High Pressure Angle spur gears for Epicyclic gear trains

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Abstract

Advanced in engineering technology have resulted in increased gearing performances. The use of high power density transmission systems such as epicyclic gear trains is the way to achieve the goal reducing the overall volume and mass compared with traditional configurations. Gears are the main component of the transmissions because they play the crucial role of transmitting the power from the input to the output with a defined ratio. In terms of gear performances, tooth geometry has a direct influence on load carrying capacity: increase the pressure angle modifies the tooth profile with a direct influence on bending and contact stress. To test the benefits of high pressure angles gears in epicyclic transmissions, four different epicyclic systems with same boundary design conditions have been modelled. The reference pressure angle have been varied from 20° to 35° and other gear parameters such as profile shift coefficient, addendum and dedendum length have been modified consequently to match the design requirements. The results show that increasing the pressure angle has a reductive effect on contact and bending stresses. Using high pressure angle gears in epicyclic transmissions has a beneficial effect on tensile stresses but is unfavourable for the compressive ones. Moreover, it has been seen that pressure angle effect might be enhanced or nullified if other modifications such as profile shift are used concurrently.

1. Introduction and Literature Review

The aim of this research work is to develop, evaluate and optimise a new design of gears to satisfy the specific requirements of high output epicyclic torque multipliers suitable for use in hand tools (figure 1).



Figure 1 – Single and multiple stages hand torque multipliers

These devices are typically used for tightening and loosening fastenings on wind turbine assemblies, oil and gas pipeline installations, and in general for applications in which an accurate and quick tightening of high number of fasteners is required.

The gear system provides the power transmission from the user to the output device achieving a mechanical advantage when a high amount of torque is required, especially in limited workspaces. In

those cases, epicyclic gear trains are the most viable solution thanks to their physical disposition with concentric axes that provide weight reduction and compactness, typical properties for applications in which high power density is required [1, 2]. Many dispositions and arrangements exist, Yu [3] has classified a vast family of epicyclic gear trains with appropriate nomenclature. In a simple system (with singular planets), the planets are engaged with the sun placed at the centre and with the ring gear. The planets are held in their relative position by a carrier connected with the output shaft. Considering a planetary configuration, with the internal gear prevented from rotating, the planets orbit around the sun, connected to the input, and provide the output torque through the carrier. The transmitted load is distributed between multiple gear pairs resulting in a highly increased torque carrying capacity [4, 5, 6].

When the first gear standards were introduced, fixed pressure angles were specified, to enable standardisation of tooling, manufacture and geometry. In an attempt to improve the performance of epicyclic gearings, this project will investigate the use of high pressure angle gears for possible use in mechanical applications, particularly for low speed-high torque operating conditions. Despite the large amount of literature available on epicyclic transmissions there is very little information about applications with low rotational speed and high levels of transmitted torque. For these specific conditions the standard AGMA 6123-B06 [7] confirms the gap, underlining the necessity of undertaking a detailed engineering study to satisfy the requirements for design of epicyclic devices. Kapelevich in [1] describes an optimization process of an epicyclic transmission modifying both the gearbox arrangement and the tooth geometry to achieve an increase of power density. A volume function that describes the compactness of the gearbox has been introduced and optimum configurations were found. A volume function, as indicator of the optimization process, is defined also by Hohn et al. [2]; they present three different epicyclic systems designed with optimized gear parameters according with ISO 6336 [8]. A Finite Element Analysis of an epicyclic transmission is performed in [9]; the study focuses on the ring gear stress distribution that has been simulated through a numerical analysis and validated by using strain gauges. The results show a good correlation between numerical and experimental results. Yang et al. [10] have investigated the dynamic behaviour of a planetary gear train and the stress propagation wave during the meshing process.

The project makes the assumption that the involute profile is most suitable for gear applications, it has advantages both in terms of manufacturing and working conditions being relatively easy to produce with cutting machines and, more important, ensures a constant transmission ratio also in presence of errors in the centre distance. More complex tooth geometry is defined through the use of additional modification coefficients such as profile shift to adapt the gear performance to the specific application. Although it is common practice to design gears with standard proportions and cut them by using standard cutters and machines, considerable performance improvements can be obtained by designing and cutting gears with non-standard proportions, including variations in pressure angle.

A study regarding high pressure angle gears has been carried out by NASA for aeronautics and space applications [11, 12]. Three combinations of 20°, 25° and ° and 35° pressure angle gears were tested and analysed with Finite Element Methods. The results show an improved performance in terms of contact and bending stress of high pressure angle gears over the traditional 20°. They also show higher efficiency and a better lubrication when lubricated with grease. Herscovici in [13] describes the advantages of High pressure angle gears compared with "silent gears"; he shows how the pressure angle variation affects the tooth geometry and the load carrying capacity. Referring to his study, high pressure angle gears can carry 16 times as much horsepower with the same volume and with the same fatigue life as the silent gears. In [14] he has designed a gear with a pressure angle of 33.7°, finding a sizeable reduction in terms of surface compressive and bending stresses compared with 25° pressure angle, with a consequent improvement in fatigue life. A more general analysis for various combinations of design, manufacturing and performance parameters of spur and helical gears are illustrated and discussed by Kawalec et al. in [15]. Kapelevich et al, [16] have shown a methodology to define the limits of selection for gear parameters defined "area of existence of involute gears" that allows to define the gear pair parameters that satisfy specific performance requirementsWith the help of Machine Design software (KISSsoft [17]), combined with Finite Element Analysis (FEA), using ANSYS [18], it is possible to identify the optimal configuration to provide the best result for a specific application. This paper is organized as follows: The first section of the paper aims to review the published literature on pressure angle and tooth profile modifications, following with the application of these concepts to epicyclic transmissions. A brief description of pressure angle is followed by the preliminary study on how the tooth shape is modified by varying the pressure angle. Then, the undertaken gear design process is described and the data of the four epicyclic systems generated are shown. The calculated results and discussions about the influence of the pressure angle, profile shift and other parameters are exhibited and followed by conclusions.

2. Preliminary study

To prove the stated effect of a large pressure angle compared with the standard 20°, a preliminary numerical study has been carried out, to investigate the resulting geometry changes.

Two different pressure angles might be defined:

- The reference pressure angle is the angle between the orthogonal to the tangent to the base circle passing through the pitch point and the pitch radius at the pitch point.
- The working pressure angle is the angle formed by the common tangent to the pitch circles and the line tangent to the base circles of the mating gears, also called Pressure Line(Figure 2).



Figure 2 – Basics of gear geometry

Reference and working pressure angles differ when a centre distance modification occurs or an unbalanced profile shift is used. Modifying the pressure angle means effecting an alteration of the tooth shape with a consequent modification of its properties.

For an increased pressure angle, the involute moves away from the base circle by an amount $(d_b=d\cos\alpha)$. This condition determines a profile modification resulting in a thicker tooth base, a reduced radius of curvature of the tooth flank, and also, reduces the occurrence of undercutting which is more evident in gears with small number of teeth. On the other hand, the top land thickness becomes smaller and as consequence, for two gears in mesh, the contact ratio is reduced.

Working pressure angle α_w is determined using the inverse involute function (1) in which $x_{\nu} x_2$ and $z_{\nu} z_2$ are profile shift coefficients and number of teeth of pinion and gear respectively.

$$\operatorname{inv}\alpha_w = \tan \alpha_w - \alpha_w = 2 \tan \alpha \left(\frac{x_1 + x_2}{z_1 + z_2}\right) + \operatorname{inv}\alpha \tag{1}$$

Since the involute function is an iterative function, the following calculation has to be performed repeatedly until the value converges:

$$\alpha_{1} = 1 + \left(\frac{\operatorname{inv}\alpha - \tan 1 + 1}{\tan 1^{2}}\right)$$

$$\alpha_{n} = \alpha_{n-1} + \left(\frac{\operatorname{inv}\alpha - \tan \alpha_{n-1} + \alpha_{n-1}}{\tan \alpha_{n-1}^{2}}\right)$$
(2)

To better understand how the pressure angle variation affects the tooth geometry, 7 tooth profiles have been created as shown in Figure 3.



35° with 2.5° increments of z=20, x=0 gear

The value of 20° has been taken as reference because it is the most commonly used and suggested by the standards ISO, AGMA. The pressure angle has been varied from 20° to 35° with steps of 2.5°. It is clearly visible in Figure 3 how the profile follows the modifications listed above while the pressure angle is increased.

3. Gear design process

For those applications in which very low speed are involved the tooth size is directly dictated by the load carried; the dynamic load would be negligible, vibration and consequent noise are not a problem so stresses dominate the design process. The idea to minimize the number of teeth deals with the necessity to achieve the smallest diameter possible combined with the highest transmission ratio achievable to satisfy the requirement of mass and volume reduction. Moreover, contact stress reduction on the drive flank and bending stress reduction at the tooth root result in a higher torque density [1, 2].

The following design process has been undertaken to model four epicyclic transmissions with a combination of non-standard parameters. Once the boundary conditions were defined, the design procedure has been guided by the pressure angle α while the number of teeth z was fixed. For each gear, undercutting, minimum top land thickness and contact ratio (when in mesh) have been considered and consequent profile modifications such as Profile shift, addendum and dedendum length have been varied to match the requirements imposed by both system boundary conditions and design limitations. As already discussed, increasing the pressure angle has a beneficial effect on gear performance thanks to a thicker tooth base and a reduced profile curvature of the tooth flank. Another main advantage is the direct effect on the condition of preventing undercutting. The following equation (3) gives the number of teeth without undercut as function of pressure angle and profile shift coefficient

$$z_{min} = \frac{2(1-x)}{\sin^2 \alpha} \tag{3}$$

The trend is shown in the figure (4) below in which only the pressure angle has been varied keeping as zero other modification parameters.





Considering a conventional industrial gear with a pressure angle (P.A.) of 20°, the minimum number of teeth without undercutting is 18. The use of non-standard parameters and correction coefficients still allow involute profiles to achieve correct meshing and overcome manufacturing limitations. For small, high reduction ratio gearboxes, as in this case, in which large loads need to be carried within the constraint of minimized diameters, it is common to have large pressure angle gears with a small number of teeth. For a 35° P.A. pinion, the bottom practical limit is 7 teeth. Nevertheless, it is possible to achieve that number of teeth even with a 20° P.A. but other modification factors have to be used as well. The profile shift coefficient, also called addendum modification, is one of the

corrections mainly used in gear manufacturing to minimize undercuts when dealing with low pressure angles and low tooth counts. The following equation allows the necessary profile shift coefficient to be determined for avoiding undercut for a given pressure angle.

х



$$=1-\frac{z}{2}\sin^2\tag{4}$$

Figure 5 - Minimum number of teeth without undercutting as function of Profile shift coefficient for fixed pressure angles α

Figure 5 shows a graph with the minimum number of teeth to avoid undercutting as function of profile shift for four given pressure angles. It can be seen that a 20° P.A. gear with 7 teeth requires a profile shift of +0.6 to avoid the condition of undercutting.



Figure 6 - Working pressure angle [deg] as function of profile shift coefficient for - $1 \le x \le 1$ for z1=20; z2=40

As shown in figure 6, the profile shift coefficient also has an effect on the working pressure angle. As the profile shift varies for the pinion or the gear, if it is not balanced with an opposite correction in the mating gear to make the total profile shift zero, it requires a variation of the reference centre distance, with a consequent modification of the working pressure angle.

Using large pressure angles and positive addendum modifications has many convenient aspects, however, there are two main limitations that have to be taken into consideration. The first, is that the reduction of the top land thickness results in pointed teeth. It is an unwanted condition particularly for hardened gears, because a hardened pointed tooth tends to be brittle at the tip. The top land thickness s_a is defined as follow:

$$s_a = d_a \left\{ \frac{\pi}{2z} + \frac{2x}{z} \tan \alpha + \mathrm{inv}\alpha - \mathrm{inv} \left[\cos^{-1} \left(\frac{zm}{d_a} \cos \alpha \right) \right] \right\}$$
(5)

and is function of pressure angle, profile shift, number of teeth and tip circle diameter.



Figure 7 - Top Land Thickness as function of pressure angle α for fixed number of teeth z and three different profile shift coefficients x

In figure 7 it is shown how the top land thickness varies as function of pressure angle for a fixed number of teeth and fixed profile shift. For positive values of addendum modification the top land thickness drops to smaller values for low pressure angles, compared with zero or negative profile shifts. That was an expected condition given that both pressure angle and addendum modification have a similar effect on the tooth shape [19]. Recommended values of top land thickness span from

0.2 to 0.6 [mm] [20]. Kapelevich [21] suggests a window of proportional values calculated as top land thickness divided by the base Pitch between 0.06 and 0.12.

The second issue is that a reduction of the contact ratio might occur. Considering two or more gears in mesh, the contact ratio c_R , defined as the average number of teeth in contact at one time, has to be taken under consideration. The contact ratio, as defined below, is function of both working pitch and base circle radiuses and working pressure angle.

$$C_{R=\frac{1}{\pi m \cos \alpha_{w}}} \left[\sqrt{(r_{a1}^{2}) - (r_{b1}^{2})} + \sqrt{(r_{a2}^{2}) - (r_{b2}^{2})} - (r_{b1} + r_{b2}) \tan \alpha_{w} \right]$$
(6)



Figure 7 - Contact ratio as function of pressure angle $\boldsymbol{\alpha}$ for two different gear pairs in mesh

For conventional gearing values of c_R are generally in the range of 1.4-1.6 with a bottom limit fixed at 1. For any smaller value the transmission ratio would be unacceptable because at some times the teeth are not in contact with a consequent pulsating torque delivery.

For conventional gears with a number of teeth above the limit of undercut, an increase in pressure angle determines a consequent reduction in contact ratio as shown by the solid line in figure 7. The gain of strength due to an increased pressure angle might compensate for a reduction of contact ratio. Nevertheless, for gears with a small number of teeth in which the undercutting condition occurs, increasing the pressure angle has a beneficial effect (dashed line in figure 7) on contact ratio that goes up to values above one and then decreases. The trend described is a balance between undercutting and pointed teeth. With the contact ratio only slightly greater than 1, contact is occurring very near the pinion tip and very near to the pinion base circle.Considering all the parameters described above and their interactions between each other, four different gear sets have been generated matching the conditions of a planetary gear system with the following characteristics:

- Gearing ratio i= 5.5 ± 5%
- 3 Planet gears
- Centre distance c=22 [mm]
- External overall diameter de=90 [mm]
- Module 2 [mm]
- Number of teeth of sun, planet and ring gear z1=8; z2=14; z3=-37
- Contact Ratio $\cong 1$

The following tables 1, 2, 3, report the main parameters used to describe the tooth profiles of sun, planet and ring gears shown in figure 8.

SUN	20	25	30	35
Dedendum	1.1	1.25	0.9	1.1
Addendum	0.8	0.8	0.9	0.8
Profile shift	0.4374	0.6	0.1543	0.3
Working pitch	16.0	16.0	16.0	16.0
Diameter [mm]				





Table 1 - Sun parameters

PLANET	20	25	30	35
Dedendum	1.25	1.25	1.1	1.1
Addendum	0.8	1	0.8	0.8
Profile shift	0.24	0.412	-0.1543	-0.3
Working pitch	28/27.98	28.0/26.78	28.0/26.78	28.0/26.78
Diameter [mm]				





Table 2 - Planet parameters

RING	20	25	30	35
Dedendum	1.25	1.25	1.1	0.95
Addendum	0.8	0.950	1	1
Profile shift	-0.320	-0.121	0.618	0.776
Working pitch	70.816	70.783	70.783	70.783
Diameter [mm]				

Table 3 - Ring parameters

Figure 8 - comparison between tooth profiles

The combinations of parameter reported in tables 1, 2, 3 have been "modelled" in the Machine Design Software KISSsoft. Moreover, 3D models of gears have been generated in KISSssoft and exported into SolidWorks to assemble the systems and check through a "motion study" to observe whether any interference was occurring during the meshing process. Once the models were ready, they have been exported into ANSYS 16.0 for the FE Analysis that will be described in the next session.

4. Finite Element Analysis

The seven models generated in the preliminary study and visible in figure 2 have been modelled in ANSYS FE software to better understand how the shape modification affects the tooth performances. The loads applied simulate the meshing forces between two mating gears when the contact occurs at the pitch point.? On each tooth flank an area has been created across the pitch circle where the force is applied as visible in Figure 9.



Figure 9 – Meshed geometry and applied force

The area has been calculated using the Hertzian contact theory in which the width of the rectangular contact area as consequence of the contact between two cylinders is described by the relation:

$$b = 4 \int \frac{F\left[\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right]}{L\pi\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$
(7)

In equation 7, F is the Force that pushes the two cylinders against each other; v_1 , v_2 and E_1 , E_2 , are the Poisson ratios and Young's modulus of the two materials in contact; R_1 and R_2 are the curvature radii of the two cylinders and L is the facewidth. As the radius of curvature changes with the pressure angle, the rectangular contact area b depends on α as is shown in Figure 10.



Figure 10 – Deformed distance b and curvature radius R as function of $\boldsymbol{\alpha}$

The tooth profiles have been constrained with a fixed support at the inner rim circle to prevent any movement at their base. The Force of 50 N has been resolved into the tangential and radial components (F_t =Fcos α ; F_r =Fsin α), and applied to the area A=L*a. The facewidth L is 5 [mm]. Results of the described analysis are discussed in section 5.



Figure 11 – Equivalent stress distribution

After the preliminary study, four planetary gear systems, designed as described in the previous section have been the research study of the Finite Element Analysis, with the aim of understanding how the stresses induced within the system were distributed among the components. The outer

face of the ring gear was defined as rigidly constrained, and the other components were defined as deformable. The material was the default structural steel specified in ANSYS. In a planetary configuration, the sun gear inputs the energy into the system which is shared among the planets and delivered to the output through the planet carrier; a reaction arm holds the ring gear fixed. The same boundary conditions were imposed for the analysis. All the displacements of the sun and planet gears were constrained except for their rotation around their own axis. The planet pins were fixed to generate resistance to the torque. A ramped moment from 0 to 50 Nm distributed in one second time step was applied to the sun.

To determine how the contacting bodies can move relative to each other and eliminate penetration phenomenon, the correct type of contact mechanics and formulation between the teeth, and on other surfaces, needs to be defined. At the interface between the planets and pins a frictionless contact has been applied to simulate the presence of a bearing. The contact between the teeth in mesh has been defined as frictional but with a zero friction coefficient. This condition allows sliding between the surfaces in contact but no separation. For the same contact the "adjust to touch" treatment interface has been used to close all the initial gaps between the teeth in contact and avoid initial impacts. For the contact between teeth in mesh the formulation method used was the Augmented Lagrange to solve the contact nonlinearities due to the nonlinear frictional contact type. After the solution, the post processing stage involves a critical analysis of the generated data.

5. Results and Discussion

FEA stress results of the preliminary study in which seven tooth profiles with different pressure angles from 20° to 35° have been generated are plotted in the figure below.



Figure 12 - FEA Contact and Bending Stresses relative to the base line α =20°

Both Contact and Bending stresses show a decreasing trend as the pressure angle is increasing. The trend proves the initial idea that a bigger p.a. has a beneficial effect on contact stress thanks to a reduced radius of curvature of the flank profile and a consequent increased area of contact. The benefit is visible also on the bending stress side due to the combination of a thicker tooth base and a more favourable inclination of the resultant force with a tangential component, responsible for bending, reduced in favour of the radial component.

The epicyclic systems generated following the gear design process described in section 3, have resulted in working pressure angles as in table 4.

P.A. [°]	SUN/PLANET	PLANET/RING
20	26.784	21.047
25	30.572	25.825
30	30	25.124
35	35	31.087

Table 4 - Workig pressure angles

The resulting working pressure angles tend to be bigger for the sun/planet mesh than for the planet/ring one. This result is in compliance with the AGMA 6123-B06 standard [7] in which it is stated that "best strength to weight ratio is achieved with high operating pressure angles at the sun to planet mesh and low operating pressure angles at the planet to ring mesh".

The results obtained from the FEA analysis of the four planetary configurations have been compared on the basis of Maximum and Minimum Principal Stresses induced by the shared load between the gears in mesh. Three specific points for the meshing gears have been chosen (Figure 13):

- Ring/Planet: stresses on the ring have been analysed
- Sun/Planet: tip loaded condition for the sun; stresses on the sun have been analysed
- Planet/sun: tip loaded condition for the planet; stresses on the planet have been analysed



Figure 13 – Meshing areas



Figure 14 - FEA Maximum and Minimum Principal Stress Vectors; tooth stress distribution chart of the ring meshing with the planet for two different pressure angles α .

Initially the full spectrum of the principal stresses along the meshing area has been recorded taking the values for each node of the mesh.Figure 14 shows the principal stress plots of the stresses induced on the ring gear at the ring/planet interface for two configurations of pressure angles. Those data give a clear indication of the positive and negative stress distribution within the members in contact. To define the nature of principal stresses is necessary to know their orientation because of their dependency on the coordinate system. In the analysis of components, these quantities allow it to be determined where the tensile and compressive stresses are induced and how they are distributed in the components. By using the plot of the vector directions it is possible to determine the nature of the stresses resulting, as in this case, positive for tensile and negative for compressive stresses. After all the data were collected, peak values of Maximum and Minimum principal stresses were compared. The peak values always occur at the tooth base both for Maximum and Minimum principal stresses.

Figure 15 shows trends of the peak values of Maximum principal stresses, for the three meshing areas. For the Ring/Planet mesh, the stress values go down going from 20° to 25° reference pressure angle corresponding to a decrease of 4.7° working pressure angle. The further decrease for the 30° reference which has a working p.a. almost equal to the previous case might be explained with the considerable increase of profile shift up to 0.6.



Figure 15 - Maximum principal stress (tensile) at the tooth root relative to the base line $\alpha {=}20^\circ$

Regarding the Sun/Planet mesh, the stress peak values "mirror" the working pressure angle trend: the stress decreases, stays almost constant between 25° and 30° and decreases again for reference p.a. of 35°. In the end, considering the stresses generated on the planet gear meshing with the sun, the expected reduction of the maximum principal stress for the second and third point in which the working pressure angle has almost a constant value, $\alpha_w \cong 30^\circ$ might have been compensated, for $\alpha=25^\circ$ by an increase of the addendum length, which increases the arm of the applied force, and, for $\alpha=30^\circ$ the reduction of profile shift with a consequent reduction of the tooth base thickness.



Figure 16 - Minimum principal stress (compressive) at the tooth root relative to the base line $\alpha{=}20^\circ$

Minimum Principal Stresses are expected to rise with the working pressure angle due to the increased radial component of the transmitted force end so an extra compressive stress induced on the gear body. More in detail, for the Ring/Planet in mesh, the compressive stress on the Ring gear increases following the working pressure angle trend. The stress on the Planet/Sun follows the linear trend as already happened for the Maximum stress but with the reverse slope up to the third point which is higher than the 25° point because of the reduction of profile shift up to negative values with a consequent reduction of the base thickness as visible in Figure 8. Minimum stress drops at 35° is attributed to the beneficial effect of a thicker tooth base due to an higher pressure angle even if the profile shift is slightly smaller than the previous case analysed. At the end, the compressive stress generated on the sun gear in mesh with the Planet follow a trend which remains constant with a small fluctuation that follows exactly the profile shift variation.

6. Conclusions

In this study, the effect of high pressure angles compared with the standard 20° has been evaluated in terms geometry modifications and generated stresses. The following observations are made from the results achieved from an initial simplified study on a single tooth model, followed by an analysis of a more complex system. The preliminary study has demonstrated how both bending and contact stresses decrease using higher pressure angles compared to the standard 20°. To prove that benefits even in a more complex environment, following a design process based on design limits, four different gear sets with non-standard parameters have been designed and assembled in epicyclic systems. It has been seen that Maximum fillet principal stress follows a decreasing trend as the pressure angle increases due to a lower Force tangential component. Minimum fillet principal stress rises as the pressure angle increases due to a bigger radial component. The combination of pressure angle with other parameters alters the stress distribution; Profile shift has a strong effect on stresses and its variation may enhance or nullify the beneficial effect of higher pressure angles. The mesh sun/planet is more stressed with tensile stresses compared with the planet/ring which is more subject to compressive stresses. In terms of contact stress, because of the different nature of contact, convex-convex for sun and planet and convex-concave for planet and ring, the are considerably higher for sun and planet in mesh.

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