurred between children and parents, which was a feature of nearly half of all the word problems completed. The final example demonstrates a rarer occurrence of a parent predominantly setting and completing goals.

6.2.2 A child's activity structure goals

The simplest examples of the activity structure shaping goals occurred when the child was able to answer a word problem without any support from their parent. In the following excerpt Sam quickly and confidently solves a problem. The goals are numbered in the column on the right-hand side of the extract and analysed below. The numbers refer to the five stage-related activity structure-linked goals, namely text (1), operation (2), form (3), computation (4) and results (5).

Vicky (parent) and Sam (child – Y3) *There are 24 biscuits in a packet. Josh ate 5 biscuits. How many were left?*				
Line	Time	Speaker	Dialogue	Goal
1	0.01	Vicky	Right, are you gonna read it?	1
2	0.04	Sam	There were 24 biscuits in a packet. Josh ate 5 biscuits. How many are... were... left? [*He counts back on his fingers one at a time whispering each number*] 24, 23, 22, 21, 20, 19.	1, 2, 3, 4, 5
3	0.32	Vicky	Mm	
4	0.33	Sam	Which I'll write it...	5
5	0.34	Vicky	Just write it in there *Vicky indicates a space in the box underneath the question where Sam writes '19'*	5

1. Text: In line 1 Vicky promotes a goal around reading the problem text. She prompts Sam to read the question. He complies and uses his reading to move onto the next step.

2. Operation: In line 2 Sam takes a cue from the text of the problem and decides upon a subtraction operation.

3. Form: Sam appears to choose a mental counting strategy to carry out the subtraction of 5 from 24.

4. Computation: To realise this he uses the counting back form to answer the subtractive function. In this case Sam counts back in 1s from 24 to 19 using his fingers to support the calculation.

5. Results: The final goal of reporting the result of the computation is shown in line 2 when Sam says the answer aloud and in lines 4 and 5 when Vicky urges Sam to provide a written answer.

6.2.3 Supporting activity structure goals

When dialogue occurred around goals it was frequently used to support children's understanding of a question, the mathematical operation, or to enable them to carry out a calculation. Whilst examples of support exist which run to many pages most supporting actions covered less than ten lines of dialogue. The following example shows Niamh supporting Connor's operation and computation goals.

Niamh (parent) and Connor (child - Y4) Katie has 159 marbles. If Josh buys 36 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?				
Line	Time	Speaker	Dialogue	Goal
			Connor reads the question then seeks clarification	1
34	5.00	Connor	Is that 36 take away a hundred and, no	2
35	5.04	Niamh	Nearly it's not thirty six take away is it? It's a hundred and fifty nine...	2
36	5.10	Connor	Oh	
37	5.12	Niamh	It is take away but you got the numbers the wrong way round	2
38	5.15	Connor	A hundred and fifty nine take away thirty six	2
39	5.18	Niamh	That's good	
			Connor writes out a column subtraction for 159-36.	3
40	5.33	Connor	Do I cross that one out or do I do it underneath?	4
41	5.37	Niamh	Well you do it how you normally do it at school yeah	4
42	5.39	Connor	I can't remember now the difference between add. Do you cross it out or put the one underneath?	4
43	5.43	Niamh	Cross it out if you, if you need to borrow to take that away. That should be an easy one shouldn't it you don't need to borrow on that one	4
44	5.50	Connor	I'm thinking about it that's why. So that's eight, seven, six, five, four, three. (Pause 4s) Two. One. One-two-three	4, 5

45	6.06	Niamh	That's good *Connor writes the answer '123' on the sheet.*	5

1. Text: Connor completes the first activity-related goal and reads the problem text.

2. Operation: As is shown in line 34 Connor is unable to complete the next stage in the model because he is unsure about the operation and subsequent calculation. Lines 34-39 show the dialogue that occurs as the Niamh attempts to support Connor's understanding of the operation. In this case Niamh confirms it is a take away and the order of the subtraction. This allows Connor, in line 38, to construct the correct number sentence.

3. Form: After constructing the number sentence 159-36 Connor decides to use a written column subtraction which he writes out on the word problem sheet.

4. Computation: Connor is unable to complete the computation and seeks clarification from his mother. In lines 40-43 we see Niamh clarify the procedures for the calculation, which Connor completes in line 44 using mental arithmetic, some of which appears to be counting down in 1s.

5. Results: The results of the computation and solution to the problem are reported by Connor as he completes the calculation in line 44 and writes the answer '123' on the word problem sheet.

6.2.4 A parent's activity structure goals

As seen earlier in Table 6A, in the majority of interactions activity structure-related goals were formed and completed by children. In many cases children were able to complete goals as a result of support from their parents. In a minority of instances parents formed and completed activity structure goals. These tended to occur when children could not complete goals, again typically operation- and computation-based, or, more rarely, when parents assumed control of goal setting and achievement and children played a more passive role. As shown in Table 6A, Ian, Julia, Lindsay and Neil were all examples of parents setting numerous activity structure goals.

The excerpt below shows Ian and Megan working together. From the beginning of the task Ian regularly set goals and supported Megan in achieving them. The example shown is a problem that followed a number of questions that Megan struggled to answer. By this point in the interaction, 17 minutes, Ian is not only setting and supporting goals but also completing them for Megan.

1. Text: Megan reads the question aloud completing the first step of problem solving.

2. Operation: In line 201 Ian recaps the main elements of the problem and then identifies the operation as an addition.

3. Form: In lines 205-207 Ian outlines the mathematical form, a counting strategy that will be used to solve the problem.

4. Computation: The majority of the excerpt, lines 209-229, concerns the computational goal of counting in tens (lines 209-218) and units (lines 219-229) to reach the answer of forty-one. Here Ian controls the calculation and works through each step with Megan.

5. Results: Ian gives the answer to the computation in line 229, which Megan then repeats.

Ian (parent) and Megan (child - Y4) Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have?				
Line	Time	Speaker	Dialogue	Goal
200	17.04	Megan	Josh and Katie have one hundred and thirteen books when they put all their books together. Josh has seventy two books. How many books does Katie have?	1
201	17.22	Ian	Right, so Josh and Katie have got a hundred and thirteen books when they put all their books together. Right so it’s like you’ve got so many and I’ve got so many and when we add them together there’s a hundred and thirteen...	2
202	17.35	Megan	Mm	2
203	17.36	Ian	...you’ve got seventy two	
204	17.38	Megan	Mm	
205	17.39	Ian	So how many do I put to it to get to a hundred and thirteen? So the best way is to count up in tens from seventy two...	2, 3
206	14.45	Megan	Mm	
207	17.46	Ian	...until you get to a hundred and thirteen	3
208	17.47	Megan	Mm so	
209	17.49	Ian	Seventy two	4

210	17.50	Megan	Eighty	4
211	17.51	Ian	Two	4
212	17.53	Megan	Eighty two, ninety two, a hundred and two and	4
213	18.02	Ian	Another ten	4
214	18.10	Megan	A hundred and twelve	4
215	18.11	Ian	Right so that's near	4
216	18.14	Megan	That's right next to it	4
217	18.15	Ian	So a hundred and twelve right so that means you've got four lots of ten, so how many is there?	4
218	18.21	Megan	Forty	4
219	18.22	Ian	Ok so how many more do you need to add on to a hundred and twelve to get to a hundred and thirteen?	4
220	18.29	Megan	I don't know	4
221	18.31	Ian	Well you've got your forty haven't you?	4
222	18.32	Megan	Yeah	4
223	18.33	Ian	So you've got a hundred and twelve and you need a hundred and thirteen	4
224	18.37	Megan	Add one	4
225	18.38	Ian	Yeah. So you've got your ten, twenty, thirty, forty...	4
226	18.44	Megan	And one	4
227	18.45	Ian	Yeah	4
228	18.46	Megan	Yeah so	4
229	18.47	Ian	Seventy two add forty one makes that number. So how many books does Katie have? Forty one	4, 5
230	18.55	Megan	Forty one	5

In summary, this section shows how the structure of the mathematical activity undertaken by parents and children influenced some of the goals that were constructed in that activity. It shows how most of the goals were 'owned' by children. It also shows the supporting role that parents played in assisting their children in completing goals. The overwhelming pattern in the data is of parents as mathematical facilitators. This is perhaps unsurprising given the findings in previous chapters concerning parents' experiences of parent-child interaction and their supportive mathematical I-positions.

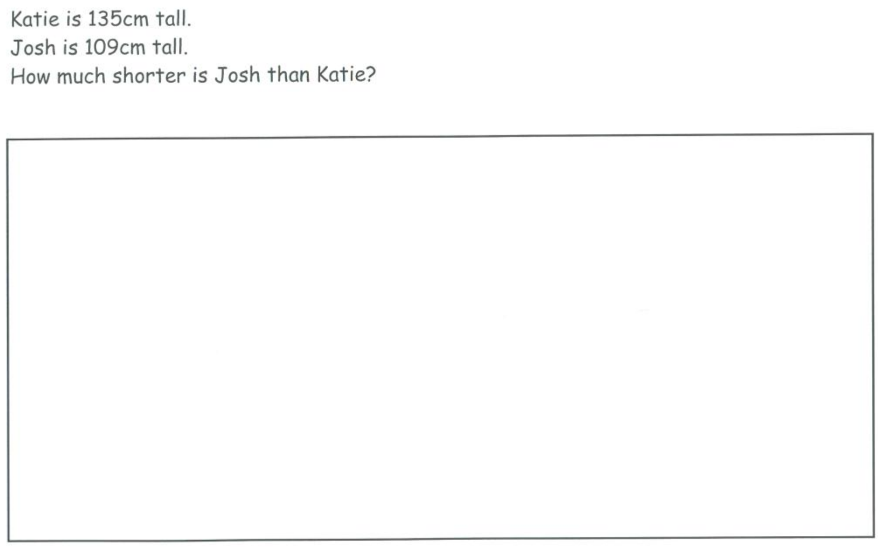
6.3 Artefacts and conventions

The next element in the emergent goal framework concerns artefacts and conventions. In this section of analysis the spotlight turns to the manner in which the artefacts that form part of the task shape activity in the task, and how the conventions inherent within the simulated school mathematical task influence individual and co-operative activity.

6.3.1 Artefacts

The task presents the dyad with a simulated school mathematical activity. The key artefact within this task is the word problem sheet. At the beginning of the activity the parent-child dyads are presented with a number of subtraction word problems of differing types and levels of difficulty. Each problem is presented on a separate sheet of paper.

Figure 6C Example word problem sheet


Katie is 135cm tall.
Josh is 109cm tall.
How much shorter is Josh than Katie?

The problem sheet contains the question and a large blank space underneath, as shown in Figure 6C. The participants are asked to treat the task like any homework session. They are not told that they have to use the space or provide any written work or answers.

The problem sheet artefact clearly influences goal construction in the dyadic interaction. All the participants interacted with the artefact in order to accomplish goals around reading the word problem. Parents often constructed goals of providing written answers to computations that they would pursue in dialogue with children.

This is shown below in the short interaction between Lindsay and Ben where Lindsay prompts Ben, in line 128, to write ‘35’ on the problem sheet.

<table>
<tr><td colspan="4">Lindsay (parent) and Ben (child - Y3)
Josh buys 12 Pokémon cards. He now has 47 Pokémon cards. How many Pokémon cards did he have in the beginning?</td></tr>
<tr><td>Line</td><td>Time</td><td>Speaker</td><td>Dialogue</td></tr>
<tr><td>125</td><td>17.47</td><td>Ben</td><td>Thirty five</td></tr>
<tr><td>126</td><td>17.49</td><td>Lindsay</td><td>So he had...</td></tr>
<tr><td>127</td><td>17.51</td><td>Ben</td><td>Thirty five</td></tr>
<tr><td>128</td><td>17.52</td><td>Lindsay</td><td>Thirty five before he wrote... brought twelve, that’s right. So write thirty five there please
Lindsay points to a place next to the calculation. Ben writes ‘35’.</td></tr>
</table>

The problem sheet was not the only artefact used in by parents and children in the task. Other pieces of paper were occasionally used for written arithmetic and a few parents used calculators to check their children’s answers, in some case leading to an incorrect answer being identified.

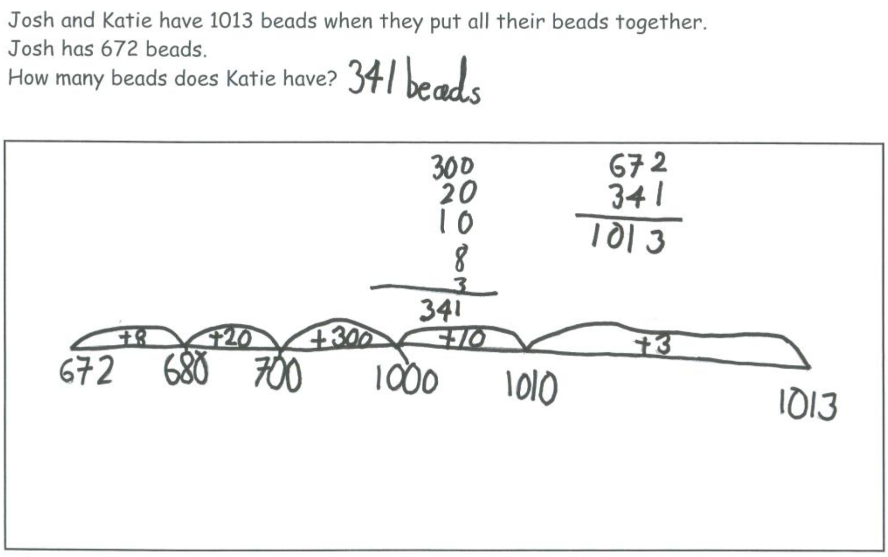
Mathematical forms

The artefact was commonly used to construct mathematical forms in order pursue calculation and computations goals. Here the artefact was elemental in helping participants to achieve mathematical goals. All twenty-four dyads constructed written mathematical forms on the artefact at some point in the parent-child task. Across the entire sample, out of 194 completed word problems, 158 (81%) incorporated a written mathematical form on the artefact.

A diversity of mathematical strategies utilised the artefact. The choice of these appeared to be defined more by personal preference of children or adults than the semantics of the word problem. The choice and use of these forms often influenced dialogue and goals within the dyadic interaction.

The space available on the artefact, the vast majority of the A4 sheet, for written mathematical forms allowed parents and children to discuss and use multiple forms to solve the same calculation. It also gave enough space for parents to check children’s computations using their own selected forms. Figure 6D shows an example of a problem sheet that was used to solve the calculation 1013-672. It includes a number line used to solve the problem and a column addition used to check the answer.

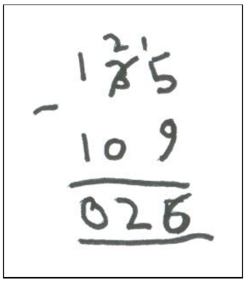
Figure 6D Word problem sheet used to complete 1013-672 (Matthew)



The artefact appeared to encourage parents to set goals around their children producing written forms. In 17 (71%) of the interactions parents prompted children to create written forms. This gave children a mechanism to answer the problem but it also gave parents an insight into children's thought-processes and understanding. It enabled parents to sometimes, but not always, spot errors in children's reasoning and better support them to get the correct answer. An interesting example of some of these points is the discussion between Beth and Scott shown next. Scott begins by appearing to choose a mental form to calculate the difference between 135 and 109. Beth instructs him, in line 5, to produce a written form. Scott complies and writes a column subtraction but reaches an incorrect answer. Beth sees this and, in line 11, attempts to clarify the conventions of the activity with the researcher. This suggests that she is unsure of the perceived expectations of the task and is behaving differently because of the researcher's presence. Eventually, in line 15, she draws Scott's attention to the error. He recognises his mistake and corrects his answer. The written mathematical form he constructed is shown in Figure 6E.

Beth (parent) and Scott (child - Y4) *Katie is 135cm tall. Josh is 109cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
4	1.55	Scott	It's the answer...
5	1.57	Beth	No don't do it in your head show your workings to there [*paper*]. That's why he's got that square there.
6	2.03	Scott	Ok *Scott writes out a column subtraction for 135-109.*
7	2.15	Scott	I think this is right anyway. Is it?
8	2.17	Beth	Yeah, that's good. Now remember what we've said mm. What do we do with your little numbers?
9	2.26	Scott	Mm fifteen... *Scott takes 10 from 30 and adds it to the 5 (in 13**5**). He works through the sum and gets the answer 25*
10	2.42	Scott	*[incorrect].*
11	2.46	Beth	Wait That's (pause). [*To researcher*] Would you prefer me to tell him if I think I've [*he's*] got it wrong or would you like him to just go on?
12	2.52	Researcher	No if you think, no if that's what you would normally do if you were working through the homework and you saw something that...
13	2.58	Beth	[*To Scott*] That's not right, five, fifteen take nine?
14	3.03	Scott	Is it six?

Figure 6E Scott's column subtraction to answer 135-109



Other ways in which the artefact influenced written forms were in terms of goals around neatness and clarity of writing, underlining key words and information in the problem text, and choices of form.

6.3.2 Conventions

The conventions that are seen to influence goal formation can be divided into general mathematical principles and the rules of the activity. Each of these were analysed in the task transcripts and are presented next.

Mathematical conventions and principles

The data from the tasks contains a huge variety of mathematical conventions and principles used by both children and parents.

Goals were constructed around mathematical operations of addition, subtraction, multiplication and division. There was also evidence of goals being formed around the following mathematical principles:

- Algebraic reasoning
- Counting and cardinality
- Decomposition (Place Value - associated with column algorithm)
- Carrying (Place Value - associated with column algorithm)
- Checking
- Estimation
- Inverse rule
- Number representations
- Ordering/sequencing numbers (prior to addition or subtraction)
- Partitioning
- Place value
- Rounding (to nearest ten and hundred)

A variety of mental calculation strategies were apparent in parent and child mathematical activity. Where children spoke as they conducted a computation or later explained their reasoning it was possible to identify the mental arithmetical form that had used.   The following is a list of the different mental calculation strategies that children operated in the tasks. The classification of forms is taken from Beishuizen (1993). Each is followed by an example to show how it differs:

- Counting on (in 1s) (e.g. 44-27= counting on in ones from 27 to 44 =17)
- Counting on (in 10s) (e.g. 44-27= counting on in tens 27 to 37 then on in ones from 37 to 44)
- Counting back (in 1s) (e.g. 44-27= counting back in ones from 44 to 27 =17)
- A10 (complementary addition e.g. 44-27 = 27+**3**=30, 30+**10**=40, 40+**4**=44, 3+10+4=17)
- 1010 (partitioning tens and units e.g. 44-27= 40-20=20, 4-7=-3, 20-3=17)

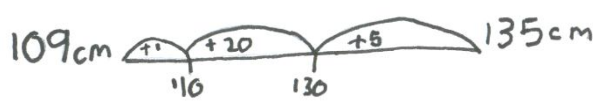
- u1010 (partitioning tens and units - units first e.g. 44-27= 4-7=-3, 40-20=20, 20-3=17)
- N10 (sequencing tens and units e.g. 44-27= 44-20=24, 24-7=17)

As mentioned earlier the word problem sheet was used to support a wide range of written mathematical goals. These included:

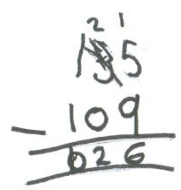
- N10 (sequencing tens and units e.g. 44-27= 44-20=24, 24-7=17)
- uN10 (sequencing tens and units - units first e.g. 44-27= 44-7=37, 37-20=17)
- 1010 (partitioning tens and units e.g. 44-27= 40-20=20, 4-7=-3, 20-3=17)
- Balancing (e.g. 44-27 = 47-30 (adding 3 to each side), 47-30=17)
- Column subtraction
- Column addition
- Number line (A10 complementary addition)
- Number line (counting on 1s)
- Counting on in 1s supported by tally marks/digits
- Expanded column subtraction

As mentioned previously, the use of a particular mathematical form did not seem to be related to the word problem but rather seemed to be a reflection of the child's, or sometimes the parent's preferred strategy. Generally when constructing written forms children in Year 3 preferred to follow counting goals, in Year 4 number lines and column subtractions were most popular, whilst in Years 5 and 6 column subtraction was the most common written form. For instance Figure 6F shows two mathematical forms that were used by different children to complete the same problem.

Figure 6F Example of different written forms used to solve 135-109



Lizzie (Year 4)



Kitty (Year 4)

The goals that children constructed in the interaction could also be seen as a reflection of their numerical understandings, as suggested by Saxe (1991). The choice of strategies, for instance simple counting back, to counting on, to an awareness of decomposition and complex written methods, are suggestive of a child's conceptual understanding of mathematics. Similarly correctly interpreting the intentions behind the semantic structure of the word problem, almost always completed successfully by older children, and the selection of operation and form gave insights into numerical understandings.

Conventions of the activity

The task matches the rules and conventions typically expected of school work that the parent and child would naturally complete at home. It also matches the activities children would complete at school. Indeed, twenty out of the twenty-four children used mathematical forms common in the classroom. For instance, out of the 158 written forms used in the task 51% (80) were column subtractions and 21% (33) were number lines. These are both conventional approaches children would be expected to use in schools.

The rules or conventions of school mathematics can be seen to shape parent-child goal construction. Parents often referred to school mathematics or homework, suggesting that this was the convention under which they were operating. This is shown in the example below as David questions Grace on the number line she has used to solve 54-19, in line 9, and on her knowledge of column subtraction, lines 13-17. David explicitly asks his daughter in line 19 what mathematics she uses at school.

David (parent) and Grace (child - Y4) *Katie had 54 stickers. She lost some of them. She now has 19 stickers. How many stickers did she lose?*			
Line	Time	Speaker	Dialogue
9	7.06	David	Yeah. Is that how you would normally do that darling?
10	7.07	Grace	Yeah
11	7.08	David	Is that because it's a big sum?
12	7.10	Grace	Yeah
13	7.12	David	Have you not learnt how to put one number under the other yet? Do you remember doing that? Do you remember how to do that?
14	7.18	Grace	Column addition?
15	7.21	David	Column subtraction actually
16	7.23	Grace	Not, not column subtraction
17	7.24	David	You don't know how to do that yet ok. Is that how you've been taught to do it?
18	7.27	Grace	No I do that in homework though, not at school
19	7.31	David	How have you been taught to do it at school?
20	7.34	Grace	Err
21	7.36	David	It is right [*reassuring Grace*]
22	7.37	Grace	We just get, we just do it on a different number line.
23	7.39	David	Oh you get a number line do you? You use a number line.

Another example of the activity mirroring school mathematics is shown in the interaction between Deborah and Caitlin. Here Caitlin is struggling to understand how to approach the first question. Deborah probes her understanding in lines 2 and 5 before, in line 9, likening the task to school mathematics.

Deborah (parent) and Caitlin (child - Y4) *Katie is 153cm tall. Josh is 118cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
1	0.26	Caitlin	Katie is...
2	0.30	Deborah	You know what to do there don't you? *Caitlin doesn't respond so Deborah refines her query*
3	0.41	Deborah	Would you write it down?
4	0.42	Caitlin	Mm
5	0.44	Deborah	Not the answer, I mean how you are going to work it out?
6	0.47	Caitlin	No
7	0.48	Deborah	Alright
8	0.49	Caitlin	Um I'm not sure
9	0.50	Deborah	What would you do if you got that at school? Say on those sheets that Mrs Hamilton gives you?

6.4 Social interaction

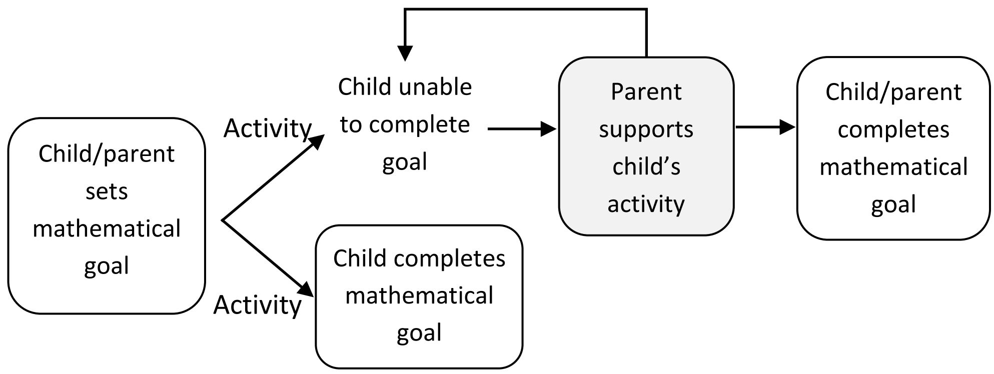
Language is not the only element of social interaction. The attitudes, expressions and emotions displayed and perceived in the task are all components of social interaction. However, it is impossible to study the role of all elements of social interaction in goal construction and learning. This analysis therefore focuses upon how goals emerge and evolve through dialogue between parents and children.

As suggested earlier, dialogue intended to support activity structure-linked goals was seen in almost half of all the word problems completed by the twenty-four dyads. When this social interaction was analysed in depth it was apparent that a number of definable dialogic phenomena existed which supported goal construction and completion by both parents and children. In every interaction mathematical goals were seen to emerge and shift in response to probes and prompts. These were generated mainly, but not exclusively, by parents rather than children.

6.4.1 Social interaction and goals: probes and prompts

When studying the formation and operation of mathematical goals in parent-child interaction a common process was observed, shown diagrammatically in Figure 6G. For the majority of goals discernible in the data the child would set the mathematical goal and complete it through some form of activity, such as reasoning about an operation or performing a calculation. In other cases the parent would set the goal, which the child would subsequently complete. These processes are shown by the lower path on Figure 6G and contained limited social interaction. Dialogue between parents and children, and the bulk of parental support, occurred where children were unable to complete a mathematical goal. As suggested in the process model, this social interaction would lead to the child being either able to complete the goal or require further support to complete the goal. In rare instances, as mentioned previously, the parent would complete the goal.

Figure 6G Process of goal-related activity visible in parent-child interactions



In order to better understand how parents and children form, negotiate and operate mathematical goals through social interaction these instances of parental support of goal-related activity were analysed. From this it was apparent that parents helped children to achieve mathematical goals through the use of two broad strategies: probes and prompts.

Probes

A probe is defined as an utterance that seeks to examine, test or explore the understanding of a goal and often occurs through questioning. In this manner parents check children's understanding and reasoning and decide whether any further support is necessary. All parents in the sample used probes to a greater or lesser extent depending on the activity of their child, the length of the task, how difficultly the child found the task, and their own interactional preferences. For instance, in the example below Charlotte probes Callum's understanding of a word problem. He does not answer her question directly but by constructing a column subtraction, in line 52, he displays sufficient understanding to satisfy his mother.

Charlotte (parent) and Callum (child – Y4) *Katie has 159 marbles. If Josh buys 36 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?*			
Line	Time	Speaker	Dialogue
50	4.33	Callum	Kate has one hundred and fifty nine marbles. If Josh buys thirty six marbles he will have the same number of marbles as Kate. How many marbles does Josh have? (Pause 2s) Piece of cake. *Callum reads the question and comments on its difficulty. Charlotte checks his understanding of the problem*
51	4.49	Charlotte	Do you understand it? *Callum doesn't respond directly to his mother's probe but begins to write out a column subtraction. He describes what he is doing as he writes*
52	4.54	Callum	A hundred and fifty nine and then you put thirty six, well the three goes under the five because it's not three hundred and six.
53	5.03	Charlotte	Ok

Where a parental probe elicited a misconception or allowed children to express a lack of understanding it was generally followed up by further parental support. In this next excerpt Charlotte's probe, in line 3, results in Callum correctly defining the operation, a subtraction, but getting the minuend and subtrahend the wrong way round. Charlotte responds in line 5 by accepting the operation but pointing out Callum's error in the order of subtraction.

Charlotte (parent) and Callum (child – Y4) *Katie is 135cm tall. Josh is 109cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
1	0.02	Charlotte	Are you ready? Have you read the question? (Pause 3s) Read it to me so I know
2	0.26	Callum	Kate is one hundred and thirty five centimetres tall. Josh is one hundred and nine centimetres tall. How much shorter is Josh than Kate?
3	0.38	Charlotte	Ok what are you going to do?
4	0.40	Callum	I'm going to take away thirty... One hundred and nine take away one hundred and thirty five and it'll leave me with the answer
5	0.51	Charlotte	Go on then. Other way round but yeah go on write it down

Prompts

Prompts are classified as dialogue that appears to urge, provoke or move another to goal-related activity. They occur principally through directions, instructions or leading questions. Prompts were widespread throughout the transcripts and used by every parent in the sample and often followed probes, as shown in the last example in line 5 where Charlotte prompts Callum to produce a written calculation. Returning to the process model shown in Figure 6G; prompts either led to successful completion of goals or, if the parent believed further support was necessary, subsequent prompts. In the next example a series of prompts are used to support Daniel in completing eleven minus seven as part of the column subtraction 113-72.

In line 203 Neil prompts his son to carry out the calculation. Daniel reaches an incorrect answer, which Neil points out. In line 206 Neil again returns to prompt Daniel to carry out 11-7. When this prompt results in further confusion Neil changes his approach and in line 209 prompts Daniel to complete 10-7, redefining the calculation as (10-7)+1. Daniel again reaches an incorrect answer and Neil simplifies his next prompt, which Daniel is able to follow and complete in line 212. This leads to another prompt from Neil to complete the calculation by adding on 1. Daniel does this and reaches the correct answer to 11-7 in line 214. He then writes '4' in his written column subtraction.

Neil (parent) and Daniel (child – Y4) *Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have?*			
Line	Time	Speaker	Dialogue
203	15.35	Neil	Now eleven minus seven
204	15.38	Daniel	Mm seven, seven
205	15.40	Neil	No that would be fourteen wouldn't it
206	15.42	Daniel	Oh yeah
207	15.44	Neil	Yeah so what's eleven minus seven?
208	16.01	Daniel	Erm I'm getting kind of muddled up here
209	16.03	Neil	Ok let's make it ten but remember we've taken one away
210	16.06	Daniel	So that's five
211	16.08	Neil	No ten minus seven
212	16.11	Daniel	Ten minus seven is (pause 2s) three
213	16.15	Neil	Now put that one back on to it
214	16.18	Daniel	Four

6.4.2 Social interaction and goals: parents' mathematical goals

As mentioned earlier the majority of the goals recorded in data set were constructed by and completed by children. On other occasions goals were constructed by parents through prompts. These goals were generally completed by the child. However on some occasions they were completed by the parent, abandoned by both parties or rejected by the child. Table 6B shows the number of goals generated and promoted by each parent.

Table 6B Parental goals generated through social interaction

Parent	Number of parent generated goals	Number of these goals completed by child	Number of these goals not completed	% successful	% un-successful
Abigail	29	28	1	97	3
Beth	14	14	0	100	0
Carl	21	21	0	100	0
Charlotte	26	25	1	96	4
Chris	3	3	0	100	0
David	9	8	1	89	11
Deborah	35	35	0	100	0
Gary	7	6	1	86	14
Gemma	42	42	0	100	0
Ian	36	31	5	86	14
Imogen	33	29	4	88	12
Jayne	9	8	1	89	11
Jennifer	2	2	0	100	0
Julia	21	16	5	76	24
Lindsay	39	26	13	67	33
Natalie	6	5	1	83	17
Neil	40	39	1	98	2
Niamh	13	12	1	92	8
Peter	3	2	1	67	33
Rebecca	28	26	2	93	7
Robert	26	23	3	88	12
Ruth	9	8	1	89	11
Suzy	28	26	2	93	7
Vicky	33	28	5	85	15

In interactions such as Chris and Lizzie, David and Grace, Jayne and Oliver, Peter and Jessica and Ruth and Michael, few parental goals were constructed because children were comfortable and able to generate and pursue mathematical goals without assistance. Jennifer and Jacob was different in that Jennifer did not generate mathematical goals even though Jacob struggled, mainly because she did not appear to identify the errors he was making. In the rest of the tasks parents generated multiple mathematical goals. In many of the interactions it was possible to see how parents reflexively amended and adjusted their goals in light of children's understanding. In interactions where children frequently struggled with the word problems, such as Ian and Megan, Imogen and Owen, Julia and Declan, Lindsay and Ben, and Vicky and Sam, the parents generated multiple goals, some of which were rejected by the child or abandoned by the parent in the face in incomprehension.

Ian (parent) and Megan (child – Y4) *Katie is 135cm tall. Josh is 109cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
4	1.33	Megan	Katie is one hundred and thirty five centimetres tall. Josh is one hundred and nine centimetres tall. How much shorter is Josh than Katie?
5	1.46	Ian	So do you understand the question?
6	1.48	Megan	No
7	1.49	Ian	Right that's somebody that's a hundred and thirty five centimetres, going to be like that, and you've got somebody that's that [*gestures with hands*]. So you want to know how much shorter is the little one than the tall one. So that's the small one and that's the tall number.
8	2.04	Megan	Mm
9	2.05	Ian	Yeah. What is the difference between those numbers? A hundred and thirty five and a hundred and nine. So we're doing like a subtraction. We're subtracting the bottom one from the top one.
10	2.15	Megan	Mm
11	2.17	Ian	Yeah. So the difference. Do you get me?
12	2.19	Megan	No
13	2.20	Ian	No? Alright you've got a hundred and thirty five, so basically it just works out as a sum. A hundred and thirty five minus a hundred and nine. So what's the difference between that?
14	2.30	Megan	Err
15	2.31	Ian	How many more would you need to add to a hundred and nine to get to a hundred and thirty five basically?
16	2.36	Megan	Err
17	2.40	Ian	Count it out if you want
18	2.53	Megan	I don't know
19	2.54	Ian	Write it down. Write it you if you want. Write it down as a sum. So you've got 135 and 109. So you add minus here.

The above example contains many of the features discussed so far in this section. Here Ian and Megan are working together on the first problem in the task. In line 5 Ian probes Megan's understanding of the question. Megan does not understand the question so Ian explains the problem and gestures with his hands physically modelling the height difference at the core of the problem. When Megan hesitates in line 8 Ian then prompts her towards the goal of finding the difference between 135 and 109 by carrying out a subtraction. Megan still does not understand even after a further simplification and prompt in lines 13 and 15. With this approach failing Ian now pursues a counting goal, it is unclear whether this is additive (counting on from 109 to 135), or subtractive (counting back from 135 to 109). Megan thinks about this goal for a while but shows, in line 18, that she is still unsure. In line 19 Ian abandons this goal and instead promotes a goal involving a written subtraction of 135-109. This example shows how goals are abandoned in the face of a lack of understanding revealed through social interaction.

This section has shown how social interaction influences the formation, operation and realisation of mathematical goals through a wide-variety of mechanisms. A model of parental support based upon probes, prompts and goals was proposed and evidence that supports these processes was discussed.

6.5 Prior understandings

The final element in the emergent goal framework is the role of prior understandings. The mathematical goals that children and parents construct can be seen as a reflection of the knowledge they bring with them into the activity. It is possible see parents' views, experiences and the roles they construct for themselves as mechanisms through which prior understandings influence goal construction. This section of analysis uses parent-child data to illuminate how prior mathematical understandings influence emergent goal construction in both children and parents. It then combines data from the parent-child tasks and parental interviews to show how parents' prior understandings of school mathematical activity, as a cultural practice, affects goals formation, construction and operation.

6.5.1 Children's prior mathematical understandings

The different levels of mathematical knowledge and understanding that children bring to the task influence the goals that they construct. Complexity and sophistication of mathematical goals appears to increase with age across the sample.

Table 6C Frequencies of children's mental and written mathematical forms together with mathematical principles displayed

	Year 3 (n=4)	Year 4 (n=14)	Year 5 (n=3)	Year 6 (n=3)
Mental mathematical forms	Counting back (1s) -9 Counting on (1s) - 8 Addition (1010) - 3 Subtraction (unknown) - 2 Subtraction (1010) - 2	Subtraction (unknown) - 20 Addition (unknown) - 11 Addition (1010) - 5 Counting on 1s - 4 Counting back 1s - 2 Addition (A10) - 2 Subtraction (N10) - 2 Addition (u1010) - 1 Multiplication (unknown) - 1	Addition (unknown) - 1	Addition (unknown) - 4 Subtraction (unknown) - 3 Subtraction (A10) - 3 Subtraction (1010) - 2 Subtraction (N10) - 1 Division (unknown) - 1 Multiplication (unknown) - 1
Written mathematical forms	Column subtraction - 4* Tally marks - 3	Column subtraction - 55 Number line (A10) - 21 Column addition - 10 Addition (A10) - 9 Subtraction (1010) - 7 Subtraction (N10) - 4 Expanded column subtraction - 4 Number line (subtraction) - 3 Tally marks - 3 Balancing - 1 Subtraction (1010) - 1	Column subtraction - 25 Number line (A10) - 3 Subtraction (N10) - 2 Subtraction (uN10) - 1 Number line (subtraction) - 1 Column addition - 1	Column subtraction - 12 Column addition - 8 Expanded column subtraction - 2 Number line (A10) - 2

Mathematical principles	Addition, subtraction Counting, partitioning, place value, ordering/sequencing *parent completed decomposition	Addition, subtraction, multiplication Algebraic reasoning, carrying 10/100, checking, counting, decomposition, estimation, inverse rule, partitioning, place value, ordering/ sequencing, rounding	Addition, subtraction Carrying 10/100, checking, decomposition, inverse rule, partitioning, place value, ordering/ sequencing, rounding	Addition, subtraction, multiplication, division Carrying 10/100, checking, decomposition, inverse rule, partitioning, place value, ordering/ sequencing, rounding

Table 6C shows the frequency of different mental and written forms used by children in Years 3, 4, 5 and 6. The brackets indicate the type of mathematical strategy (e.g. 1010), which were discussed earlier in 6.3.2. Next to each form is a number indicating its frequency across the age group. It was impossible to identify some mental forms. These are labelled as ‘unknown’.

Table 6C highlights the mathematical principles that were evident in each year group. Whilst this should be treated carefully because of the low numbers of children in Years 3, 5 and 6, it does appear to show that the mathematical goals of younger children, and the mechanisms they use to operate these goals, are simpler than those of older children. Year 3 children arranged goals predominantly around counting, both mentally and using written tally marks. These strategies are still present in Year 4, where they are complemented by a vast array of other approaches, due in part to the size of this element of the sample. By Years 5 and 6 counting has been replaced by more complex written and mental forms of calculation. Similarly, Table 6C also shows that the mathematical principles evident in children’s dialogue and written communication become increasingly complex over time, for instance as the inverse rule and decomposition are better understood.

It is possible to say that children’s prior understandings both facilitate and restrict the formation and operation of mathematical goals. It has already been evident in previous excerpts used in this chapter that children often cannot understand a goal set by their parent because of its complexity. This was particularly noticeable in Year 3 and 4 children where a parent would promote a written form, typically a column algorithm, which the child did not understand.

Children’s prior understanding also appeared to influence the amount of support they received and the number of goals that were set by parents in the task. Table 6D shows the number of mathematical goals in each age group by parents.

Table 6D Parental goals generated through social interaction by age group

Year group	Total number of parent generated goals	Total number of these goals completed by child	Total number of these goals not completed	Mean number of parent generated goals	Mean number of these goals completed by child	Mean number of these goals not completed
Year 3 (n=4)	126	99	27	32	25	7
Year 4 (n=14)	324	309	15	23	22	1
Year 5 (n=3)	18	15	3	6	5	1
Year 6 (n=3)	16	14	2	5	3	1

Year 4 had many more goals because there were many more children in this group. However it is evident that the four children in Year 3 had far more goals set by their parents than the three children in Year 5 or the three children in Year 6. Taking this argument further and looking at the mean scores for the four groups, shown in Table 6D, it is apparent that the mean number of goals set for Year 3 (32 goals) and Year 4 (23 goals) children is much higher than was set for Year 5 (6 goals) and Year 6 (5 goals) children. This could be a function of the difficulty of the task, with younger children requiring more support. Even so this supports the fact that a lack of prior knowledge meant that the children required more goal-setting by their parents.

6.5.2 Parents' prior mathematical understandings

The mathematical knowledge that parents brought to the task undoubtedly influenced the goals they constructed for their children. In a large number of instances parents constructed goals that required written forms of computation that their child was unaware of or had little experience using. Many parents set goals because they favoured and understood their own mathematics more than their child's.

The influence of a parent's prior understandings is illustrated in the next example. In line 169, Karen is beginning to draw a number line in order to solve 238-112. She gets confused and asks for help but Carl, in line 172, is unable to help because he does not understand number lines and prefers to use column subtraction. Karen continues but makes an error in her complementary addition. Carl points out the correct answer in line 176 but Karen continues her number line. When Karen adds up the jumps on her number line she reaches the incorrect answer of 226, line 183. In line 184 Carl again points out the incorrectness but is unable to support his daughter in her number line goal because of his prior understandings. He therefore prompts her towards the goal of a written column subtraction. Karen completes this form reaching the correct answer quickly.

Carl (parent) and Karen (child - Y4) *Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have?*			
Line	Time	Speaker	Dialogue
169	12.53	Karen	Yeah, Ok two hundred and thirty eight goes here
170	13.00	Carl	Yeah
171	13.02	Karen	And a hundred and twelve coins here and we've got to work out that amount. So if you add a hundred that equals two hundred and twelve. And then you need to add (pause 4s) add erm (pause 6s) six more and that would get you to two hundred and eighteen, would it?
172	13.44	Carl	I don't know, I don't like using these number lines to be honest, I'd just prefer a straight take away. So you have a look at it
173	13.51	Karen	I'll carry on
174	13.53	Carl	You carry on
175	13.55	Karen	Add another, add six equals a hundred and eighteen and then you have to add twenty and that gets you to a hundred and a hundred and twenty... a hundred and twenty eight
176	14.16	Carl	A hundred and twenty six isn't it?
177	14.18	Karen	Eight
178	14.20	Carl	Ok
179	14.23	Karen	Oh thirty eight and then you just add another hundred and that takes you to...
180	14.32	Carl	Oh I see right
181	14.33	Karen	... two hundred and thirty eight
182	14.37	Carl	So what's the answer then?
183	14.39	Karen	So that means that the answer is two hundred and twenty six
184	14.45	Carl	No, I don't know where you've gone wrong there (laughs). I'd just do, do a take away. I don't about... You're giving Richard [*the researcher*] some useful information on this one. Dads aren't good at number lines I think (laughs).
185	15.06	Karen	Two from eight is six
186	15.08	Carl	Yes
187	15.10	Karen	One from three is two
188	15.12	Carl	Yeah
189	15.14	Karen	One from two is one
190	15.16	Carl	Yes
191	15.17	Karen	A hundred and twenty six

As was expected given the earlier discussion in 4.3.1, parents' mathematical knowledge and understanding acted as a barrier impeding parental support and parent-child mathematical interaction. On several occasions this barrier led to confusion or frustration. An example of this is the following interaction between Gary and Shaun. They correctly identify the question as a two-step problem (lines 13-15). Shaun decides to work out 307-43 first which he gets incorrect, as shown in line 16. Gary does not know whether this is correct and appears unable to check mentally. This confusion leads him to query the order of subtraction in lines 17 and 19 and suggest that it "doesn't look right" in line 21. Shaun now writes a column subtraction for 307-118, which he attempts to answer but gets wrong (line 26). Again Gary is unable to help because he does not fully understand the written form and cannot complete the calculation mentally. In lines 27-29 they appear to both agree that the answer is probably incorrect but cannot see why and reach an impasse where their knowledge is not sufficient to avoid confusion.

Gary (parent) and Shaun (child – Y6) *Josh had 307 stamps. He gave 118 stamps to Katie. He lost another 43 stamps. How many stamps does Josh have now?*			
Line	Time	Speaker	Dialogue
13	4.06	Gary	Oh right so you've got more than one calculation to do now haven't you?
14	4.12	Shaun	Mm
15	4.13	Gary	So which one do you think you should do first? *Shaun starts to writes 10 above the 307 on the question. He then writes 307-43=168. He does produce any workings and appears to answer mentally. He gets an incorrect answer of 168 (rather than 264).*
16	5.42	Shaun	One-six-eight
17	5.44	Gary	I would have to check that to be honest. Why did you decide to do it that way round?
18	5.47	Shaun	That way?
19	5.48	Gary	Yeah so why did you take forty-three off first?
20	5.53	Shaun	Because it, because I find, I find it easier [*118 would require decomposition when subtracting from 307 whilst 43 does not*] because three hundred (pause). Actually I don't know.
21	6.01	Gary	That doesn't look right to me.
22	6.02	Shaun	[*Abandons his previous calculation and writes 307-118*] So three-oh-seven minus one-one-eight.
23	6.18	Gary	There's no panic Shaun don't rush it.
24	6.22	Shaun	Seven minus eight you can't do that.
25	6.25	Gary	So what do you do?
26	6.26	Shaun	So I can't borrow anything from there [*there are no tens in 307, he cancels the hundreds from 3 to 2, and changes the units to 17*] so that makes it back to seventeen minus

			eight. That equals nine. Zero minus one you can't do that so (pause). Oh you can zero isn't it? [*He takes 1 from the hundreds adds it to the tens to make 10 he then subtracts 1 to give 9. He writes an answer of 99. The correct answer is 189*]
27	6.56	Gary	(*sighs*) I don't, I don't actually remember doing them…
28	7.00	Shaun	I'm rubbish at column subtraction (pauses 8s). It can't be ninety-nine.
29	7.12	Gary	No it can't can it

6.5.3 Parents' prior understandings of parent-child school mathematical activity

What parents understood to be the practice of school-related mathematical activity and therefore the role they constructed for themselves appeared to influence how they helped and supported their children's goals, and set their own goals. For instance, in chapters 4 and 5 it was apparent that many parents constructed a supportive role for themselves that they used when assisting their child with school work. An example of this was Gemma. As the following quote shows she saw herself as supportive and was heavily involved in school mathematics at home.

Parent participant: Gemma
I give the time for their homework as well usually it is here where we are sitting and I will sit down, sometimes with both at the same time if they've got erm a similar activity. And I will always read it with Kitty, I will check she understands what she's got to do and before she even picks up her pencil I will say to her, "Right, tell me what you think they are asking you to do and how are you going to do it?" And if she can give me that feedback I'm quite happy or if she can't I'll address what she's got to do.

Gemma (parent) and Kitty (child - Y4) *Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?*			
Line	Time	Speaker	Dialogue
237	16.48	Gemma	Go on then go again
238	16.50	Kitty	Josh buys forty, I mean thirteen Pokémon cards. He now had one hundred and eighty one Pokémon cards. How many Pokémon cards did he have in the beginning? (Pause 4s) Josh buys Pokémon cards. He now has. So Josh buys thirteen Pokémon cards, he now has, (pause 4s) did he have that at the beginning? *Kitty reads the question and underlines "buys 13 Pokémon card" and "now has 181 Pokémon cards"*
239	17.26	Gemma	He's just bought that
240	17.28	Kitty	Oh! He now, right you have to take thirteen from one hundred and eighty one *Kitty identifies operation and calculation, Gemma discusses reasoning*
241	17.37	Gemma	Yeah because it's like saying, it's like saying he had that but we've no idea...
242	17.41	Kitty	Yeah
243	17.42	Gemma	...he bought that, (pause 2s) and he's now got?
244	17.48	Kitty	One hundred and eighty one
245	17.50	Gemma	That's it. So we're trying to find the?
246	17.52	Kitty	Missing number
247	17.53	Gemma	Yeah so, so you do exactly what you said (pause 2s) because it's a bit like...
248	17.57	Kitty	No but you've got to take away, can I do the take away
249	17.59	Gemma	Yeah, yeah, yeah there *Kitty writes 181-13*

This attitude was clearly observable in the goals Gemma constructed in the parent-child task. She often explored Kitty's understanding and supported her goal-related behaviour. An example of Gemma's supportive behaviour is shown in the above example. Here Gemma begins by directing Kitty towards reading the question (line 237) and checking her understanding (lines 241, 243 and 245) before Kitty writes the correct calculation 181-13.

Natalie provides less direct support and more autonomy to her daughter Daisy. This approach is justified by her belief in Daisy's mathematical ability and the time constraints in her daily life.

Parent participant: Natalie

She has homework every week. She has one lot every week and she's given the week to get it done and during that week she will just take herself off and do it. I'll say, "Have you done it yet? Any problems? Yes or No?" Sometimes it's a big, lengthy thing I'll go through it but if I say, "Was it ok?" [And Daisy replies] "Yes it was easy". Then I think well, and I've never had any come back [from school]. Obviously with three of them [children] I'm not checking everything that their doing all the time, and I usually work, see, 5-8 so that's the time that they're here doing it.

Natalie (parent) and Daisy (child – Y5) *Katie is 153cm tall. Josh is 118cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
1	0.51	Natalie	I don't normally help you with your homework do I...
2	0.52	Daisy	No
3	0.53	Natalie	...because you just go upstairs and do it. So read the question and do it.
4	0.56	Daisy	Now
5	0.57	Natalie	Yeah now
			Daisy writes 153+5=158 then draws a number line from 158 to 168
6	1.25	Daisy	Can I cross that out?
7	1.27	Natalie	Yeah just pretend it's your homework
			Daisy rubs out the number line
8	1.38	Natalie	Now it's all been rubbed out
9	1.41	Daisy	Except for that
10	1.42	Natalie	Don't worry darling. (Pause 8s) Don't press on so hard.
11	1.54	Daisy	Ok
			Daisy now writes 118+2=120 then underneath draws a number line from 120 to 153
12	2.15	Natalie	Would you do it this way for every question?
13	2.18	Daisy	Mm
14	2.20	Natalie	Ok, only asking
			Daisy completes the number line calculation.
15	2.42	Natalie	Done

This is mirrored in her interactions with Daisy. In the above extract, at the beginning of the task, Natalie sets out the 'ground rules' and how she expects Daisy to work, shown in lines 1, 3 and 5. When Daisy raises a query Natalie directs her to treat it like a school mathematical task. Daisy corrects an error in her calculation and proceeds to use a number line to find the difference between 120 and 153. In line 12 Natalie questions her approach suggesting a lack of understanding of her daughter's methods.

The prior understandings that parents and children brought to the task could be seen to influence the emergence of mathematical goals in the task and how those goals evolved and were more often than not accomplished. It certainly appeared that children's mathematical understandings played a much larger role than parents' understandings in the goal formation, negotiation and operation. Parents' previous experience of the cultural practice, and their views and opinions, also appear to influence interaction and goal construction.

6.6 Contingency shifts

Within the participant data many parents were seen to vary the amount of support they offered based on the child's previous performance. This was mainly evident where children struggled to complete word problems. In such cases the parent typically took a stronger role, usually instructional or directive with an increased frequency of prompts.

This phenomenon is seen in the following example taken from the interaction between Vicky and Sam. In the task Sam completed the first two word problems without any assistance from his mother. During the third question he requested help and so Vicky explained the question and prompted him towards an operation, allowing Sam to complete the problem successfully. In the fourth problem Sam struggled with 44-9 requiring a great deal of support from his mother. This leads to a contingent shift at the beginning of problem five, shown below. From the start of this problem Vicky is reading the question for Sam and straightaway explaining the task. Again Sam struggled to complete the computation without a great deal of intervention.

Vicky (parent) and Sam (child – Y3) *Katie has 59 marbles. If Josh buys 16 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?*			
Line	Time	Speaker	Dialogue
68	6.36	Vicky	Right. Katie has fifty-nine marbles... Go on.
69	6.42	Sam	Josh buys sixteen marbles will have...
70	6.47	Vicky	He will have the same number of marbles as Katie. How many marbles does Josh have? So again...
71	6.56	Sam	Yeah
72	6.57	Vicky	If he buys sixteen...
73	6.58	Sam	Yeah
74	6.59	Vicky	...he'll get to fifty-nine.
75	7.01	Sam	Will he?
76	7.01	Vicky	Yeah.
77	7.02	Sam	Mm
78	7.03	Vicky	Right. So how many did he have before?
79	7.07	Sam	We don't know because it doesn't tell us.
80	7.09	Vicky	No it doesn't but you've got to work it out. So if he buys sixteen he'll then have fifty-nine. So if you take the sixteen away from fifty-nine that will tell you how much he... how many he has. So come on, do you want to write it out? Put fifty-nine... minus

When the next question started, shown below, Vicky had taken over reading the question and directed the operation and mathematical form. Subsequent to this excerpt she talked Sam through each of the steps he needed to undertake to complete the calculation. Again she displays contingent intervention.

Vicky (parent) and Sam (child – Y3) *Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?*			
Line	Time	Speaker	Dialogue
109	9.14	Sam	Josh had...
110	9.19	Vicky	A hundred and forty-seven stamps. He gave thirty-three stamps to Katie. And he lost another thirty-six stamps. How many does he have now? So write down your hundred and forty-seven. *Sam writes down 147*
111	9.35	Sam	Add?
112	9.36	Vicky	No take away... your thirty-three [*Sam writes 147-**33***]. Right so equals...
113	9.45	Sam	[*Writes an equals sign*] Equals...
114	9.46	Vicky	Right so...

As contingent intervention occurs goals change. The origin of the goal changes from child to parent, how the goal is negotiated and progressed alters, and the actual strategies also change. These assertions are all visible in the example of

Vicky and Sam. Sam's subtractive goals are supported by counting but when Vicky sets the subtractive goals they become a written form.

Contingency in the opposite direction, where a parent reduced support based on a child's success was less visible. A reason for this may be that the activity was designed with questions that got progressively harder the further the child got. Therefore support tended to increase through the task rather than decrease.

6.7 Scaffolding elements

In order to address the research question concerning the extent to which parents 'scaffold' learning and conceptual development in mathematics, scaffolding within the parent-child interaction was investigated using the framework for scaffolding analysis produced by van der Pol et al. (2010). This proposes that scaffolding processes can be divided into strategies and intentions. Strategies consist of feedback, hints, instructing, explaining, modelling and questioning. Intentions involve direction maintenance, cognitive structuring, reduction of degrees of freedom, recruitment and contingency management/frustration control.

Goal-related dialogue of each parent was coded for these eleven scaffolding elements. Table 6E shows the presence of these features across the sample and how the presence of scaffolding elements varies between parents.

Whilst the notion of probes and prompts was able to show linkages to goals the multiple elements of scaffolding provided a more opaque picture. The interaction and association of the eleven elements and patterns of success or failure with numerical goals was far harder to ascertain. Because of this absence of clarity the analysis focused on individual scaffolding elements and goals, rather than scaffolding as a coherent multi-part process.

The existence and frequency of features of scaffolding, shown in Table 6E, was not necessarily a reflection of successful goal completion. It also did not appear to be directly connected to what might be thought of as 'good practice' or an 'ideal' model of parental instruction. Typically the longer transcripts showed more evidence of scaffolding elements as more dialogue occurred so more opportunity for scaffolding existed. For instance the longest transcripts, Gemma and Deborah, both showed ten out of the eleven scaffolding elements. Their children found some of the questions difficult but the parents often appeared to scaffold or promote goals and dialogue that the child could probably have completed without assistance. Conversely, highly instructive parents, using mostly closed dialogue, who completed many goals for their children, such as Lindsay and Ian, also used ten and eleven of the components respectively. This supports the finding that scaffolding elements were not necessarily related to overall parent interactional style or how difficult the child found the task.

Table 6E Scaffolding elements present in parental goal-related dialogue

Parent	**Scaffolding strategies**						**Scaffolding intentions**				
	Feedback	**Hints**	**Instructing**	**Explaining**	**Modelling**	**Questioning**	**Direction maintenance**	**Cognitive structuring**	**Reduction of degrees of freedom**	**Recruitment**	**Contingency management / frustration control**
Abigail	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓
Beth	✓		✓	✓	✓	✓	✓		✓		✓
Carl	✓	✓	✓			✓	✓		✓		✓
Charlotte	✓	✓	✓	✓		✓	✓	✓	✓		✓
Chris	✓			✓		✓					✓
David	✓	✓	✓	✓		✓	✓		✓		✓
Deborah	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
Gary	✓		✓			✓	✓				✓
Gemma	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
Ian	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Imogen	✓	✓	✓		✓	✓	✓		✓	✓	✓
Jayne	✓	✓		✓		✓		✓			✓
Jennifer	✓		✓	✓		✓	✓				
Julia	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓
Lindsay	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓
Natalie	✓	✓	✓	✓		✓	✓		✓		✓
Neil	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Niamh	✓	✓	✓	✓		✓	✓		✓		✓
Peter	✓			✓		✓	✓	✓			✓
Rebecca	✓	✓	✓		✓	✓	✓		✓	✓	
Robert	✓	✓	✓	✓		✓	✓		✓	✓	✓
Ruth	✓		✓			✓					✓
Suzy	✓	✓	✓	✓		✓	✓	✓	✓		✓
Vicky	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

All the parents used questioning and feedback to a greater or lesser extent. These elements, analogous to probes and prompts, shaped goal construction as discussed extensively earlier. Hints were used by most of the parents to direct children towards an activity or answer but without completing the activity for the child. A common occurrence was beginning a sentence and expecting the child to finish it. This was used frequently to support children completing addition and subtraction calculations. An example of this is shown as Niamh probes Connor's understanding and prompts him towards finding the difference in line 6. She again hints in line 10 but does not get a response from Connor so she finishes the sentence.

Niamh (parent) and Connor (child – Y4) *Katie is 135cm tall. Josh is 109cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
4	0.55	Niamh	Right so what are you going to do to work that out?
5	1.00	Connor	Don't know
6	1.11	Niamh	What do you need to find between those two numbers? The?
7	1.14	Connor	Difference
8	1.15	Niamh	Yeah, so how would you find the difference?
9	1.20	Connor	Take away
10	1.21	Niamh	Take one away from? (Pause 2s) From the other yeah?

Instruction was used as a strategy by twenty-one parents. This represents all the interactions where consistent amounts of dialogue occurred. The most common instructions were linked with parents directing children to read the question, complete a certain computation or provide a written answer on the word problem sheet. Here Ruth instructs Michael to use the problem sheet to read the question and state a computation that can be written down on the problem sheet. Michael agrees to these goals and reads the question, in line 2, and begins to construct a number sentence, in line 4.

Ruth (parent) and Michael (child – Y5) *Katie is 153cm tall. Josh is 118cm tall. How much shorter is Josh than Katie?*			
Line	Time	Speaker	Dialogue
1	1.03	Ruth	Right go on then you read the question and I'll read it
2	1.06	Michael	Katie is one hundred and fifty-three centimetres tall. Josh is one hundred and eighteen centimetres tall. How much shorter is Josh than Katie?
3	1.16	Ruth	Right you tell me what you want to write
4	1.19	Michael	Err one hundred and fifty three

The majority of parents explained the rationale behind a particular goal, e.g. the workings of a computation, to support their children in successfully completing that goal. In the following example Alex interprets the question incorrectly producing an answer of 13. Robert explains why this is incorrect then negotiates goals for Alex to calculate and answer the problem.

Robert (parent) and Alex (child – Y4) *Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?*			
Line	Time	Speaker	Dialogue
153	17.26	Alex	Josh buys thirteen Pokémon cards. He now has one hundred and eighty one Pokémon cards. How many Pokémon cards did he have in the beginning? (Pause 2s) Easy thirteen!
154	17.42	Robert	No, you've... He buys thirteen, once he's bought those thirteen he's got a hundred and eighty one. So you need to find out before he'd got those, bought those thirteen, how many? So how are you going to do it? Are you going to do a line again?

Modelling was used by ten of the parents to assist their child in understanding or completing a goal, typically a calculation using a physical mathematical form such as counting. As a strategy it was restricted to Year 3 and Year 4 parents. This is possibly because these children generally required more assistance from their parents. The next excerpt shows modelling by Julia as she supports Declan in finding the difference between 19 and 44. Declan has been struggling to count in tens and ones to reach his answer. The excerpt starts in line 38 where Julia is holding up three fingers to indicate the tens from nineteen to thirty-nine. She then again uses her fingers to model counting in ones from thirty-nine to forty-four. Julia refers to the fingers she is holding up as representing the tens and units. Her modelling, direction and simplification enables Declan to complete the calculation and answer the problem.

Julia (parent) and Declan (child – Y3) *Katie had 54 stickers. She lost some of them. She now has 19 stickers. How many stickers did she lose?*			
Line	Time	Speaker	Dialogue
53	7.02	Julia	[*Holding up three fingers on one hand to represent three tens then counts with him on her fingers on the other hand from 49 to 54*] Fifty, fifty-one, fifty-two, fifty-three, fifty-four [*holding up five fingers*] So they're all ten each [*referring to the three fingers she is holding up on one hand*] so what's that? Ten...
54	7.15	Declan	Twenty, Thirty
55	7.17	Julia	Thirty add five?
56	7.21	Declan	Thirty-one, thirty-two, thirty-thee, thirty-four, thirty-five
57	7.26	Julia	Right then there's your answer.

Scaffolding intentions were less straightforward concepts to code. Direction maintenance, defined as keeping the child on track to achieve a goal, was seen in all the interactions that contained consistent amounts of dialogue. In the next illustration Peter maintains Jessica's attention to the task of subtracting 7 from 25 after her initial incorrect answer of 19. He directs her focus back to this in lines 11 and 13. Eventually, in line 16, she completes the computation using a column subtraction.

Peter (parent) and Jessica (child – Y5) *Josh is in the kitchen. There are 25 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table?*			
Line	Time	Speaker	Dialogue
8	3.39	Jessica	Is that right [*points to her answer '19 forks'*], is that right?
9	3.45	Peter	It's a bit doubtful Jess.
10	3.49	Jessica	Well you work it out too then.
11	3.50	Peter	No Jess you do it.
12	3.52	Jessica	But I already did.
13	3.54	Peter	What's fifteen take away seven.
14	3.56	Jessica	Fifteen take away seven... nine
15	4.00	Peter	No it's not that.
			Jessica crosses out her sum and answer and writes another column subtraction
16	4.07	Jessica	Twenty five, seven, that'll be fifteen and ok. Ten take away seven is three plus five equals eight. So eighteen.

Cognitive structuring was displayed by ten of the twenty-four parents in the sample. Following van der Pol et al. (2010) this was defined as parents providing explanations that involved mathematical principles in pursuit of a particular goal. The nature of the task meant that the majority of such structuring was around mathematical principles of addition, subtraction and place value. A number of parents pursued checking goals during the interaction that they linked to the inverse principle. Here Kitty has completed 54-19 and obtained the correct answer of 35. In line 145 Gemma encourages her daughter to check her answer by using the inverse, which she later explains as the opposite to subtraction. In line 148 we see Kitty understand addition as the opposite of subtraction.

Gemma (parent) and Kitty (child – Y4) *Katie had 54 stickers. She lost some of them. She now has 19 stickers. How many stickers did she lose?*			
Line	Time	Speaker	Dialogue
145	9.18	Gemma	Yeah well you've done that, put thirty-five there because that's what you've found out, and how would you check that fifty-four take away nineteen is thirty-five? What could you do to those three numbers to check?
146	9.32	Kitty	Erm
147	9.35	Gemma	If you've got a take away what's the opposite of a take away to do a checking system?
148	9.38	Kitty	Add oh!
149	9.40	Gemma	So which two would you add, it's called an inverse, which two would you add the opposite way?

Explanation, modelling and, mainly, simplification were all seen to be utilised by many parents in order to reduce the degrees of freedom in the task. In this manner parents channelled their children towards goal completion. We have seen evidence of the reduction in degrees of freedom earlier in Neil's simplification of 11-7, Ian prompting Megan towards understanding a word problem and a calculation of 135-109, Robert's direction for Alex to construct a number line for 181-13, and Julia assisting Declan with 54-19.

Table 6E suggests that recruiting the child's interest in the task was only used as a scaffolding intention by a minority of parents. This is not because parents did not see the value of the task but because the children generally pursued the activity without the need for encouragement. However in several interactions the parent had to recruit the child's interest because the child was bored or did not want to complete the activity. The example below shows such an interaction between Julia and Declan. Here Julia has to recruit Declan's interest, possibly because he is finding the task difficult or he is disinterested in it, when he begins to play with his toy cars. Through simplification of the problem and asking him to stop playing with the cars Julia recruit Declan's attention to the task and he begins to count in ones from fifty-four to nineteen.

Julia (parent) and Declan (child – Y3) *Katie had 54 stickers. She lost some of them. She now has 19 stickers. How many stickers did she lose?*			
Line	Time	Speaker	Dialogue
15	3.38	Julia	Katie had fifty-four stickers. She lost some of them.
16	3.43	Declan	What?
17	3.44	Julia	She's lost some of her stickers. She had fifty-four. And now she has nineteen stickers. How many stickers has she lost?
18	3.55	Declan	I have no idea!
19	3.57	Julia	Fifty-four. She had nineteen left and she had fifty-four so what's the difference between those two numbers? (Pause 4s) Can you do fifty-four take away nineteen? *[Declan starts playing with some toy cars that are on the table]* Come on.
20	4.22	Declan	I don't know!
21	4.24	Julia	Fifty-four...
22	4.25	Declan	I don't know!
23	4.27	Julia	...take away nineteen. Stop it [*talking about toy cars*].
24	4.32	Declan	Right fifty-four, fifty three...

Contingency management and reduction in frustration mainly occurred through praising children's success rather than punishing them for non-completion. However, in the above example Declan appears frustrated, saying, "I have no idea" in line 18 and, "I don't know!" in lines 20 and 22. His mother manages this frustration by simplifying the task and telling him not to play with his cars.

Following an approach focusing on scaffolding and goals shows the many different features of parental support that can be observed in dyadic interaction. Certainly, elements of scaffolding were prevalent in the parent-child mathematical simulated-school mathematical task.

6.8 Discussion

In this chapter a range of findings were produced with regard to parent-child school mathematical activity. Primarily these were directed towards a greater understanding of goals and goal-related mathematical activity. Additional conclusions were also reached with respect to contingency and scaffolding, though these were less emphatic.

6.8.1 RQ6: How do parents and children form, negotiate and operate mathematical goals?

By adopting Saxe's (1991) four parameter model of emergent goal formation it was possible to study mathematical goals associated with the structure of the parent-child mathematical activity, the artefacts and conventions inherent within the activity, the forms of social interaction adopted by parents and children during mathematically-related dialogue, and the prior knowledge and understanding children and parents brought to the activity. Since the model has not been previously used to study parent-child school mathematical activity this analysis was therefore unique in its subject. It nevertheless showed the strength of the model and its applicability to the study of parent-child school mathematical activity. Indeed, it was possible to show that how parents and children formed, negotiated and operated mathematical goals was directly influenced by the four parameters.

As with prior research the structure of an activity was found to influence goal construction. This occurred in five separate ways, associated with reading the question, defining the operation, selecting a form, performing a computation and reporting results of this calculation. This appeared to authenticate the task as a reflection of parent-child school-related mathematical activity.

The pattern in which parents and children constructed goals around these elements showed that children were the main originators and accomplishers of numerical goals, with parental goal-setting mainly fixed upon supporting computational problems. Individual differences were seen in goal-related activity with parents differing in the number of activity structure goals they constructed for their children.

It is evident that, as Saxe (1991) contended, activity structure is linked to the motives driving the activity. The motive of completing a calculation or supporting a child's mathematics is reflected in the goals required to complete a mathematical problem. Time and again these were shown to follow the problem solving approach shown in Figure 6B, rather than the simpler model proposed by Greer (1997).

The analysis supported the notion that parents' and children's emergent goals were influenced by the artefacts and conventions inherent within the cultural practice. Participation in the cultural practice required the adoption of certain motives, and hence goals, that were framed by the artefacts and conventions within the activity.

Just as dominoes influence players goals (Nasir, 2000b) and coins and currency shaped candy sellers mathematics (Saxe, 1991), so the main artefact in the task, the word problem sheet, clearly influenced the mathematical goals of parents and children. The problem sheet facilitated the use of a range of written mathematical forms. These approaches were used by children to pursue certain mathematical goals, but they also allowed parents an insight into children's reasoning and thereby influenced parents' mathematical goals.

A vast array of school mathematical conventions was used by children in the tasks. These different forms unmistakably influenced goal formation. The choice and use of goal-related forms were also a reflection of children's numerical understandings, a point further enhanced in studying how children's prior understandings influenced goal setting.

Social interaction, in the form of dialogue, was seen to have highly-visible influence on how parents and children promoted, rejected, negotiated and accomplished mathematical goals.

Parental support was mainly constructed though social interaction. The use of probes to investigate understanding and prompts to stimulate activity were seen to impact upon parents' and children's mathematical goals. The number of mathematical goals parents constructed was typically linked to difficulties children were experiencing and parents' individual interactional styles. Parent-child social interaction not only encouraged and supported goal formation but also led to reflection and negotiation of mathematical goals.

The division of labour between individuals engaged in a cultural practice influences mathematical goals (Guberman & Saxe, 2000; Saxe, 2002) and activity (Leont'ev, 1981). Through studying goals emerging through social interaction, in this case between parents and children, it is possible to see these processes. Whilst the activity was co-operative there were clear roles and expectations held by both parties. Through social interaction these motives were expressed as goal-related activity.

Table 6F gives an indication of the manner in which the parameters of activity structure, artefacts and conventions, and social interaction can be seen to influence emergent goal construction. This shows the relationship between elements of activity structure, forms or artefact, types of social interaction and different types of mathematical goal. Different goals evolve in different elements of the task aligned to certain artefacts and mathematical understandings.

The prior understandings that parents and children bring to the task, in terms of the knowledge and conceptual understanding of mathematics and experience of the cultural practice, clearly influences participation in the practice, motives, and the construction and pursuit of mathematical goals. As expected, this supports the findings of several other studies (e.g. Nasir 2000a, 2000b, 2002, Saxe, 1991, Saxe & Guberman, 1998).

The age of children and therefore awareness of different forms of school mathematics influenced the goals children pursued. As children aged the mathematical goals they pursued generally became more sophisticated, from finger counting to written algorithms. Children's goals could be linked to the prior knowledge they acquired in the classroom and reflected their progress through school. Parents' own knowledge of mathematics similarly influenced how they approached the task in terms of the mathematical goals they selected. On occasion parents' knowledge differed, thereby altering motives and the balance of activity.

The amount of support offered by parents in terms of goal generation and completion reduced with as children aged. This showed how guided participation, as parents' structure problem solving, alters with age as children become more knowledgeable and parents desire to give them more autonomy. In this sense, as Rogoff (1990) supposed, children become more active learners gaining more control of their goal-related activity and, therefore, mathematical learning.

Table 6F Activity structure, artefacts and conventions, social interaction and numerical goals

Activity structure	Artefacts and conventions	Social interaction (with parent)	Emergent mathematical goals
TEXT Objective: Read and comprehend word problem text	Word problems sheet	e.g. probes and prompts around reading question correctly	Number representation
OPERATION Objective: Select operation required to answer problem	Mathematical operations (+, -, x and ÷)	e.g. probes and prompts concerning operation and understanding problem	Addition, subtraction, multiplication and division
FORM Objective: Choose a mathematical form to enable computation	Word problem sheet; personal knowledge mental and written mathematical forms	e.g. probes and prompts around suitable mathematical forms or understanding of forms	Addition, subtraction, multiplication and division (Awareness of potential goals around decomposition, estimation, ordering/sequencing numbers, inverse operations, and partitioning may be considered together with number representation and operation)
COMPUTATION Objective: Carry out computation	Word problem sheet; personal knowledge of mental and written mathematical forms / strategies	e.g. probes and prompts around mathematical calculations and errors	Addition, subtraction, multiplication and division. Decomposition (place value - associated with subtraction, particularly column algorithm); carrying ten/hundred (place value - associated with addition, particularly column algorithm); checking and comparison of results/answers; rounding (to nearest 10/100), mathematical reasoning (approaches, forms/strategies and answers); estimation; ordering and sequencing numbers (often prior to addition or subtraction); inverse operations; algebraic reasoning; partitioning; number representation; counting; cardinality
RESULTS Objective: Report results of computation and answer the problem	Word problem sheet	e.g. probes and prompts about written answers or checking validity of results	Checking and comparison of results/answers; mathematical reasoning (approaches, forms/strategies and answers); ordering and sequencing numbers (resulting from operation of forms); inverse operations; number representation

Note: Layout of table taken from Saxe, 1991, p.64, Table 7.1

Without pre-testing it is impossible to ascertain when and where parents and children were operating within the zone of proximal development, the difference between assisted and unassisted performance. However the parental support of children's goal-related activity shown in this analysis, particularly when addressing difficulties and misconceptions, shows 'expert' and 'novice' roles. These roles, as Rogoff (1990) argues, on occasion appear allow participation in the cultural activity that would otherwise be inaccessible for a child, or novice.

6.8.2 RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?

Through this analysis it was possible to identify instances of contingency shifts arising as a consequence of children's difficulties during the task, matching the original findings of Wood and Middleton (1975). Shifts resulted in more direct parental support and, typically, a reduction in degrees of freedom for the child.

Contingency shifts were similar to those observed in other parent-child mathematical work (e.g. Pratt et al., 1992; Saxe et al., 1997) and appeared a common form of parental involvement.

The manner in which mathematical goals and goal-related behaviour affect contingency was not clear in previous research. Also it was not clear how the cultural practice of mathematical activity in the home supports a contingent strategy. The findings produced in this analysis shed light on this area. Contingent intervention was seen to influence goals in the parent-child interaction. As shifts occurred parents typically took control of goal setting, and sometimes even goal completion, away from their children. This often involved parents' selecting the mathematical strategy that the children would adopt. In the opposite direction shifts to reduce support after children's success were not particularly noticeable in the sample. It is suggested that this arose as a function of the progressive difficulty of the task, so support tended to increase through the task rather than diminish.

6.8.3 RQ8: To what extent do parents 'scaffold' learning and conceptual development in mathematics?

The results of this analysis show that many of the different strategies and intentions that make up the concept of scaffolding were present in the parent-child interactions. Furthermore the eleven elements of scaffolding coded in the data set could be linked to goal-related activity. This supports the ideas of Rogoff (1990) with regard to guided participation and Tharp and Gallimore (1988) concerning scaffolding, goal-directed action and the zone of proximal development.

Rogoff (1990) did not show how the individual components of scaffolding interact with goal-related activity. In this analysis it was difficult to address this point. However, numerical goals could be seen in conjunction with elements of scaffolding. The findings here support Rogoff's assertions but do not further them.

The most interesting result of the approach taken is that it is clearly evident that parents differ in the extent to which they scaffold learning and development in mathematics. Similarly to Hyde et al. (2006) and Pratt et al. (1988), individual differences were found between parents' scaffolding. For instance Ian and Vicky had over twice the number of scaffolding elements as David and Ruth. The reasons behind this are unclear. Scaffolding elements did not appear to be directly related to children's age or ability, as Connor and Cross (2003) found, nor parents ability, as suggested by both Hyde et al. (2006) and Neitzel and Stright (2003). Unlike Laakso (1995), but similar to Pratt et al. (1988) and Mattanah et al. (2005), no clear differences emerged between the existence of scaffolding elements in mother- and father-child interaction. Neither did there appear to be differences based upon the gender of the child, as Lindberg et al. (2008) likewise showed.

The next chapter draws together the results of the analyses of mathematical goals and identity to investigate the way in which identity shapes activity.

Chapter 7

Identity and goals

7.1 Introduction

This chapter draws together ideas and findings generated in the two preceding chapters to investigate the link between identity and goals. It seeks to produce an inquiry into the role parental mathematical identity plays in supporting children's conceptual understanding of mathematics. It does this though analysis of mathematical goals alongside the 'self' and 'other' positioning discussed previously. This allows exploration of the role of identity in dyadic interaction, the subject of the following research question.

RQ5: How does mathematical identity influence parent-child school-related mathematical activity?

Nasir (2002) proposed the following model, shown earlier and repeated in Figure 7A, connecting goals, identity and learning.

Figure 7A The relationship between goals, identity and learning

Note: Adapted from Nasir, 2002, p.239, Fig 4

In her model the three elements of goals, identity and learning enjoy bidirectional relationships where, for instance, goals shape identity and identity shapes goals. This thesis has already separately investigated mathematical identity and mathematical goals. If the model offered by Nasir (2002) is accurate it should be possible to connect these analyses and study the link between identity and goals.

Dialogical self theory (Hermans, 1996; Hermans & Kempen, 1995; Hermans et al., 1992) has been used here to analyse identity, specifically positioning the 'self' and positioning the 'other'. If there is a connection between identity and goals then positions should be seen to give rise to mathematical goals. Similarly the mathematical goals analysed in the previous chapter, using the approach of Saxe (1991), should influence identity. This is also expected since part of dialogical self theory is the idea that I-positions arise, change and are discarded as a result of social and cultural activity (Hermans et al., 1992).

Measuring changes in identity would be difficult to address without further data collection so is not the objective of this chapter. However, given the analyses already undertaken it was possible to address the above research question and study how mathematical identity influences parent-child school-related mathematical activity.

7.2 General patterns

Through analysing the data set commonalities can be observed between parents' self-described identities, the identities they construct for their children and goal construction in mathematical activity. It is however difficult to directly connect a particular mathematical goal operated during dyadic interaction to a particular position. Whilst many goals appear to reflect positioning of the 'self' and positioning of the 'other', they cannot be expressly connected because of the myriad of potential stimuli in the sociocultural environment. Nevertheless the theoretical connection between identity and activity can begin to be borne out by showing how positioning is a potential cause of activity in dyadic interaction.

The data set as a whole appears to show that some positions are more important than others in determining goal formation and operation in parent-child interaction. Rather than a neat linear relationship between identity and goals it is possible to see a more complex association of different elements of the 'self' and 'other' influencing goal-related activity.

The pattern that emerges from studying parental positioning and goals is shown in Figure 7B. It is apparent that parental activity is driven more by positioning of the 'other', i.e. the child, than by positioning the 'self'. In other words the perceived mathematical identity of the child seems to reflect more closely parental behaviour than the parent's own mathematical identity. Whilst each parent-child pair is unique, and some dyads varied in the visibility of different categories of positioning and goal-related activity, the behaviours, competencies and emotions ascribed to children in the parental interview more closely match parental activity than the parents' own mathematical I-positions, especially those linked to competencies and emotions.

Figure 7B The influence of positioning of the 'self' and 'other' on parental goal-related activity

Generally in higher- and lower-mathematically confident parents when I-positions and 'other' positions were compared to activity the patterns presented above were repeated. If anything the lower confidence parents displayed less of their competency- and emotionally-related positioning of the 'self' than more confident parents. This was frequently seen as parents who felt competent appear competent, but those who did not feel competent also sought to appear competent in interactions with their children.

In cases where parents were more heavily involved in goal formation and completion (e.g. Ian-Megan, Imogen-Owen, Julia-Declan, Lindsay-Ben and Neil-Daniel) there was strong supportive and proactive positioning of the 'self' but a range of competency- and emotionally-related positions. In these cases the 'other' was often seen as competent but lacking motivation. Prior experiences of parent-child mathematical interaction may well be the key determiner driving parental goals here, but again this relates more closely to positioning of the 'other' rather than competencies and emotional components of the 'self'. Where parents were less involved in goal-related activity (e.g. Chris-Lizzie, David-Grace, Jayne-Oliver, Jennifer-Jacob, Natalie-Daisy, Peter-Jessica and Ruth-Michael) it was much harder to see links beyond parent's positioning their children as 'competent' or 'good' at mathematics and therefore not requiring much support.

There are nevertheless exceptions and differences to the general pattern described in Figure 7B. In some circumstances nervous or apprehensive behaviour (e.g. Rebecca- Zoë and Lindsay-Ben) did play a larger role in activity. These competency positions influenced how able these parents felt to interact mathematically. However even these parents with mathematical difficulties attempted to shape the activity to the needs or positioning of the 'other'.

Whilst the relative visibility of the four strongly influential categories of positioning shown in Figure 7B varied, they constantly appeared to be the most important 'self' and 'other' positions in goal-related activity. In order to support and illustrate these assertions an in-depth analysis from a single parent-child dyad is presented in the chapter. Because of the wealth of data each participant-pair provides from the two data streams, parental interview and parent-child task, and the need to appreciate both to see reflections of individual identities in mathematical activity, a single parent-child dyad enables a detailed analysis to be presented in the limited space available within this thesis. However, to bolster certain general points or patterns across the twenty-four dyads, supplementary examples are occasionally used from other parent-child pairs.

7.3 An example of identity and goals: Abigail and Zach

The pair chosen for this analysis, Abigail and Zach, are first described before moving on to a discussion of Abigail's positioning of herself and her son. These 'self' and 'other' positions are then compared to goals operated and formed in the parent-child task.

7.3.1 Participant details

Abigail and Zach were chosen for this in-depth analysis for a variety of reasons. In many ways they represented a norm in the group: a working-class mother, a Year 4 child of average to slightly above average attainment, and a range of parental experiences of mathematics and mathematical activity. Abigail also presented both typical and unique positions of the 'self' and the 'other'. The parent-child interaction contained elements of goal generation across all four emergent goal parameters. There were some word problems that both parent and child found difficult and others that appeared to be easier.

Abigail

Abigail was in her mid-thirties. She was married to Paul and had two boys, Zach aged 8 and Nicky aged 4. The family lived in a working-class suburb of a large city in the north of England. Abigail worked part-time in the public sector. Part of her work involved dealing with cash transactions. Abigail left school with GCSE mathematics grade C.

Zach

Zach attended Year 4 at the local primary school. When talking to the researcher, Zach spoke about enjoying mathematics at home and at school. He suggested that he was competent at mathematics and found some areas, particularly multiplication, difficult. His performance in the task and Abigail's recall of conversations with his teachers suggested he was average to slightly above average in terms of mathematical attainment.

Zach's school was a large suburban primary with over 500 pupils on roll. The typical attainment of pupils in mathematics, English and science was broadly in line with the national and local average.

7.3.2 The mathematical 'self' and the mathematical 'other'

This section presents the analysis of the mathematical 'self' as formed by Abigail and the mathematical 'other' she provided Zach.

Abigail: the mathematical 'self'

Abigail co-constructed sixteen mathematically-related I-positions during her interview with the researcher. These positions are shown in Table 7A and separated into the three categories of behaviourally-, competency- and emotionally-related positions.

Table 7A Abigail's mathematically-related I-positions

Positions	Type of position
I as assertive regarding my child's mathematical development I as involved in my child's mathematical development I as organised regarding mathematical activity I as practical regarding mathematical activity I as playing a proactive role in my child's mathematical development I as supporting my child's mathematical development	Behaviourally-related
I as a competent user of mathematics I as finding mathematics difficult I as not good at mathematics I as a novice and learning mathematically from my child I as struggling with mathematics	Competency-related
I as feeling vindicated through mathematical activity I as negative towards mathematics I as not enjoying mathematics I as pressured by mathematics I as regretful of mathematical activity	Emotionally-related

The positions in Table 7A show that Abigail perceived herself to be supportive and proactive but not necessarily confident towards mathematics. The following examples shows a range of behavioural positioning Abigail used to construct her supportive and proactive mathematical self. During the interview she frequently built I-positions around involvement in her son's education and homework. The first example shows her organised and supportive approach to homework, specifically set routines and the use of 'we' to describe the activity.

Abigail (parent) – Interview	
Dialogue	Mathematical I-position
Because I have to like say, <u>(1) "Oh come on, let's do your homework". [Ethan says] "Oh do I have to". And I'll say, "Well once we've done it's out of the way and you've got a whole weekend to play".</u> <u>(2) I make sure, where possible, we do it at the weekend, get it over and done with, because we've got that much going on during the week.</u>	<u>(1) I as organised regarding mathematical activity</u> <u>(2) I as organised regarding mathematical activity</u>

Abigail appeared to value mathematics and saw it as important to support and be involved with her son's mathematical development.

Abigail (parent) – Interview	
Dialogue	Mathematical I-position
(1) And it's important to understand what your child is doing at school. Erm (pause) (2) So I mean there's only so much you can help them but you can be involved with, I like to know what he's doing	(1) I as supporting my child's mathematical development (2) I as involved in my child's mathematical development

She portrayed herself as proactively involved in her son's education. On numerous occasions, such as the one below, she described contacting teachers to discuss homework queries, be apprised of Zach's mathematical progress and attainment, and any other worries she might have about his schooling or development.

Abigail (parent) – Interview	
Dialogue	Mathematical I-position
(1) I've been in a couple of times, it's me approaching them, I've been in a couple of times asking questions.	(1) I as playing a proactive role in my child's mathematical development

Abigail's competency- and emotionally-related positioning of the 'self' was more complex. When asked what she thought of when somebody says 'mathematics' she responded by briefly mentioning her schooling then repeating the view of herself at school as a position she currently held.

Abigail (parent) - Interview	
Dialogue	Mathematical I-position
Err (1) "Oh god not maths". "I can't do it". (2) I didn't like it. (3) I struggled, that was the (pause) that was the main thing when you say maths (4) I think I'm not good at maths. That is, when somebody says maths, I'm not good at maths.	(1) I as not good at mathematics (2) I as not enjoying mathematics (3) I as struggling with mathematics (4) I as not good at mathematics

Abigail's positioning regarding mathematics showed some chronological and spatial shifts, as well as the influence of social positioning. Her social positions, both specific and general, are shown in Table 7B.

Table 7B Abigail's mathematically-related social positions

Social position	Type of position
I as more successful at mathematics than others	General
I as reflecting the influence of my parents I as replicating the mathematical practices of my parent I as competent at mathematics compared to my sibling I as mathematically unsuccessful compared to my sibling I as similar to my child I as mathematically successful compared to my friends I as competent at mathematics compared to my peers	Specific

Her negative attitude towards her own mathematical ability appeared to be at least partially driven by her experiences with her mother and comparisons to her older sister. At different points she perceived herself as 'I as a competent user of mathematics', 'I as negative towards mathematics' and 'I as struggling with mathematics'. Such mathematical I-positions and their related social positions are shown in the following interview excerpt.

Abigail (parent) - Interview		
Dialogue	Mathematical I-position	Social position
(1) I always felt that I was ok at maths but erm *(i) I probably had a bit of a downer on myself because Sally [sister] was so good at maths. And we had the same teacher, you know, and I always felt that, not that he thought that I wasn't, but a lot of the teachers remembered Sally. She was good at science, she was good at maths and I wasn't amazing at either*. (2) I was passable. *(ii) I was in the top group erm for it all (iii) but I always felt that I had something to prove and I never quite lived up to it.* (3) I felt very, very down, it was very negative I suppose around the time of the mock exams. (4) I was struggling then and I do remember the months leading up to it being very negative for me because I was struggling and I knew I couldn't do it, you know if you know what I mean. *(iv) I was struggling with it but yet my mother was pushing me, "You can do it. You can do it. If Sally can do it then you can do it".*	(1) I as a competent user of mathematics (2) I as a competent user of mathematics (3) I as negative towards mathematics (4) I as struggling with mathematics	*(i) I as mathematically unsuccessful compared to my sibling* *(ii) I as more successful at mathematics than others* *(iii) I as mathematically unsuccessful compared to my sibling* *(iv) I as mathematically unsuccessful compared to my sibling*

Zach: the mathematical 'other'

Abigail utilised a variety of positions when constructing a mathematical identity for Zach, as shown in Table 7C. These show variation in other positioning with Zach at different times being seen as 'good' or 'struggling'. The predominant 'other' positions visible in Abigail's narrative suggested that she perceived him to struggle at maths and have limited self-confidence.

Table 7C Abigail's positioning of Zach

Position	**Type of position**
My child as defensive about mathematics	Behaviourally-related
My child as doing well at school mathematically My child as good at mathematics My child as growing in mathematical self-confidence My child as slow at mathematical activity My child as struggling with mathematics	Competency-related
My child as lacking self confidence in mathematics My child as not enjoying mathematics My child as panicked by mathematics My child as scared by mathematics	Emotionally-related

The feelings and emotions that she perceived in her son are clearly apparent in the positions Abigail constructed in the following example.

Abigail (parent) - Interview	
Dialogue	Other positioning
<u>(A) I think he's scared of if, erm he's in a class or in a year at his school,</u> I know we are going to go onto that in a bit, where there are, there are a lot of high achievers. And whilst he was in the top group last year, because we did have an issue with it last year so it's quite, you know, apt. <u>(B) He was moved down a group purely because he used to panic.</u>	<u>(A) My child as scared by mathematics</u> <u>(B) My child as panicked by mathematics</u>

Table 7D Abigail's social positioning of Zach

Social Position	**Type of position**
My child as comparing himself/herself to his/her peers My child as progressing well in mathematics as defined by teachers	Specific

The social positions Abigail formed for Zach are shown in Table 7D. These show that Abigail perceived her son to be comparing himself to his classmates, leading to him panic because he was not as 'fast' as them. The next example shows 'My child as lacking self confidence in mathematics' and 'My child as slow at mathematical activity' linked to a social position of 'My child as comparing himself/herself to his/her peers'.

Abigail (parent) - Interview		
Dialogue	Other positioning	Social positioning
I mean the main thing we get out of him is, **(A) (I) "I don't like maths". That is the, his big thing. It's not the case that I don't think he doesn't like maths, he doesn't like the fact that (B) he can't do maths as quick as everybody, you know other people. I think that's he's main thing. He just doesn't like it. "I can't do it",** I think that's his main feelings on it, yeah.	(A) My child as lacking self confidence in mathematics (B) My child as slow at mathematical activity	**(I) My child as comparing himself/herself to his/her peers**

Abigail also perceived Zach to be progressing well in his mathematical learning. Part of this identification, and the reason she gives for his position of 'My child as panicked by mathematics', is through comparison to his peers, as shown in the following example.

Abigail (parent) - Interview		
Dialogue	Other positioning	Social positioning
And it was that, the fact that there are a lot of high achievers and whilst (A) Zach is very good at maths (B) **(I) he's not fast, that fast and because when they were doing, like, writing them down off the board he was finishing slightly, taking slightly longer than the others** (C) it was panicking him and, you know, upsetting him. So what they did is they moved him down and (D) **(II) he's now at the top of the second group** and it's given him confidence.	(A) My child as good at mathematics (B) My child as slow at mathematical activity (C) My child as panicked by mathematics (D) My child as growing in mathematical self-confidence	**(I) My child as comparing himself/herself to his/her peers** **(II) My child as progressing well in mathematics as defined by teachers**

7.3.3 Analysis of identity and goals

The positioning outlined above suggests that Abigail saw herself as supportive, involved and competent but still slightly insecure mathematically. Zach was positioned as mathematically able but easily panicked by mathematics and lacking in self-confidence. These and other dialogical positionings are next compared to activity in the parent-child task. Specific positions cannot be matched to specific goals but general trends in goal-related activity can be seen to be symptomatic of 'self' and 'other' positioning. Often each excerpt shows mathematical goals and a mathematical 'self' consistent with several dialogical positions. The three main categories of 'self' and 'other' positioning, related to behaviours, competencies and emotions, are investigated in turn.

Reflections of the social positions shown in Table 7B and Table 7D were less visible because the dialogue did not typically compare or refer to others outside the activity. Some positions related to emotions and competencies could be linked back to social positions. For example Abigail's positioning of Zach as 'My child as panicked by mathematics', which could be seen as a cause of her goal-related activity, was linked to prior reflection on the social position of 'My child as comparing himself/herself to his/her peers'.

Behaviourally-related positions and goals

The dyadic interaction showed Abigail operating in a supportive manner in which she attempted to let Zach complete the questions, and restricted her assistance to probing understanding and highlighting problems.

Abigail was proactive about her involvement with her son's education. Her previous difficulties helping Zach led her to seek support from her son's school. Abigail was then able to use this knowledge of modern primary mathematics to help and support Zach. Some of this was gained from her attendance at a mathematics workshop at Zach's school which covered number lines and, as the quote below attests, chunking and partitioning.

Parent participant: Abigail
We had a maths workshop and I've actually got my notes that I took up on there [*shelf*] erm and also I think it's linked to the website as well they've got some teachers, they've videoed the teachers doing the lessons showing partitioning, chunking and things like that and you can actually refer to it. I haven't had to yet because I remember from, from the workshop. That was really important because they showed us what they were doing and I actually said at the end of the workshop, "I wish we'd had this in Y2 or I wish we'd have had this six months ago because we wouldn't have had the tears, the tantrums and everything that we had".

Abigail's behaviour-related positions are evident in the following example. Here Zach has been trying to solve problem 5. He began with a column subtraction which was incorrect since he did not fully understand the process of decomposition needed in 114-36. He then switched to a number line, a strategy he used successfully in previous questions. He has added 30 to 36 to make 66 but is trying to now add 38 and is getting confused. As a consequence Abigail suggests using a school-based strategy, one that she is aware of from her attendance at a mathematics workshop at her son's school. She encourages him, in lines 105 and 107, to first round to the nearest ten (i.e. 66+4=70) before adding 30. Zach does this and is able to complete the rest of the calculation on his own adding 30+4+30+14=78.

Abigail (parent) and Zach (child - Y4) *Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?*			
Line	Time	Speaker	Dialogue
98	12.59	Abigail	Well why don't you, go up in a few, round it up first?
99	13.02	Zach	Hold on
100	13.03	Abigail	Ok
101	13.15	Zach	Mm
102	13:17	Abigail	I think you are trying to jump too much there. (Pause 4s) Ok so what does that make then? (Pause 10s) Zach why don't you, like Mr Darcy [*teacher*] showed you what to do right
103	13.59	Zach	Mm
104	14.00	Abigail	Put sixty-six there yeah. Why don't you round that up to your nearest ten? Yeah do you see what I mean? *Zach rubs out part of the number line*
105	14.13	Abigail	So what could you round that up to? *Zach draws a jump to 70 and writes +4 inside*
106	14.25	Abigail	Ok and now carry on. (Pause 3s) What's the next thing that you could round it up to? Because you've got units, tens and...
107	14.34	Zach	Eighty

108	14.35	Abigail	Or you've got your
109	14.36	Zach	Hundred
110	14.37	Abigail	Right ok
			Zach next draws +30 to 100 then +14 to 114

Positions such as 'I as supporting my child's mathematical development', 'I as assertive regarding my child's mathematical development' and 'I as playing a proactive role in my child's mathematical development' all appeared to give Abigail the necessary tools to support school mathematical goal construction. This is evident in her desire to access school mathematical forms and support Zach at home, shown in the above context of rounding and in Abigail's knowledge and understanding of number lines. Abigail's behaviour-related positioning of the 'self' clearly influenced the goal she constructed and, more consistently, enabled her to support the goals Zach formed and operated.

Zach did not appear defensive, Abigail's behaviourally-related positioning of him, during the task, though this was just a single event of parent-child mathematical activity. Whilst she supported his goals and often challenged him, as in lines 102 and 104 above, she never criticised his mathematical or suggested her mathematics over school mathematics. A reason for this may be that she positioned him as 'My child as defensive about mathematics'

If we look at the wider sample then we see other examples of behaviourally-related positioning of the 'self' and 'other' influencing mathematical goals. For example, Robert positioned himself as 'I as co-operative during mathematical interaction', 'I as encouraging my child's mathematical development' and 'I as supporting my child's mathematical development'. He positioned his son Alex as 'My child as lacking motivation regarding mathematics' and 'My child as struggling to maintain concentration during mathematical activity'. In this next excerpt it is possible to see these positions in both parent and child. Firstly, in line 33 Alex gets distracted by his hamster and then, in line 35, breaks his pencil. Robert tries to maintain his son's focus on the task at several points. He encourages him to start the problem in line 34. Robert then prompts the calculation in lines 36 and 38. He also praises Alex in line 43. When Alex appears to question how many more he has to do, suggesting a lack of motivation, his father, in line 44, focuses back on the task and prompts him to continue the activity by placing the next question in front of him.

Robert (parent) and Alex (child - Y4) *Katie has 44 stickers. She lost some of them. She now has 9 stickers. How many stickers did she lose?*			
Line	Time	Speaker	Dialogue
33	3.33	Alex	Katie had forty four stickers. She lost some of them. She now has nine stickers. How many stickers did she lose? [*Alex gets distracted by his hamster*] Rocky wants to play.
34	3.50	Robert	Come on then we'll do this one
35	3.59	Alex	Whoops [*pencil breaks*]
36	4.01	Robert	It's alright if that one's broke use one of the others. Come on now have you got? (Pause 2s) How many did she have to start with?
37	4.09	Alex	Forty four so that's...
38	4.12	Robert	How many has she got left now? *Alex draws a number line from 9 to 44 then jumps back from 44 -30 to 14, -4 to 10, then -1 to 9*
39	4.45	Alex	Thirty, four, thirty five
40	4.50	Robert	How many?
41	4.51	Alex	Thirty five
42	4.52	Robert	Right good boy.
43	4.59	Alex	How many more have I got?
44	5.01	Robert	No, no [*Robert places next question in front of Alex*]

Throughout the ten word problems they completed in the task, Robert maintained a very structured co-operative approach, producing goal-related activity that maintained his son's focus and narrowed his opportunities for distraction. His behaviourally-related of the 'other' shaped and defined parent-child activity.

Competency-related positions and goals

The majority of mathematical goals in the interaction between Abigail and Zach were formed and completed by Zach. This was not uncommon in the overall sample, as discussed in the previous chapter. On the surface this suggests that Zach was well acquainted with the mathematical practice of problem solving and was competent enough to complete the questions with limited support. However, looking in greater depth at the dialogue from the task it was evident that Zach often needed assistance to solve computational goals, usually the most complex elements of the problem solving process.

Abigail's behaviour of assisting and supporting Zach's mathematical goals, and ensuring he completed the goals rather than concluding them herself, corresponds with her mathematical I-position of ‘I as supporting my child’s mathematical development’. Her facilitating behaviour, often accompanied by praise following Zach’s completion of activity structure-linked goals, could be seen as a reflection of her positioning of Zach as ‘My child as lacking self confidence in mathematics’ or even ‘My child as panicked by mathematics’.

Abigail (parent) and Zach (child - Y4) Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table?			
Line	Time	Speaker	Dialogue
37	4.50	Zach	To a draw another, do I draw another number line?
38	4.53	Abigail	Err I don’t know (laughs) let’s have a read of it first. (Pause 6s) So what do you have to, so it’s asking you how many?
39	5.06	Zach	Forks on the table
40	5.08	Abigail	Right so how many spoons are on the table to start with?
41	5.10	Zach	Twenty-eight
42	5.12	Abigail	Ok, so, what are you doing then? Are you minusing?
43	5.18	Zach	I’ve got an idea
44	5.21	Abigail	Do you know what you are doing then?
45	5.22	Zach	Yeah *Zach begins to write out a column subtraction for 28-7=21*
46	5.24	Abigail	Mmhuh
47	5.29	Zach	Mm *Zach works on the column subtraction.*
48	5.41	Zach	Twenty-one!
49	5.42	Abigail	Mm right. (Pause 4s) I’m impressed with that.

These positions are all visible in the above example. Here Zach has read the question but is unsure about the form. At this stage it has not been possible to ascertain whether he has selected an operation, though given the manner in which he only uses number lines for complementary addition he probably has this in mind. The example begins, in line 37, with Zach checking his approach with his mother. She is unsure so reads the question and summarises the key points for him. She then probes his understanding and in line 42 checks whether he is doing a subtraction. Rather than responding to her prompt he decides upon a column subtraction which he then pursues. Abigail is content to give him autonomy to follow this goal. Zach completes the task and Abigail praises his efforts, supporting his self-confidence, in line 49.

Abigail's supportive behaviour is not necessarily the key point to take from this exchange, rather that her behaviour in this example and throughout the task reflects her positioning of Zach's competencies, for instance 'My child as growing in mathematical self-confidence', far more than Abigail's own competency positions of 'I as finding mathematics difficult', 'I as not good at mathematics', 'I as a novice and learning mathematically from my child' or 'I as struggling with mathematics'.

When looking at Abigail's activity in the task there is no evidence of the majority of competency-related positions in Table 7A. The only position apparent in the task is of 'I as a competent user of mathematics'. The negative competencies are not displayed, possibly because Abigail does not want to show this to Zach or because she believes it would hinder his activity, especially given her positioning of him as struggling and not enjoying mathematics and his previous tantrums that she recounted.

Considering the competencies ascribed to Zach, within the interaction it is possible to see a child who has limited self-confidence in mathematics and approaches the task in a steady, methodical manner. Of the positions listed earlier in Table 7C it is possible to see 'My child as good at mathematics' in the manner in which he complete goals within the activity and 'My child as slow at mathematical activity' in the time it takes him to methodically work through problems.

His ability to complete the majority of goals in the task unassisted supports Abigail's view of him as doing well and being good at mathematics, not him struggling at the subject. In the parent-child task Abigail routinely encouraged and praised Zach and tells him not to worry about time (shown in a later example in this chapter). This reflected her positioning of him as panicked by mathematics because he was not as quick as other children.

In terms of competency-related positions, it is positioning the 'other' that drives goal-related activity more than positioning the 'self'. This was particularly apparent in the case of Jennifer and Jacob. Here Jennifer created strong positions for Jacob as highly-able at mathematics and gave him a great deal of autonomy in the task. He got many problems incorrect but his mother's faith in his ability appeared to lead to her paying only cursory attention to checking his answers.

This gave the impression that she limited her goals because of the competency-related positions she formed for Jacob. This is shown in the following observation made by the researcher during the parent-child task.

Jennifer (parent) and Jacob (child - Y4) Katie has 159 marbles. If Josh buys 36 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?
00:05.50 – 00:07.35 Jacob seems to read then think about this question for half a minute before he starts to write out an expanded column decomposition for 159-36. Again he makes an error in the subtraction of the units. He swaps 6 with 9 but still writes the correct element 3. Under the sum he writes 100+20+3 but does not complete the calculation and write the 123. Jennifer is still sat at the table.

Emotionally-related positions and goals

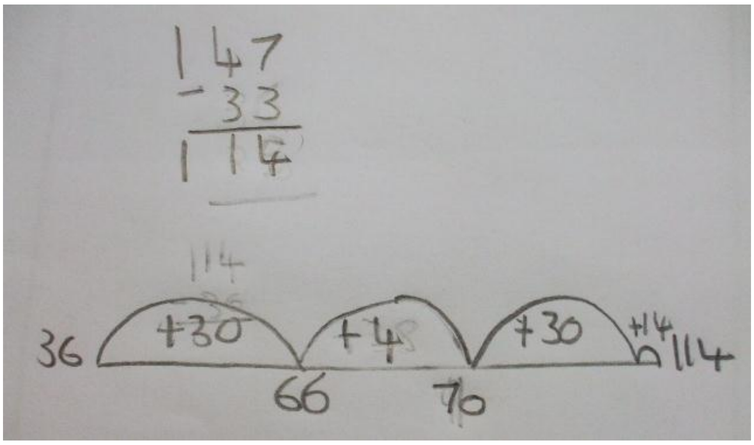
There was little evidence of Abigail's I-positions linked to feelings and emotions shaping goal construction. She did not appear negative towards mathematics, pressured or antagonistic towards the activity. Whether she felt these things was uncertain but if anything she responded in a manner opposite to her emotionally-related I-positions.

Similarly, Zach did not appear to get despondent or experience other negative emotions during the task. This could have been due to his mother's supportive behaviour attempting to avoid these reactions, or his ability to complete the majority of goals without parental intervention.

This lack of negative emotions is apparent in excerpt presented shortly, where Abigail helped Zach overcome a difficultly where his chosen mathematical form did not match his knowledge and understanding.

During the task Zach used two written mathematical forms. Number lines (using complementary addition) were used in problems 1, 3, 5, 6, 7 and 8. Column subtraction was used in problems 2, 4 and 5. In problem 5, the latter part of which was presented earlier, Zach used both a column subtraction and a number line, shown below in Figure 7C.

Figure 7C Column subtraction (147-33) and number line (114-36) used by Zach to solve 147-33-36



Generally, Zach's written mathematical forms were conventional approaches common in primary classrooms. He followed goals in line with these mathematical understandings, for instance his preference for number lines for subtraction due to a lack of understanding of column subtraction. This could be expected given his age and the typical focus upon number lines in early Key Stage 2.

In the following example Zach has written out a column subtraction 147-36-33, possibly suggesting an insecure knowledge of this form. In line 68 Abigail suggests it might be more efficient or easier to break down into two parts. Zach does not get discouraged or upset by this and in agrees to follow his mother's goal erasing 33 to leave 147-36. She directs him to look at the question again in line 73. His mistake does not unduly perturb him and he replaces 36 with 33 (to make 147-33). Next he calculates an incorrect answer, which Abigail asks him to check. He corrects it without comment. By line 80 Zach appears to realise that he cannot subtract 6 from 4 (11**4**-3**6**) suggesting a lack of understanding concerning decomposition. Abigail sees his difficultly and probes his understanding about how he could tackle the calculation. He initially appears unsure but eventually decides to draw a number line, the completed version of which is shown in Figure 7C.

Abigail (parent) and Zach (child - Y4) *Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?*			
Line	Time	Speaker	Dialogue
68	9.10	Abigail	Well do you think that you might need to break it down? Because it's like two questions isn't it so if you look at it like that
69	9.15	Zach	Mm
70	9.17	Abigail	So could you do that one first do you think?
71	9.19	Zach	Ok
72	9.21	Abigail	You don't need to rub it all out do you? *Zach rubs out 33 leaving 147-36*
73	9.30	Abigail	Look, look at the question again. Look at the first half of the question.
74	9.35	Zach	Yeah one hundred and forty-seven, oh *Zach sees his error and rubs out 36 then replaces it with 33 giving 147-33.*
75	10.18	Abigail	Count up from three *Zach continues with the calculation writing an answer of 115 [incorrect]*
76	10.26	Abigail	Now is that right? *Zach checks then rubs out the 5*
77	10.38	Abigail	Ok *Zach replaces the 5 with a 4 to read 114*
78	10.46	Abigail	Ok so then you can do the second question can't you? We'll see, sorry go on *Zach writes a column subtraction 114-36*
79	10.59	Abigail	Mm
80	11.10	Zach	Oh
81	11.13	Abigail	So what do you want to do for that one then?
82	11.15	Zach	Err
83	11.19	Abigail	To have to work it out, can you do, can you do that one?
84	11.21	Zach	No
85	11.22	Abigail	Right so is there another way you could do
86	11.24	Zach	Erm, number line
87	11.26	Abigail	Ok *Zach rubs out the column subtraction 114-36.*
88	11.28	Abigail	You don't have to rush. *Zach draws a number line from 36 to 114.*

Throughout this example Zach had to repeatedly alter and amend his mathematical goals and activity. He did not display any of the negative emotional 'other' positions he was associated with. This was possibly because of the supportive mathematical 'self' visible in Abigail's activity and even, including in line 88, her positioning of Zach as 'My child as slow at mathematical activity' and 'My child as panicked by mathematics', when he felt he had to rush. Another possible explanation is that the presence of the researcher caused Zach to behave in a manner different to previous school-related mathematics sessions with his mother. The emotionally-related positions assigned to Zach, the 'other', did not drive his goal construction but wariness of them may have driven his mother's activity. Abigail, like many of the parents in the sample, had negative experiences with mathematics. However like other similar parents she did not want her experiences and attitude to influence her son. She did not want him to experience the same feelings and emotions that she associated with mathematics. Here the positioning of the 'other' is again a stronger reflection of parental mathematical activity than positioning the 'self'.

Across the sample, as suggested earlier in Figure 7B, there appeared strong influences between emotionally-related 'other' positions and goal-related activity, and weak influences between emotionally-related I-positions and goal-related activity. Another example of this was the interaction between Imogen and Owen.

Imogen positioned herself as 'I as apprehensive of mathematics', 'I as not enjoying mathematics' and 'I as scared of mathematics'. Her emotionally-related positions for Owen included: 'My child as frustrated by mathematics', My child as lacking self-confidence in mathematics', 'My child as not enjoying mathematics' and 'My child as panicked by mathematics'. The next example shows the pair working together on the second word problem in the task. It shows Imogen ignoring her own emotionally-related positioning and shaping her activity towards mitigating the negative positions she holds for Owen. She regularly praises and supports his self-confidence. She simplifies the task and even completes goals, which she deems acceptable such as defining the operation but not concluding the calculation, preventing frustration or other emotionally-negative consequences.

In line 67-74 Imogen supports Owen's reading of the question, prompting him when he hesitates or struggles. Once this goal is completed, in line 77, she probes his understanding and identifies the operation, again simplifying the task for Owen. After he correctly selects the subtractive element she praises his efforts in line 79. Imogen then prompts him to write the calculation on the problem sheet. When he appears to struggle with 28-7, in line 82, Imogen's goal alters towards simplifying the calculation by promoting a number of different forms. Eventually, in line 100, after support and encouragement Owen is able to complete the calculation. This is followed by heavy praise from his mother.

Imogen (parent) and Owen (child - Y4) *Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table?*			
Line	Time	Speaker	Dialogue
67	10.17	Imogen	Right are you ready
68	10.18	Owen	Josh is in the kitchen. There are twenty spoo...
69	10.35	Imogen	Spoons
70	10.36	Owen	...spoons on the table. He puts seven of them out...
71	10.50	Imogen	Away
72	10.51	Owen	...away so there would be the s...
73	10.59	Imogen	Same
74	11.00	Owen	...same number of spoons as forks on the table. How many forks are on the table?
75	11.23	Imogen	So how many, so they've got the same amount...
76	11.26	Owen	Yeah
77	11.27	Imogen	...of forks and spoons and he had twenty eight spoons. So what have you got to do? What have you got to take away from the twenty eight?
78	11.36	Owen	Seven
79	11.37	Imogen	Good boy. Right so do you want to put twenty eight take away on there *Owen writes out 28-7=*
80	11.47	Owen	[whispering] Twenty eight take away seven
81	11.58	Imogen	Do you know the answer?
82	12.02	Owen	Err
83	12.04	Imogen	Do you want to separate it out again? What about doing it again like this? We've got twenty, how many tens are in twenty?
84	12.15	Owen	Two
85	12.16	Imogen	So we've got two tens and we've got an eight right? So do you want to take the seven away from the eight, take what's that seven from eight?
86	12.26	Owen	[whispering] Seven, seven, six...
87	12.33	Imogen	What are you left with?
88	12.38	Owen	[whispering]...five, four, three, two, one, zero
89	12.50	Imogen	No you're taking seven. So what did, what did you say? You said the number
90	12.57	Owen	One
91	12.58	Imogen	Good boy, right so we've got ten and ten and one so can you add those together?
92	13.08	Owen	Ok
93	13.09	Imogen	So that's ten add ten
94	13.12	Owen	I know
95	13.13	Imogen	You know
96	13.15	Owen	Ten, ten
97	13.19	Imogen	It's your ten times table, ten
98	13.23	Owen	Ten
99	13.23	Imogen	Ten
100	13.25	Owen	I know, twenty one
101	13.27	Imogen	Good boy, put it on there then fantastic (pause 3s). So

			it's, there's twenty one, you're here, so there's twenty one.

7.4 Discussion

This chapter sought to investigate the link between identity and goals by combining the analyses undertaken earlier on dialogical identity and mathematical activity. It produced a number of findings highly-relevant to understanding how identity influences parent-child mathematical interaction.

7.4.1 RQ5: How does mathematical identity influence parent-child school-related mathematical activity?

By comparing the findings of the analysis of both dialogical identity and mathematical goals it was possible to see that elements of positioning the 'self' and 'other' were clearly visible in parental goal-related activity. In other words certain features of mathematical identity influence parent-child school-related mathematical activity. These include behaviourally-related features of the parental mathematical 'self', along with perceived behaviourally-, competency- and emotionally-related components of mathematical identities extended to children.

This is not to say that parental competency- and emotionally-related positions do not influence goals, just that these are not as visible in parent-child interaction. Likewise parental competency-related positions, how parents see themselves, should not be confused with actual mathematical competency. Ability to do mathematics is clearly going to influence mathematical activity. The suggestion here is that how a parent sees the competency of a child more closely reflects parental activity than how they see themselves.

The general pattern described in Figure 7B, and shown specifically in the examples included in this chapter, is typical for the sample as a whole. Identities created by parents for themselves and their children cannot be directly linked to specific mathematical goals, but goals and goal-related activity can be seen to echo positions.

The results presented here complement and deepen understanding of mathematical identity and activity shown by other authors investigating teacher-pupil and pupil-pupil interaction (e.g. Boaler & Greeno, 2000; Esmonde, 2006, 2009). For instance, Boaler and Greeno (2000) showed how identities brought to the classroom influenced activity in the classroom. The analysis here shows the influence upon activity of the different positions that constitute identity.

Returning to the model proposed by Nasir (2002), and shown in Figure 7A, it is possible to state that there is a clear relationship between *elements* of identity and goals. It is clear that goals can be seen to form in response to some identity positions. Logically, given the theory behind the dialogical self and the construction of 'self' and 'other' seen in this thesis, the success or failure of certain goals, perhaps in exam or other highly-valorised task, could result in a spatiotemporal position shift thereby influencing identity. This supports the bi-directional relationship between identity and goals proffered by Nasir (2002).

Moving onto the other elements in the model shown in Figure 7A, firstly goals and learning. If how someone positions themselves, or others, influences their goals in an activity then it also therefore influences their learning. This is because goals themselves reflect learning (Saxe, 1991) and are linked to participation in cultural practices (Rogoff, 1991). It is through such participation that the external becomes internalised. Likewise, learning influences goals, as shown in the role of prior experiences in emergent goal formation. A two-way relationship between identity and learning is also logical. Certainly, the analysis shown here and in chapter 5 links experiences of learning mathematics with the mathematical I-positions people assume. The findings in this chapter also affirm this link between learning and identity proposed by Ligorio (2010). She discussed the relationship between identity and learning by drawing together the sociocultural tradition and dialogical self theory. From this she suggested that:

> Learning is not only a cognitive and social experience, but also an identity experience. Who we are, what we are able to do, and what we will be, based on what we learn, are constantly challenged when we attend learning situations.
>
> Ligorio, 2010, p .97

Without doubt further in-depth analysis of the bidirectional nature of the relationships between identity, learning and goals would shed further light on the strength of the model in Figure 7A. However, the results presented here, and the theories underlining this project, show how mathematical identity influences parent-child school-related mathematical activity.

In evaluating the findings presented in this chapter it is pertinent to reflect upon the role of the researcher and the method of data collection. Since observations occurred on a single occasion it cannot be categorically stated that the interactions analysed here represent 'typical' interactions. Similarly, it is possible that the presence of the researcher altered the typical activity of parent and child.

Next, the final chapter of this dissertation presents a summary of the key findings of this study and its implications for the work of researchers and professionals involved in parental involvement and primary education. It also suggests several limitations of the project and lays out potential further research that could heighten understanding of experiences, identity and activity associated with parent-child school mathematical interaction.

Chapter 8

Conclusion

8.1 Introduction

This final chapter concludes this thesis by providing a summary of the key findings of the research project. It then discusses the implications of these findings in terms of contemporary research and supporting parental involvement in children's mathematical development. Then, the limitations of this project are presented and evaluated. Finally, the debate shifts to potential areas for further research and investigation involving mathematical experiences, identity and activity.

8.2 Summary of key findings

This project set out to answer eight research questions surrounding parental experiences of mathematics and mathematical activity, mathematical identity, and parent-child school mathematical interaction. A summary of key findings relating to each of these research questions is presented below.

8.2.1 RQ1: What techniques, strategies and mechanisms do parents use to support their children's mathematical development?

Thematic analysis of twenty-four parental interviews showed six common strategies were used to support children's mathematical development: propinquity; promoting autonomy; evaluating understanding; challenging; demonstration, modelling and explanation; and research. Many of these mechanisms have also been found, to a greater or lesser extent, by other authors in the UK and USA (Civil & Andrade, 2002; Civil et al., 2008; Hoover-Dempsey et al., 1995; Solomon et al., 2002). As in these similar studies, it was found that not all parents supported their children in the same manner or used the same strategies.

The most common technique appeared to be propinquity. This idea of nearness in time, space or relation, was clearly driven by parents own experiences of mathematical interactions and the valorisations of mathematical practices. This is a new way to conceptualise support.

Through the thematic analysis a number of themes emerged relating to parents' valorisations of past, present and future mathematical activity. These support a number of findings elsewhere (e.g. Civil & Andrade, 2002; Civil et al., 2008; McMullen & Abreu, 2011). In this case valorisations could be seen to be related to both parental experiences and knowledge and understanding of mathematics. Valorisations influenced the strategies, techniques and mechanisms parents used. This was certainly clear when parents held particularly strong views on their own mathematics, which impeded them from supporting their children. It could also be seen in the value parents placed upon involvement in their children's education. Valorisations were seen to have both positive and negative influences on parental behaviour.

8.2.2 RQ2: What barriers do parents face in supporting their children's mathematical development?

A range of barriers emerged from the thematic analysis. These were categorised as: parents' mathematical knowledge and understanding; children's mathematical knowledge and understanding; children's rejection of parents' mathematics; children's tiredness and motivation; and parents not wanting to confuse children. These impediments were often widespread and had a clear influence on parental involvement.

Given the analysis in chapter 4, it can be easily argued that the most important of these was parents' mathematical knowledge and understanding. The findings showed that curricular changes since parents' own education, such as the *National Curriculum* (Department for Education and Employment, 1999a) and *National Numeracy Strategy* (Department for Education and Employment, 1999b), meant that they had insufficient knowledge to support their children. This impeded parental involvement in children's school mathematical education. This supported similar research undertaken in the UK (Abreu & Cline, 2005; Baker et al., 2006; McMullen & Abreu, 2011; Street et al., 2008) and USA (Civil & Bernier, 2006; Hoover-Dempsey et al., 1995; Jackson & Remillard, 2005; Remillard & Jackson, 2006). Narrative structural analysis highlighted the negative emotional connotations of a lack of parental knowledge and understanding, which would be expected to itself impede parent-child school mathematical interaction. It also showed the intertwining of themes and the negative emotional consequences of barriers and impediments. It was evident that parents who had received information on contemporary mathematics teaching were more confident and experienced fewer barriers to parental involvement.

Other significant barriers included parents' not wanting to confuse children by teaching them their own mathematics rather than modern primary school mathematics. This supports existing research (e.g. Abreu et al., 2002; Abreu & Cline, 2005; McMullen & Abreu, 2010). Children's rejection of parents' mathematics was a barrier for many participants. This showed children producing higher valorisations for school mathematics than for their own parents' knowledge. Both these themes were linked to divergent understandings caused by differences in parents' and children's mathematical understandings.

These various findings are importance since, as was discussed at the beginning of this thesis, parent-child interaction appears to be a significant influence on attainment in primary school (Desforges & Abouchaar, 2003). If interaction and hence involvement, which again was shown to be important in mathematical attainment (Duckworth, 2008), are hampered then parents are not fully able to support their children and children are not fully able to realise their mathematical potential.

8.2.3 RQ3: How are parental teaching practices shaped by, and through, communication with schools?

Parental teaching practices were shaped by communication with schools in a number of modes. Access to information appeared to be crucial. Parents valued good quality information regarding homework, children's progress and mathematical strategies and approaches. When schools actively engaged parents through seeking their views, opinions and promoting parental involvement in children's mathematical development, then parents were given more agency to support their children. Conversely, a lack of these elements led to an inhibition of parental teaching practices and involvement. These findings replicated the kind of experiences of many parents involved in large-scale projects improving home-school communication (e.g. Hughes & Pollard, 2006) and significantly extend large-scale questionnaire studies (e.g. Peters et al., 2008). Interestingly, the findings in this thesis appear to show that increasing the quality of involvement does not require large-scale interventions. Some parents in this sample engaged in simple, low-cost, school-run initiatives and felt that they benefited enormously. A little knowledge appears to go a long way in terms of facilitating parental mathematical agency.

8.2.4 RQ4: How do parents dialogically construct identities for themselves and for their children?

Analysing parental narratives using dialogical self theory showed that parental mathematical identity, expressed in the idea of a mathematical 'self', was made up of a multiplicity of positions that varied across space and time. These positions drew upon experiences of mathematical activity from childhood and adulthood. The large number of positions that made up the mathematical 'self' could conflict or oppose other I-positions. Furthermore, it was possible to divide parental mathematical I-positions into behaviourally-, competency-, and emotionally-related positions.

Within parental narratives it was also visible how parents created identities for their children, positioning the 'other'. These positions again met dialogical self theory tenets of multiplicity and spatiotemporal influences. Positioning the other could also be divided into behaviourally-, competency- and emotionally-related positioning.

These results built upon studies involving components of mathematical identity in parent-child, teacher-pupil and pupil-pupil interaction (e.g. Abreu, 2002; Abreu & Cline, 2003; Crafter & Abreu, 2010; Boaler & Greeno, 2000; Esmonde, 2006, 2009; Esmonde et al., 2011; Gorgorió & Prat, 2011; Mullen & Abreu, 2011).

The key contribution to academic knowledge of this analysis was that for the first time mathematical identity of parents and projected identities of children was exhaustively studied and documented using dialogical self theory. This approach provided a more detailed, fine-grained approach to understanding mathematical identity than currently exists in contemporary research.

In terms of research using dialogical self theory, the findings here support research on the fundamental elements of the self (Hermans, 1996; Hermans & Gieser, 2011; Hermans & Hermans-Konopka, 2010; Hermans & Kempen, 1995; Hermans et al., 1992) and work on social positioning (Akkerman & Meijer, 2010;

Aveling & Gillespie, 2008), particularly how social positions become incorporated into the mathematical 'self' and 'other' through a process of reflexivity. In this manner, akin to the process of internalisation where the interpsychological becomes the intrapsychological, the individual reflects on the positions afforded to it by others in the sociocultural sphere and some of these become absorbed into the self. Whilst not discussing reflexivity directly, Gorgorió and Prat (2011) nevertheless clearly show this process in how positions provided to students in the mathematics classroom are absorbed or rejected into students' identities.

Mathematical identities are dynamic and context-related, as would be expected in both sociocultural theory and dialogical self theory. They are constructed through positions founded upon experience and participation in social and cultural activity. Therefore they are often unique and display a degree of individual difference.

8.2.5 RQ5: How does mathematical identity influence parent-child school-related mathematical activity?

Mathematical identity, in the form of positioning the 'self' and 'other' was seen to influence goal-related activity during parent-child interaction. Interestingly, certain components of identity influenced activity more than others. Behaviourally-, competency- and emotionally-related 'other' positions were seen to influence the mathematical goals constructed by parents. This was seen in the way parental behaviour reflected: children's feelings and emotions regarding mathematics, children's perceived competencies at mathematics, and children's attitude towards mathematical activity. Parental mathematical identity was much less important than children's perceived identities. Only parents' behaviourally-related positioning (e.g. 'I as supporting my child's mathematical development') were visible in the interactions. Overall, in this sample it appears that how a parent perceives the mathematical identity of their child is more important in determining mathematical activity than the parent's own mathematical identity.

Deepening the understanding of the influence of mathematical identity on activity, the analysis supported the triad of bi-directional relationships between goals, learning and identity proposed by Nasir (2002). In particular it produced strong findings showing the links between *elements* of identity and goals. This division of stronger and weaker influences on activity is absent in much research on mathematical identity and therefore deepens understanding in this field.

8.2.6 RQ6: How do parents and children form, negotiate and operate mathematical goals?

The analysis of the parent-child simulated school mathematical task showed that parents and children form, negotiate and operate mathematical goals in accordance with the four parameter model of emergent goal construction put forward by Saxe (1991). The model had never before been used to study parent-child school mathematical interaction. The findings presented here support the work of others who have similarly used Saxe's model to study culture and cognitive development in mathematics (e.g. Nasir, 2000a, Guberman et al., 1998).

The structure of school mathematics influenced the activity of the dyad through shaping goals towards the five stages necessary to solve school mathematical word problems: reading the problem text; defining the required operation; selecting a mathematical form; carrying out a computation; and answering the problem. This model adapts and extends the common model of Greer (1997). Children generally set and achieved these goals whilst their parents supported them. Occasionally parents would set and complete goals, generally around selecting the operation, if they felt the child needed such support.

The artefacts and conventions intrinsic to the task effected how parents formed, negotiated and operated mathematical goals. The primary artefact was the problem sheet. This shaped goals around written mathematics and stimulated parents to compare the activity to school mathematics. The conventions, particularly mathematical conventions children brought from their own schooling, influenced the form and calculation goals pursued by children and reflected their numerical understandings. Such conventions also allowed parents to monitor children, which acted to moderate parents' goal-related behaviour.

Social interaction through parental probes and prompts was a clear source of, and influence upon, both children's and parents' mathematical goals. Probes and prompts facilitated reflection and negotiation of goals and were the prime conduit for parental goals-related activity. There was diversity within the sample in terms of parental interactional style with some using more dialogue, probes and prompts and other promoting greater autonomy and offering less direct support.

The prior experiences and mathematical understandings that were brought to the activity by both parents and children influenced goals and goal-related activity. This was visible in how children's goals reflected their level of primary schooling, increasing in sophistication with age. It could also be seen in the goals adopted by parents and how these related to their own school experiences of mathematics.

As with the findings of others using the Saxe (1991) approach, the four parameters were heavily interconnected meaning that goals, and the motives behind them, often emerged due to one or more of the parameters. As in other research on parent-child interaction (e.g. Hyde et al., 2006; Solomon et al., 2002) the task showed that individual differences were present in parental behaviour during parent-child interaction. In this case these differences were observed in goal-related activity. Certainly elements of the five parenting styles detailed by Solomon et al. (2002) - no support, unconditional support, promoting autonomy, proactive involvement, and monitoring – could be seen in the task activity. This was shown in the lack of support provided by Natalie, the unconditional support of Chris, the autonomy promoted by Jennifer, the proactive involvement of Robert and the monitoring of Niamh. However these behaviours did not necessarily occur to the exclusion of others. For instance, Robert could be observed providing autonomy, monitoring and proactive involvement at various times during his interaction with Alex. As might be expect such types of behaviour, as with the contingency and scaffolding elements discussed shortly, appeared to influence goals and goal-related activity. Children given more autonomy created more goals for themselves. Parents who were proactively involved often set goals for their children to accomplish.

8.2.7 RQ7: Is there evidence for contingency shifts in parental behaviour in parent-child interaction?

As with other similar research on parent-child interaction (e.g. Pratt et al., 1992; Saxe et al., 1997; Wood & Middleton, 1975) analysis showed evidence of contingency shifts. These shifts occurred primarily following children experiencing difficulties in the task.

In this case the interesting, and novel, findings were around the link between contingency and goal-related activity. When shifts occurred, ownership or control of mathematical goals moved from children to parents. This included both goal formation and completion.

8.2.8 RQ8: To what extent do parents 'scaffold' learning and conceptual development in mathematics?

The analysis undertaken on scaffolding supported a great number of studies of scaffolding in adult-child interaction. In particular it showed the extent to which parents scaffold learning and conceptual development in mathematics using the eleven scaffolding means and intentions set out by van der Pol et al. (2010). As with previous research, individual differences were seen in how parents scaffold learning, particularly in the way they utilised the different elements of scaffolding. These differences did not appear to be related to mathematical confidence or ability.

It was difficult to directly link scaffolding and goal-related activity but an attempt was made to show how the two were intertwined as suggested by Rogoff (1990) and Tharp and Gallimore (1988). Certainly, elements of scaffolding, for instance reducing degrees of freedom or cognitive structuring, could be seen to be enacted through the setting of appropriate mathematical goals. However the unpicking the links between goals and scaffolding was a complex undertaking, the length of which was outside the scope of this research study that was already focused upon the triad of experiences, identity and activity.

8.3 Implications

This thesis began by discussing the crucial role of parental involvement in children's attainment in primary school mathematics. It showed that factors such as parent-child interaction, supporting schoolwork at home, communication with school, and parental values and aspirations were key mechanisms of involvement, and therefore attainment.

Several of these factors have been studied in this thesis. The outcomes of this have several implications in terms of experiences, identity and activity.

8.3.1 Implications in terms of mathematical experiences

The main implications of this work in terms of understanding parental experiences of mathematics and school mathematical activity, in-line with research elsewhere concerning minority groups in the UK (Abreu & Cline, 2005; Abreu et al., 2003; Baker et al., 2006; McMullen & Abreu, 2011; Street et al., 2008), is that parental involvement is hampered by parents' knowledge and understanding of primary mathematics. This was shown in a sample of twenty-four parents drawn from fourteen schools across five local education authorities, strengthening the suggestion that the pattern is potentially representative of a much larger population.

Negative experiences, with particularly strong emotional connotations, were recalled by many parents in the sample. Whilst this is not a new phenomenon an interesting finding here, which has implications for both policy makers and those involved in primary education, is the impact of 'better' communication from school. This communication provides confidence and agency for parents and facilitates parent-child mathematical interaction. Parental involvement has long been promoted in the UK (Cockcroft, 1982; Department for Children, Schools & Families, 2008; Department for Education & Skills, 2007; Plowden, 1967; Rose, 2009, Taylor, 1977; Williams, 2008). However, aside from individual interventions, often sponsored or funded through local authorities or central government (e.g. Department for Children, Schools & Families, 2007) many schools have limited support for parents. The few parents in this sample who had experienced high-quality interaction with school, in the form of parental workshops on primary mathematics and easy access to teachers, reported more confidence and a heightened ability to support their children. These parents often had negative mathematical identities but were given the tools and ability to support their children.

Potentially minimal investment by schools, both financially and in terms of teachers' time, could have an impact on parental involvement and therefore academic outcomes. This would tackle a great many of the barriers and impediments parents face, as well as potentially positively influencing valorisations. Support could be in the form of face-to-face activities or alternatively printed or web-based resources. A small commitment by schools could both help parents and further support children conceptual understanding in mathematics.

8.3.2 Implications in terms of mathematical identity and activity

Research directed towards the influences of identity on learning is growing. Following in the tradition of sociocultural theory, situated cognition and, now, dialogical self theory this project implies that it should be possible to study mathematical identity across a range of contexts and cultural practices. A polyphony of positions leads to a multitude of potential identities, which come into play during participation in cultural activity. The mathematical identities of the parents in this sample are shown to be different at school, in the home and at work, for instance the negative identity created by Gary at school and the home contrasted with his successful career as an account, a highly numerate discipline. Context influences the positions individuals assume.

Many parents held negative positions because of their experiences of learning mathematics, parent-child interaction and poor home-school communication. If, as discussed above, experiences and communication were improved then mathematical I-positions could also change. Negative competency- and emotionally-related positions would then be replaced, or supplemented by more positive ones. This research shows that parents' behaviourally-related I-positions influence activity. Therefore building positive behaviourally-related positions, where parents felt more confidence and agency, should then lead to more successful parental involvement. Parents who were supported by school highly-valued this support and felt empowered to support their children.

8.3.3 Implications in terms of methodology

In terms of methodological innovation, this study has generated two main implications. Firstly, the qualitative analysis of specific dialogical positions in narratives using dialogical self theory presents an alternate mechanism to those used elsewhere. In particular it contrasts with psychological assessments and ratings scales used in some personality research. Secondly, combining dialogical and goal analyses provides a way to study identity and activity. This has application beyond research in mathematical learning into numerous fields embracing the fundamental sociocultural principles of Vygotsky (1978) and Leont'ev (1981).

8.4 Limitations

As with any academic study, there are several limitations regarding the methodology, analysis and results of this project that should be recognised.

With respect to methodology, a number of points should be reflected upon. Firstly, the sample of parents was largely self-selecting. In other words they were happy to take part in the study, talk about their experiences and be observed undertaking the mathematical task. Because of this it could be argued that the study did not attract participants with low levels of confidence or mathematical fears. This was reflected in the characteristics of the sample, where children tended to be average to above average in terms of mathematical attainment. This could be why these children were confident and happy to take part, whilst low-attaining and lower-confident children may have rejected the opportunity.

Originally, as set out in chapter 3, the study was aimed at 7-9 year olds but then had to be expanded to include children in Years 5 and 6 to establish a larger sample size. Because of this the numbers of children across the sample are heavily weighted towards Year 4, rather than Years 3, 5, and 6. In fact with four dyads in Year 3 and three dyads in each of the Year 5 and 6 groups the results from these pairs do not give overwhelming evidence for comparison of goal-related behaviour. This is particularly the case when considering the influence of prior understanding on emergent goal construction.

The study investigated identity and activity associated with school mathematics in the home using a simulated school mathematical task. Whilst this mimicked the kind of activity parents and children regularly completed as homework it did not originate from school. Similarly, a researcher observing mathematical activity did not normally accompany school mathematics in the home. Even though the analysis suggested that parents and children behaved as if they were completing school mathematics the simulated task is still a possible limitation. Possible ways to mitigate this would be to observe the use of an actual piece of homework or use less directly invasive techniques to record activity, for instance self-recording.

Unlike authors similarly studying mathematical activity (e.g. Saxe et al., 1987; Saxe & Guberman, 1998; Guberman & Saxe, 2000) it was not possible to judge learning and cognition in mathematics. These operate from a quantitative or mixed quantitative-qualitative paradigms and use pre- and post-testing of children to give fixed beginning and end points for conceptualisation. This therefore allows the interpretation of learning in interaction. Because of this, in this project it was only possible to infer current understandings and not development over time (i.e. ontogenesis). Similarly, without pre-testing it is difficult to judge parental goals and guided participation in the zone of proximal development, even though 'expert' and 'novice' roles and scaffolding elements were present.

8.5 Further research

The findings of this project suggested several potential areas for future academic research around the areas of mathematical experiences, identity and activity.

8.5.1 Further research on mathematical experiences

As discussed previously, many similar themes to those reported in chapter 4 have been found in other studies looking at different populations in the UK. In this and other studies, children's motivation and rejection of parents' mathematics have both been reported from the parental perspective (i.e. through parental narratives). They have not been investigated from the child's perspective then, a step further, compared to parents' views. This comparison of valorisations would enlighten further understanding into the reasons for rejection of parents' mathematics and valuing of school mathematics. Does this higher valorisation come about through the physical location of 'school'? Is it due to particular teacher or characteristics of their activity? Is it related to social dynamics or group/peer factors? Is it because of different conceptualisations? If so what counts as 'different'? These and other such questions could be addressed by a deeper investigation into the rejection of parents' mathematics.

Another interesting experience related to parents perceiving children to be 'teachers' of school mathematics. What are the factors and conditions that facilitate this process? In this case it appeared to be related to the flexibility of the role parents formed in parent-child interaction. It could logically also be connected to mathematical confidence in children? Are there ways to incorporate activities into mathematical school learning that allow children to take more control and ownership of their home-based learning? Given the importance of involvement, children's role in communicating school mathematics and homework, and divergent understandings, this could be a source of interesting research.

8.5.2 Further research on mathematical identity

The emphasis in this project was very much on parents' mathematical 'self' and the identities that they constructed for their children. This was enabled by the collection of multiple narratives concerning values, aspirations and experiences connected to mathematics. It would be useful to find out more about children's own co-constructed mathematical identities, and even how they position their parent as a mathematical 'other'. This would certainly provide for a richer understanding of how identity influences activity. Such a focus was discounted early in the formulation of the project given the difficulties of obtaining long narratives from children. However, other methods such as the Personal Position Repertoire (Hermans, 2003) or the Personality Web (Raggatt, 2000) do not necessarily require narratives and can involve ranking tasks. Simplified versions of these, or something similar but designed specifically for 7-11 year old children, could be used to study the mathematical 'self' and 'other' of children. This could impact not just on the study of mathematical identity, but also on mathematical activity, potentially in a variety of sociocultural contexts.

The model linking identity, goals and learning produced by Nasir (2002) was considered as part of this thesis. However not all these linkages could be analysed in this project. Further research, involving a longitudinal design with different data collection time points, could further increase knowledge of the relationship between these elements. For instance, goals could be observed and compared to changes in mathematical learning and conceptual understanding as a child progressed through primary school. Evolution in dialogical identity via I-positions and social positions could be assessed through regular interviewing. This would probably require repeated observation and interviews with parents and children, as well as testing of children to track learning.

8.5.3 Further research on mathematical activity

Following the findings concerning parents who had received little support and those that had attended school-based mathematical workshops, a number of research questions naturally emerge. How do parental goals and goal-related activity, and also potentially identity, alter before and after such workshops. Does the schoolwork completed at home, and the overall mathematical attainment of children, improve if parents are engaged by schools? How does this work in terms of activity? Such a study could involve both qualitative and quantitative measures. It could occur in contrasting socioeconomic or ethnic settings, since parental involvement does appear to differ across different groups in the UK (Peters et al., 2007).

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Appendix A: Participant information sheet

Participant Information Sheet

Principal Researcher: Richard Newton
Institute for Research in Child Development, B1.01 Buckley, School of Social Sciences and Law, Oxford Brookes University, Gipsy Lane, OXFORD, OX3 0BP.
Phone: 01865 483776. Email: richard.newton@brookes.ac.uk
Website: http://psych.brookes.ac.uk/research/maths

Dear Parent,

Parent-child interactions on school-related mathematics

You are being invited to participate in the above research study. Before you decide whether or not to take part, it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully.

What is the project's purpose?

This project has been designed to study how parents and children work together on school-related mathematical tasks and support children's understanding of concepts around subtraction.

Why are my child and I being invited to take part?

A great deal of research shows that parental involvement and support is an important influence on children attainment in primary school. However, little research has taken place into how parents and children work together in the home to support children's learning of subtraction in primary school. Therefore, this project looks at how parents and children work on school mathematics in the home and how this interaction is influenced by communication between parents, teachers and schools.

What will happen if my child and I take part?

Firstly, you and your child will be asked to complete a mathematical task on subtraction. This activity should last no longer than 20 minutes and will ideally take place in your home. The activity will be audio recorded by the researcher. Afterwards you will both be invited to talk about your experiences, views and opinions of the task. An audio recording of this will also be made. A transcript of the activity and discussion can be provided to you later.

Secondly, you will be asked to take part in a 60 minute interview with the researcher in your home, or at an agreeable neutral venue. In the interview you will be asked questions about your own experiences of mathematics, communication with your child's teacher and school, and how you support your child's learning at home. This interview will be audio recorded. A transcript of the interview can be provided to you.

All the information that is collected will be kept strictly confidential and made anonymous. You and your child will not be identified in any reports or publications. In accordance with the University's policy on Academic Integrity, the data used during the course of this project will be kept securely in paper or electronic forms for a period of five years after the completion of the project. At any time you can request a copy of the data held on, and supplied by you and your child. At any time you can withdraw your permission for you and your child to take part, at which point any data concerning you both will be destroyed.

What are the possible benefits of taking part?

Whilst there are no immediate benefits for those people participating in the project, it is hoped that the results of the study can be published and disseminated to support parents, pupils and schools in improving mathematical understanding and attainment.

What are the possible disadvantages of taking part?

The disadvantages for the participants are minimal. Participants will be asked to commit to providing around 60 minutes for the interview and 30 minutes of their time for the mathematics activity and discussion afterwards. Whilst the interview requires interviewees to dwell on past experiences it is not intended to be an upsetting or distressing experience. The participant always controls whether or not to answer a question. The activities completed by you and your child are similar to those encountered as part of regular school mathematics, and therefore should not cause unexpected stress. If it appears that you or your child is experiencing stress, or is likely to become upset from attempting the activity, then the session will be terminated. Whilst you and child will not be named in any publication, it may be possible for others aware of your participation to be able to identify views and comments you have provided.

Do my child and I have to take part?

Taking part in the study is entirely voluntary. You are free to withdraw from the research at any point without having to offer any reason for doing so.

What should I do if my child and I wish to take part?

In order to confirm this you and your child will be asked to complete a participant consent form. Please discuss the project with your child to ensure that they understand what their participation entails and that they consent to being audio recorded whilst completing the activity with you.

What will happen to the results of the research project?

The results of the project will be published as part of a PhD thesis. They may also be disseminated in the form of published articles or books. A copy of the completed PhD will be available to be viewed through the Oxford Brookes University library by the end of 2012.

Further information

This project is funded by Oxford Brookes University and supervised by Prof. Guida de Abreu, gabreu@brookes.ac.uk, Department of Psychology, and Dr. Alison Price, aprice@brookes.ac.uk, Westminster Institute of Education. The researcher is completing the project as part of a doctoral qualification. The project has been ethically reviewed, and approved by the University Research Ethics Committee, Oxford Brookes University (Reg. 100457).

If you would like to take part in the study, or have any further questions please contact me via the details provided at the top of this leaflet. If you have any concerns about the way this study is being conducted you may contact the Chair of the University Research Ethics Committee on ethics@brookes.ac.uk.

Thank you for taking the time to read this information sheet.

Appendix B: Participant consent form

Participant Consent Form – Parents and Children

Parent-child interactions on school-related mathematics

Principal Researcher: Richard Newton
Institute for Research in Child Development, B1.01 Buckley,
School of Social Sciences and Law, Oxford Brookes University,
Gipsy Lane, OXFORD, OX3 0BP

Please initial box

1. I confirm that I have read and understand the information sheet for the above study and have had the opportunity to ask questions. ☐
2. I understand that our participation is voluntary and that we are free to withdraw at any time, without giving reason. ☐
3. I agree to take part in the above study. ☐

Please tick box

	Yes	No
4. I agree to the home tasks being audio recorded	☐	☐
5. I agree to any interview being audio recorded	☐	☐
6. I agree to the use of anonymised quotes in publications	☐	☐

Name of parent participant	Date	Signature
Name of child participant	Date	Signature
Name of researcher	Date	Signature

Appendix C: Parent interview schedule

Do you have any questions before we start?

1. Can you tell me a little bit about yourself?
2. What do you associate with the word ‘mathematics’?
3. How does this word make you feel? Can you think of a time when you felt this way?
4. Do you think learning mathematics at school was a positive or negative experience? Can you describe or give an example of why you felt this way?
5. Did you feel it was important to be good at maths when you were at school?
6. Did your parents support your maths when you were at school?
7. Has your view of mathematics changed as you get older?
8. Do you think it is important to be good at maths?
9. What do you think your child feels about maths? Can you think of something they have said or done which shows this?
10. Does your child often talk about what he/she has being learning in class? Can you recall the last time your child spoke to you about their mathematics work in school?
11. How often do you do school mathematics with your child?
12. Does your child enjoy doing maths at home with you? What makes you think this?
13. Do you think it is important for parents to help their children with their school mathematics? Why?
14. Do you feel that the way your child is learning mathematics is the same or different to the way you learned mathematics?
15. Is there a time you have struggled to help your child with their school mathematics? Can you tell me why you felt this way?
16. How does your child react if you show them a way of solving a problem that is different to the way they have learnt in school? Can you remember a situation where you and your child have disagreed about how to tackle a problem?
17. Do you feel you are given enough information to be able to complete school work at home with your child?
18. How often do you contact the school regarding your child’s mathematics? Can you remember a particular case when you have done so?

19. Do you, or your child, receive feedback on the work you have completed at home?
20. Do you feel the school gives you enough information about your child's progress in mathematics? Are you happy with his/her progress?
21. Can you think of a situation where you have discussed your child's maths with their teacher/school?
22. Do you feel you views and opinions on your child's education are taken into account by their teacher/school? Do your priorities differ? Can you think of a time when this was a problem?
23. When you visit the school do you feel welcome? Can you recall a time you have felt particularly welcome/unwelcome?

Is there anything else you would like to say, perhaps you felt something was missing from the interview? Is there something you wanted to say, or perhaps clarify anything we spoke about earlier?

<u>Short Questions</u>

How many children do you have? What school years are they in?

Which age group do you fall into? 20-29, 30-39, 40-49 or 50+

What is your highest formal qualification in mathematics? None, CSE, O-Level, GCSE, A-Level, Degree or other

Do you use maths is your daily life/work? If so when do you use maths?

Appendix D: Word problems used in the parent-child mathematical task

Each child was first given five questions (Set 1) depending on their year group (e.g. Year 3). Their performance dictated the next set of five word problems (Set 2, Set 3 etc.). For instance, as shown below, in Year 3 children would first answer five problems pitched at a Year 3 level (Department for Children, Families and Schools, 2010a). Questions 6 to 10 would either (a) follow the expectations for that year group and get gradually more difficult, or (b) be slightly easier, for children operating below age-related expectations. The former were used if the child had no problems with the first five questions and the latter if they struggled. In Years 5 and 6 a further set of questions set well-below expectations were produced (labelled Set 4 here) but were not used. Children did not necessarily answer all the questions in each set or complete them in a set sequence. They were able to leave or return to a problem later.

Year 3 Word Problems

Set 1. (Questions 1-5)

Katie is 135cm tall. Josh is 109cm. How much shorter is Josh than Katie? Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table? Katie had 44 stickers. She lost some of them. She now has 9 stickers. How many did she lose? Katie has 59 marbles. If Josh buys 16 marbles he will have the same number of marbles as Katie. How many marbles does Josh have? Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?

Set 2. (Questions 6-10: Age-related expectation)

Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have? Josh has 143 Lego pieces. Katie has 59 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie? Josh has 261 counters. He has 134 more counters than Katie. How many counters does Katie have? Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have? Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Set 3. (Questions 6-10: Below age-related expectation)

Josh and Katie have 41 books when they put all their books together. Josh has 19 books. How many books does Katie have? Josh has 28 Lego pieces. Katie has 15 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie? Josh has 67 counters. He has 21 more counters than Katie. How many counters does Katie have? Katie has 53 coins. Josh has 25 coins fewer than Katie. How many coins does Josh have? Josh buys 12 Pokémon cards. He now has 47 Pokémon cards. How many Pokémon cards did he have in the beginning?

An additional set was created for Sam as his mother indicated, when first contacted about the study, that he was operating well below the level expected in Year 3.

Set 4. (Sam) (Questions 1-10)

There were 24 biscuits in a packet. Josh ate 5 biscuits. How many biscuits are left? Josh is nine years old today. Katie is twelve years old today. How many years older than Josh is Katie? Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table? Katie had 44 stickers. She lost some of them. She now has 9 stickers. How many stickers did she lose? Katie has 59 marbles. If Josh buys 16 marbles he will have the same number of marbles as Katie. How many marbles does Josh have? Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now? Josh and Katie have 41 books when they put all their books together. Josh has 19 books. How many books does Katie have? Josh has 28 Lego pieces. Katie has 15 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie? Josh has 67 counters. He has 21 more counters than Katie. How many counters does Katie have? Katie has 53 coins. Josh has 25 coins fewer than Katie. How many coins does Josh have?

Year 4 Word Problems

Set 1. (Questions 1-5) Katie is 135cm tall. Josh is 109cm. How much shorter is Josh than Katie? Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table? Katie had 54 stickers. She lost some of them. She now has 9 stickers. How many did she lose? Katie has 159 marbles. If Josh buys 36 marbles he will have the same number of marbles as Katie. How many marbles does Josh have? Josh had 147 stamps. He gave 33 stamps to Katie. He lost another 36 stamps. How many stamps does Josh have now?

Set 2. (Questions 6-10: Age-related expectation)

Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have?
Josh has 143 Lego pieces. Katie has 109 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 261 counters. He has 134 more counters than Katie. How many counters does Katie have?
Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have?
Josh buys 43 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Set 3. (Questions 6-10: Below age-related expectation)

Josh and Katie have 41 books when they put all their books together. Josh has 27 books. How many books does Katie have?
Josh has 55 Lego pieces. Katie has 23 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 161 counters. He has 34 more counters than Katie. How many counters does Katie have?
Katie has 153 coins. Josh has 25 coins fewer than Katie. How many coins does Josh have?
Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Year 5 Word Problems

Set 1. (Questions 1-5)

Katie is 153cm tall. Josh is 118cm. How much shorter is Josh than Katie?
Josh is in the kitchen. There are 25 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table?
Katie had 84 stickers. She lost some of them. She now has 29 stickers. How many did she lose?
Katie has 259 marbles. If Josh buys 116 marbles he will have the same number of marbles as Katie. How many marbles does Josh have?
Josh had 307 stamps. He gave 118 stamps to Katie. He lost another 43 stamps. How many stamps does Josh have now?

Set 2. (Questions 6-10: Age-related expectation)

Josh and Katie have 1013 beads when they put all their beads together. Josh has 672 beads. How many beads does Katie have?
Josh has 843 Lego pieces. Katie has 359 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 1261 counters. He has 134 more counters than Katie. How many counters does Katie have?
Katie has 1553 coins. Josh has 1275 coins fewer than Katie. How many coins does Josh have?
Josh buys 63 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Set 3. (Questions 6-10: Below age-related expectation)

Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have? Josh has 143 Lego pieces. Katie has 109 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie? Josh has 261 counters. He has 134 more counters than Katie. How many counters does Katie have? Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have? Josh buys 43 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Set 4. (Questions 6-10: Well below age-related expectation)

Josh and Katie have 41 books when they put all their books together. Josh has 27 books. How many books does Katie have? Josh has 55 Lego pieces. Katie has 23 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie? Josh has 161 counters. He has 34 more counters than Katie. How many counters does Katie have? Katie has 153 coins. Josh has 25 coins fewer than Katie. How many coins does Josh have? Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Year 6 Word Problems

Set 1. (Questions 1-5)

Katie is 153cm tall. Josh is 118cm. How much shorter is Josh than Katie? Josh is in the kitchen. There are 25 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table? Katie had 84 stickers. She lost some of them. She now has 29 stickers. How many did she lose? Katie has 259 marbles. If Josh buys 116 marbles he will have the same number of marbles as Katie. How many marbles does Josh have? Josh had 307 stamps. He gave 118 stamps to Katie. He lost another 43 stamps. How many stamps does Josh have now?

Set 2. (Questions 6-10: Age-related expectation)

Josh and Katie have 1013 beads when they put all their beads together. Josh has 672 beads. How many beads does Katie have?
Josh has 1843 Lego pieces. Katie has 1359 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 2261 counters. He has 1134 more counters than Katie. How many counters does Katie have?
Katie has 4553 coins. Josh has 2275 coins fewer than Katie. How many coins does Josh have?
Josh buys 163 Pokémon cards. He now has 281 Pokémon cards. How many Pokémon cards did he have in the beginning?
A fence has three posts, equally spaced. Each post is 15 centimetres wide. The length of the fence is 157 centimetres. Calculate the length of one gap between two posts.

Set 3. (Questions 6-10: Below age-related expectation)

Josh and Katie have 413 books when they put all their books together. Josh has 272 books. How many books does Katie have?
Josh has 843 Lego pieces. Katie has 359 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 1261 counters. He has 134 more counters than Katie. How many counters does Katie have?
Katie has 1553 coins. Josh has 1275 coins fewer than Katie. How many coins does Josh have?
Josh buys 63 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Set 4. (Questions 6-10: Well below age-related expectation)

Josh and Katie have 113 books when they put all their books together. Josh has 72 books. How many books does Katie have?
Josh has 143 Lego pieces. Katie has 109 Lego pieces. How many Lego pieces does Josh have to lose to have as many as Katie?
Josh has 261 counters. He has 134 more counters than Katie. How many counters does Katie have?
Katie has 238 coins. Josh has 112 coins fewer than Katie. How many coins does Josh have?
Josh buys 43 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?

Appendix E - Procedure for coding scaffolding processes

The manner in which parent-child mathematical interactions were studied for the presence of scaffolding processes follows the example of van der Pol et al. (2010, 2011). Their framework originated primarily from the work of Wood et al. (1976) and Tharp and Gallimore (1988). It divided scaffolding process into means (or strategies) and intentions.

Scaffolding means comprise feedback, hints, instructing, explaining, modelling and questioning. Scaffolding intentions include direction maintenance, cognitive structuring, reduction of degrees of freedom, recruitment, and contingency management/frustration control. Parent-child dialogue was studied for each of these eleven aspects of scaffolding.

Means / Strategy	Definition	Example
Feedback	Parent evaluates child's behaviour / work	*Right, ok next, right* - Robert *Well done that's good. Ok that's good* - Suzy
Hints	Parent gives hint concerning problem or does not supply entire answer / instruction	*So you cross that out, then what do you do? That number changes to?* - Lindsay
Instructing	Parent provides information about 'what' the child needs to do and 'how' the child could do it	*So we're doing like a subtraction. We're subtracting the bottom one from the top one* - Ian
Explaining	Parent provides information on why an action / process / solution etc. is appropriate	*Because now look you've got thirteen when you put them together, cross that out, put zero and put a one there and it's still a thirteen isn't it* - Charlotte
Modelling	Parent demonstrates a concept / action / solution etc.	*Thirty-one. Right do you want me to hold my hand up? A hundred and thirty-one...* - Imogen
Questioning	Parent probes understanding, prompts reasoning, requests information etc.	*So break it down then. What have you got to do?* - Chris

The table defines each of the means and intentions and gives at least one example from data collected as part of this study. The definitions draw heavily upon the work of van der Pol et al. (2011 p.56-57) and where appropriate have only been slightly altered to fit with this specific study.

Intentions	Definition	Example
Direction maintenance	Parents takes over elements of the approach to the activity e.g. through hints or instruction	*Read it first then and see if you can do it and if not I'll help you* - Niamh
Cognitive structuring	Child's examples are structured by the parent in terms of the mathematics required for the task. Parent offers general mathematical principles to provide structure for the child's activity	*So you change that into a four and put your ten over there* - Deborah
Reduction of degrees of freedom	Parent takes over elements of activity itself e.g. through modelling or explanation	*So if there were twenty-eight and he put seven away so that there's the same number of forks. So that means that there's seven less* - Vicky
Recruitment	Parent motivates child to engage in activity	*Come on then we'll do this one* - Robert
Contingency management / frustration control	Parent praise, rewards or punishes child's actions. Parent attempts to reduce child's frustration	*There you go. Marvellous. Spot on* - Neil

PARENT-CHILD INTERACTIONS ON PRIMARY SCHOOL-RELATED MATHEMATICS

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This paper reports some initial results and findings of a research project investigating parent and child interaction when completing primary school-style mathematics. It suggests that through using a sociocultural lens and a theoretical and analytical structure based on activity goals we can study how parents and children interact and co-construct learning and conceptual development in primary school-related mathematics. The paper also sketches out how a wider study into the milieu of parent-child interaction on primary school-related mathematics could reap interesting and insightful findings in the UK context.

INTRODUCTION

How a parent supports their child's learning impacts upon that child's attainment in primary school (Morrison, Rimm-Kauffman, & Pianta, 2003). A comprehensive review of contemporary literature on parental involvement, carried out for the UK government by Desforges (2003), showed that the quality and character of parent-child interaction plays a significant role in attainment in primary school. This is supported by large-scale statistical studies (Duckworth, 2008; Peters, Seeds, Goldstein & Coleman, 2007) and UK government policy (DfES, 2007). These suggest that in the UK, attainment at the end of primary school is more closely correlated to types and qualities of parental involvement than social class, income, maternal educational level, or the school attended. Whilst some research in the UK has focused on school- and home–mathematics practices (Abreu & Cline, 2005; Street, Baker & Tomlin, 2006), a limited amount has addressed the dynamics of primary school-related mathematics in the home contexts. In order to address this gap a research project was formed to specifically investigate parent-child interactions on primary school-related mathematics in the UK.

This paper presents a theoretical framework emerging from the study and an initial analysis of a single event of parent-child interaction on primary school-related mathematics. It sets out to begin to answer two research questions: (1) How do parents and children interact and co-construct learning on primary school-related mathematics? (2) How do parents support children's development of conceptual understanding of primary school mathematics? The paper tackles this by analysing an event of parent-child interaction. It concludes by proposing a wider study into some of the factors influencing parent-child interaction.

Theoretical framework

In this project the parent-child interaction is located in sociocultural theories of learning and development, primarily within the work of Vygotsky (1978), Leont'ev (1981), Wertsch (1985) and Saxe (1991). Such a position assumes that learning takes place on the social plane before it is reproduced within the individual. This viewpoint has been used successfully to study the social interaction between parents and children and the resultant co-construction of mathematical knowledge (Anderson & Gold, 2006; Hyde, Else-Quest, Alibali, Knuth, & Romberg, 2006; Saxe, Guberman, & Gearhart, 1987).

Vygotsky's (1978) ideas on mediation, internalization and a zone of proximal development are relevant to developing a theoretical framework to study parent-child interaction. Vygotsky rejected the idea that development was driven by any single factor, and so can not be explained by any single corresponding principle (Wertsch, 1985). The idea that psychological processes develop through 'culturally mediated' activity is at the heart of Vygotskian theory (Cole, 1996). Vygotsky (1978) was primarily preoccupied by the role of language in mediation. He saw language as facilitating social connections and cultural behaviour (Vygotsky, 1997). It is social connection, interaction and transmission of culture that allows the internalization of higher psychological functions. Internalization is not just a mental function it is the formation of a mental plane (Leont'ev, 1981). This formation occurs through cooperation and social interaction (Tharp & Gallimore, 1988). The process of internalization is critical in Vygotsky's (1978) 'general law of cultural development', which states that learning takes place on the social plane before it is reproduced within the individual. In order to ascertain learning and development Vygotsky (1978) developed the concept of the 'zone of proximal development' (ZPD). This allows us to study the difference between assisted and unassisted performance, in other words processes which are advancing or maturing but have not yet been finalised or completed. Because the ZPD is a social and contextual concept it involves some form of negotiation (McLane, 1987). This negotiation takes place between the more capable 'expert', and a less capable 'novice'. Using and interpreting theories of mediation, internalization and the ZPD to study development in cultural contexts is not new, however it is a difficult proposition.

A solution to this problem can be found by utilising elements of Activity Theory (AT) which has been used to operationalise both Vygotsky (Wertsch, 1985) and sociocultural studies in mathematical understanding (Beach, 1995). AT can be traced to the work of Leont'ev (1981). It is a complex entity and difficult to apply in its entirety. Of the many elements within AT, Leont'ev argued for a focus on goal-directed activity as a mechanism for understanding culture and cognition (Nasir, 2002). The centrality of goals to AT is expounded by Nasir and Hand (2006, p.460)

“Activity theory presupposes that all activity is goal directed. These goals, or objectives, manifest differently depending on the level of analysis; taking the activity as the fundamental unit of analysis, these objectives appear as motives. Moving to an individual or group level, motives become directly aligned with conscious goals. Although often explicit, these goals generally emerge over the course of activity”

Saxe et al. (1987) studied the relationship between numerical goals and social and cultural processes. Their basic assumption, which is adopted by this paper, is that

“…children’s numerical understandings are their goal-directed adaptations to their numerical environments, therefore, the study of number development should entail coordinated investigations of children’s emerging abilities to generate numerical goals and the shifting sociocultural organization of their numerical environments”

Saxe et al., 1987, p. 4

This supports the idea that negotiation, interaction and goal-construction plays an important role in emergent and situated cognition. Saxe (1991) shows cognitive developments are enacted through efforts to accomplish numerical goals. He developed a framework for studying the components of these emergent goals at the microgenetic scale. Goals are emergent in the sense that they alter and shift in response to: (1) activity structures, the goals that are formed in the practice; (2) social interactions, where goals are modified and though negotiation take form; (3) artefacts/conventions; and (4) prior understandings. This is termed the four parameter model. This approach has been used in a number of research studies (Guberman & Saxe, 2000; Nasir, 2000, 2002; Saxe, 2002; Saxe and Guberman, 1998). If we accept, as Saxe does, that goals are a reflection of situated cognition, then by studying the goals of parents and children we can study co-construction of knowledge and conceptual understanding in mathematics.

METHODOLOGY

In this paper an instance of parent-child interaction is analysed using the earlier theoretical framework. The participants were a 40 year-old British female and her 10 year-old son. The dyad completed a 30-minute mathematics task which involved a number of subtractive calculations and word problems. This topic was chosen as a particular focus since professional experience, and academic research (Barmby, Bilsborough, Harries, & Higgins, 2009), suggests that children can struggle with different elements of subtractive understandings. Teaching of subtraction has evolved greatly over the past 10-15 years, which means parents may well have different experiences and mathematical representations to their children. The word problems tackled different elements in subtraction and presented different subtractive structures in order to elicit a range of conceptualisations. The task was similarly designed to allow elements of ‘expert-novice’ communication and co-construction of mathematical knowledge. Research on word problems informed the production of the task (Fuson, 1992) as did research on calculation (Anghileri, 2006). The task was designed to replicate the schoolwork parents and children regularly complete together. Whilst this is not a study of actual homework practices, it does look at how parents and children negotiate and co-construct mathematical understanding, and begins to highlight how this interaction is shaped by social and cultural forces. The dyad was video recorded as they completed the task. This video was then transcribed and analysed qualitatively using NVivo 8.

ANALYSIS

The video recording presented a highly complex and rich corpus of data that could have been investigated from a range of directions. This analysis concentrates on the co-construction of mathematical learning evidenced by language use and behaviour. It approaches this from three tiers of complexity. These progressively narrow the focus on the analysis, but in doing so lose elements of their wider interconnectedness. This approach was both emergent, in the sense that it was informed by the data, and theoretical, in the sense that it was informed by relevant research literature.

First tier: Descriptive analysis of mathematical operations and thinking

This first tier of analysis looks at the interaction globally to begin to address the research questions of this project on parent-children co-construction of learning and understanding. In this case it interprets the utterances of the dyad in accordance with theories of goal-directed activity and mathematical principles and understandings. For example, in the following passage the dyad is trying to find the difference between 86 and 64, M refers to the mother and C to the child.

C: Okay, so, count on from 64 to 86 because you add 6 it gets to 70 another 10 so that’s 16.

M: Sorry?

C: 16 I think.

M: You think 16.

C: What do you think?

M: 64, she's got 64 but she had 86…

C: Yeah, Yeah.

M: …so I would kind of...I'd look at my 64 and I probably turn it...I...I'd add it up rather than try to take that figure away.

C: I know that's what I just did.

M: So that's what you've done. So if you have your 64 how many do you need on...4 to make 6?

C: 2 [M writes down on sheet]

M: How many from 6 to make 8?

C: 2 [M writes down 22 on sheet]

The child has a number of different strategies available given the operation and the numbers involved. He decides to use a complementary addition and add, in steps, from 64 to 70 and 70 to 80. However he does not use a third step and add from 80 to 86. This means he reaches an answer of 16 rather than 22. He then follows a procedural objective of seeking M’s confirmation of his correctness. His mother confirms the appropriateness of his strategy and that she would similarly use complementary addition. However, whilst he appears to use a mental number line to count in steps between 64 and 86, she seems to use a mental imaging of a column subtraction. This entails counting the difference between 4 and 6 (64 and 86) then writing 2, and 6 and 8 (64 and 86) writing another 2 to make the number 22. This shows that the two have a different understanding of what it means to ‘add up’ to ‘find’ a difference. This could be linked to contrasting school experiences.

Second tier: Evidence of practice-linked goals through the analysis of emergent goal construction

This second tier looks deeper to try to highlight the parameters linked to the ‘emergent’ goals (Saxe, 1991) formed in this cultural practice. It uses Saxe’s four parameter model to study and explain the practice-linked goals constructed by the dyad. In this case instances of each parameter in the transcript were coded using a simple framework and linked to potential explanations.

The **prior understandings** that are brought to a cultural practice both enable and constrict emergent goals (Saxe, 1991). So children and parents could be expected to construct different goals since they are utilising different mathematical experiences and representations. This assertion is supported by data from the parent-child task. M had a very different primary mathematics experience to her son. This is displayed in the strategies she uses in the task and the barriers she appears to face regarding a familiarity and understanding of the mathematical methods that her son uses. Of the four parameters prior understanding is perhaps the most difficult to determine through the study of interaction alone, initiatives to address this shortcoming are discussed later.

Cultural practices, in this case the **activity structure** of the task, are defined by the motives required to complete them. The goals of one practice may be different from the goals of another. Within the interaction it is possible to see a great deal of evidence of the role that the activity setting has on practice-linked goal formation and the objectives the mother and child pursue. This is shown below in the following subtraction calculation activity. Here the dyad answered a question by following the practices ingrained within the school mathematics-related activity: reading the question, answering the question, and explaining reasoning.

M: Alright… right, let's have a look. Have we read the question?

C: Yeah. Can you solve these subtraction calculations, show your workings. 40 minus 21 equals… 20 away from 40 is 20, and take away 1 is 19.

M: Well ok. Start by writing that out then, so how did you get to that? So how did you first do it? [C writes an explanation, M checks]

C: That's super… right… just pop your answer there, so you got 19.

The task gave a great deal of information about the way in which **social interaction** impacts upon goal construction. There were several cases when one participant suggested a procedure which altered the goal of the other. This usually led to a phase of negotiation around the appropriateness of the procedure. These examples showed mother and child playing out of Vygotskian roles of 'expert' and 'novice' in setting and amending practice-linked goals. The task also provided numerous examples of M explaining or modelling strategies and concepts to scaffold onto C's mathematical understandings.

The dyads' practice-linked goals constructed within the activity are also influenced by the **artefacts and conventions** enmeshed within this cultural practice. Calculations and word problems, similar to those used in the classroom, triggered a certain style of response and practice-linked goal structure (as evidenced in the above example). There was evidence that mathematical artefacts, such as algorithms for subtraction, influenced goal construction in the dyad.

Third Tier: Evidence of the negotiation of mathematical goals

This final tier delves deeper into the interaction to observe how mathematical goals are negotiated, formed and operated. Within the task the dyad appeared to operate through negotiation. There was little conflict or disagreement. There was however several instances of M prompting different mathematical goals and of C needing to reason and justify his choices. A coding framework, informed by the background literature and instances within the transcript, was used to study these negotiation processes within the social interaction.

Table 1 Codes used to study the negotiation of mathematical-linked goals

Code	*Description*	*Code*	*Description*
C1	Agreement with a statement	C11	Abandoning a previous answer or approach
C2	Disagreement with a statement	C12	Appearing to mathematically reason
C3	Probing understand/action	C13	Setting a new mathematical goal
C4	Prompting understanding/action	C14	Accepting the mathematical goal of the other party
C5	Confusion	C15	Rejecting the mathematical goal of the other party
C6	Checking the reasoning of the other party	C16	Abandoning their mathematical goal
C7	Suggesting an answer to a mathematical operation	C17	Suggesting a different mathematical goal
C8	Providing an explanation or model		
C9	Responding to a question or prompt		
C10	Asking the other party whether an argument is right or wrong		

These codes break down the interaction into smaller components in order to view the building blocks of the co-construction of mathematical goals. From this we saw that the parent M tended to allow her child to formulate and operate his own mathematical goals, but she would intervene if she thought his reasoning was flawed or a more efficient method existed. In the following episode, which includes my reflections, we see M and C negotiating how to solve a two-step word problem: *Josh had 307 stamps. He gave 118 stamps to Katie. He lost another 43 stamps. How many stamps does Josh have now?* The problem can be solved in two different forms: 307 – 118 = 189, 189 – 43 = 146; or 118 + 43 = 161, 307 – 161 = 146. M favoured following a goal leading to the first, whilst C preferred the latter goal. Through probing and prompting, giving and answering questions and apparent reasoning we can see how one goal was accepted and another rejected.

Dialogue	Interpretation	Codes
C: How many stamps does Josh have now. So it's 307 take away 118.	*C recognises that the answer can be found by 307-118=x then x-43=y.*	C12
M: Yeah C: So basic...so then he lost 43 so... 118 add 43.	*C sees that he could add the two subtractive elements (118 and 43) then subtract the answer from 307.*	C1 C11 C12
M: I'll tell you what...what we'll do...yeah, you can add your...you could start with the sum and take the 118 from the 307 couldn't you	*M agrees with C but proposes 307-118=x then x-43=y suggesting that C could break it down or (partition) prior to subtraction.*	C15 C4
C: Yeah. M: And you could say...could say right well we'll break that down we'll take the hundred off the 300 first and then we'll take the 18 off... C: That'd be 161 mum wouldn't it? [C points to sum on the paper?]	*C suggests the answer to 118+43 and seeks confirmation.*	C1 C12 C15 C7 C10
M: ...and then we could add the 7 back, yeah. Or you can do it...yeah...you can do it this way, do you find it...	*M recognises that compensation and partitioning does not work well with these numbers. This leads her to think through C's method.*	C12
C: That's 161 [C points to calculation on paper] M: If he's lost 43 and he's given this amount away as well, add those together, and then take that figure off the 307.	*M discusses C's method and recognises that it would work.*	C7 C12 C16 C14 C11
C: So add those two its 161...	*C seeks confirmation of his answer.*	C7
M: Well we'll add it, we'll work it out here, write it	*M accepts C's goal and*	C1

down...write it down. Put your 118 and put your 43 underneath, do it as a sum write it as a sum like you would do at school. Yeah.	*rejects her own*.	C4 C13

Conclusion and ideas further research

In this paper we can see different mechanisms inherent within parent-child interaction, even though these can be difficult to untangle and classify. By operating from a sociocultural viewpoint in terms of study design and analysis we can attempt to answer our two research questions regarding interaction, co-construction and the development of conceptual understanding. This paper, nonetheless, only presents a single story limited to two characters. A much wider study of more parent-child dyads is needed to see if the ideas and findings from this one case are comparable to others and whether any similarities or differences exist. In addition, a focus on the interaction alone is not enough to explain the interaction. Whilst we understand a great deal about how children are taught mathematics we have little indication of parental experiences or mathematical identities. Nasir (2002) has shown how prior experience, motive and identity are important in goal construction. Her model allows the paralleling of the microgenetic study of goal-directed activity with an ontogenetic study of identity and motive. This can be incorporated by episodic interviewing (Flick, 2000) of parents, allowing a greater awareness of the milieu of the parent-child interaction and richer answers to our main research questions. Since research shows that parent-child co-construction of school-related mathematical knowledge is also influenced by factors such as the level and quality of communication between home and school (Hughes & Pollard, 2006) this should also be taken into account. This too can be accomplished through interviewing parents.

These points present a model for the next stages of this inquiry and a way forward to further study some of the elements of parental involvement which have been shown to play such a key role in attainment.

NOTES

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Analysing goals in parent-child mathematical activity

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Aims and Focus:

Parents are commonly involved in supporting their children's learning in two mathematical practices. In home-mathematics parents are involved in integrating informal learning, games and activities involving mathematics into everyday life. In school-mathematics parents are involved in supporting their children's formal education in mathematics. In the UK, a number of research projects have investigated the relationship between these two forms of practice (Abreu & Cline, 2005; Hughes & Pollard, 2006; Street, Baker & Tomlin, 2008). However, in the UK at least, little research has focused on the actual interactions that take place when parents facilitate children's learning in home and school mathematics. This paper aims to directly address the latter of these two practices. It presents the interim findings of a research project exploring how parents and children interact and work together at home when accomplishing school mathematics. It describes how conceptual understanding and development displayed by parents and children can be linked to a number of different social and cultural factors.

Theoretical Framework:

In this report the study of parent-child interaction is located in the sociocultural theories of learning and development promoted by Vygotsky (1978) and expanded by Leont'ev (1981), Wertsch (1981, 1985) and Saxe (1991). Vygotsky dismissed the notion that development could be generated by any single factor or explained by any single corresponding principle (Wertsch, 1985). He supposed that psychological processes develop through culturally mediated activity. Vygotsky (1978) was primarily preoccupied by the role of language in mediation. It is language that facilitates social connections and cultural behaviour (Vygotsky, 1997), and supports the internalization of higher psychological functions through co-operation and social interaction (Tharp & Gallimore, 1988). In this sense internalization,

the formation of a mental plane (Leont'ev, 1981), is fundamental to learning. Vygotsky's (1978) 'general law of cultural development' describes how learning occurs on the social plane (interpsychological) prior to its reconstruction within the individual (intrapsychological plane).
However, such theoretical reasoning around learning is difficult to research practically. Leont'ev (1981) attempted to address this through generating activity theory, which comprises the study of action, activity, goals, objects, operations and motives, in order to understand development. Whilst these elements are all intertwined, both Leont'ev (1981) and Wertsch (1981) argue strongly for a focus on goals in activity to understand learning and development. A goal can be defined as a conscious action carried out as a result of a particular motive (Leont'ev, 1981).

Saxe, Guberman and Gearhart (1987) studied the relationship between goals and social and cultural processes in mathematical activity. They supposed that numerical understanding is a result of goal-directed adaptation to the surrounding environment. This supports the idea that cognitive development is enacted through efforts to accomplish goals (Saxe, 1991).

Saxe (1991) analysed goals that emerged, formed, shifted and evolved in response to four parameters: the structure of the activity, the artefacts and conventions within the practice; the social interaction taking place between those involved; and the prior understandings people bring to a practice. If we accept that goals are a reflection of cognition, then by studying the goals of parents and children we can study their conceptual understanding and development in mathematics. This approach has been used in a number of research studies outside the UK (Guberman & Saxe, 2000; Nasir, 2000, 2002; Saxe, 1991, 2002; Saxe and Guberman, 1998).

Methodology:

This short paper presents some interim findings from the first author's Ph.D. research. The examples used in this paper are taken from a sample of 18 parent-child pairs. The children are all aged between 7 and 11, UK school Years 3-6. The participants all reside in the UK.

Each parent-child pair was observed completing a 20-minute simulated school mathematics task focusing on addition and subtraction word problems. The task was designed to replicate homework that parents and children would regularly complete. The observation was audio-recorded and field notes were taken by the researcher. The transcripts were then analysed in terms of Saxe's (1991) four parameters of: activity structure, artefacts and conventions, social interaction, and prior experiences and understandings. Parents also took part in episodic interviews (Flick, 2000) where they were asked to recount their views and experiences of learning mathematics and of supporting their child's learning of mathematics. Data from the interviews is used here to understand how prior experiences and attitudes may have shaped the behaviour of the parents in the task.

Results:

This section reports results arising from the interim analysis of the manner in which parents construct goals in order to facilitate their children's mathematical learning. Saxe's (1991) analytical themes of activity structure and social interaction are only briefly discussed. A much deeper discussion follows on the themes of artefacts and conventions and prior experiences and understandings. Given the space available in this paper a small number of excerpts are used to illustrate some key points.

<u>Activity structure</u> - Analysis of the parent and child behaviour showed that the practice, or activity structure of the simulated school-mathematics task, shaped the mathematical goals of both parents and children. Parents and children form and carry out goals associated with completing a mathematics task. These motives and actions were seen, in many instances, to attempt to replicate perceptions of learning in schools. In other words, parents and children commonly used school-mathematics procedures and strategies to solve the homework-style task.

<u>Social interaction</u> - Through studying the parent-child dyads it was evident that parents operated a number of different styles of social interaction, such as negotiation, direct instruction, scaffolding, modelling, probing and prompting. These can be seen to operate with differing levels of success. In

some instances they appeared to further understanding in mathematics whilst in others they led to misconceptions. Generally, the type of social interaction employed by the parent and child appeared to reflect the roles they constructed for themselves in the task and their prior experiences and attitudes towards mathematics.

Artefacts and conventions – Within the task the primary artefact is the word problem sheet. Here we see the artefact, which replicated the kind of activities parents and children complete regularly as school-mathematics, influence how parents and children behaved in the home, in terms of providing written answers and written explanations.

Conventions refer to the rules and roles that define participation in an activity. Here it was highly noticeable that parents and children conceived different roles for themselves, taking varying levels of responsibility for setting and facilitating mathematical goals. These roles themselves evolved in response to success or failure. Within the data collected there is no ‘standard’ role, operated by either parents or children. Each parent and child are different. In fact analysis shows that within each interaction the roles children construct change depending on the problem and the on-going social interaction with their parent. The role a parent constructs influences their goal construction and consequently mathematical facilitation.

In Excerpt 1 and 2 we see parent and child roles evolve in response to mathematical difficultly. In Excerpt 1 we see the child (Adam) following the expected role of attempting to answer the question, logically influencing goal construction. We see the parent (Vicky) performing a secondary role, leaving the child to complete the calculation since she does not see a necessity to intervene. In Excerpt 2 we see the roles change, the child (Adam) steps back and hands control to the parent (Vicky). She highlights the operation and calculation in line 14, allowing the child to answer the question correctly.

Excerpt 1: Vicky (parent) and Adam (child aged 7, UK school Year 3).
Parent-Child Task

Problem: *There were 24 biscuits in a packet. Josh ate 5 biscuits. How many were left?*

1. Adam:	There were 24 biscuits in a packet. Josh ate 5 biscuits. How many are... were... left? [*He counts back on his fingers 1 at a time whispering each number*] 24, 23, 22, 21, 20, 19.
2. Vicky:	Mm
3. Adam:	Which I'll write it...
4. Vicky:	Just write it in there [*she indicates a space in the box underneath the question where Adam writes his answer*].

Excerpt 2: Vicky (parent) and Adam (child aged 7, UK school Year 3).
Parent-Child Task

Problem: *Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so that there would be the same number of spoons as forks on the table. How many forks are on the table?*

11. Adam:	Josh... Josh is in the kitchen. There are 28 spoons on the table. He puts 7 of them away so there would be the same number of spoons as forks on the table. How many forks are on the table? [*He taps his pencil on the table as he thinks*]. Can you help me?
12. Vicky:	Yeah. So if there were 28 and he put 7 away so that then there's the same number of forks. So that means that there's 7 less.
13. Adam:	So shall I?
14. Vicky:	So if there's 28 and there's 7 less, how many is there?
15. Adam:	[*He counts down on his fingers*] 27, 26, 25, 24, 23, 22, 21. [*He writes 21 on the sheet]*

<u>Prior experiences and understandings</u> – The prior experiences and understandings parents and children brought to the mathematics task were

shown vividly in the contrasting mathematical conceptions they hold regarding addition and subtraction. These differences sometimes led to confusion and misconceptions. This supports other research in which parental interviews have highlighted parents' difficulties in understanding and operating mental and written mathematical representations that are currently taught in UK primary schools (McMullen and Abreu, 2011). Curricular changes in the UK since the advent of the National Numeracy Strategy (Department for Education and Employment, 1999) and subsequent Primary National Strategy (Department for Education and Skills, 2006) have meant that children are taught a range of strategies that are different to their parents' conceptions of mathematics.

An example of this is shown below in Excerpt 3 where a parent (Rebecca) discusses the problem she faces helping her daughter (Zoë) with addition. The parent sees addition as a column algorithm whereas the child sees it as partitioning and number lines.

Excerpt 4: Rebecca (parent). Parental Interview
The very first time Zoë was sat down doing this partitioning and I just said "Well if you do it like this", like I said "putting the numbers underneath each other". And she just looked at me like I was an alien or something! And I just said "Well that's how, that's how you add Zoë". [*Zoë said*] "No we do this!" and started drawing all these hills [*number line*] to get to numbers and I'm like "well ok" (laughs).

Rebecca had little confidence in mathematics. She had found learning mathematics a negative experience at school, a point she develops in Excerpt 5 when asked how she feels about mathematics.

Excerpt 5: Rebecca (parent). Parental Interview
It makes me think urgh (laughs). I just was never any good at maths at school. And I think because I knew I wasn't, and I don't think you get the help (pause) you got the help then like you do now. And I knew I couldn't do it I didn't (pause) want to do it. So it didn't interest me.

We could argue that Rebecca's prior understanding and experience influence the goals she constructed in Excerpt 6. Here Zoë is trying to find the difference between 72 and 113. She has been trying to count up on her fingers but has already failed once. Rebecca knows her daughter uses more efficient methods, such as number lines and partitioning, but here she does not construct a goal or facilitate her child to use these. Consequently we see Zoë continue to struggle with an inefficient counting goal.

Excerpt 6: Rebecca (parent) and Zoë (child aged 8, UK school Year 4). Parent-Child Task

Problem: *Josh and Katie have 113 beads when they put all their beads together. Josh has 72 beads. How many beads does Katie have?*

144. Rebecca: No, no you're going from 72, you've already got 72 so that's 73 [*points to a mark Zoë made earlier on the paper when she was writing tally marks to support her counting from 72 to 113*] Yeah.

145. Zoë: 74, 75, 76, 77, 78 [*making marks on the paper*]...

146. Rebecca: Yeah 78

147. Zoë: 78, 79 (pauses). I forgot (laughs) (pauses) 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 [*continues making marks on the paper]*.

148. Rebecca: Right so that's 100 yeah. So what's the tens after a hundred there?

149. Zoë: 101?

150. Rebecca: No what's that there? [*pointing at '113' in the question on the sheet*]

151. Zoë: 13

152. Rebecca: Right so add that to them

153. Zoë: 100, 101, 102, 103 [*continues to draw marks but then stops*]

154. Rebecca: (pause) Did you count 13 then?

155. Zoë: No. I am going to count because I lost track and I...

Parents who were more confident with mathematics and had a better understanding of mathematics teaching appeared to find it easier to support their children and assist them in producing mathematical goals. Below, in Excerpt 7, a more confident parent (Robert), who uses mathematics regularly in his work, discusses how his own schooling in mathematics was different to his son's (Thomas). Nonetheless he appears to understand these 'new' methods and tries to support his child's use of them.

Excerpt 7: Robert (parent). Parental Interview

It was done very different to how it's done now [*the teaching of mathematics*] and that's sometimes where I have problems with Thomas. Like the number lines, it's taken me a long time (pause) it doesn't feel right for me but I don't want to sort (pause) My temptation was, "No you do it like this", but I don't want to destroy what the school's doing and so I go along with it even if sometimes I think, "Well, I can't see how this is going to develop later". But I except that often they [*the school*] know best.

In the parent-child task Robert suggested goals and strategies that allowed his son to build on his prior knowledge. In Excerpt 8 Thomas has misinterpreted a word problem. Robert points out the error and then prompts him to write a number line. We also see him encourage his son when Thomas wants to avoid answering the problem.

Excerpt 8: Robert (parent) and Thomas (child aged 8, UK school Year 4). Parent-Child Task

Problem: *Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning?*

140. Thomas: Josh buys 13 Pokémon cards. He now has 181 Pokémon cards. How many Pokémon cards did he have in the beginning? Easy 13!

141. Robert: No, you've... He buys 13, once he's bought those 13 he's got 181. So you need to find out before he'd got those, bought

	those 13, how many? So how are you going to do it? Are you going to do a line again?
142. Thomas:	Do I have to do this one?
143. Robert:	Yeah, so come on, it's a very big number line

Conclusion:

The sociocultural theories of Vygotsky (1978), Leont'ev (1981), Wertsch (1981, 1985) and Saxe (1991) provide us with a lens through which we can study learning and development. Specifically, Saxe's (1991) emergent goal framework provides insights into mathematical learning and the manner in which parents facilitate their children's mathematics.

The results outlined in this report begin to show that the roles parents define for themselves influence goal construction and learning. Parental facilitation is flexible depending on the context. Both parents and children seem to modify their behaviour based on the mathematical situation.

A great deal of research supports the notion that current mathematics pedagogy in the UK acts as a barrier preventing parents from supporting their children's school-mathematics learning (Abreu, 2008; Abreu & Cline, 2005; McMullen and Abreu, 2011; Street, Baker and Tomlin, 2008). This finding was replicated in the interviews carried out in this study. But, uniquely, here we are able to see how this problem influences interaction and facilitation. Using Saxe's (1991) approach we can see how a parent's prior understanding of modern school-mathematics appears to influence how they interact with their child. The barrier provided by contemporary pedagogy is not the same for every parent. Parents with more mathematical confidence, or those who construct highly supportive roles for themselves, appear better able to bridge the gap between their own mathematical knowledge and the mathematical understanding of their children.

This research project has just reached its mid-point and its findings are at an interim stage. More parent-child data will be collected through the summer of 2011. Subsequent analysis will focus on parent-child goals, mathematical representations and strategies, and parental experiences and mathematical identities. It is hoped that by understanding parents' experiences of school-mathematics in the home we can better support parents as mathematical facilitators.

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