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Energy Optimal Control for Formula One Race Car

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Abstract

Formula One (F1) is considered to be the forefront of innovation for the automotive and motorsport industry. One of the key provisions has been towards the inclusion of the Energy Recovery System (ERS) since 2014 in F1 regulations. ERS comprises Motor Generator Unit-Heat (MGU-H), Motor Generator Unit-Kinetic (MGU-K) and an Energy Storage (ES). This has not only converted the conventional powertrain into a hybrid power-split device, but also imposed constraints on the fuel energy available, energy recovered and deployed by MGU-K, and charge stored in ES, along with various other parameters. Although the objective for a F1 race is to minimize lap-time, it is obvious that there is no unique control path or decision to meet this objective. This builds up needs to optimally control the power-split and energy of the system.

In this study, we propose an energy optimal control strategy for a F1 car by constructing a detailed force-balanced mathematical model of the F1 powertrain, identifying state-space variables, as well as regulated constraints and weighted-cost functions and then solving for minimizing cost function based on model-based optimization inside GT-Suite© using Discrete Optimization and Genetic Algorithm. The obtained optimal trajectory is compared to the global optimum obtained by Dynamic Programming. Finally, the results are validated over in our high-fidelity GT-Drive based F1 powertrain simulator and also compared against conventional rule-based controls for added advantage to race performance and energy minimization. The result is the optimal strategy that results in minimal energy consumption for the provided speed trajectory over a single lap.

Introduction

Limited thermal efficiency, high fuel consumption and inefficient energy usage have pushed powertrain manufacturers to look for more efficient, robust and environment friendly options. The discussion is further heated for high performance F1 hybrid electric vehicles (HEV), with a strict ban on fuel consumption of 100kg/hr during race and amount of energy deployment and recovery of 4MJ and 2MJ per each lap [1]. Although options like energy recovery system (ERS) have been permitted by FIA regulations [2], the optimum control strategy for either when to recover or deploy energy is not well discussed in published domain. There is a need to understand and develop an optimal control logic for high performance powertrains which could eventually increase the thermal efficiency, fuel economy and energy efficiency of the vehicle, above all providing a competing edge on track.

Energy management system (EMS) is responsible for the efficient and even optimal deployment or recovery of energy. In passenger vehicle applications, EMS has been extensively used to control power-flow between engine and electric motor. Engine is capped by a certain fuel mass flow, as well as tail -pipe emissions. The electric motor, on other hand, is limited by the battery capacity. The choice of choosing the power source is thus based on calculation against a defined cost or objective. This cost could be fuel consumption or available battery capacity.

In high-performance powertrains, like the F1, the energy available in the car is restricted by the regulations. This forms an interesting control problem which requires a better understanding of the power flow in the powertrain, in addition to the knowledge of the optimization theory. Only then can a car succeed to provide the desired performance over the lap.

The F1 powertrains have a few unique features: Firstly, instead of a turbocharger, they contain a Motor Generator Unit- Heat (MGU-H). This makes it act as a supercharger, heat energy generator or a simple turbocharger as needed during the operation of the car. The other component is the Motor Generator Unit-Kinetic (MGUK). This helps supply a power boost or regenerative braking as needed. The electric energy is provided by the battery called as Energy Storage (ES).

Acknowledging these restrictions as well as the complex hybrid structure, it is apparent that for any constructor, a careful consideration towards the development of controller is essential. Above all, to deploy and recover energy efficiently for maximum performance, the choice of when to use what power source is also essential. Such a control system is called the Energy Management System (EMS) [3].

EMS functions to have a supervisory control over component level controllers like engine, battery, and motor controllers. This enables a robust and, in some cases, nonlinear control strategy which may not be possible in conventional control systems. Here it becomes important to understand the different approaches for EMS, namely rule-based and optimization-based control.

Rule-based control is dependent on heuristics which are set by the designer with common examples of thermostat (on/off) control, fuzzy logic control and power-follower control. On the other hand, optimization-based control aims to optimize the system characteristics based on the system dynamics. This is done by constructing a cost-function and minimizing/maximizing the cost function based on the set constraints and end-point conditions. Some optimization-based control can be done off-line from real-time system and commonly include Linear Programming (LP) [4], Dynamic Programming (DP) [5] and Non-linear Programming (NLP) [6]. Off-line optimal controls are also known for their ability to give global-optimum solution and thus are used as benchmarks. Whereas some optimization-based controls can be run on-line and make possible real-time optimal control. These control types include Model Predictive Control (MPC) [7] and Equivalent Consumption Minimization Strategies (ECMS) [8], to name a few. Not only this, but there are also a lot more types of EMS strategies developed for computationally solving the optimization and/or control problems like use of Neural Networks and Machine Learning, Particle Swarm Optimization (PSO), Genetic Algorithm (GA), in addition to conventional methods like Trial and Optimize using numerical tools, etc. As suggested by the numerosity in types, each energy management strategy has different applications, each with a different caveat, as already described in detail in published domain [9]. A summary of this discussion in provided in Figure 1.

One of the many applications of EMS include implementation of optimal power-split for Prius by using stochastic DP and ECMS to optimize engine operation benchmarked against DP [10]. Sciarretta *et* *al.* [11] describe the development of ECMS with main work on development of instantaneous cost function leading to 30% reduction in fuel. Similarly, more recent work has been done towards passenger hybrid electric vehicles with concern towards fuel economy and emissions[12][13]. Yet next to little implementation and development has been done in context to F1 in published domain.

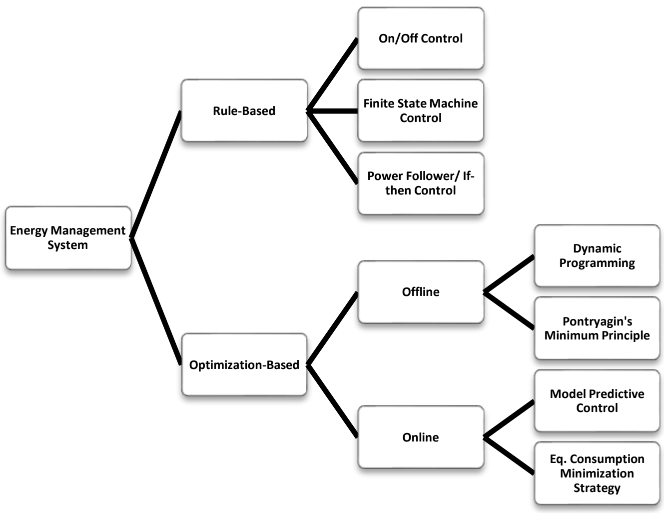

Figure 1 Summary of the Energy Management System techniques used mostly in published domain.

One of the earlier validations of Kinetic ERS (KERS) in published domain has been done by [14]. In their work, they have investigated the benefit of KERS to "fuel economy and lap time" by optimization using Dynamic Programming. Bengolea *et al.* [15] have studied choices of optimum technology for maximum benefit to fuel consumption, emissions, and energy for F1 vehicles by comparing different technologies in terms of benefit to the road vehicle using drive cycles. Some researchers have developed MGU deployment strategies based on deployment speed by comparing different vehicle speed profiles from racetrack data [16].

One of the few other series of work for high performance Hybrid Electric Vehicle (HEV) has been done towards lap-time minimization by using Pontryagin Minimum Principle (PMP) [17], Convex Optimization [18] and MPC [19]. Although the work is explicit and one of its kind, there are a few caveats to this approach: Firstly, it is based on lap time-based optimization, which although is a key objective of the race, however, does not guarantee that system energy is consumed optimally. This is important because the plant (i.e., powertrain) optimal behavior is not guaranteed [20]. Secondly the use of energy optimization approach enables to use drive cycles developed from actual race-track data. This ensures that ERS has more energy to be deployed before reaching 4MJ limit and relying solely on engine for power. This back-up energy will help towards faster lap-time as well. Thus, an optimal split of energy between engine and battery is a key objective of this work.

The present study proposes an energy optimal control strategy for F1 car by constructing and validating a force-balanced mathematical model of F1 powertrain, identifying the state-space variables, as well as constraints regulated by F1 regulations and weighted-cost functions and then solve for minimizing the cost function based on model-based optimization inside GT-Suite using EMS. The results are compared to those obtained by DP and benchmarked against the energy consumption by a rule-based control strategy inside GT-Suite.

The findings of this study will have two-fold of impact to the existing literature. Firstly, it will provide a detailed and validated mathematical model of a F1 powertrain which can be extended to use in future concepts and applications. Secondly, the study will propose a strategy to achieve maximum performance of a high-performance race car, without compromising on the limits of the energy flow as imposed by regulations.

The paper is structured in the following way: Methodology section describes the tools and structure of analysis using three different approaches namely, GT-Drive Rule Based Controls which serve as benchmark for the optimization results, GT-Drive Optimization Based Controls which involve optimizing inside GT-Suite for higher-fidelity calculations, and lastly Dynamic Programming in MATLAB. The optimization is done based on mathematical model development. Following this, Results and Discussion describe the validation of mathematical model is performed along with comparison of different control strategies and cost function.

Methodology

GT-Drive Modelling

Here the pre-validated Formula One powertrain model in GT-Drive in earlier work [21] has been used as the baseline. This model is illustrated in Figure 2. The critical parameters for the model have been listed in Table 1.

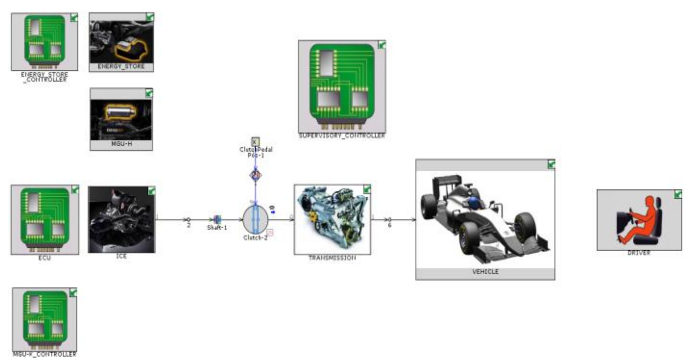

Figure 2 Overview of the model of F1 powertrain inside GT-Suite used for the work.

Rules Based Controls

Rule based control is the conventional control mostly involving "if this, then that" behavior [23]. In context to the application, rule-based control is done inside **GT-Drive**. The major benefit of this method is that extensive powertrain simulation can be done while the downside is that the control is not optimal, but rather sub-optimal i.e., there is no guarantee that the vehicle is performing optimally or not. However, this approach is used to evaluate the benefit of optimization in terms of *percentage improvement in energy consumption* or recovery. The model is split as below.

Table 1 Important parameters based on GT-Drive vehicle model.

Parameter	Unit	Value
Engine Capacity	L	1.6
Vehicle Mass	Kg	728
Passenger Mass	Kg	70
Vehicle Frontal Area	m^2	1.5
Drag Coefficient	-	0.78
Battery Capacity	A-h	20
Tyre Rolling Radius	m	0.331
Tire Rolling resistance factor	-	0.01
MGU-K gear ratio	-	4
Final Drive ratio	-	3.8
Initial SOC	-	0.6
Simulation Circuit	-	Monza
Lap Duration	s	80

Supervisory controller is responsible to key calculations including selecting engine mode, the power required from MGU-K based on power demand and finally the total power loss in the system. Primarily, it has two controllers i.e. MGU-K and Energy Storage.

MGU-K controller shown in Figure 3 is responsible to request power to/from MGU-K. Also, it has constraints for energy deployment and recovery as set by FIA regulations. It receives the energy request from Supervisory controller and decides how much of energy it can provide based on those constraints.

The Energy Storage controller is made to calculate the total energy deployed and recovered in MJ and difference in SOC, a critical parameter to be monitored during the race. This ensures that overall energy constraints in FIA regulations are fulfilled. This is shown in Figure 4.

Optimization Based Control

In order to achieve the best performance on track, an energy optimization approach is followed. This involves laying out the mathematical model of F1 powertrain, evolving into the state space model and finally optimizing for energy consumption. In order to achieve a computationally efficient while optimal solution, results from three different optimization techniques are compared.

Mathematical Modelling

The mathematical model is constructed based on the description of powertrain in regulations as shown in Figure 5. Torque required to drive the vehicle can be estimated and therefore, acceleration capability of the vehicle at any given state can be obtained from a complete powertrain model.

$$\dot{\omega}\,(I) = T_{excess} = T_{availalbe} - T_{required} \qquad (1)$$

In this way (1) can be used to describe the evolution of kinetic energy of the system. The steps involved to achieve mathematical formulations for F1 power-split are summarized below.

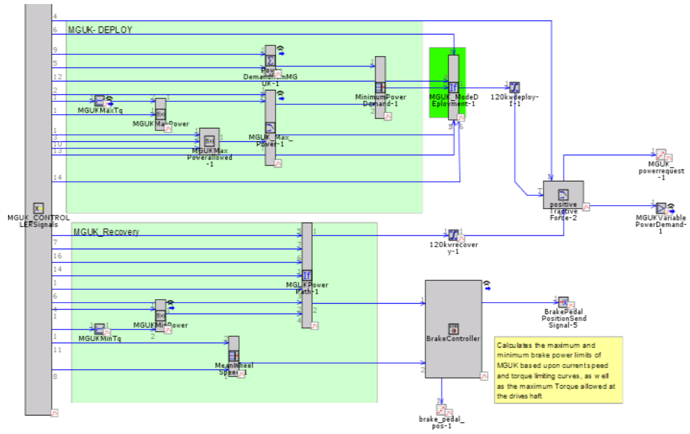

Figure 3 Snapshot of MGU-K controller inside GT-Suite.

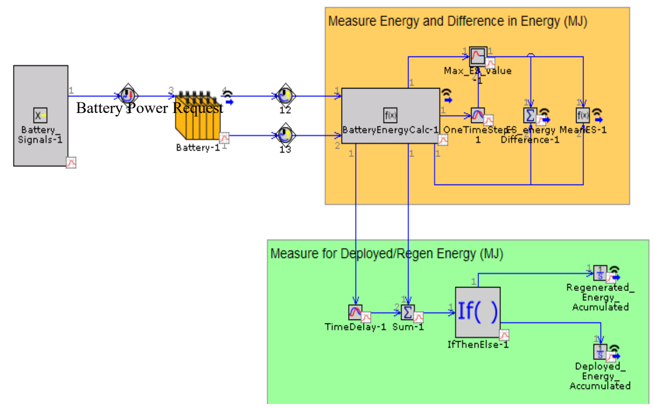

Figure 4 Summary of the model used in GT-Suite for the energy storage calculations.

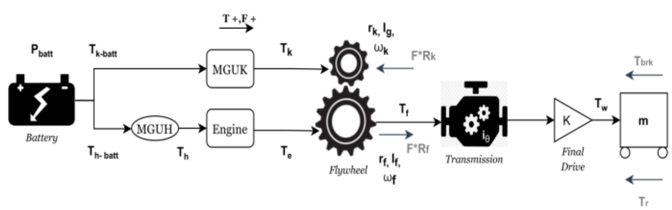

Figure 5 Schematic of the powertrain model describing th key forces and arrangemenet of system.

The power required to move the car at a certain speed is provided by the engine, MGU-K, and brakes (negative power), with some component lost to the losses in the system as in (2).

$$P_r = P_e + P_k + P_{brk} - P_{loss} \qquad (2)$$

Usually, power balance is useful as it does not depend on gear ratios or speed difference. However for powertrain, it is useful to include a torque-based model which describe how the system accelerates over time. The power balance at flywheel gives us (3).

$$T_f\omega_f = T_e\omega_e + T_k\omega_k \qquad (3)$$

Where, $\omega_f = \omega_e$ i.e., engine and flywheel speed are the same. Thus, it can simplify to get (4).

$$(T_f - T_e)\omega_f = T_k\omega_k \tag{4}$$

Where, i_k could be defined as the MGU-K gear ratio as,

$$i_k = \frac{r_f}{r_k} = \frac{\omega_k}{\omega_f} \tag{5}$$

Hence, we get (6) from (5).

$$T_f = T_k i_k + T_e \tag{6}$$

From force balance, we get (7) and (8),

$$\dot{\omega}_k(I_k + I_g) = T_k - Fr_k \tag{7}$$

$$\dot{\omega}_f I = (T_e + Fr_f) - T_r + \frac{1}{i_\theta K}(T_{brk}) - T_{loss} \tag{8}$$

Here, T_r assumes no acceleration part and is solely based on the resistance forces, since acceleration is taken out into $\dot{\omega}_f I$. Upon further simplification we get (9). $\dot{\omega}_f$, flywheel acceleration is related to car acceleration by (10). This finally gives (11).

$$\dot{\omega}_f I = T_e + T_k i_k + \frac{1}{i_\theta K}(T_{brk}) - T_{loss} - \cdots \\ \frac{r_w}{K i_\theta \eta_t}(0.5\rho C_d A v^2 + mgf_r) \tag{9}$$

$$\dot{\omega}_f = \frac{K i_\theta}{r_w}\frac{dv}{dt} \tag{10}$$

$$\frac{dv}{dt} = \frac{r_w}{K i_\theta I}[T_e + T_k i_k + \frac{1}{i_\theta K}(T_{brk}) - T_{loss} - \cdots \\ \frac{r_w}{K i_\theta \eta_t}(0.5\rho C_d A v^2 + mgf_r)] \tag{11}$$

The total inertia is composed of the following components as in (12).

$$I = \frac{mr^2}{(i_\theta K)^2} + \frac{I_w}{(i_\theta K)^2} + \frac{I_t}{i_\theta{}^2} + I_e + i_k\,(I_k) \tag{12}$$

Note that MGU-H is not involved in the direct power supply. The behavior is in fact non-linear. Hence, in this model development, the impact of MGU-H is split into two parts, engine, and battery. The impact on engine is directly modelled into the engine parameter i.e., BMEP to give the resulting power from MGU-H. On the electrical side, map develop by [21] are used to give the mechanical power based on MGU-H speed and throttle input. The numerical basis is summarized in (13) to (16).

$$\dot{\omega}_h(I_h) = T_h = T_{ex} - T_{tc} \tag{13}$$

$$P_h = T_h\omega_h \tag{14}$$

$$P_h = f(\omega_e, \tau) \tag{15}$$

$$\omega_h = f(\omega_e, \tau) \tag{16}$$

Note that, maps are used for extracting BMEP of the engine. MGU-K power is not determined by the maps and is directly computed based on (2).

Similarly, the evolution of energy in the battery E_{batt} can be described as (19) where SOC has been used based on formulation (17) to (18).

$$Q(t) = SOC(t) \times Q_{max} \tag{17}$$

$$E(t) = VOC \times Q(t) = VOC \times SOC(t) \times Q_{max} \times 3600 \tag{18}$$

$$P_{batt} = \frac{dE_{batt}}{dt} = V_{oc}\,.\,Q_{max}\,.3600.\frac{dSOC(t)}{dt} \tag{19}$$

The power of battery P_{batt} is shown in (20). P_{k-batt} and P_{h-batt} show MGU-K and MGU-H power at battery. The individual components are shown in (21).

$$P_{batt} = P_{k-batt} + P_{h-batt} \tag{20}$$

$$P_{batt} = T_k\omega_k\eta_{ek}{}^S\eta_{ik}{}^S + T_h\omega_h\eta_{eh}{}^S\eta_{ih}{}^S \tag{21}$$

Where, $S = \{-1\,||\,1\}$ based on if the battery is charging or discharging respectively. Here, the deduction of using $P_{batt} = V_{oc} \times I_{batt}$ leads to the evolution of SOC equation as shown in (22).

$$\frac{dSOC(t)}{dt} = -\frac{I_{batt}}{Q_{max}\,.3600} \tag{22}$$

Where negative sign is for the decay in the value of SOC over consumption. The State of Charge of battery, as defined by F1 regulations, is the difference between the initial energy $E_{batt}(0)$ and the instantaneous energy $E_{batt}(t)$ as (23).

$$\Delta E_{batt} = E_{batt}(t) - E_{batt}(0) \tag{23}$$

Finally, the evolution of fuel energy E_{fuel} in powertrain is related by equation (24).

$$P_{fuel} = \frac{dE_{fuel}}{dt} = \dot{m}_{fuel}Q_{HV} \tag{24}$$

The mass flow $\dot{m}_{fuel}$ is restricted by regulations based on engine speed as shown in (25). Where the engine speed ω_e can be shown to vary linearly in simple model as in (26).

$$\dot{m}_{fuel} = \begin{Bmatrix} 100 \quad ,\omega_e > 10500 \\ 0.009\omega_e + 5.5 \quad ,\omega_e \le 10500 \end{Bmatrix} \tag{25}$$

$$\omega_e = \frac{\omega_{e,max} - \omega_{e,idle}}{100}\tau + \omega_{e,idle} \tag{26}$$

Here two additional states are defining to control the limits on regeneration E_{rec} and deployment $E_{depletion}$ as in FIA regulations. These are given in (27) and (28).

$$\frac{dE_{depletion}}{dt} = \max(0, P_{batt}) \tag{27}$$

$$\frac{dE_{rec}}{dt} = min\,(0,\ P_{k-batt}) \tag{28}$$

Finally, the cost function can be constructed for the problem. This is done in (29). Here $\propto_1$ and $\propto_2$ are constants for assigning weights to the terms of the cost function, provided their sum is equal to 1.

$$J = \int_{t_0}^{t_f} (\propto_1 P_{batt} + \propto_2 P_{fuel})\, dt = \propto_1 E_{batt} + \propto_2 E_{fuel} \tag{29}$$

From (29), a normalized cost function can help with balancing the differences in magnitudes of the two energies as provided in (30).

$$J = \int_{t_0}^{t_f} (\propto_1 \hat{P}_{batt} + \propto_2 \hat{P}_{fuel})\, dt = \propto_1 \hat{E}_{batt} + \propto_2 \hat{E}_{fuel} \tag{30}$$

Where '^' over each quantity denotes normalized quantity with the general form for variable '**V**' given in (31).

$$\hat{V} = \frac{V - V_{min}}{V_{max} - V_{min}} \tag{31}$$

Here V_{min} is the minimum value of the variable and V_{max} is the maximum value of the variable.

State-Space Formulations

It is important to note that only three equations, (11), (21) and (24), can fully describe the evolution of powertrain characteristics in a car. Of all variables, some are dependent while other are the fundamental (minimum number of) variables to describe the state, and hence called state variable *x(t)*. Whereas some variables are needed to be defined to be able to solve the equations according to degree of freedom of system and are called input variables *u(t)*. These variables are essential to define control problems and are illustrated for defined equations as in Table 2. This enables to construct the optimizations problem now which has been constructed in Table 3.

Note that of all inputs, throttle τ is the most fundamental as it not only controls the engine but also MGU-K and MGU-H as identified in the equations earlier. Hence, τ is mainly used as the only input for proceeding optimizations, where the T_{brk} and i_θ are assumed to be controlled by the driver in a rule-based strategy for the simulation.

On the argument that how effective throttle is over other possible inputs say $\boldsymbol{P_k}$, throttle is more explicit in implementation by the driver than any other input. The mathematical model presented earlier validates that throttle is a common input to almost all variables on car, including $\boldsymbol{P_k}$. Studying throttle would also give a better insight to the driving behaviour required to achieve close to optimal results on actual track.

Table 2 Summary of the state variables and input variables defined for state-space equation.

State Variable x*(t)*	Input Variable *u(t)*
$v, E_{batt}, E_{fuel}, E_{depletion}, E_{rec}$	τ, T_{brk}, i_θ

Table 3 Summary of the Optimisation problem along with the objective function and constraints.

$x = \{v, E_{batt}, E_{fuel}, E_{depletion}, E_{recovery}\} \in \mathcal{X}^5$ $u = \{\tau, T_{brk}, i_\theta\} \in \mathcal{U}^3 \quad \mathcal{X}, \mathcal{U} \in \mathbb{R}$
$\min_u \int P(x,u,t)dt \ \ OR \ \ \min_u E(x,u,t)$ **Where,** $E(x,u,t) = \propto_1 E_{batt} + \propto_2 E_{fuel}$
S.t., Dynamics, $\frac{dv}{dt} = \frac{r_w}{Ki_\theta I}T_e + T_k i_k + \frac{1}{i_\theta K}(T_{brk}) - T_{loss} - \cdots$ $\frac{r_w}{Ki_\theta \eta_t}(0.5\rho C_d A v^2 + mgf_r)$ $P_{batt} = \frac{dE_{batt}}{dt} = V_{oc} \,.\, Q_{max} \,.\, 3600 \times 10^{-6} . \frac{dSOC(t)}{dt}$ $P_{fuel} = \frac{dE_{fuel}}{dt} = \dot{m}_{fuel} Q_{HV}$ $\frac{dE_{depletion}}{dt} = max\,(0,\ P_{batt})$ $\frac{dE_{recovery}}{dt} = min\,(0,\ P_{k-batt})$ **S.t., Constraints,** $-50,000\ rpm \le \omega_k(t) \le 50,000 rpm$ $-125,000\ rpm \le \omega_h(t) \le 125,000\ rpm$ $0\ MJ \le \Delta E_{batt}(t) \le 4MJ$ $0 \le \dot{m}_{fuel} \le 100 \frac{kg}{hr}$ $E_{depletion} \le\ 4\ MJ$ $E_{recovery} \le -2\ MJ$ $P_k(t) \in [-120, 120]\ \ kW$ **And,** $\tau(t) \in [0, 100]$ $T_{brk}(t) \in [0, T_{brk,max}]\ \ N.m$ $i_\theta \in [i_{\theta,1st\ gear}, i_{\theta,8th\ gear}]$

Solving optimization in GT-Suite

The approach to solve optimization problem in GT-Suite is based on the work by [24]. Here the state space equations developed before are used to construct the cost function as provided in Figure 6. The cost function model has three stages i.e., for fuel, MGU-K and MGU-H energy calculation.

The model in Figure 6 is discretized into N cases, where N is the number of seconds in a lap drive cycle. In the optimizer the output of the cost function block is minimized while the throttle is assigned to be the input varied between 0 and 100. The optimizer runs calculation over each case and results in optimal throttle for each case. For the purpose of this study, both Discrete-grid [25] and Genetic algorithm [26] methods are used in the GT-optimizer settings to get optimal throttle. For base simulations, weightage parameters $\propto_1$ and $\propto_2$ are set to be 0.95 and 0.05, respectively. This will minimize battery consumption more than the fuel, which is useful as fuel flow energy is not restricted while the battery energy is limited by the regulations.

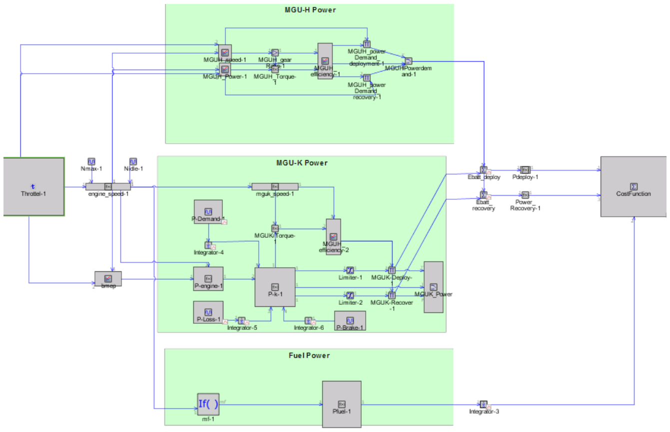

Figure 6 Cost function GT-Suite file used to calculate the impact of input on cost at each time step.

The final step is implementation, which involves using the optimal throttle as input to the 'optimized model' as shown in Figure 7. This model also ensures that vehicle dynamics components which are not explicitly handled like maximum speed at corners does not exceed limits.

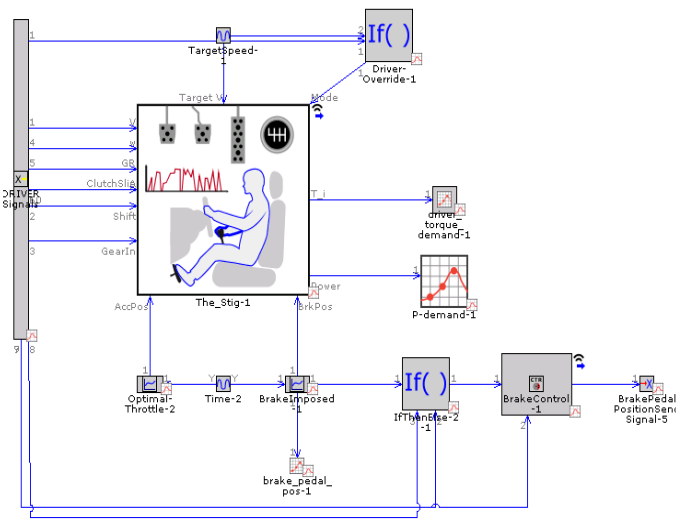

Figure 7 Implementation of optimal throttle in the GT-Suite model by imposing throttle and brake pedal positions.

Dynamic Programming in MATLAB

Dynamic Programming is an exhaustive search method [5]. The optimization problem is solved by constructing a grid of points. The cost from moving from one node to another is thus called cost-to-go 'Y'. The action of moving from one node to another is solely determined by the input variable 'u'. The input variable which results in minimum cost are called optimal inputs 'u^*' and the cost is called as optimal cost 'J^*' as in (32).

$$J^* = \min(J) = \min(M|x_0, t_0 , x_f, t_f| + \int_{t_0}^{t_f} L|x, u, t|\, dt) \qquad (32)$$

And the optimal policy or control inputs which result in optimal cost can be summarized as in (33).

$$u^* = \underset{u \in U}{\operatorname{argmin}}(J) = \underset{u \in U}{\operatorname{argmin}} (M|x_0, t_0 , x_f, t_f| + \cdots \int_{t_0}^{t_f} L|x, u, t|\, dt) \qquad (33)$$

Where *argmin* represents that the argument that J is minimum over the input u equates to the optimal u^*. The path, comprising of nodes, which leads to minimum cost is called as the optimal path.

Table 4 Summary of calculations done in MATLAB to achieve dynamic programming-based optimization.

Step 1: Initialize and Import data

%Problem definition, assign constants, load data and initialize output matrix

Step 2: Solve Dynamic Programming problem

```
For T = Ts:  N
  For t = tmin:  tmax
    For i = 1:  101
      %Calculate Pk, Ph
      %Calculate mdotfuel
      func1 = (Ph +Pk)
      X1(T, i) = Ts. *func1 + X1(T-1, i).
      func2 = (mdotf. *44 000. /3600);
      X2(T, i) = Ts. *func2 + X2(T-1, i);
      % Calculate Cost function like:
      C= X1(T, i) + 0.05.*X2(T, i);
      %Store minimum C in J (T, t+1)
    End
  End
End
```

Step 3: Sort data and Export

```
For j= 1:1:801.
  [minval, index] =min (J(j,:));
  throttle(j,1) =index 1.
  value(j,1) =minval.
End
```

Dynamic programming problem is constructed and solved in MATLAB. The structure of code is divided into three steps as shown in Table 4. The first for-loop is responsible to run calculation for the duration of drive cycle. The second for-loop varies the throttle from 0 to 100, while the third for-loop is responsible for calculating cost-to-go. The minimum for each throttle is stored in a cost matrix which has cost at each stage along the rows and throttle value from 0 to 100 along the columns. The final part of code is responsible to sort the cost matrix and to select the minimum cost at each stage.

Implementation and Analysis of EMS

Validation of Mathematical Model.

The mathematical model developed for the current work shows a great correlation with the high-fidelity powertrain model developed in GT-Suite as in Figure 8 for torque output, velocity profile and power deployed by MGU-K.

Optimization inside GT-Suite

The optimization inside GT-Suite is done based on Discrete-Grid (DG) optimization and Genetic-Algorithm (GA) over the same cost function as discussed before with results shown in Figure 9 (b) and (c).

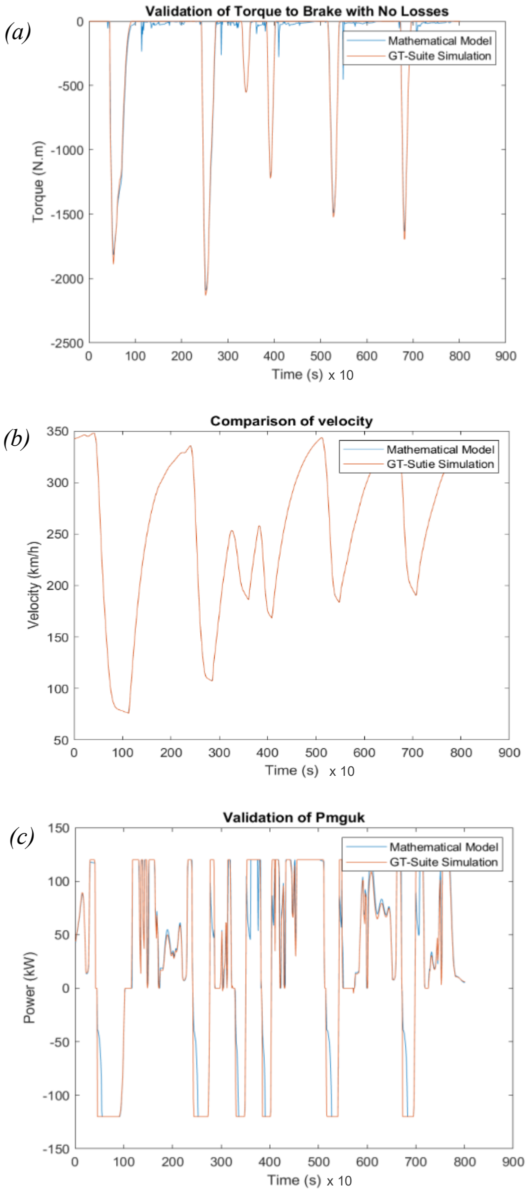

Figure 8 (a) Plot showing the validation of torque required to brake between the mathematical model and GT-Suite simulation when no Torque losses are considered. (b) Comparison of velocity calculated by mathematical model and the GT-Suite model (c) Comparison of Power to MGU-K from mathematical model and GT-Suite results.

In terms of computational effort, it is found that DG finds the points that correspond to minimum cost in lesser time than the GA However, the statement is not conclusive as GA gives lesser energy deployment of 1.2 MJ. The reason is because GA runs simulation over several generations, selecting the best and proceeding with it forward for each time step. Note that the vehicle velocity of the optimization follows the target speed for the lap by being less than 1% average error in all cases.

Since GA gives better results over higher generations or number of runs, it is worthwhile to examine the behavior of results for GA by varying the number of generations from low to high. This will indicate the sensitivity of the optimization problem and possibility of faster iterations by going with lesser no. of generations. Each generation involve running almost 10 iterations in GT-Suite optimizer, with each iteration taking an average of 51 seconds. This would roughly equate to 8.5 min for each generation. The summary of this analysis is given in Table 5 which was stopped after 15 generations. The comparison suggests that to save computational effort, even 5 generations of iterations are enough to achieve satisfactory results. This conclusion will vary if the model complexity or the cost function is changed.

Table 5 Comparison between different GA runs based on number of generations. * Simulation success is marked if the resulting throttle profile can achieve the target speed and performance on implementation.

No. of Generations (designs)	Computational Time (min)	Simulation Success? *	Energy Deployed (MJ)
2 (20)	17	Yes	2.15
5 (50)	45	Yes	1.196
10 (100)	93	Yes	1.203
15 (150)	136	Yes	1.189

DP in MATLAB

The results of optimization after applying dynamic programming suggest significant improvement in the deployed energy while still maintaining similar velocity profile (within 1%). The deployed energy can be seen to be of mere 1.13 MJ or 72% reduction of energy as in Figure 9 (d) compared to Rule-based method Figure 9 (a).

Comparing Optimal Profile and Search Algorithms

Here the 4 throttle profiles are compared obtained by namely, Rule based, DG, GA-5 (i.e., till 5th generation), and DP. Performing percentile analysis on the optimal profile results in Figure 10. This follows with a few important deductions towards the actuation of throttle in a real-world scenario:

- RB strategy follows a mostly bang-bang control with either 0% or 100% throttle and only around 15th percentile of throttle points are as part-throttle (in between the extremes). Also, the 50th percentile or most common operating point is 100% throttle.
- DG algorithm dominantly forces part-throttle for around 60th percentile of operating points. This is seen as a very gradual throttle profile with an approximate 50/50 split between throttle points that are above and below 50% throttle.
- GA-5 and DP follow approximately similar trajectory. The 50th percentile of operating point is around 98% throttle for both GA-5 and DP. However, the DP one is observed to have the highest percentile among all optimization profiles to have the 0% and 100% throttle. This is fundamentally true as DP appears to reduce the power consumption of MGU-K the most.
- The throttle percentile increases exponentially for GA- 5 and DP for more than 70% throttle. This corresponds to high variations of throttle up till 100% throttle.

From the discussion above, optimization results in improved savings for energy deployment by MGU-K. In general, gradual increment in throttle from 0% to 100%, optimized maximum throttle value and varying throttle based on model optimization, collectively result in reduction of energy deployed by the battery. DP being a global search method results in the minimum energy consumption.

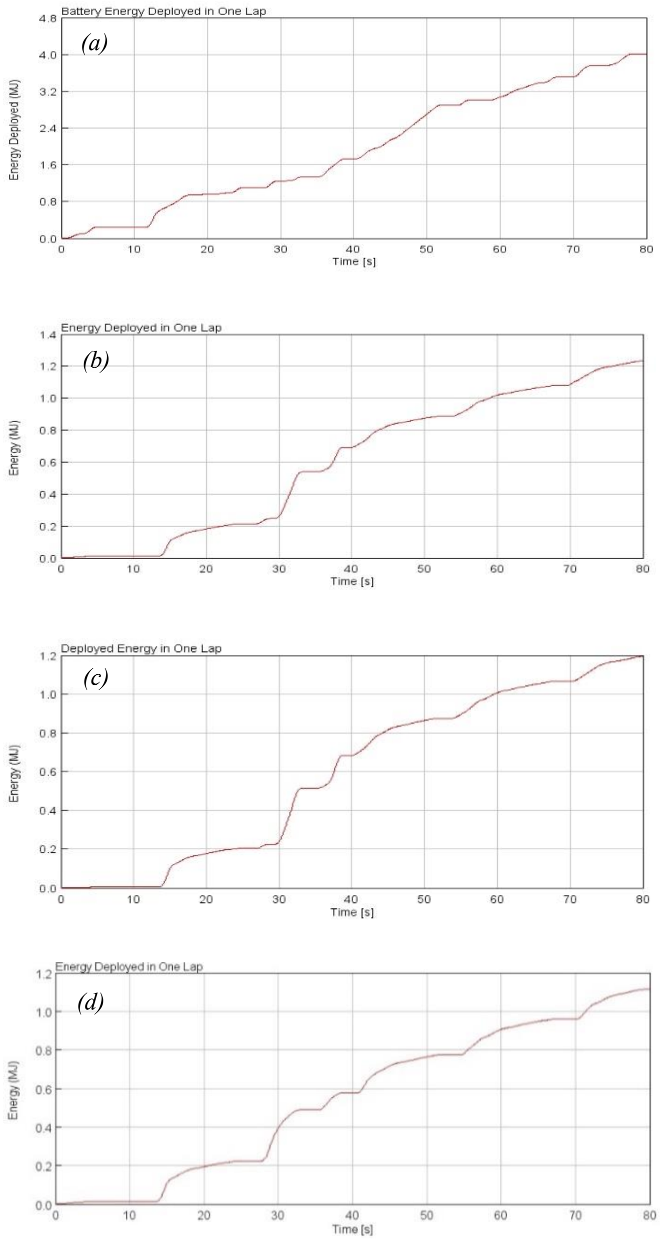

Figure 9 Comparison of energy deployment results for different EMS.(a) Rule-Based, (b) Discrete Grid , (c) Genetic Algorithm and (d) Dynamic Programming. All based on under 1% error to vehicle speed target.

The summary of the search algorithms and their usefulness has been summarized as a merit matrix in Table 6.

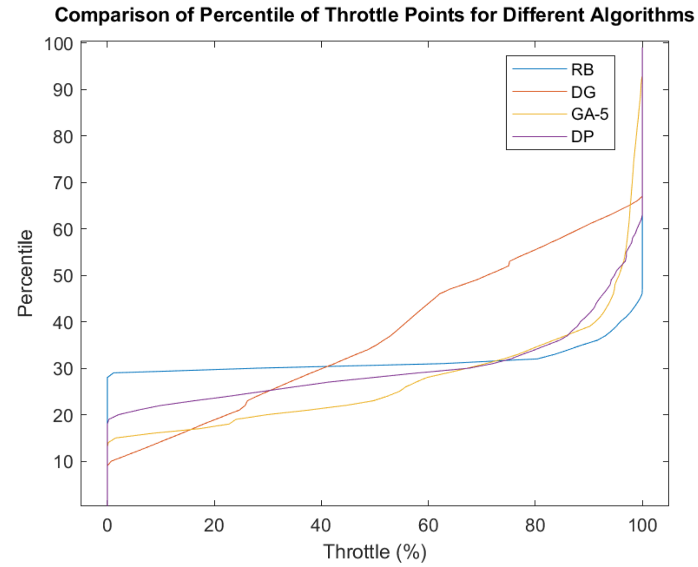

Figure 10 Comparison of percentile of throttle points for various control strategies.

Table 6 Merit matrix of the three optimal search algorithms compared. *Higher value means better.

Search Method	Compu-tational Effort*	Energy Savings*	Complexity of Model*	Overall Score*
Discrete Grid	5.0	8.0	4.0	17.0
Genetic Algorithm	3.0	8.5	4.0	15.5
Dynamic Programming	7.0	9.0	6.0	22.0

Variations in Power-split Ratio

In all the preceding discussion and development of model itself, the cost function was left unchanged. The weightage parameters $\propto_1$ and $\propto_2$ were kept 0.95 and 0.05 before for all the calculations. It is apparent that more weightage to certain component (battery energy or fuel energy) will result in more consumption of either of the two components. For example, changing $\propto_1$ and $\propto_2$ to 0.5 each will mean equal weightage to reduction in energy of fuel and battery to achieve the same vehicle speed over the lap. In fact, the combination of the weightage parameters $\propto_1$ and $\propto_2$ decide the *power-split ratio* between the battery and engine (fuel power). A similar concept has been previously explored in literature to achieve optimal torque split based on cost function being optimized [27].

Note that the power supplied by battery is limited to ±120 kW, hence there will be a threshold for the weightage parameter values above which model will no longer be able to optimize. Also note that here only DG-based results will be studied as the rest would not add more to the understanding of power-split in cost function.

The results from optimization are presented in Table 7. The results indicate that the weightage ratios can be set as a tunable parameter on the steering wheel or as pre-defined modes. This can enable driver to tune car to use more or less MGU-K or engine based on unpredictability on track, for instance to gain competitive advantage in a lap.

Table 7 Comparison of different weightage parameter values to see relation with consumption values after optimization.

α_1	α_2	Consumption Values	
		Battery Energy (MJ/Lap)	Fuel (g/Lap)
1	0	1.638	1531
0.8	0.2	2.23	1462
0.6	0.4	2.77	1366
0.4	0.6	3.015	1350
0.2	0.8	4	1283
0	1	4	1285

Implementing Optimal Throttle on Track

The following methods could be used to implement optimal throttle on track. Firstly, Optimal throttle profile relates to battery energy consumption. Since battery energy and SOC are directly proportional, the optimal SOC profile can be implemented by PID controller on track to ensure vehicle battery depletion follows similar trajectory as the optimally computed one. For instances where non-optimal design would be feasible, like change in track conditions, the gain of PID can be tuned. This can enable driver to switch from optimal and non-optimal in real-time. Secondly, an electronic circuitry which enables modification of bang-bang throttle input to a gradual one in real-time can be used. Last, yet not least, the aforementioned options can have an additional toggle on steering to control the power-split between MGU-K and engine. This can enable driver to respond to uncertainty on track and gain competitive advantage as needed.

Conclusions

In this study, an energy efficient control strategy was developed using energy management system. During this process, a mathematical model for F1 powertrain was developed and validated. Thereafter, energy management strategies including discrete-grid, genetic algorithm, and dynamic programming were implemented in a validated GT-Suite based F1 powertrain model.

The use of energy management strategies led to a significant reduction in energy deployed by the battery compared to conventional rule-based strategy by implementing optimal throttle input in high-fidelity GT-Suite based F1 powertrain simulator. All the optimal strategies were found to follow the target vehicle speed within 1% error. Among different strategies, dynamic programming was found to give the minimum consumption in energy while still being computationally efficient, while discrete-grid and genetic algorithm being directly implementable inside GT-Suite. The weightage factors in cost function were found to be directly responsible for power-split between engine and battery, and tunable by the driver based on track-side conditions.

In terms of implementation of the optimal strategy by F1 teams, optimal throttle could be fed into controls using PID controller, while the relaxation of bang-bang type throttle could be added using electronic circuitry. Additionally, adjustable power-split between engine and motor could be implemented to balance the energy deployment based on trackside conditions. Since powertrain architecture would be more or less similar between different teams, similar approach could lead to faster model development using their data for vehicle, engine maps etc. The real-world implementation of strategy would obviously vary with inclusion of several other constraints like noise, the procedure highlighted in the work would regardless be implementable with better flexibility by added tunable constraints like power-split.

Overall, the developed method and findings were found to contribute to achieving better performance on track whilst reducing energy consumption compared to non-optimal or rule-based control strategy.

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Nomenclature

$\hat{E}$	Normalized Energy
$\hat{P}$	Normalized Power
$\dot{m}_{fuel}$	Mass flow of Fuel
r_f	Radius of Flywheel
r_k	Radius of MGU-K Gear
r_w	Radius of Wheel
$\dot{\omega}$	Angular Acceleration
C_d	Drag Coefficient
E_{batt}	Energy of Battery
Q_{HV}	Heating Value of Fuel
V_{oc}	Open Circuit Voltage
f_r	Rolling Resistance
i_k	MGU-K Gear Ratio
i_θ	Gear Ratio
r	Radius
ω	Angular Speed
u	Input Variable
x	State Variable
A	Frontal Area
I	Current
I	Inertia
K	Final Drive Ratio
P	Power
Q	Charge of Battery
SOC	State of Charge
T	Torque
V	Voltage
g	Gravitational Acceleration
m	Mass
t	Time
$\boldsymbol{v}$	Velocity
η	Efficiency
ρ	Density