Polymer Composite Belleville Springs for an Automotive Application

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Abstract

This paper investigates mathematical modelling and manufacturing of polymer composite Belleville springs, and their potential application. The original expression for load carrying capacity developed for metal springs is refined by considering the variation of elastic modulus and Poisson's ratio of laminates in polar coordinates. A novel series spring stacking arrangement is proposed to achieve complex stiffness variation by progressive action. The experimental results show consistent effect of number of plies on the spring rate and compare well with the theoretical predictions. Although handmade, the variations in load carrying capacity is very small (~10%) confirming manufacturing viability. It is shown that a smooth, variable spring rate curve can be produced by reducing slip-stick frictional forces with the use of spacers within the spring stacks. In one application, the composite springs are shown to offer significant vehicle dynamic performance improvement through reduction of the tyre contact patch force variation and vehicle body acceleration.

Keywords: Polymer Composite Spring; Belleville Spring; Nonlinear Suspension Stiffness; Ride Comfort; Spring Stack; Vehicle Dynamics; Light-weighting

1. Introduction

The stiffness and characteristics of springs plays a vital role in determining the performance of vibration isolation systems such as in vehicle suspension. Traditionally, helical coil springs are used. In applications where system parameters vary and loading conditions change the use of helical springs is a limitation as the availability of springs with small stiffness variations can be a significant advantage in optimising the performance, which is something helical coil springs cannot provide. Belleville springs offer this ability to provide better non-linear stiffness characteristics than coil springs and stacked in different configurations can achieve precise or varying stiffness characteristics. Further to this they can be relatively easily manufactured out of carbon fibre which offers the possibility of considerable mass reduction. Therefore, to show these potential benefits this paper provides both experimental and theoretical models for the polymer composite Belleville spring and stacks along with a manufacturing procedure that is suited to road vehicle applications.

Many different types of helical springs have been designed to offer variable spring rates, such as the hourglass, funnel and bell springs [1]. However, these springs are often harder to package and sometimes less durable. However, Belleville springs allow a large range of non-linear load-deflection curves and spring rates to be achieved [2]. This is done by stacking of Belleville springs in series and parallel, offering further means to obtain desired stiffness variation [3]. This nonlinearity and control of spring stiffness offers significant advantages for motorsport applications where it is often desirable to be able to quickly change the springs to achieve different vehicle handling characteristics. The use of composite Belleville springs also eliminates a major disadvantage of helical coil springs [4], as coil springs produce torsional side loads when loaded axially which cause friction within the seals and bearings of the shock absorber assembly. In contrast, the Belleville springs simply flatten out under axial loading conditions [5].

Composite springs could therefore have great potential for both automotive and motorsport applications, as they can offer considerable benefits in terms of dynamic performance, weight reduction and sustainability. In both automotive or motorsport applications, reduction of wheel hub assembly mass, which has contribution from suspension springs, is extremely beneficial improved vehicle dynamic performance [6]. Also, the current requirement for vehicles to meet tightening emissions and fuel economy targets, weight reduction through composite springs can help achieve these targets [7]. There is also potential for the composite springs to offer better dynamic properties when compared to steel, specifically flexural damping over the range of frequencies seen in automotive applications, as found by Gibson [8]. All of which reinforce the value of the research into their performance and modelling.

In terms of existing research, Belleville springs made of polymer composite have been analysed by Yang *et al* [9]. For the theoretical development, the average of radial component of the elastic modulus, over angles between 0 and 360 was used to calculate the stiffness coefficient. However, the effect of Poisson's ratio variation was ignored, which can have a significant impact on the accuracy of stiffness prediction. Recently, Patangtalo *et al* [10] developed a numerical method based on classical thin shell theory to analyse load-deflection characteristics of composite Belleville springs. In depth analysis was performed of the effect of height to thickness ratio on the nonlinear load-deflection behaviour. A further research by Patangtalo *et al* [11] investigated the complex effect of fibre orientation combined with conical geometry of Belleville springs. A numerical model was developed in conical coordinate system. The study concludes that for symmetrical layup of plies the kinematic behaviour, for practical purposes, being a symmetric.

Looking at applications of the technology, a variable or progressive spring rate can be used on road vehicles to give an initial softer rate for good ride comfort and a stiffer rate when under more corning force to give better handling. Thus, a good compromise can be made between comfort and handling. Currently, in many industrial applications the same type of Belleville springs are used in the stacks, which may not result in the desired characteristics required for racing applications [12]. In this study, a novel approach is proposed where different sized Belleville springs can be stacked to arrive at a very finely-varying stiffness. As they also exhibit damping properties with their hysteresis curves, a major downside of variable rate springs, the difficulty of keeping within an effective damping range [13], can be greatly improved.

Combining the potential benefits and areas for new research discussed above, this paper contains several original contributions. To fully understand these possible applications for Belleville springs a refined analytical model has been developed, which uses the calculation of average stiffness coefficient considering the variation in elastic modulus and Poisson's ratio, in contrast to Yang *et al* [9] where Poisson's ratio was considered constant. To develop the use of Belleville springs in a stack, to achieve the variable spring rates that are desired, the paper proposes a novel stacking arrangement where spacers are used between stacks. This arrangement allows the use of different spring sizes in series which creates a progressively acting spring. This also helps reduce frictional slip-stick behaviour which is crucial as the friction due to contact between the Belleville springs, specifically connected in series, can be a major concern, as it can create non-smooth characteristics.

Springs were manufactured using a similar approach to that of Yang *et al* [9], however the manufacturing variability was assessed, as the fibre orientation [14], curing time and temperature [15] have large influence on the stiffness. The theoretical predictions were also validated. Apart from initial softness seen in the springs, the stiffness prediction model compares very well with the experimental results. This paper then looks to accommodate this initial softness at lower deflection by using an updated empirical model. Belleville stacks have also been configured so that they can be used as vehicle suspension springs, as this is such a potential benefit. A quarter car model has been used to analyse the effects of using these stacks and compared against a model with linear helical spring to show the potential dynamic improvements. The material used is prepreg and a mould was developed to obtain the conical shape and free height of the Belleville springs. The details of theoretical, manufacturing and experimental aspects are given in this paper followed by the discussion on important results.

2. Polymer Composite Belleville Springs

The theoretical and experimental aspects of isotropic Belleville springs is well established [3], however the availability of similar details of orthotropic laminate Belleville springs is limited. This section provides the basis of theoretical stiffness calculation of polymer composite Belleville springs, which is based on the refinement of an approach proposed recently by Yang *et al* [9]. Further to this, the brief details of manufacturing, testing and analysis of polymer composite Belleville springs are given.

2.1. Stiffness Calculation

The material stiffness coefficient of Belleville springs plays a vital role in determining the stiffness variation in the spring. For an isotropic material, the calculations are straightforward and are well established. For the springs made of polymer composites, the variation of material properties in polar coordinates is required as both elastic modulus and Poisson's ratio vary depending on the material layup.

2.1.1. Load-deflection Characteristics

A typical Belleville spring with geometrical variables is shown in Figure 1. The axial load is applied on the inner circumference whereas the outer circumference is supported on a rigid surface. A polar coordinate representation can be used in deriving an expression for stiffness. An elemental area is considered at an angle θ, away from the primary axis.

The relation between load applied, P, and the deflection, δ, in terms of both material and geometric parameters for the polymer composite can be obtained using the theory ([3]) developed for the metal springs but by using an appropriate stiffness coefficient. The expression relating the applied load with deflection and geometric parameters is below.

$$P = \frac{4S_{av}\delta}{MD^2}\left[(h-\delta)\left(h-\frac{\delta}{2}\right)t+t^3\right] \tag{1}$$

The variables used are as follows: D is the outer diameter, d is the inner diameter, t is the thickness and h is the free height. M is given by:

$$M = \frac{6(\alpha-1)^2}{\pi\alpha^2\ln(\alpha)} \tag{2}$$

Here, α is by the ratio of outer to inner diameters, (D/d). The average stiffness coefficient, S_{av} can be calculated by averaging material properties over angles between 0 and 2π; it is given by:

$$S_{\mathrm{av}} = \frac{1}{2\pi}\int_0^{2\pi}\frac{E(\theta)}{1-\nu(\theta)^2}d\theta \tag{3}$$

Here, the effect of conical shape is ignored which may result is some errors, but results are validated using experiments on prototypes on appropriateness of this assumption. Patangtalo *et al* [11] have shown that for symmetric layup plies, in spite of complex conical geometry the displacement behaviour being axisymmetric. The expressions developed here are with the intention of using them with symmetric arrangements – if used in other cases errors are expected. The material properties that vary as a function of angle are calculated by transformation of bending compliance matrix. Using the effective bending properties in X and Y direction, the Young's modulus and Poisson's ratio at any angle can be calculated. The related expressions for finding angle dependent Young's modulus and Poisson's ratio are given in Equations 4 and 5.

$$\frac{1}{E(\theta)} = \frac{c^4}{E_x} + \left(\frac{-2\nu_{xy}}{E_x} + \frac{1}{G_{xy}} \right) s^2 c^2 + \frac{s^4}{E_y} \tag{4}$$

$$\nu(\theta) = \frac{-E(\theta)}{E_x} \left[-\nu_{xy}\left(c^4 + s^4\right) + \left(1 + \frac{E_x}{E_y} - \frac{E_x}{G_{xy}}\right) c^2 s^2 \right] \tag{5}$$

$c = \cos(\theta); \quad s = \sin(\theta)$

The effective bending properties are given by:

$$E_x = \frac{12\left(D_{11}D_{22} - D_{12}^2\right)}{t^3 D_{22}}; \quad E_y = \frac{12\left(D_{11}D_{22} - D_{12}^2\right)}{t^3 D_{11}};$$
$$G_{xy} = \frac{12 D_{66}}{t^3} \quad \text{and} \quad \nu_{xy} = \frac{D_{12}}{D_{22}} \tag{6}$$

The elements of bending stiffness matrix, D_{11}, D_{22}, D_{12} and D_{66}, for symmetric laminates can be obtained from classical lamination plate theory [16].

$$D_{11} = \frac{1}{3}\bar{Q}_{11}\left(h_k^3 - h_{k-1}^3\right); \quad D_{12} = D_{21} = \frac{1}{3}\bar{Q}_{12}\left(h_k^3 - h_{k-1}^3\right)$$
$$D_{22} = \frac{1}{3}\bar{Q}_{22}\left(h_k^3 - h_{k-1}^3\right); \quad D_{66} = \frac{1}{3}\bar{Q}_{66}\left(h_k^3 - h_{k-1}^3\right). \tag{7}$$

The values of transformed reduced stiffness matrix coefficients based on the orientation of laminates are found using the following equations:

$$\bar{Q}_{11} = Q_{11}m^4 + Q_{22}n^4 + 2\left(Q_{12} + 2Q_{66}\right)m^2n^2;$$
$$\bar{Q}_{12} = \left(Q_{11} + Q_{22} - 4Q_{66}\right)m^2n^2 + Q_{12}\left(m^4 + n^2\right);$$
$$\bar{Q}_{22} = Q_{11}n^4 + Q_{22}m^4 + 2\left(Q_{12} + 2Q_{66}\right)m^2n^2;$$
$$\bar{Q}_{22} = \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)m^2n^2 + Q_{66}\left(m^2 + n^4\right); \tag{8}$$

where $m = \cos(\phi)$; $n = \sin(\phi)$ and ϕ is the ply orientation. The reduced stiffness matrix coefficients from plane stress problem Q_{11}, Q_{22}, Q_{12} and Q_{66} are given by:

$$Q_{11} = \frac{E_1}{1-\nu_{21}\nu_{12}}; \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{21}\nu_{12}}; \quad Q_{22} = \frac{E_2}{1-\nu_{21}\nu_{12}}; \quad Q_{66} = G_{12} \tag{9}$$

The material properties used above can be obtained either from manufacturer's data or through series of tensile tests. Table 1 shows the material properties of the prepreg used in this study, which are based on the tensile tests. The manufacturer's data sheet for the material can be found at the link given in Ref [17], which gives more details.

2.1.2. Stacked Belleville Springs

The stacking arrangement of Belleville springs determines the stiffness curve. In an assembly, the individual Belleville springs that are arranged in opposite direction are in series connection; this arrangement reduces the spring rate and hence results in increased deflection for a given load. When Belleville springs are assembled nested inside each other, the resulting connection is parallel, henceforth referred to as a 'unit', as shown in Figure 2. In parallel connection, the force needed for a given deflection increases, thus increasing the stiffness. Often a combination of both stacking methods is used to achieve appropriate stiffness.

2.1.3. Novel Spring Stacking Arrangement

The springs stacked in the classical way must be of similar geometrical dimensions, especially the inner and outer diameter. This can be a restriction in achieving a finely varying, but differing stiffness characteristic. Furthermore, the springs experience friction due to many contacting surfaces; the stacked springs experience relative motion between the surfaces, which can have a significant effect on the performance of the overall system. The springs of different stiffness connected in series have frictional contact between the surfaces and the different level of deflections for a given load in the parallel connected springs will result in relative motion. There is a stick slip behavior, which can cause a non-smooth overall stiffness curve. The springs require uniform contact at the boundary between the springs and the manufacturing variations can also have significant effect.

The use of a mild steel spacer between the spring units in series is proposed here to reduce the effect of friction and allow for progressive action of different springs. The arrangement used for one such stack is shown in Figure 2, where the springs are stacked in parallel and each unit contains springs of similar stiffness. The use of spacers allows stacks to be formed from combination of springs with different outer diameters which results in progressively acting springs providing refined load-deflection curves, which is crucial for high performance applications such as motor racing. The economic benefits can be huge as number of Belleville springs needed to potentially obtain numerous load-deflection curves can be very small. The arrangement of the stack shown in Figure 2 requires development of new mathematical model to represent the behaviour, which is discussed next.

2.1.4. Modelling of Belleville Spring Stack

Arranged in parallel, when all the springs in the unit are of the same ply specification and then as with metal springs, individual stiffness can be multiplied by the number of springs in the unit (Wahl, 1944 [3]). If the springs in parallel are of differing ply specification, then the individual spring load carrying capacity is added to obtain the total load carrying capacity of the stack. For springs that show no negative stiffness or very small negative simple formulae of approximate load carrying capacity as function of deflection can be developed. For applications that are envisaged, this restriction is practical as springs with large negative stiffness introduce instability. These type of springs, in the proposed application, can only be is used in their positive stiffness deflection range. Therefore,

$$P_s(\delta)=\sum_{i=1}^{k}P_i(\delta) \qquad (10)$$

where P_s is the total load carrying capacity of spring stack, P_i is the load carrying of spring unit *i* and *k* is the number of spring units in the stack. A huge advantage can be gained by combining springs of different stiffness in series; this aspect is exploited in this study. In the proposed arrangement, the contribution of individual units to the total deflection increases sequentially in order of stiffness, with stiffer units providing a progressively larger contribution. The exact calculation of deflection involves complex terms as softer springs are expected to reach maximum deflection i.e. flatten out, hence can be considered solid after the limiting deflection during the process. The deflection of the stack of springs can, therefore, be written as:

$$\delta_s(P)=\{\delta_1(P) \quad \delta_2(P) \quad .... \quad \delta_{k-1}(P) \quad \delta_k(P)\} \qquad (11)$$

where, $\delta_1(P)=\sum_{i=1}^{k}\delta_i(P)$ for $P \leq P_1$, $\delta_2(P)=\sum_{i=2}^{k}\delta_i(P)$ for $P_1 < P \leq P_2$,

$\delta_{k-1}(P)=\sum_{i=k-1}^{k}\delta_i(P)$ for $P_{k-2} < P \leq P_{k-1}$, $\delta_k(P)$ is for $P_{k-1} < P \leq P_k$,

P_1 is the load where the softest Belleville spring in the stack becomes flat, corresponding value for the second softest spring is P_2 and so on. For springs that show large negative stiffness the expression of load carrying capacity can be written in terms of stiffnesses.

2.2. Manufacturing Methods

The woven prepreg material is used to make the springs. Each ply is made up of one whole layer of the material, cut to the correct size in terms of inner and outer diameters. As Belleville springs are simple shapes variations in thickness should not occur [18]. A two-part mould is used to form the shape of the spring, as shown in Figure 3a. The specification for the springs and thus mould design was determined from a combination of the required *h*/*t* ratio, testing apparatus so that they were at least comparable in size to a helical coil spring. The mould consisted of the male bottom half onto which the carbon plies would be laid, with the female top half fitting over the spring and providing a surface onto which the pressure required for curing could be applied. To ensure that equation (4) and the assumption of quasi-isotropic properties would be valid the correct orientation for layup was used. For example, the 4 ply springs used a 0/90/90/0 fibre orientation. If two plies are used then the predictions from equation (4) are approximate, involving some errors.

The plies were placed on to the mould, which was covered in release film. The weights positioned on top applying the curing pressure to ensure the lamina were pressed together and any air pockets removed [19]. The springs were then cured at 120°c, the recommended specification from the carbon manufacturer. After curing inner and outer diameter were machined to obtain the required dimensions.

2.3. Testing Process

All springs were compression tested individually to check their performance. Figure 4 shows typical setup used in testing Belleville spring load-deflection behaviour. This ensured that the loading was vertical with no side loads [20]. A compression test speed 1 mm/minute was used with a compression limit of 2.2mm. This limit is the maximum deflection where 80mm diameter 4 ply springs show zero stiffness and then thereafter are expected to show negative stiffness depending on the ratio, $\frac{h}{t}$. The stacks of springs were tested using a specially designed test setup which facilitates the stacking of up to 15 springs in both parallel and series configurations, where each unit can be separated by a steel spacer. Deflection profiles can then be extracted for a range of stack configurations and comparisons drawn with the theory.

2.4. Case Study: Quarter Car Model with Belleville Springs

The benefits of composite Belleville springs were ascertained using numerical models that were created to analyse the effect on key vehicle suspension performance criteria. 5 quarter car model was used, which is shown in Figure 4.

The equations of motion describing motion of mass elements are given by:

$$\begin{aligned} m_u\ddot{x}_u + c_s\dot{x}_u - c_s\dot{x}_s + k_s\left(x_u - x_s\right) + k_t x_u &= k_t y \\ m_s\ddot{x}_s - c_s\dot{x}_u + c_s\dot{x}_s - k_s\left(x_u - x_s\right) &= 0 \end{aligned} \tag{12}$$

If only one type of Belleville spring is used in a series arranged stack, we can write nonlinear suspension stiffness, k_s, as

$$k_s = \frac{1}{n}\left\{\frac{4tS_{av}}{MD^2}\left(t^2+h^2\right) - \frac{6htS_{av}}{MD^2}\left(x_u - x_s\right) + \frac{2tS_{av}}{MD^2}\left(x_u - x_s\right)^2\right\} \tag{13}$$

The other variables are: m_u is the unsprung mass, m_s is the sprung mass, c_s is the suspension damping coefficient, k_t is the tyre stiffness and y is the road input. A road input of swept sine is used to generate the frequency response functions. Three different Belleville spring stacks were analysed along with a comparable linear spring. The Motion of mass elements is used to define performance especially, the tyre-ground contact load variation and the sprung mass acceleration. The resonant behaviour introduces oscillatory changes in these parameters; the aim is to reduce the amplitude of oscillations so that the performance parameters vary as little as possible with respect to the excitation frequency. This means in the frequency domain the response should be as flat as possible. The suspension parameters of vehicles can be found using inverse methods, for an example, as given in a research published on a frequency domain approach [21]. The vehicle parameters used in the model are: unsprung mass of 35kg, sprung mass of 230kg, suspension stiffness of 30kN/m, suspension damping coefficient of 1500Ns/m and tyre stiffness of 300kN/m, which are typical of a small hatchback car.

The equations of motion given in equation (12) were translated in to a Matlab-Simulink model where the suspension stiffness made from the Belleville spring stack is represented as a look-up table. The input used was a sine-sweep (chirp) of 0 to 30 Hz with a constant velocity input. This type of input represents typical road profiles. The responses were then obtained using a fixed time step option of 0.001 sec, using Ode4 solver.

3. Results and Discussion

3.1. Belleville Spring Samples

Four type of Belleville springs were manufactured where 2, 3 and 4-plies were used. Table 2 shows the geometrical details of such springs. These geometrical dimensions enable the use of positive nonlinearity. Some of the springs show very large negative stiffness but these are used in a stack such that operating range doesn't encounter these deflections. One the spring (small OD) shows zero and nearly zero stiffness in the negative stiffness region which is later exploited in the case study presented. Examples of the finished springs are shown in Figure 3b. There were small manufacturing variations in geometrical parameters, which may influence the achieved stiffness to some degree. For example, the target OD in spring number 1 in Table 2 was 80mm but could achieve 81.2mm. As will be shown in the next section these geometrical variations haven't influenced the load-deflection behaviour that significantly.

3.2. Manufacturing Consistency and Effect of Ply Number

The geometrical parameters of the springs were expected to vary due variability in manufacturing. This can influence the load carrying capacity as well the deflection behaviour. An analysis of consistency was undertaken by producing several springs of a set specification. The springs produced were 2-ply in thickness with the specification as given in Table 2.

Figure 6a shows the load-deflection curves. The performance spread of these springs was within a small tolerance considering that these were hand produced parts. The curves for these 9 test springs varied by only around 5% over the full stiffness curve. This again indicates that the manufacturing process is reliable and robust, as well as the spring behaviour being predictable.

The spring rate depends on the number of plies in the layup construction of the springs. For the three springs, the thickness was varied from 2 to 4-ply, with all other Belleville spring parameters kept the same as shown in Table 2. Figure 6b shows the performance of each layup configuration. As expected, the thicker the spring, the higher is the stiffness. These results were found using an average of 5 springs' load vs deflection behaviour. Directly comparing each spring, the 2-ply spring showed around 50% of the spring rate when compared to the 4-ply spring. The 3-ply spring performance was between the two as expected.

3.3. Theoretical Stiffness Prediction and Validation

In this section, the load carrying capacity calculated using laminate theory given in Equation 1 is compared with the experimental results. As formulated in Section 2.1, the angular variation of elastic modulus and Poisson's ratio has a huge impact on the load carrying capacity of Belleville springs; Figures 7a and 7b show, respectively, the variation of these parameters as a function of angle θ. As expected, there is a large reduction is elastic modulus at 45 degrees and contrarily the Poisson's ratio increases. The stiffness coefficient (shown in Figure 7c), $\frac{E(\theta)}{1-\nu(\theta)^2}$ which is dependent on both elastic modulus and Poisson's ratio also decreases but the rate of decrease is reduced due to combined effect. Also, shown in Figure 7c is the stiffness coefficient as estimated using formulation given in the work of Yang *et al*, (2014 [9]) where the decrease in stiffness coefficient significantly overestimated, due to the effect of Poisson's ratio not being considered.

Figure 8 shows predicted load carrying capacity for a spring made of 4 plies with 60mm outer diameter i.e. 4th spring listed in Table 2 using (1). The estimate based on expression given by Yang *et al* (2014 [9]) is also shown in the figure. Due to under estimation of stiffness coefficient (Figure 7c), the load carrying capacity is predicted as much smaller than that from the refined model of this study.

Figure 9 shows experimental load carrying capacity of the concerned spring. The theoretical stiffness at small deflections is observed to be larger compared with the experimental results (compare initial values in Figure 8 refined model and Figure 9). The manufactured springs, for the given dimensions, are softer for up to ~ 0.3mm deflection. After this initial phase the load carrying capacity appears to follow a similar pattern to the predictions based on equation (1). The initial phase, therefore, may be treated as a consequence of manufactured composite spring geometry and can be accounted by fitting a polynomial to the initial softer part of the curve and a predictions-based equation used (1) for larger deflections. The load variation curve calculated using this refined formula is given in Figure 9. The band corresponding to expected manufacturing variation based on manufacturing variability analysis of Section 3.2 is also shown and there is a small deviation at larger deflections. The difference however, is not expected to increase as the load carried becomes almost constant for further deflection, for the springs considered. Overall, the stiffness predicted compares very well with the experimental data and the curve is well within the manufacturing variation range.

3.4. Stacked Belleville springs behaviour

Figure 10 shows performance of a stack of springs. The springs are arranged in series i.e. springs are connected in opposite directions. The theoretical estimate based on equation (11) is also shown which agrees reasonably well with the experimental curves. The smaller

OD spring as in Table 2 forms the softest unit in the stack. In the load carrying capacity curve, there is an abrupt transition from initial softer spring rate to that of higher spring rate. This transition may be due to surface contact variations expected when two packs of springs are in contact, leading to slip-stick frictional behaviour. This frictional influence seen in stack performance can be reduced by the use of spacers between the packs of springs such as suggested in Section 2.1.4. There is a difference in the arrangement – the springs are not connected in opposite directions. The spring performance improves significantly; the transition between the springs rates is very smooth which is essential to produce the variable spring rate characteristics desired for motorsport and automotive springs, either for ride comfort/NVH or vehicle handling performance. In the stacking arrangement, when one stack is significantly softer than the other, it will almost flatten out before the next stack becomes active and the stiffer spring rate is seen. The results shown earlier limited the extension to 2.2mm where the stiffness was positive for all deflections. Here, however, the softest spring is deflected until flat so that some negative stiffness is also used. The negative stiffness encountered is very small as seen in Figure 10 (blue, bold line). At ~4.5mm the softest spring becomes flat. Overall deflection is much larger than the individual springs as the stacks are arranged in series connection.

3.5. The effect of hysteresis on Belleville spring performance

Under rebound conditions, the Belleville springs show levels of hysteresis and damping both individually and when in a stack. Figure 11a shows the difference in loading and unloading part for single spring. The difference can come from two sources: a) the internal damping in the spring material and b) frictional force due micro sliding of outer edge of the Belleville springs. Figure 11b shows hysteresis effect in stacked springs. The hysteresis increases with increasing deflection. The amount of hysteresis is directly dependent on the number of springs and how close to maximum deflection they have been loaded. The stacks, which are described in Table 3, are made up of two units arranged in series using the individual springs shown in Table 2.

This hysteresis, essentially a form of damping, is potentially very useful for automotive and motorsport applications as it will reduce the rate at which the wheel oscillates when hitting kerbs or potholes, thus increasing tyre contact patch forces and ride comfort when optimised along with a matching damper [22]. The theoretical benefits to having damping within the spring is with very high frequency inputs where the damper effectively becomes solid; here, if the composite Belleville spring stack can provide some damping it will be of a large benefit.

3.6. Case Study Vehicle suspension simulation results

The springs of Table 2 that make stacking units of Figure 11b can be used to generate stiffness characteristics, for potential use in vehicle suspension. Figure 12 shows load carrying capacity of three such spring where number of stacked units used vary from 10 to 17. The load-deflection curves are based around typical linear, helical spring's stiffness values of 30kN/m. The curves are such that they provide varying stiffness at low and high deflections. Stiffness at higher deflections is expected to influence the vehicle body motion i.e. the motion of m_s in Figure 5.

Figure 13 shows normalised force and acceleration responses that influence handling and ride comfort of a vehicle. The velocity normalised tyre force in Figure 13a shows two peaks – one due to the vehicle bounce mode at lower frequency and the second one, the wheel hub mode. Ideally, the flatter the curve the more consistent is the tyre force. The Belleville stacks produce massive improvement by reduced force variation at the vehicle bounce mode, but show some deterioration nearer wheel hub mode. There is a huge potential in tuning nonlinear stiffness to optimise performance throughout the frequency range. Figure 13b shows the velocity normalised acceleration of vehicle body. Here the improvements at the vehicle bounce mode are maintained while the performance difference at wheel hub mode is minimal. Despite of limited number of options explored, overall the Belleville stacks are seen to provide significant ride comfort improvement and to a smaller extent, improvement in vehicle handling. The different Belleville stacks show varying amount of improvements and there is a scope for further optimisation. The idea of tailoring the stiffness works well, as one can optimise both vehicle handling and the ride comfort.

4. Conclusions

Modelling and manufacturing of polymer composite Belleville springs is investigated with the aim of an application of vehicle dynamics in order to reduce mass while improving the dynamic performance. The Belleville springs can be a viable alternative to helical coil springs for automotive and motorsport vehicles with considerable potential benefits. The following conclusions can be drawn based on this study:

1. A theoretical model was developed relating the load and deflection in terms of material and geometrical parameters, considering variation of both elastic modulus and Poisson's ratio in laminates. Using the proposed model, the load carrying capacity of individual springs with 4-ply layups was predicted, the differences seen are well within the manufacturing variation range.
2. The thickness of the composite Belleville affects the spring rate performance by the margin predicted by the theoretical calculations. Therefore the 4 ply springs are around double the stiffness when compared to the 2-ply Belleville springs, with 3-ply midway between the two.
3. Variable spring rates are achievable using composite Belleville spring stacks and using the theory outlined in this paper, the behaviour can be predicted so they can be implemented at the design stage.
4. Composite Belleville springs clearly show hysteresis characteristics and this behaviour can be used to adding damping when large wheel inputs are seen which can overload the shock absorber.
5. Adding a spacer between each unit of Belleville springs allows the use of springs with different dimensions and also reduces the slip-stick based frictional forces resulting in smooth the transition.
6. Composite Belleville springs have been theoretically proven to improve vehicle handling and comfort by reducing tyre contact patch force variation and vehicle body

acceleration. As part pf further research, the suspension system based on this stacking arrangement is being produced and tested in assembly to confirm the predicted improvements.

Overall, the polymer composite Belleville springs are a viable, potential alternative to current suspension springs as they offer improved dynamic performance and significant mass savings.

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Data Availability Statement

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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Table 2: Material properties for VTM-264 prepreg.

E_1 (GPa)	**E_2 (GPa)**	**ν_{12}**	**G_{12} (GPa)**
48.5	50.4	0.1	2.3

Table 2: Belleville spring specifications.

Spring Number	**Ply Number**	**Height (mm)**	**OD (mm)**	**ID (mm)**	**Thickness (mm)**
1	2	7.3	81.2	26.6	1.04
2	3	7.6	82.0	26.4	1.44
3	4	8.5	83.2	26.4	1.94
4	4	5.0	61.0	25.8	1.88

Table 3: Arrangements of Stacks 23, 24 and 25

Stack	Unit 1	Unit 2
23	4 x Spring 3	1 x Spring 4
24	6 x Spring 3	1 x Spring 2
25	3 x Spring 3	1 x Spring 4

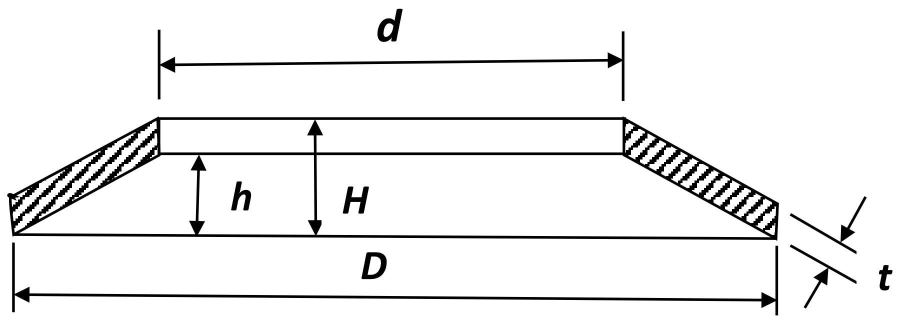
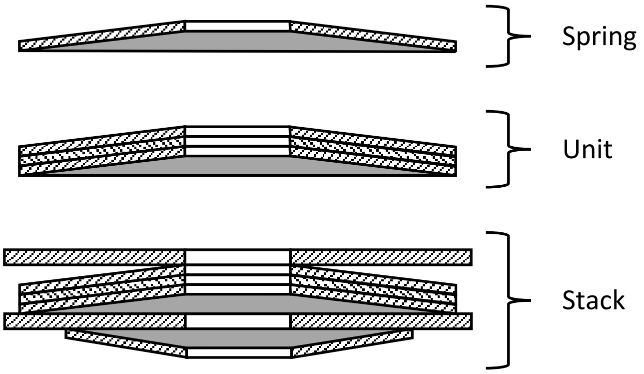


Figure 1: Geometrical parameters of typical Belleville spring.

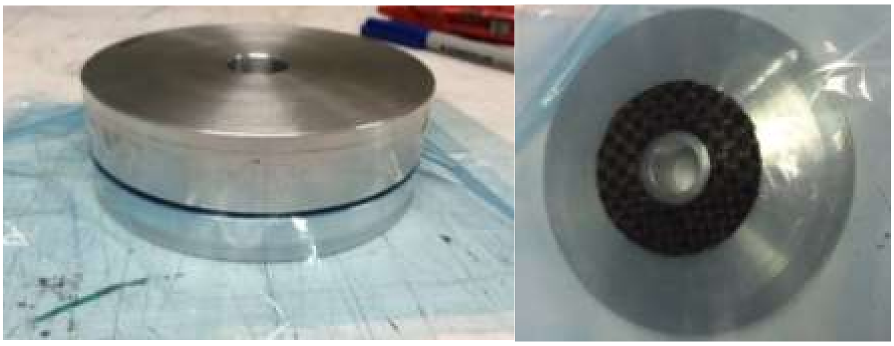
a)



b)

Figure 2: Belleville spring stacking – a) A stacking arrangement for different size Belleville springs with spacers in-between and the notations used and b) Example of the testing equipment used for the stack testing. Note the use of spacers in between each stack.

a)



b)

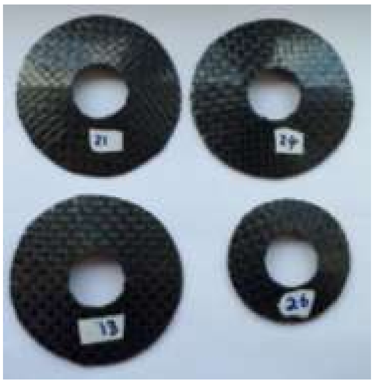


Figure 3: a) Composite Belleville spring layup and mould and b) Samples of manufactured springs.

Figure 4: Example setup of compression testing on a Belleville Spring.

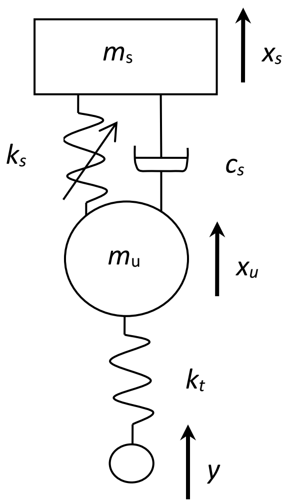
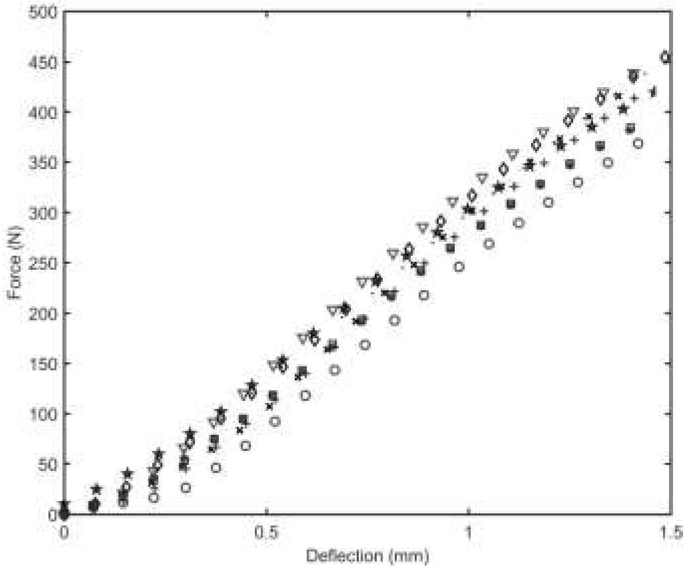


Figure 5: An application in the form of a vehicle suspension system – quarter car model

a)



b)

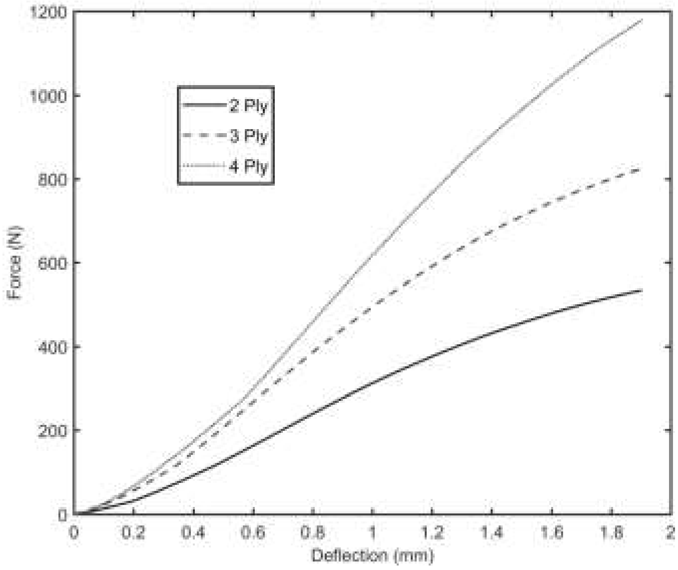


Figure 6: Performance of manufactured springs. a) manufacturing variation of spring load capacity of 2 Ply springs and b) load capacity of different ply configurations with 80mm outside diameter.

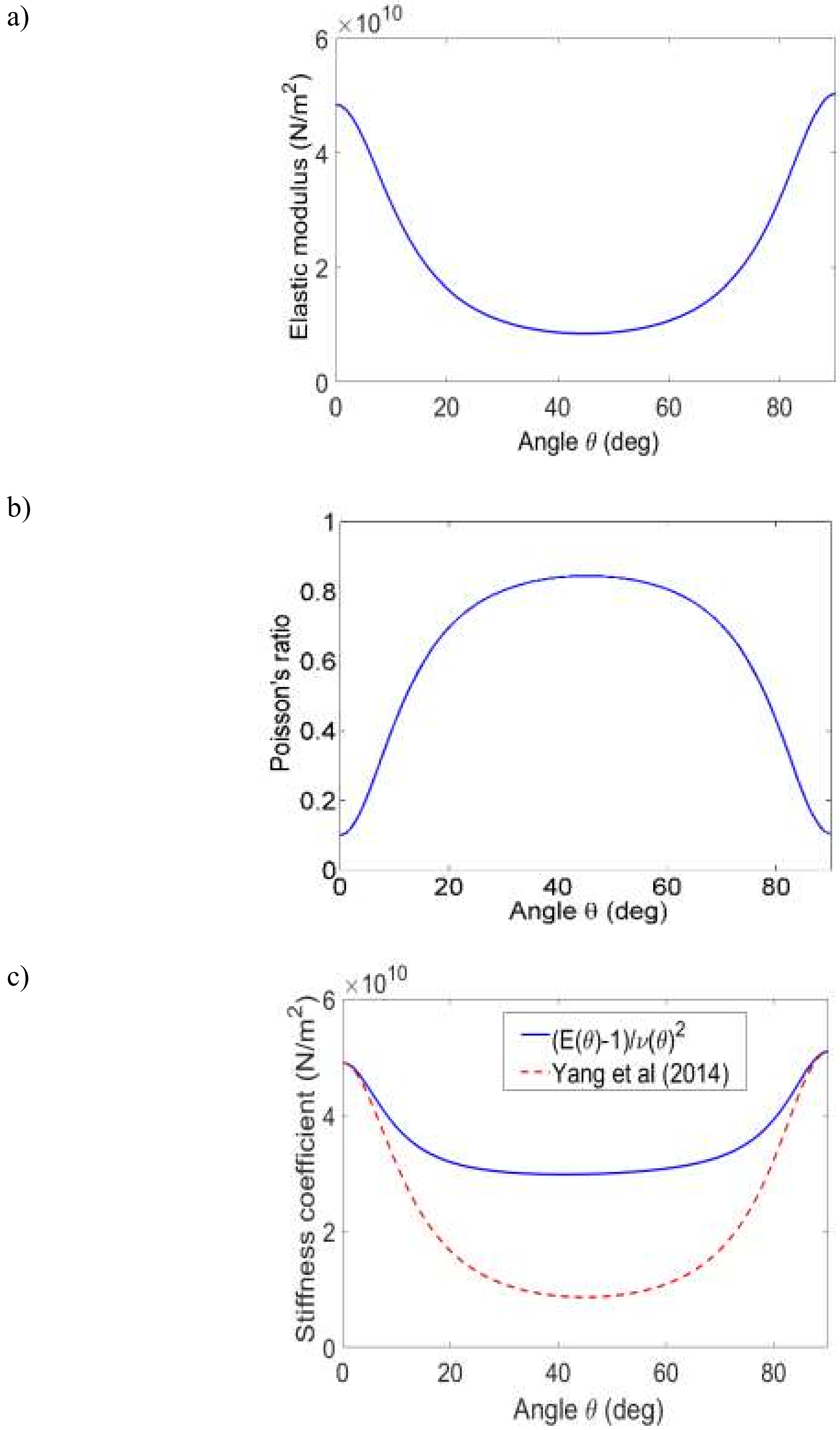


Figure 7: Variation of laminate properties as a function angle. a) Radial component of elastic modulus, b) Poisson's ratio and c) Stiffness coefficient.

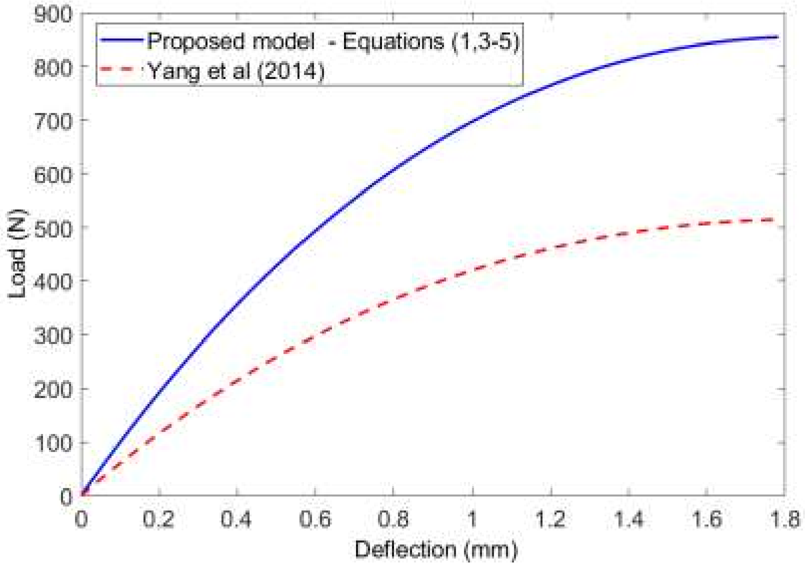


Figure 8: Theoretical estimates of load carrying capacity of 60mm outside diameter spring (No. 4 in Table 2) based on Equation (1) and from Yang et al, 2014 [9].

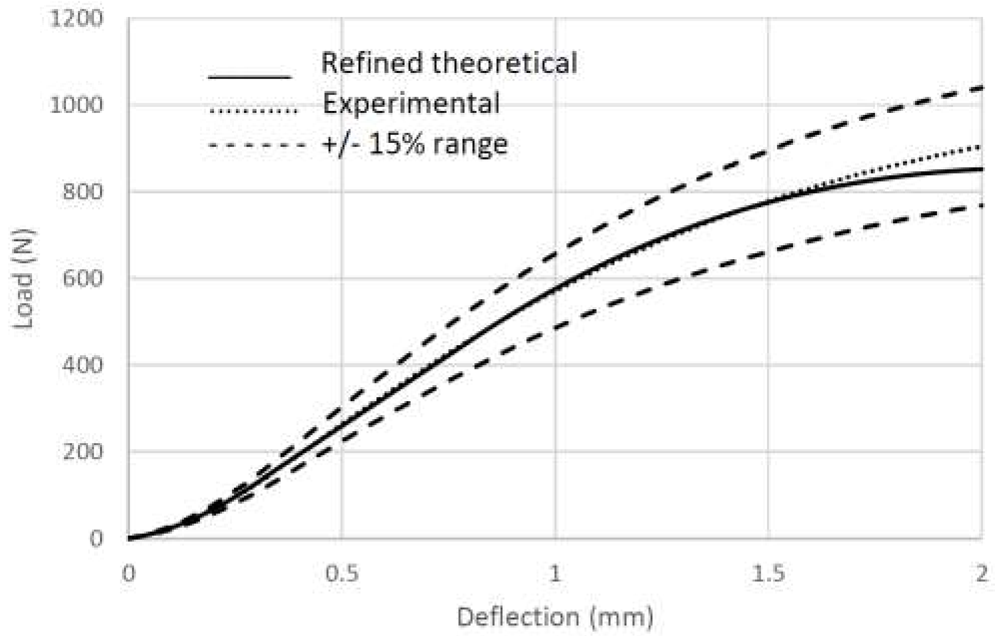


Figure 9: Comparison of theoretical and experimental spring rates for a 4-ply spring with 60mm OD.

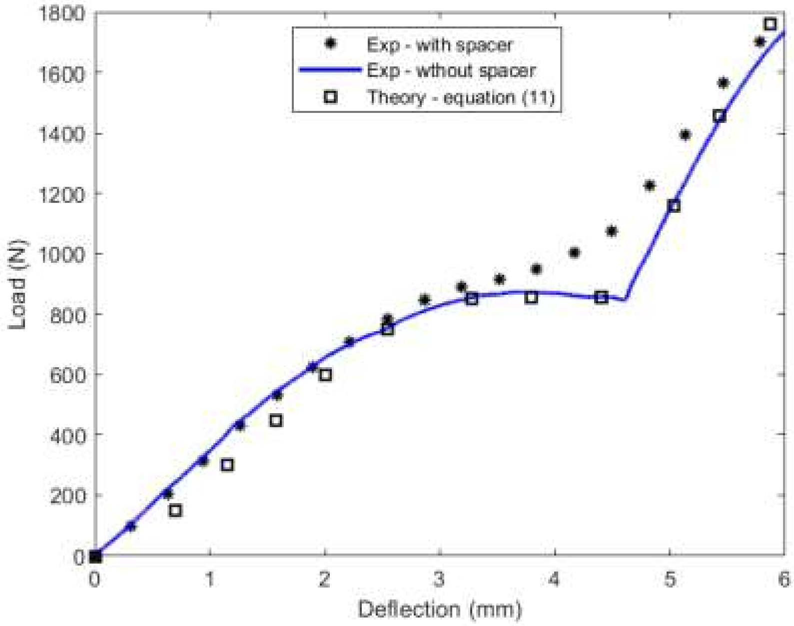
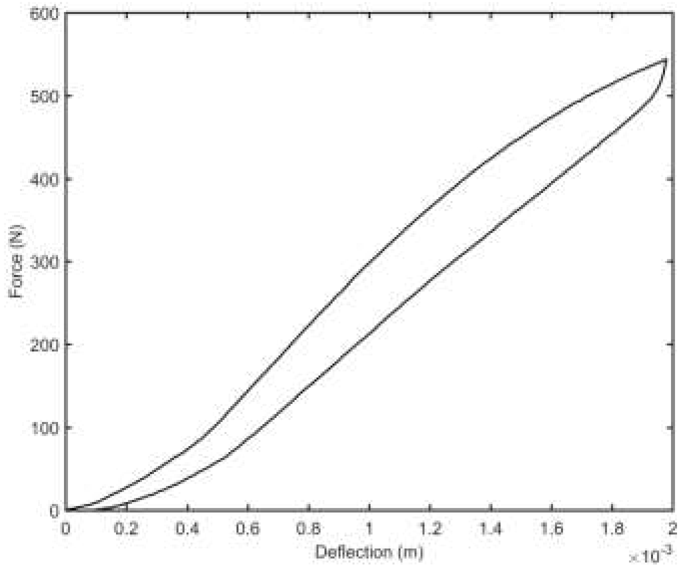


Figure 10: Stack performance showing the load-deflection curve with/without spacers.

a)



b)

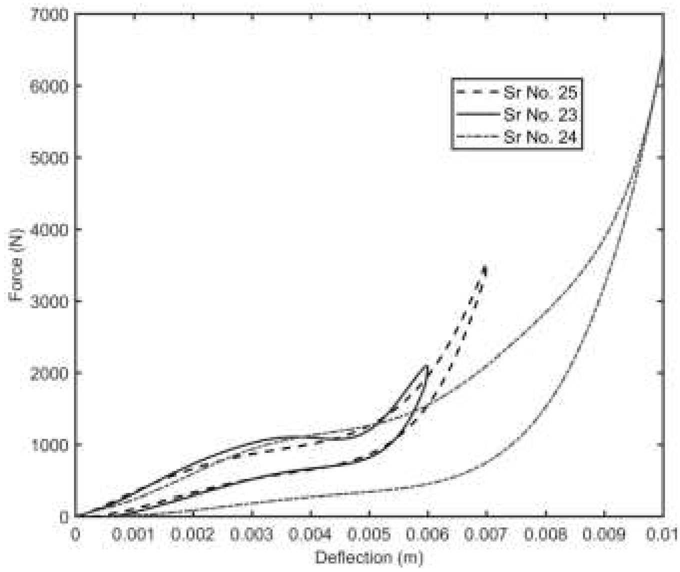


Figure 11. Example hysteresis curve showing the loading and unloading cycles. a) Individual spring and b) three different Belleville stacks.

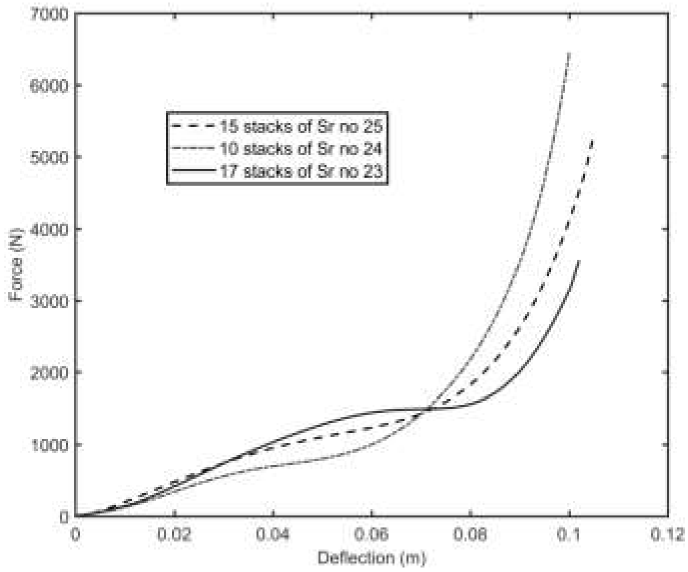
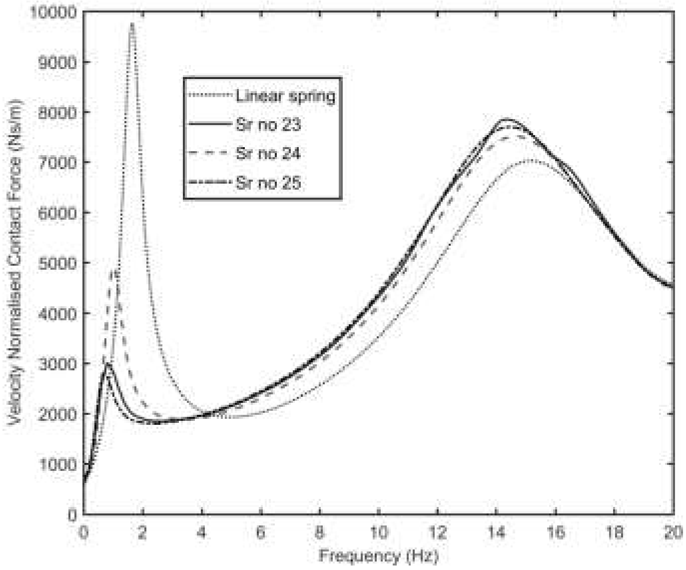


Figure 12: The stiffness characteristics of Belleville spring stacks developed

a)



b)

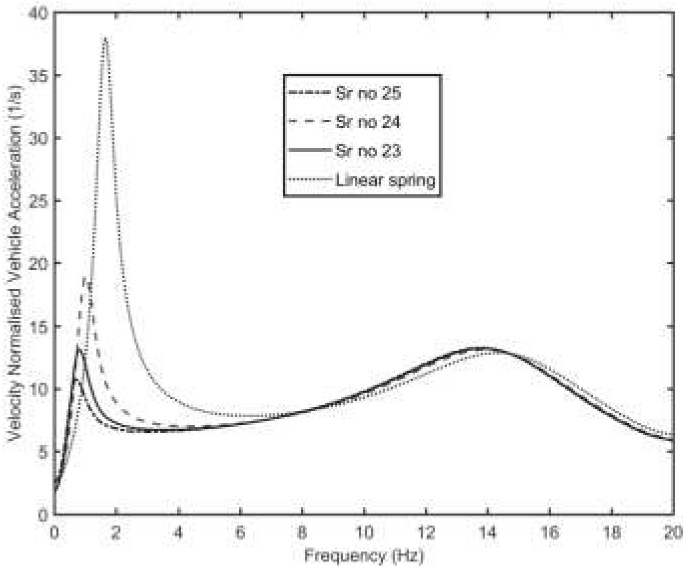


Figure 13: Example frequency dependent performance comparison of various springs. a) Contact force variation and b) vehicle body acceleration.