Dragon Slaying with Ambiguity: Theory and Experiments*

Sara le Roux†
Department of Economics,
Oxford Brookes University, England.

David Kelsey
Department of Economics,
University of Exeter, England.

2nd December 2015

Abstract

This paper studies the impact of ambiguity in the best shot and weakest link models of public good provision. The models are first analysed theoretically. Then we conduct experiments to study how ambiguity affects behaviour in these games. We test whether subjects' perception of ambiguity differs between a local opponent and a foreign one. We find that an ambiguity safe strategy, is often chosen by subjects. This is compatible with the hypothesis that ambiguity aversion influences behaviour in games. Subjects tend to choose contributions above (resp. below) the Nash equilibrium in the Best Shot (resp. Weakest Link) model.

Keywords: Public goods; Ambiguity; Choquet expected utility; strategic complements; weakest link; best shot.

JEL Classification: C72, **C**91, **D**03, **D**81, **H**41

*Financial support from the University of Exeter Business School is gratefully acknowledged. We would like to thank Jürgen Eichberger, Zvi Safra, Scott Mcdonald, Dieter Balkenborg, Miguel Fonseca and some anonymous referees for their comments and suggestions.

†**Address for Correspondence:** Sara le Roux, Department of Economics, Oxford Brookes University, Wheatley Campus, Oxford OX33 1HX. E-mail: drsaraleroux@gmail.com Tel: +44 (0) 1865 485918

1 Introduction

This paper reports some theoretical results on how ambiguity affects behaviour in the best shot and weakest link models of public good provision. We then proceed to study experimentally the impact of ambiguity in these models.

Public goods are goods which can be consumed by everybody. We study situations where individuals make voluntary contributions to the provision of the public good. Due to the collective nature of the good everybody enjoys the same amount of it, irrespective of their own contribution. The usual assumption is that the amount of the public good available is a function of the *sum* of all individual contributions. Ambiguity in the standard public goods model has been previously studied in Eichberger and Kelsey (2002) and Bailey, Eichberger, and Kelsey (2005). These models have been tested experimentally by Di Mauro and Castro (2011).

An alternative, is the best-shot model, where production of the public good is determined by the *maximum* contribution made by an individual in the community. This may be represented as: $u_i(x_i, x_{-i}) = \max\{x_1, ..., x_n\} - cx_i$, where x_i denotes the contribution of individual i and c denotes the marginal cost of a contribution. In this case, making a large contribution, x_i results in a large cost, cx_i, but the benefit accrues to all members of the group. This model may be illustrated by a medieval village that is besieged by a dragon. It is only the knight endeavouring to slay the dragon, who bears the cost - in this case, the chance that he will be burnt to a crisp by the dragon. However, once the dragon is slain, the benefits of a dragon-free village are enjoyed equally by all the village folk! A "whistle-blower" may be seen as a modern-day dragon-slayer. He bears the burden that comes with the act of exposing corruption or incompetence, though the benefit of his act accrues to the general public.

A third possibility is the weakest link model, in which provision of the public good is a function of the *minimum* of the individual contributions. It may be represented as: $u_i(x_i, x_{-i}) = \min\{x_1, ..., x_n\} - cx_i$. It may be noted that making a large contribution, x_i, would have a large cost, cx_i, but does not guarantee a large payoff, since the minimum contribution made within the group of individuals would determine the level of the public good.

This model can be illustrated by the example of a small island community that must build sea defences to protect itself from flooding. The success in holding back the storm waters will depend on the minimum height or strength of the different sections of the dyke. As such, it is the weakest dyke that will succumb to the storm first, resulting in the entire island being flooded. Similarly, a weakest-link problem may be observed when trying to prevent the spread of infectious diseases such as Ebola, combating the entry of illegal drugs into a country, or controlling illegal immigrants. The weakest link model is also relevant for environmental problems. Consider a global pollutant such as CO_2 where the damage to the environment depends on total emissions. If industries can relocate easily, then the level of pollution would depend on the country with the weakest environmental regulation.

Our analysis shows that although both models have multiple Nash equilibria (henceforth NE), when ambiguity is sufficiently high, Equilibrium Under Ambiguity (henceforth EUA) is unique. Ambiguity-aversion will cause people to choose the highest effort level in the Best-Shot model and the lowest in the weakest link model. We proceed to test our results in the laboratory. Our experimental hypothesis is that ambiguity will decrease individuals' contributions in the weakest-link model, whereas it will increase them in the best-shot case.

Kilka and Weber (2001) conducted an experiment where they asked German subjects to rate their competence when judging stock price changes of Deutsche Bank (Germany's largest banking group) and Dai-Ichi Kangyo Bank (one of Japan's largest banks). A majority of their subjects (51 of the 55 studied) reported that they felt less competent when judging stock price changes of the foreign security as opposed to the domestic security.

We test whether players in games feel a similar lack of competence when dealing with foreign opponents. If the analogy were to hold, a subject would feel more anxious when faced by a foreign opponent, than when he is faced by a local one. The rationale behind this hypothesis is that he believes the local opponent has been raised in a similar sociocultural background as himself. Thus the behaviour of a foreign opponent, about whom there is limited knowledge, is less predictable.

We find that behaviour of the subjects is consistent with our hypothesis and that ambiguity does indeed lead subjects to decrease (resp. increase) contributions in the weakest-link (resp. best-shot) game. However, though subjects display ambiguity aversion on the whole, the level of ambiguity does not become more pronounced when they are matched against a foreign opponent.

Organisation of the Paper In Section 2 we describe our framework and definitions. The public goods models are analysed theoretically in Section 3. In Section 4 we describe our experimental design and the next section discusses the data we find. Section 6 compares our findings with related literature and Section 7 provides a summary of our results together with future avenues of research.

2 Framework and Definitions

In this section we explain how we represent ambiguity in public good models. If a public good is provided by voluntary contributions the individuals concerned are playing a non-cooperative game. The pay-off of any given individual will depend on the contributions of all other individuals as well as his/her own contribution. Thus to understand the impact of ambiguity we need a theory of ambiguity in games.

In a Nash equilibrium, players behave in a manner that is consistent with the actual behaviour of their opponents. Players can perfectly anticipate the actions of their opponent and can thus choose a best response to it. If beliefs are non-additive, we need to modify the idea of having consistent beliefs and the ability to play a best response. We assume that players choose pure strategies. In equilibrium, the support of a player's beliefs must be best responses for the opponent, given his/her beliefs.

Our notation for games is as follows. A 2-player game $\Gamma = \langle \{1,2\}; X_1, X_2, u_1, u_2 \rangle$ consists of players, $i = 1, 2$, finite pure strategy sets X_i and payoff functions $u_i(x_i, x_{-i})$ for each player. Both players have the same strategy set, $X_1 = X_2 \subseteq \mathbb{N}$, which consists of all integers between a lower bound $\underline{x}$ and an upper bound $\bar{x}$.[1] The notation, x_{-i}, denotes the strategy chosen by i's opponent and the set of all strategies for i's opponent is X_{-i}. The space of all strategy profiles is denoted by X. We shall adopt the convention that female pronouns (she, her etc.) denote player 1 and male pronouns denote player 2.[2]

A player has a possibly ambiguous belief about what his/her opponent will do. These beliefs are represented by *capacities,* which are similar to subjective probabilities except that they are not required to be additive over disjoint events. Formally capacities are defined as follows.

Definition 2.1 *A* capacity *on X_{-i} is a real-valued function ν_i on the subsets of X_{-i} such that $A \subseteq B \Rightarrow \nu_i(A) \leqslant \nu_i(B)$ and $\nu_i(\varnothing) = 0,\ \nu_i(X_{-i}) = 1$.*

The expected payoff obtained from a given act, with respect to a non-additive belief, ν_i, can be found using the Choquet integral, defined below.

Definition 2.2 *The* Choquet integral *of $u_i(x_i, x_{-i})$ with respect to capacity ν_i on X_{-i} is:*

$$V_i(x_i) = u_i\left(x_i, x_{-i}^1\right) \nu_i\left(x_{-i}^1\right) + \sum_{r=2}^{R} u_i\left(x_i, x_{-i}^r\right) \left[\nu_i\left(x_{-i}^1, ..., x_{-i}^r\right) - \nu_i\left(x_{-i}^1, ..., x_{-i}^{r-1}\right)\right],$$

where the strategy profiles in X_{-i} are numbered so that $u_i\left(x_i, x_{-i}^1\right) \geqslant u_i\left(x_i, x_{-i}^2\right) \geqslant ... \geqslant u_i\left(x_i, x_{-i}^R\right)$.

[1] The restriction to integer values enables us to apply the equilibrium concept from Eichberger and Kelsey (2014). It is not essential to require effort levels to be integers. It could be any other finite set of real numbers.

[2] Of course this convention is for convenience only and bears no relation to the actual gender of subjects in our experiments.

Schmeidler (1989) axiomatised preferences which may be represented by maximising a Choquet integral with respect to a capacity. Chateauneuf, Eichberger, and Grant (2007) introduced the neo-additive capacity model, which is a special case of Schmeidler's theory. In this model, the decision-maker has beliefs based on an additive probability distribution π_i. However, these beliefs are ambiguous. The confidence in the belief is represented by $(1-\delta_i)$, with $\delta_i = 1$ corresponding to complete ignorance and $\delta_i = 0$ denoting no ambiguity. The decision-maker's attitude to ambiguity is measured by α_i. The higher the α_i, the more ambiguity-averse the decision-maker will be. These preferences are defined as follows.

Definition 2.3 *A* neo-additive-*capacity* ν_i *on* X_{-i} *is defined by* $\nu_i\left(X_{-i}|\alpha_i,\delta_i,\pi_i\right)=1$, $\nu_i\left(\varnothing|\alpha_i,\delta_i,\pi_i\right)=0$ *and* $\nu_i\left(A|\alpha_i,\delta_i,\pi_i\right)=\left(1-\alpha_i\right)\delta_i+\left(1-\delta_i\right)\pi_i\left(A\right)$ *for* $\emptyset \subsetneqq A \subsetneqq X_{-i}$, *where* $0 \leqslant \delta_i < 1$, π_i *is an additive probability distribution on* X_{-i}, *and* $\pi_i(A)=\sum\limits_{x_{-i}\in A}\pi_i(x_{-i})$.[3]

Chateauneuf, Eichberger, and Grant (2007) show that the Choquet expected value of a pay-off function $u_i\left(x_i,\cdot\right)$ with respect to the neo-additive-capacity ν_i is given by:

$$V_i\left(x_i\right)=\delta_i\alpha_i\min_{x_{-i}\in x_{-i}}u_i\left(x_i,x_{-i}\right)+\delta_i\left(1-\alpha_i\right)\max_{x_{-i}\in X_{-i}}u_i\left(x_i,x_{-i}\right)+(1-\delta_i)\mathbf{E}_{\pi_i}u_i\left(x_i,x_{-i}\right),$$

where $\mathbf{E}_{\pi_i}$ denotes conventional expectation with respect to the probability distribution π_i. This expression is a weighted average of the highest payoff, the lowest payoff and an average payoff. The response to ambiguity is partly optimistic represented by the weight given to the best outcome and partly pessimistic. We define the support of a neo-additive capacity to be the support of the additive probability on which it is based.[4]

Definition 2.4 *The* support *of a neo-additive capacity* $\nu_i\left(A|\alpha_i,\delta_i,\pi_i\right)=\alpha_i\delta_i+\left(1-\delta_i\right)\pi_i\left(A\right)$ *is given by* $\operatorname{supp}\nu_i=\operatorname{supp}\pi_i$.

[3]Where convenient we shall suppress the arguments α,δ and π and simply write $\nu\left(A\right)$. Chateauneuf, Eichberger, and Grant (2007) write the neo-additive capacity in the form $\nu\left(A\right)=\delta\alpha+\left(1-\delta\right)\pi\left(A\right)$. In the main text we have modified the definition of a neo-additive capacity to be consistent with the majority of the literature where α is the weight placed on the minimum expected utility.

[4]For a justification of this definition and its relation to other support notions see Eichberger and Kelsey (2014).

In games, π_i is determined endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, α_i and ambiguity, δ_i, as exogenous. Define the best-response correspondence of player i given that his/her beliefs are represented by a neo-additive capacity ν_i by $R_i(\nu_i) = \operatorname{argmax}_{x_i \in X_i} V_i(x_i)$.

Definition 2.5 (Equilibrium under Ambiguity) *A pair of neo-additive capacities (ν_1^*, ν_2^*) is an Equilibrium Under Ambiguity (EUA) if for $i = 1, 2$, $\operatorname{supp}(\nu_i^*) \subseteq R_{-i}(\nu_{-i}^*)$.*

Equilibrium strategies are given by the supports of the capacities, which are required to be best-responses. If these are singleton sets, we have a *pure equilibrium*. Otherwise we shall say that an equilibrium is *mixed*.[5] In an EUA each player perceives ambiguity about the strategy of his/her opponent. This is represented by an ambiguous belief, in the form of a capacity over the opponent's strategy space. However the support of a player's beliefs is itself an ambiguous event. This reflects some uncertainty about whether or not his/her opponents play best responses. Players respond to this ambiguity partly in an optimistic way by over-weighting the best outcome and partly in a pessimistic way by over-weighting the worst outcome. In this context the best (resp. worst) outcome is one's opponent playing the most (resp. least) favourable strategy. Consistency between beliefs and actions is achieved by requiring that all strategies in the support of a player's beliefs be a best response for his/her opponent.

A common interpretation of NE is that each player chooses a strategy which maximises his/her utility given the strategy of the other players. However it is also possible to view NE as an equilibrium in beliefs. From this viewpoint each player has a subjective belief about the actions of his/her opponents and chooses a best response to this belief. Definition 2.5 extends the interpretation of NE as an equilibrium in beliefs, by allowing these beliefs to be

[5] This definition of equilibrium comes from Eichberger and Kelsey (2014). It is based on earlier work by Dow and Werlang (1994).

non-additive. We interpret the deviation from additivity as representing ambiguity about the opponent's strategy choice.

We aim to extend the concept of NE by allowing for possibility that a player may view his/her opponents' strategy choice as ambiguous. If we added to Definition 2.5 a requirement that the capacities ν_1^* and ν_2^* were additive, it would be an alternative definition of NE. Hence we believe we have just extended NE to allow for ambiguity.

3 Public Goods Theory

In this section we study the effect of ambiguity in the Best-Shot and Weakest-Link public goods models. We find that ambiguity-aversion tends to reduce contributions in the weakest link model but increases them in the best shot case. In both cases, the equilibrium strategies are unique if there is sufficient ambiguity.

3.1 Best Shot

If players are ambiguity-averse, we show that for high levels of ambiguity, both of them will choose the highest possible strategy. To understand this, recall that in NE, one player will provide the highest possible effort level and the other will free-ride by supplying the lowest effort. However with high ambiguity each player becomes concerned about the worst scenario, which is his/her opponent supplying low effort. This can cause both players to choose the highest effort level. Thus ambiguity encourages the individuals to supply more effort. These effort levels would be inefficiently high in the corresponding situation without ambiguity. It is less clear that they are inefficient when ambiguity is present, since they do protect the agents against ex-ante utility losses due to ambiguity-aversion.

If both players are ambiguity-loving, then one may get an equilibrium where each provides the lowest possible effort level. Ambiguity-loving causes a player to over-weight the best outcome, which occurs when the opponent chooses the highest effort. This over-weighting reduces the given player's perceived marginal benefit, which results in the choice of the lowest effort level. Effectively both are attempting to free ride on the effort of the other. If one player is ambiguity-loving and the other is ambiguity-averse then we can get an equilibrium where the ambiguity averse player uses the highest strategy and the other uses the lowest strategy. As above, ambiguity-aversion increases the incentive to choose a high strategy and ambiguity-loving increases the incentive to play a low strategy.

Finally if ambiguity is low then there are multiple EUAs. In one Player 1 plays the highest strategy and Player 2 plays the lowest strategy and in the other the roles are reversed. This is similar to the standard Nash equilibria.

Proposition 3.1 *Assume that both players are ambiguity averse i.e.* $\alpha = 1$*. The pure equilibria under ambiguity of the Best-Shot game are as follows:*

1. *if* $(1-\delta_1)+\delta_1\alpha_1 < c$ *and* $(1-\delta_2)+\delta_2\alpha_2 < c$*, the equilibrium strategies are unique and are equal to* $\langle \underline{x}, \underline{x} \rangle$.

2. *if* $(1-\delta_1)+\delta_1\alpha_1 < c$ *and* $(1-\delta_2)+\delta_2\alpha_2 > c$*, the equilibrium strategies are unique and are equal to* $\langle \underline{x}, \bar{x} \rangle$.

3. *if* $(1-\delta_1)+\delta_1\alpha_1 > c$ *and* $(1-\delta_2)+\delta_2\alpha_2 < c$*, the equilibrium strategies are unique and are equal to* $\langle \bar{x}, \underline{x} \rangle$.

4. *if* $(1-\delta_1)+\delta_1\alpha_1 > c > \delta_1\alpha_1$ *and* $(1-\delta_2)+\delta_2\alpha_2 > c > \delta_2\alpha_2$*, there are two possible pairs of equilibrium strategies* $\langle \bar{x}, \underline{x} \rangle$ *and* $\langle \underline{x}, \bar{x} \rangle$.

5. *if* $\delta_1\alpha_1 < c$ *and* $\delta_2\alpha_2 > c$*, the equilibrium strategies are unique and are equal to* $\langle \underline{x}, \bar{x} \rangle$.

6. *if* $\delta_1\alpha_1 > c$ *and* $\delta_2\alpha_2 < c$*, the equilibrium strategies are unique and are equal to* $\langle \bar{x}, \underline{x} \rangle$.

7. *if* $\delta_1\alpha_1 > c$ *and* $\delta_2\alpha_2 > c$*, the equilibrium strategies are unique and are given by,* $x_1 = x_2 = \bar{x}$.

Proof. Consider Player 1. Suppose that she believes that Player 2's effort will be $\tilde{x}_2$, where $\underline{x} \leqslant \tilde{x}_2 \leqslant \bar{x}$. Then 1's (Choquet) expected pay-off conditional on this belief will be:

$$V^1(x_1|\tilde{x}_2) = \begin{cases} \delta_1(1-\alpha_1)\bar{x} + (1-\delta_1)\tilde{x}_2 + (\delta_1\alpha_1 - c)x_1, & \text{if } x_1 < \tilde{x}_2; \\ \delta_1(1-\alpha_1)\bar{x} + [(1-\delta_1) + \delta_1\alpha_1 - c]x_1, & \text{if } x_1 \geqslant \tilde{x}_2. \end{cases} \tag{1}$$

By similar reasoning the pay-off of Player 2 is given by:

$$V^2(x_2|\tilde{x}_1) = \begin{cases} \delta_2(1-\alpha_2)\bar{x} + (1-\delta_2)\tilde{x}_1 + (\delta_2\alpha_2 - c)x_2, & \text{if } x_2 < \tilde{x}_1; \\ \delta_2(1-\alpha_2)\bar{x} + [(1-\delta_2) + \delta_2\alpha_2 - c]x_2, & \text{if } x_2 \geqslant \tilde{x}_1. \end{cases} \tag{2}$$

Part 1 Clearly this implies $\delta_1\alpha_1 - c < 0$. Hence, by equation (1), $V^1(x_1|\tilde{x}_2)$ is a strictly decreasing function of x_1. The only possible best response is $x_1 = \underline{x}$. Similarly the best response for Player 2 is $\underline{x}$ and thus the equilibrium strategies are $\langle \underline{x}, \underline{x} \rangle$.

Part 2 As in part 1, $V^1(x_1|\tilde{x}_2)$ is a strictly decreasing function of x_1 and the only possible best response is $x_1 = \underline{x}$. From equation (2) we see that $V^2(x_2|\underline{x})$ is increasing in x_2. Hence Player 2's best response is $\bar{x}$ and thus the equilibrium strategies are $\langle \underline{x}, \bar{x} \rangle$.

Part 3 By similar reasoning to that used in part 2, we may show that $\langle \bar{x}, \underline{x} \rangle$ is the unique equilibrium strategy profile.

Part 4 By equation (1), $V^1(x_1|\tilde{x}_2)$ is decreasing for $x_1 < \tilde{x}_2$ and increasing for $x_1 > \tilde{x}_2$. Consequently the only possible best responses are $x_1 = \underline{x}$ or $x_1 = \bar{x}$. Since Player 2 is in a similar position if $\tilde{x}_2$ is a best response we must have $\tilde{x}_2 = \underline{x}$ or $\tilde{x}_2 = \bar{x}$. If $\tilde{x}_2 = \bar{x}$, then $V^1(x_1|\bar{x})$ is decreasing, hence Player 1's best response is $\underline{x}$ and thus the equilibrium strategies are $\langle \underline{x}, \bar{x} \rangle$. If $\tilde{x}_2 = \underline{x}$, then $V^1(x_1|\underline{x})$ is increasing, Player 1's best response is $\bar{x}$ and hence the equilibrium strategies are $\langle \bar{x}, \underline{x} \rangle$.

Part 5 Note that $\delta_2\alpha_2 > c$ implies $(1-\delta_2) + \delta_2\alpha_2 > c$. From equation (2) we see that

that $V^2(x_2|\tilde{x}_1)$ is strictly increasing in x_2 for all values of $\tilde{x}_1$. Hence Player 2's best response is $x_2 = \bar{x}$. By equation (1), $V^1(x_1|\bar{x})$ is decreasing in x_1. Thus $x_1 = \underline{x}$ is Player 1's best response. Consequently the equilibrium strategies are $\langle \underline{x}, \bar{x} \rangle$ and are unique.

Part 6 Follows by similar reasoning to part 5.

Part 7 Then by parts 5 and 6 the best responses of player 1 and 2 are $x_1 = \bar{x}$ and $x_2 = \bar{x}$ respectively. This holds regardless of the player's belief about the behaviour of his/her opponent. Consequently the equilibrium strategies are $\langle \bar{x}, \bar{x} \rangle$ and are unique. ■

Next we analyse the mixed equilibria in the Best Shot game. We find that there is an interval of values of marginal cost, c, for which a mixed equilibrium exists. However this interval becomes small as ambiguity, δ, increases and eventually disappears in the limit where $\delta \to 1$. The following result states this formally.

Proposition 3.2 *The Best Shot Game has a mixed equilibrium in which both players have two best responses $\bar{x}$ and $\underline{x}$ provided, $1 - \delta_1(1 - \alpha_1) \geqslant c \geqslant \delta_1\alpha_1$ and $1 - \delta_2(1 - \alpha_2) \geqslant c \geqslant \delta_2\alpha_2$.*

Proof. We look for a mixed strategy equilibrium in which both $\bar{x}$ and $\underline{x}$ are best responses. Then, by definition, the support of the equilibrium beliefs must be $\{\underline{x}, \bar{x}\}$. In the neo-additive model of ambiguity this implies that $\pi(\bar{x}) = \bar{\pi} > 0, \pi(\underline{x}) = (1 - \bar{\pi}) > 0$ and $\pi(x) = 0$ for $x \notin \{\underline{x}, \bar{x}\}$.

Consider Player 1. Her (Choquet) expected utility from playing these strategies are:

$$V^1(\underline{x}) = \delta_1(1 - \alpha_1)\bar{x} + \delta_1\alpha_1\underline{x} + (1 - \delta_1)[\bar{\pi}\bar{x} + (1 - \bar{\pi})\underline{x}] - c\underline{x},$$

$$V^1(\bar{x}) = \delta_1(1 - \alpha_1)\bar{x} + \delta_1\alpha_1\bar{x} + (1 - \delta_1)\bar{x} - c\bar{x}.$$

For both $\underline{x}$ and $\bar{x}$ to be best responses we must have: $V^1(\underline{x}) = V^1(\bar{x})$ hence,

$$(\delta_1\alpha_1 - c)\underline{x} + (1 - \delta_1)[\bar{\pi}\bar{x} + (1 - \bar{\pi})\underline{x}] = (\delta_1\alpha_1 - c)\bar{x} + (1 - \delta_1)\bar{x}.$$

Thus $(\delta_1\alpha_1 - c)(\bar{x} - \underline{x}) + (1 - \delta_1)[(1 - \bar{\pi})\bar{x} - (1 - \bar{\pi})\underline{x}] = 0.$

Hence $(\delta_1\alpha_1 - c) + (1 - \delta_1)(1 - \bar{\pi}) = 0$, which implies $(1 - \bar{\pi}) = \frac{c - \delta_1\alpha_1}{1 - \delta_1}$.

For such an equilibrium to exist it is necessary that $1 \geqslant 1 - \bar{\pi} \geqslant 0$. This is equivalent to $1 - \delta_1 (1 - \alpha_1) \geqslant c \geqslant \delta_1 \alpha_1$. From a similar analysis of Player 2's choice we need to have $1 - \delta_2 (1 - \alpha_2) \geqslant c \geqslant \delta_2 \alpha_2$. ■

3.2 Weakest Link

In the weakest link model, provision of the public good is equal to the *minimum* of the individual contributions; Cornes and Sandler (1996)). This model is also known as the minimum effort coordination game; Huyck, Battalio, and Beil (1990). In the absence of ambiguity, any situation where both players choose the same effort level is a NE.

Below we show that if there is sufficient ambiguity, then equilibrium will be unique. If players are sufficiently ambiguity-averse, they will both use the lowest possible strategy. In contrast, they will choose the highest possible strategy if they are sufficiently ambiguity-loving. The intuition is that ambiguity-aversion can cause a given individual to be concerned that somebody else will make a low contribution. In which case his/her own effort will be wasted. As a result the given individual will make a low contribution. However if all think likewise, the only equilibrium is where everybody makes the smallest possible contribution. For lower degrees of ambiguity the EUA is similar to the equilibrium without ambiguity. Coordinating on any of the possible effort levels constitutes an equilibrium.

The following result describes the impact of ambiguity in the weakest link public good model. As one might expect, increases in ambiguity-aversion make it more likely that a player will provide the lowest effort. If there is a high degree of ambiguity and players are very ambiguity loving, then it is possible that both will choose the highest possible strategy. However we believe that this outcome is unlikely to be observed in practice. For low levels of ambiguity there are multiple EUA. In contrast the equilibrium is unique when ambiguity is high.

Proposition 3.3 *Assume that both players have neo-additive preferences. The equilibria under* *ambiguity of the weakest link model are as follows:*

1. *if either* $c > 1 - \delta_1\alpha_1$ *or* $c > 1 - \delta_2\alpha_2$*, the equilibrium strategies are unique and are equal to* $\langle \underline{x}, \underline{x} \rangle$*;*

2. *if* $1 - \delta_1\alpha_1 > c > \delta_1(1-\alpha_1)$ *and* $1 - \delta_2\alpha_2 > c > \delta_2(1-\alpha_2)$*, then* $\langle \hat{x}_1, \hat{x}_2 \rangle$ *is a pair of EUA strategies if and only if* $\hat{x}_1 = \hat{x}_2$*;*

3. *if* $\delta_1(1-\alpha_1) > c$ *and* $1 - \delta_2\alpha_2 > c$*, the equilibrium strategies are unique and are equal to* $\langle \bar{x}, \bar{x} \rangle$*;*

4. *if* $1 - \delta_1\alpha_1 > c$ *and* $\delta_2(1-\alpha_2) > c$*, the equilibrium strategies are unique and are equal to* $\langle \bar{x}, \bar{x} \rangle$*.*

Proof. Consider Player 1. Suppose that she believes that Player 2's effort will be $\tilde{x}_2$, where $\underline{x} \leqslant \tilde{x}_2 \leqslant \bar{x}$. Then 1's (Choquet) expected pay-off conditional on this belief will be:

$$V^1(x_1|\tilde{x}_2) = \begin{cases} \delta_1\alpha_1\underline{x} + (1 - \delta_1\alpha_1 - c)\,x_1, \text{ if } x_1 < \tilde{x}_2; \\ \delta_1\alpha_1\underline{x} + (1-\delta_1)\,\tilde{x}_2 + (\delta_1 - \delta_1\alpha_1 - c)\,x_1, \text{ if } x_1 \geqslant \tilde{x}_2. \end{cases} \tag{3}$$

Similarly if Player 2 believes that Player 1's effort will be $\tilde{x}$, then 2's (Choquet) expected pay-off will be:

$$V^2(x_2|\tilde{x}_1) = \begin{cases} \delta_2\alpha_2\underline{x} + (1 - \delta_2\alpha_2 - c)\,x_2, \text{ if } x_2 < \tilde{x}_1; \\ \delta_2\alpha_2\underline{x} + (1-\delta_2)\,\tilde{x}_1 + (\delta_2 - \delta_2\alpha_2 - c)\,x_2, \text{ if } x_2 \geqslant \tilde{x}_1. \end{cases} \tag{4}$$

Part 1 Clearly $1 - \delta_1\alpha_1 > \delta_1 - \delta_1\alpha_1$. If $c > 1 - \delta_1\alpha_1$ then V^1 is strictly decreasing in x_1. Thus Player 1 has a unique best response, $\underline{x}$. Given this, Player 2's (Choquet) expected pay-off is decreasing in x_2, hence his best response is $\underline{x}$. The proof for the case where $c > 1 - \delta_2\alpha_2$ is similar.

Part 2 Let $\langle \hat{x}_1, \hat{x}_2 \rangle$ be a pair of EUA strategies. Suppose, if possible, that $\hat{x}_1 > \hat{x}_2$. Then by equation (3), V^1 is strictly decreasing in x_1 for $x_1 > \hat{x}_2$. This establishes that $\hat{x}_1$ is not a best response, contrary to the original hypothesis. Thus we must have $\hat{x}_1 \leqslant \hat{x}_2$. A similar argument establishes that $\hat{x}_2 \geqslant \hat{x}_1$, which implies that $\hat{x}_2 = \hat{x}_1$.

To prove the converse assume that $x' \in X$. We need to show that $x_1' = x_2' = x'$ are EUA strategies. Substituting $\tilde{x}_2 = x_2'$ into equation (3) we find that V^1 is increasing in x_1 for $\underline{x} \leqslant x_1 \leqslant x'$ and decreasing in x_1 for $x' \leqslant x_1 \leqslant \bar{x}$. This implies that $x_1 = x_1'$ is Player 1's best response. By similar reasoning $x_2 = x_2'$ is a best response for Player 2. It follows that $\langle x_1', x_2' \rangle$ is a pair of equilibrium strategies.

Part 3 Note that $\delta_1 (1 - \alpha_1) > c$ implies $1 - \delta_1 \alpha > c$, hence V^1 is strictly increasing in x_1 for any beliefs she has about Player 2. Thus $x_1 = \bar{x}$ is her best response. Given that $\tilde{x}_1 = \bar{x}$, V^2 is strictly increasing in x_2, which implies that Player 2's best response is $x_2 = \bar{x}$. Hence there is a unique pair of equilibrium strategies, $\langle \bar{x}, \bar{x} \rangle$.

Part 4 Follows by similar reasoning to part 3. ■

The intuition for this result is as follows. In part 1 a given player i, perceives a high degree of ambiguity and is ambiguity-averse thus $\delta_i \alpha_i$ is close to 1. This causes him to overweight the possibility that his opponent will supply minimal effort. He decides to supply minimal effort since anything more is wasted. His opponent responds by also supplying low effort. Parts 3 and 4 describe a situation where one individual perceives a high degree of ambiguity and is also extremely ambiguity-loving. This causes him/her to choose the highest possible strategy. His/her opponent chooses the highest possible strategy in response.

Part 2 is similar to the situation without ambiguity, since any situation in which both players choose the same effort level can be an equilibrium. However as ambiguity increases, the range of parameter values in this case shrinks and eventually it disappears altogether. One may show in general the equilibrium is unique provided ambiguity is sufficiently high.

The next result finds mixed strategy equilibria in the weakest link game. It is similar to the corresponding result for the best shot game. There exist mixed strategy equilibria provided the marginal cost, c, lies in a certain interval. As ambiguity increases this interval becomes shorter and eventually disappears.

Proposition 3.4 *Let $\tilde{x}$ and $\hat{x}$ be two strategies such that $\hat{x} > \tilde{x}$, then the Weakest Link Game has a mixed strategy equilibrium in which both $\tilde{x}$ and $\hat{x}$ are best responses provided,* $1 - \delta_1\alpha_1 \geqslant c \geqslant \delta_1 (1 - \alpha_1)$ *and* $1 - \delta_2\alpha \geqslant c \geqslant \delta_2 (1 - \alpha_2)$ *.*

Proof. We are looking for a mixed equilibrium in which the support of the equilibrium beliefs is $\{\tilde{x}, \hat{x}\}$. In the neo-additive model of ambiguity this implies that $\pi(\hat{x}) = \hat{\pi} > 0, \pi(\tilde{x}) = 1-\hat{\pi} > 0$ and $\pi(x) = 0$ for $x \notin \{\tilde{x}, \hat{x}\}$.

Consider Player 1, her (Choquet) expected utility from the two strategies is given by:

$$V^1(\hat{x}) = \delta_1 (1 - \alpha_1) \hat{x} + \delta_1\alpha_1 \underline{x} + (1 - \delta_1) (\hat{\pi}\hat{x} + (1 - \hat{\pi}) \tilde{x}) - c\hat{x}$$

$$V^1(\tilde{x}) = \delta_1 (1 - \alpha_1) \tilde{x} + \delta_1\alpha_1 \underline{x} + (1 - \delta_1) \tilde{x} - c\tilde{x}.$$

If both $\tilde{x}$ and $\hat{x}$ are best responses, in equilibrium they have to yield the same expected utility, which implies $[\delta_1 (1 - \alpha_1) - c] (\hat{x} - \tilde{x}) + (1 - \delta_1) \hat{\pi} (\hat{x} - \tilde{x}) = 0$ thus $\hat{\pi} = \frac{c - \delta_1(1-\alpha_1)}{1-\delta_1}$. For $\hat{\pi}$ to be a probability we require, $1 \geqslant \frac{c-\delta_1(1-\alpha_1)}{1-\delta_1} \geqslant 0$, which implies $c \geqslant \delta_1 (1 - \alpha_1)$ and $1 - \delta_1 \geqslant c - \delta_1 (1 - \alpha_1)$ thus $1 - \delta_1\alpha_1 \geqslant c$. Putting this together, the following inequalities must be satisfied in a mixed strategy equilibrium: $1 - \delta_1\alpha_1 \geqslant c \geqslant \delta_1 (1 - \alpha_1)$. A similar analysis of Player 2's choice shows that existence of a mixed strategy equilibrium requires $1 - \delta_2\alpha_2 \geqslant c \geqslant \delta_2 (1 - \alpha_2)$. ■

The result applies for any pair of strategies $\hat{x}$ and $\tilde{x}$ such that $\hat{x} > \tilde{x}$. Thus if c is in the specified range there are many mixed strategy equilibria, all of which disappear as ambiguity increases.

4 Public Good Experiments

4.1 Experimental Model

In this section we describe our experiments and the predicted behaviour. We aim to test the hypothesis that ambiguity has opposite effects in games of strategic complements and substitutes; Eichberger and Kelsey (2002).

The task given to the subjects was to choose an effort level from the set $X = \{100, ..., 150\}$. The marginal cost of effort was kept constant at 50% of the effort exerted, i.e., $c = 0.5$. In the weakest-link game, the payoff of the subject would thus be: $u_i(x_i, x_{-i}) = \min\{x_i, x_{-i}\} - 0.5x_i$, where x_i denotes the contribution of individual i and $c = 0.5$ is the marginal cost of a contribution. In the best-shot scenario, the payoff of the subject was: $u_i(x_i, x_{-i}) = \max\{x_i, x_{-i}\} - 0.5x_i$. The final payoff matrices (after subtracting costs) for the games can be seen in Figures 1 and 2.

Figure 1: Two Player Representation of the Best Shot Game

	Column Player						
		100	110	120	130	140	150
Row Player	100	50, 50	60, 55	70, 60	80, 65	90, 70	100, 75
	110	55, 60	55, 55	65, 60	75, 65	85, 70	95, 75
	120	60, 70	60, 65	60, 60	70, 65	80, 70	90, 75
	130	65, 80	65, 75	65, 70	65, 65	75, 70	85, 75
	140	70, 90	70, 85	70, 80	70, 75	70, 70	80, 75
	150	75, 100	75, 95	75, 90	75, 85	75, 80	75, 75

By Proposition 3.3, the NE of the weakest-link game is for both players to coordinate on any one of the six effort levels available, thus $\{(x_1^*, x_2^*) \in X \mid x_1^* = x_2^*\}$. As a result, there are multiple NE. Given this, it is understandable that there would be ambiguity among the subjects about which effort level they should attempt to coordinate on. The equilibrium action with

Figure 2: Two-Player Representation of the Weakest Link Game

		Column Player					
		100	110	120	130	140	150
Row Player	100	50, 50	50, 45	50, 40	50,35	50, 30	50, 25
	110	45, 50	55, 55	55, 50	55, 45	55, 40	55, 35
	120	40, 50	50, 55	60, 60	60, 55	60, 50	60, 45
	130	35, 50	45, 55	55, 60	65, 65	65, 60	65, 55
	140	30, 50	40, 55	50, 60	60, 65	70, 70	70, 65
	150	25, 50	35, 55	45, 60	55, 65	65, 70	75, 75

a high level of ambiguity would be for a subject to choose an effort level of 100, which gives him an ambiguity-safe payoff of $50ECU$ (See Figure 2), irrespective of his opponent's choice. Selecting an effort level of 100 thus frees the subject from having to depend on his opponent's choice and/or having to achieve perfect coordination in their chosen effort levels.

By Proposition 3.1, the best-shot game has two pure NE: $\langle x_1^*, x_2^* \rangle = \langle 100, 150 \rangle$ and $\langle x_1^*, x_2^* \rangle = \langle 150, 100 \rangle$.[6] NE predicts that one of the players will exert the highest effort level (in our case 150) , while the other will free-ride and choose the lowest effort available (in our case 100). Here again, we have multiple NE and it is expected that subjects would perceive ambiguity about which one to choose. If the level of ambiguity about the opponent's choice is high, the equilibrium action under ambiguity is to choose the highest effort level, i.e., 150, since this provides the player with a constant payoff irrespective of the opponent's decision.

By identifying the monetary payments to subjects with utilities we are implicitly assuming risk neutrality. We believe this is not problematic since the qualitative results only depend on ordinal properties of the pay-offs and so would be unchanged if we assumed risk aversion. Risk aversion would alter the position of the boundaries between the different regimes. However it would not change the comparative statics of ambiguity or ambiguity-attitude, which was the

[6]This prediction can be obtained from Proposition 3.1 by setting $\delta_i = 0$, for $i = 1, 2$.

issue focused on in the present experiment.

4.2 Experimental Design

The games described above were used in paper-based experiments, conducted at St. Stephen's College in New Delhi, India, and at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK. We recruited undergraduate students from St. Stephen's College and the University of Exeter as our subjects. All the subjects recruited at St. Stephen's College were Indian nationals, who (by assumption) had an Indian sociocultural upbringing. While sending out the invitations to recruit subjects at the University of Exeter, we took particular care to weed out any foreign students who were Indian. As such, the subjects recruited at FEELE were non-Indian nationals, who had a completely different sociocultural upbringing. We expected this difference would create ambiguity on the part of Exeter subjects.

The experiments were conducted with three different treatments. In Treatment I, subjects were matched with locally recruited subjects - this included two experimental sessions where Indian subjects played other locally recruited Indian subjects, and one session where Exeter subjects played other Exeter subjects. In Treatment II, Exeter subjects were matched with subjects from India. The Exeter subjects were informed that they would be matched with an Indian subject whose responses we had already collected. In Treatment III, subjects were told that their opponent might either be an Indian subject (whose response we had already collected) or a subject from Exeter. Subjects were allowed to choose different effort levels against the two opponents.[7]

Subjects first read through a short, comprehensive set of instructions at their own pace, following which the instructions were also read out to all the participants in general.[8] The

[7] The first two sessions (40 subjects) were run in India, where locally recruited Indian subjects played against each other. The remaining seven sessions (141 subjects) were run in Exeter, where Exeter subjects either played each other or against the foreign subject.

[8] Experimental protocols are available on-line at: http://saraleroux.weebly.com/experimental-protocols.html.

subjects were asked to fill out practice questions to check that they understood the games correctly. Each subject was then asked to choose one effort level for the weakest-link game, followed by an effort level for the best-shot game. In the case of Treatment III, subjects played the weakest-link and best-shot games once against the local subject and once against the foreign subject. As such, subjects were asked to make two choices in Treatments I and II and four choices in Treatment III. Subjects played each game *exactly once.* We believe that this made the games effectively one-shot, as subjects received no feedback between the rounds and could not incorporate any learning.

Once subjects had made both decisions, a throw of dice determined *one* game round for which they would be paid. Subjects in India were paid a show-up fee of $Rs.200$ (£2), together with their earnings from the chosen round, where $100ECU = Rs.200$. Exeter subjects were paid a show-up fee of £3, together with their earnings from the chosen round, where $100ECU = £2$.[9] We picked one round at random for payment in order to prevent individuals from self-insuring against payoff risks across rounds; see Charness and Genicot (2009). If all rounds count equally towards the final payoff, subjects are likely to try and accumulate a high payoff in the first few rounds and then care less about how they decide in the following rounds. In contrast, if subjects know that they will be paid for a random round, they treat each decision with care. Players' decisions were matched according to a predetermined random matching, and pay-offs were announced.

Treatments I and III consisted of 60 subjects each and Treatment II had 61 subjects. In total there were 181 subjects who took part in the experiment, 81 of whom were males and the remaining 100 were females. Each experimental session lasted a maximum of 30 minutes including payment.

[9] These experiments were conducted in the period of November 2010 - February 2011. The exchange rate at that point was 1 GBP = 80 INR. The idea was that the average earnings from our experiment which lasted a maximum of 30 minutes, should be able to afford our subjects (university students) the chance to go out for a meal and a non-alcoholic drink. The purchasing power parity that we were aiming for was a burger meal.

In our experiments, subjects always made a decision for the weakest-link game followed by the best-shot game. This order was not varied. According to Harrison, Johnson, McInne, and Rutstrom (2005), an order effect occurs when having participated in one task, a subject's behaviour in subsequent tasks is affected by his/her prior experience. In particular, participating in a low-payment choice before making a high-payment choice, magnifies the scale of the utility received/payoff from the subsequent task. As such, there may be some "order-effects" in the decisions made by our subjects. These order-effects can be mitigated if the order in which subjects see the two decision choices is varied, or if subjects are asked to only make one choice, i.e., either take part in the weakest-link game or the best-shot game, not both. Moreover, in our experiments, we do not control for individual's risk aversion as done by Holt and Laury (2002), who used a menu of ordered lottery choices to elicit risk attitudes under various payment conditions. In future experiments, it may be interesting to elicit an independent risk-attitude of subjects and check whether there is any correlation to their ambiguity-attitude. However, this is beyond the scope of the present paper.

5 Data Analysis and Results

Treatment I In this treatment, subjects were matched against other locally recruited subjects only.[10] In the weakest link experiment, we find that 22% (13) of the subjects chose an effort level of 100, (See Figure 3). This is the effort level at which the subject has a constant payoff, which is independent of the opponent's action. Moreover, 65% (39) of subjects chose an effort level between 100 and 120, i.e., the lower end of the spectrum of effort choices. This confirms the theoretical prediction that ambiguity would lead to subjects reducing their effort levels. Some

[10] This treatment included two experimental sessions where Indian subjects played other locally recruited Indian subjects, and one session where Exeter subjects played other locally recruited Exeter subjects. Since subjects always play other locally recruited subjects in this treatment, we have collated the data from the three sessions, without loss of efficiency. For a country-specific breakdown of choices under Treatment I, please see Appendix Table 4.

subjects (9), chose the maximum effort level, 150, however, they were in a very small minority. Previous experiments conducted by us, Kelsey and le Roux (2015), have found that a minority of subjects display ambiguity-seeking behaviour in Ellsberg-urn type decision problems. Thus, the present experiment confirms that similar ambiguity-seeking behaviour is also observed in games. In the best-shot round, we find that 47% (28) of the subjects chose the effort level 150 (the equilibrium action under uncertainty). Moreover, ambiguity may be seen as the reason for subjects increasing their effort levels - with 67% (40) of the subjects choosing an effort level in the high range of $130 - 150$.

Figure 3: Crosstable Frequency of Effort Levels: Weakest Link vs. Best Shot in Treatment I

		Best Shot Game						
		100	110	120	130	140	150	Σ
Weakest Link Game	100	5	0	1	0	0	6	12
	110	1	1	1	0	0	1	4
	120	2	1	1	4	1	13	22
	130	1	1	2	1	2	4	11
	140	1	0	0	1	0	0	2
	150	2	0	0	2	1	4	9
	Σ	12	3	5	8	4	28	60

While analysing the manner in which people switch effort levels between the two scenarios, we find that 55% (33) of our subjects switched from a low effort level in the weakest-link round, to a higher effort level in the best-shot game (See Table 3). These subjects display ambiguity-averse behaviour, which is in line with Eichberger and Kelsey (2002). Interestingly, we find that 25% (15) of subjects display a preference for ambiguity, choosing a high effort level in the weakest-link game and then switching to a low effort level in the best-shot round. We also note that 20% (12) of subjects did not change their chosen effort levels between the two rounds - these subjects could be displaying ambiguity neutral behaviour.[11]

[11] Alternatively, unchanged effort levels might be caused by subjects were trying to be consistent. Another trivial reason could be that, there are subjects who having chosen an effort level in the previous round, do not want to go to the trouble of thinking again and stick with their previous decision.

Table 1: Switching Effort Levels between Weakest Link and Best Shot Game

	Treatment I		Treatment II	
Low Effort to High Effort	33	55%	28	46%
High Effort to Low Effort	15	25%	20	33%
Constant Effort Level	12	20%	13	21%
$\sum$	60		61	

Treatment II In this treatment, Exeter subjects were matched with the foreign opponent only. In the weakest-link round, only 8% (5) of the subjects chose effort level 100 (See Figure 4). Even though the constant-payoff effort level has been chosen by a small minority, a sizeable 59% (36) of subjects have chosen low contribution levels in the range $100 - 120$. In the best-shot game, 43% (26) of the subjects chose an effort of 150, which is the equilibrium action under ambiguity, while 59% (36) of the subjects chose high contribution levels in the range of $130 - 150$. This provides an indication that ambiguity is resulting in efforts being concentrated at the lower end of the set of efforts in the case of the weakest-link game; and at the higher end, in case of the best-shot game.

Figure 4: Crosstable Frequency of Effort Levels: Weakest Link vs. Best Shot in Treatment II

		Best Shot Game						
		100	110	120	130	140	150	Σ
Weakest Link Game	100	1	0	0	0	0	4	5
	110	0	2	0	0	0	1	3
	120	4	4	4	3	2	11	28
	130	1	0	0	2	0	6	9
	140	3	0	1	0	1	1	6
	150	3	1	1	1	1	3	10
	Σ	12	7	6	6	4	26	

Table 1, summarises the manner in which people switch effort levels between the two scenarios. We find that 46% (28) of the subjects who chose a low effort level in the weakest-link experiment switched to a higher effort level in the best-shot game. This is compatible with ambiguity-averse behaviour. Moreover, 33% (20) of subjects display ambiguity-seeking behaviour, choosing a high effort level in the weakest-link round followed by a lower effort level in the best-shot game, while 21% (13) of subjects do not change their chosen effort levels between the two rounds. It is interesting to note from Table 1, that more subjects displayed ambiguity-seeking behaviour when faced by the foreign subject (compared to Treatment I). We had expected subjects to be more ambiguity-averse/feel more anxious when matched against the foreign subject, but our data points in the opposite direction.

Treatment III In Treatment III, subjects were told that their opponent might either be an Indian subject (whose response we had already collected) or a subject from Exeter. Subjects were allowed to choose different effort levels against the two opponents. As such, subjects in this treatment made four choices, i.e., one, in the weakest-link game against the local subject; the second, in the weakest-link game against the foreign subject; third, in the best-shot game against the local subject; and finally, in the best-shot game against the foreign subject. This was done to check whether the level of ambiguity perceived by them or their ambiguity-attitude depended on the type of opponent. Figures 5 and 6, provide a summary of subject behaviour in this treatment.

Figure 5: Crosstable Frequency of Effort Levels against Local Subject: Weakest Link vs. Best Shot in Treatment III

		Best Shot Game						
		100	110	120	130	140	150	Σ
Weakest Link Game	100	5	0	0	0	2	9	16
	110	0	1	0	0	1	1	3
	120	5	1	3	2	1	4	16
	130	3	0	1	0	0	6	10
	140	1	0	2	0	0	4	7
	150	6	0	0	0	0	2	8
	Σ	20	2	6	2	4	26	

In the weakest-link round, 27% (16) of Exeter subjects chose an effort level of 100 against a local opponent while 28% (17) chose it against the foreign opponent. The difference in the number of people choosing the lowest effort level vs. the foreign opponent is very marginal. On the whole, 58% (35) of the subjects chose a low effort level between $100 - 120$, against the local opponent, while 53% (32) chose an effort in that range against the foreign opponent. We find (as in Treatment II) fewer people behaving in an ambiguity-averse manner against the foreign subject.

In the best-shot game, we find that 40% (24) of the subjects chose the maximum effort 150 against the local opponent, while 43% (26) chose it against the foreign opponent. Moreover, 55% (33) and 57% (34) of subjects chose in the high effort range of $130 - 150$, against the local and foreign opponents respectively. We find that 28% (17) and 35% (21) of subjects chose effort level 100 against the local and foreign opponents respectively, in the hope of free-riding. Intuitively, these subjects are over-weighting the probability of a good outcome (that their opponent has chosen effort level = 150), and are thus displaying an optimistic attitude to ambiguity. Again, it can be noted that more subjects are displaying this optimistic attitude toward uncertainty against the foreign opponent!

Even though we do not see a huge disparity in the effort choices versus the local and foreign opponent, ambiguity does explain (most of) the deviations from Nash equilibrium. In the case of the weakest link game, most responses are concentrated towards the lower end of the spectrum between $100 - 120$, while in the best shot case, responses are concentrated towards the high end, i.e., at 150.

Figure 6: Crosstable Frequency of Effort Levels against Foreign Subject: Weakest Link vs. Best Shot in Treatment III

		Best Shot Game						
		100	110	120	130	140	150	Σ
Weakest Link Game	100	6	1	0	0	0	10	17
	110	2	0	0	1	0	0	3
	120	3	2	1	2	2	2	12
	130	0	2	0	1	1	4	8
	140	2	0	0	1	1	3	7
	150	6	1	0	1	1	4	13
	Σ	19	6	1	6	5	23	

We ran McNemar's test to check whether subjects responded to ambiguity in a pessimistic way. The null hypothesis was that the probability of choosing a low effort in the weakest-link round, followed by a high effort in the best-shot round (henceforth, labelled ambiguity-averse behaviour/decision), equalled the probability of choosing a high effort in the weakest-link round, followed by a low effort in the best-shot round (henceforth, labelled ambiguity-seeking behaviour/decision). The alternative was that the number of ambiguity-averse decisions was not equal to the number of ambiguity-seeking decisions (in particular, the ambiguity-averse decisions > the ambiguity-seeking decisions). The statistical data follows a chi-squared distribution with one degree of freedom. When looking at the data as a whole (all three treatments/nine sessions), we reject the null at a 1% level of significance. The number of ambiguity-averse decisions made, on the whole, was twice the number of ambiguity-seeking decisions (See Table 2). When looking at the different treatments individually, we reject the null at a 1% level of significance for Treatment I, and at a 5% level of significance for Treatment II. We fail to reject the null for Treatment III, for decisions against both the local subject and the foreign subject. The McNemar Test thus demonstrates that on the whole, there was a difference between the number of ambiguity-averse and ambiguity-seeking choices made.

Table 2: Summary of Decision Choices

	W-L Low Effort, B-S High Effort (Ambiguity-averse)	W-L Low Effort, B-S Low Effort	W-L High Effort, B-S High Effort	W-L High Effort, B-S Low Effort (Ambiguity-seeking)
T I	25	13	15	7
T II	21	15	15	10
T III LS	20	15	11	14
T III FS	18	14	17	11
Total	84	57	58	42

In order to investigate further, whether the number of ambiguity-averse decisions exceeded the number of ambiguity-seeking decisions, we ran a Wilcoxon signed-ranks test. The null hypothesis was that the number of ambiguity-averse decisions made equalled the number of ambiguity-seeking decisions; while the alternative was that ambiguity-averse behaviour exceeded ambiguity-seeking behaviour. We ranked behaviour in all the different sessions, and found that we could reject the null at a 1% level of significance. Furthermore, if we consider a more simple sign test, with the same null and alternative as before, we find that we can reject the null at a 5% significance level. Thus, we find that subjects showed ambiguity-averse behaviour significantly more often in our experiments, than ambiguity-seeking behaviour.

6 Related Literature

This section reviews previous experimental studies of similar games. Typically the authors have found results which appear paradoxical when viewed from the perspective of Nash equilibrium. We believe that many of these apparent paradoxes can be resolved by using the notion of equilibrium with ambiguity.

Experiments on the weakest-link game were previously studied by Huyck, Battalio, and Beil (1990). They study tacit coordination in this game, and conclude that it is unlikely that a payoff-dominant equilibrium would be chosen in a one-shot game or in repeated play. Moreover, they find that when there are a large number of players attempting to coordinate, the equilibrium is secure but inefficient. Our results in the weakest-link round are consistent with their conclusions. We find that 59% (142) of subjects chose an effort level in the range $100 - 120$, which would result in a payoff-dominated equilibrium. Furthermore, even though our game consisted of only two subjects coordinating (and not a large number of players), we found that 21% (51) of subjects chose an effort level of 100, which would have resulted in a secure but inefficient equilibrium.

Harrison and Hirshleifer (1989), compare contributions to a public good in a sealed bid as well as a sequential game. They use a repeated game scenario which implements all three possible versions of the game - standard summation, weakest-link and best-shot, in order to ascertain which of the three formats results in the greatest free-riding. They find that both sealed bid as well as the sequential game treatments, confirmed their hypothesis that the under-provision of the public good expected under the standard format, is mitigated under the weakest-link format, but aggravated under the best-shot version. In contrast we found that 55% (33) and 46% (28) of subjects in Treatment I and II switched from a low effort level in the weakest-link round to a higher effort level in the best-shot game (See Table 1). This may be because the one-shot nature of our games increases ambiguity and prompts players to increase contributions.

Goeree and Holt (2001) (henceforth GH) study, a set of games which initially conform to Nash predictions when tested experimentally. However, they show that in each case a change in a parameter, which is not relevant according to Nash equilibrium leads to a large change in observed behaviour and failure of Nash predictions. In particular, they study the weakest link model where subjects could choose an effort from the set $E = \{110, ..., 170\}$ at a marginal cost of either $c = 0.1$ or $c = 0.9$. Recall that in Nash equilibrium subjects coordinate on the same effort level $\{(e_1^*, e_2^*) \in E^2 \mid e_1^* = e_2^*\}$. GH find that for low marginal costs ($c = 0.1$), subjects choose high effort levels and for high marginal costs ($c = 0.9$), a majority of subjects choose low effort levels. They conclude that this concentration of choices at the lower (resp. higher) end of the effort spectrum is caused by the high (resp. low) marginal cost of effort. Eichberger and Kelsey (2011) argue that GH's results can be explained by ambiguity.

In our experiment we chose a value of marginal cost, ($c = 0.5$), between the two values used by GH. We find that an intermediate proportion of subjects (22%) choose the lowest effort level. We find that subjects' effort choices depend on the effect of ambiguity, given the strategic nature of the game being played. This can be seen in Table 1, where even though the marginal cost of the effort is constant at $c = 0.5$, subjects switch their effort levels depending on whether it is the weakest-link or the best-shot game being played.

7 Conclusions

In this paper we have developed a theoretical model of the effect of ambiguity in the best-shot and weakest-link models of public good provision. We then proceeded to test the theory experimentally. Subject behaviour was found to be consistent with our hypothesis. We find that in the presence of ambiguity, subjects choose low effort levels in the weakest-link game and high effort levels in the best-shot game. Moreover, we find that on average, 51% (61) of the subjects who took part in Treatments I and II, display ambiguity-averse behaviour; 29% (35) of subjects display ambiguity-seeking behaviour; and 20% (25) of subjects do not change their chosen effort levels between the two rounds.[12]

We expected the subjects to display a greater level of ambiguity-averse behaviour when faced by a foreign opponent. However, although we observe ambiguity-averse behaviour on the whole, we find a significant minority of subjects who display an optimistic attitude towards ambiguity. This is quite a curious finding, as one would expect that the ambiguity-safe option would be chosen more often against the foreign subject. Our findings contrast with those of Kilka and Weber (2001), who found that subjects are more ambiguity-averse when the returns of an investment are dependent on the performance of foreign securities than when they are linked to domestic securities.

There may be several other reasons that might explain why the level of ambiguity when facing a foreign subject may remain unchanged. One may be that the subjects wanted to be consistent in their choices. If this was the case, they would put extra effort into choosing the same action against both opponents. In addition, the returns of a bank depend in a complex way on financial markets and hence present a more difficult decision problem than finding the right strategy in a game with a relatively small strategy space. This may explain part of the heightened "ambiguity" captured by Kilka and Weber (2001), where the subjects were presented with the option of an investment dependent on foreign securities. It is easier for subjects to conceptualise another person whom they may be faced with, than investments in known/unknown financial markets. It may be interesting to run a follow-up experiment, where subjects are given the choice of either facing a foreign opponent or investing in a foreign security.

Moreover, the missing effect in Treatment II may be attributed to the closeness of UK

[12] Ambiguity-averse (resp. ambiguity-seeking) behaviour was displayed by subjects who chose a low (resp. high) effort level in the weakest link game followed by a higher (resp. lower) effort level in the best shot game.

and India that developed over the years of India being a British Colony, Indian immigrants coming to the UK and Indian food becoming national British dishes... Perhaps using students as subjects might be ambiguity diminishing - as subjects viewed the foreign student as akin to any other local student. This is not that difficult to understand. Globalisation and increased media awareness, together with the spreading tentacles of social networking and escalating international student numbers, have ensured that a foreign subject (in this case from India) is not an unknown quantity any more. There are not many parts of the world, that hold the kind of ambiguity for us today, as there were in the past.

A Appendix

Table 3: Decision Choices by Session

Treatment	Notional Session	Ambiguity-Averse Decisions	Ambiguity-Seeking Decisions
T1	1	8	2
T1	2	9	2
T1	3	8	3
T2	4	4	5
T2	5	11	3
T2	6	6	2
T3 LS	7	9	3
T3 LS	8	3	6
T3 LS	9	8	5
T3 FS	10	7	2
T3 FS	11	5	5
T3 FS	12	6	4

a

Table 4: Country Specific Decision Choices in Treatment I

Treatment I						
	India (2 sessions)		India (Avg.)		Exeter (1 session)	
Effort	W-L	B-S	W-L	B-S	W-L	B-S
100	10	8	5	4	3	4
110	3	2	1.5	1	1	1
120	16	5	8	2.5	6	0
130	6	5	3	2.5	4	3
140	1	2	0.5	1	1	2
150	4	18	2	9	5	10
$\sum$	40	40	20	20	20	20

References

Bailey, R. W., J. Eichberger, and D. Kelsey (2005): "Ambiguity and Public Good Provision in Large Societies," *Journal of Public Economic Theory*, 7, 741–759.

Charness, G., and G. Genicot (2009): "Informal Risk Sharing in an Infinite-Horizon Experiment," *The Economic Journal*, 119(537), 796–825.

Chateauneuf, A., J. Eichberger, and S. Grant (2007): "Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities," *Journal of Economic Theory*, 137, 538–567.

Cornes, R., and T. Sandler (1996): *The Theory of Externalities, Public Goods and Club Goods*. Cambridge University Press, Cambridge, UK., 2nd edn.

Di Mauro, C., and M. F. Castro (2011): "Kindness Confusion or ... Ambiguity," *Experimental Economics*, 14, 611–633.

Dow, J., and S. R. C. Werlang (1994): "Nash Equilibrium under Uncertainty: Breaking Down Backward Induction," *Journal of Economic Theory*, 64, 305–324.

Eichberger, J., and D. Kelsey (2002): "Strategic Complements, Substitutes and Ambiguity: The Implications for Public Goods," *Journal of Economic Theory*, 106, 436–466.

——— (2011): "Are the Treasures of Game Theory Ambiguous?," *Economic Theory*, 48, 313–339.

——— (2014): "Optimism and Pessimism in Games," *International Economic Review*, 55, 483–505.

Goeree, J. K., and C. A. Holt (2001): "Ten Little Treasures of Game Theory and Ten Intuitive Contradictions," *American Economic Review*, 91, 1402–1422.

Harrison, G. W., and J. Hirshleifer (1989): "An Experimental Evaluation of Weakest Link/Best Shot Models of Public Goods," *Journal of Political Economy*, 97(1), 201–225.

Harrison, G. W., E. Johnson, M. M. McInne, and E. Rutstrom (2005): "Risk Aversion and Incentive Effects: Comment," *American Economic Review*, 95(3), 900–904.

Holt, C., and S. Laury (2002): "Risk Aversion and incentive effects," *American Economic Review*, 92, 1644–1655.

Huyck, J., R. Battalio, and R. Beil (1990): "Tacit Coordination Games, Strategic Uncertainty and Coordination Failure," *American Economic Review*, 80, 234–248.

Kelsey, D., and S. le Roux (2015): "An Experimental Study on the Effect of Ambiguity in a Coordination Game," *Theory and Decision*, 79, 667–688.

Kilka, M., and M. Weber (2001): "What Determines the Shape of the Probability Weighting Function under Uncertainty?," *Management Science*, 47, 1712–1726.

Schmeidler, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 57, 571–587.