

**Residual Stress Determination in Advanced Silicon
Carbide Fibre Reinforced Aluminium Matrix
Composites**

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A thesis submitted for the degree of

Doctor of Philosophy

at

Oxford Brookes University

December 2021

ABSTRACT

Metal matrix composite (MMC) materials can provide very desirable material and component properties. MMCs can be divided into two groups; those that are reinforced by long continuous fibres (such as carbon, alumina, silicon, boron and carbide) or those reinforced by discontinuous fibres (particulates). The addition of fibres and particulates reinforcements enhance desirable properties of the base metal (matrix). A key benefit of MMCs is the ability to tailor component behaviour via fibre orientation or through variation of fibre dispersion through the component. The strength properties of MMC's are known to be heavily influenced by the manufacturing process, caused by changes in the matrix microstructure and through the introduction of residual stresses. Residual stresses are introduced into the composites due to the thermal expansion mismatch between the fibre reinforcements and the matrix. It is very difficult to determine the internal residual stresses within MMCs. This thesis presents an indirect method to predict the thermal residual stresses, through matching numerical and experimental curvature. This research focuses on predicting manufacturing residual stresses to improve the performance of silicon carbide fibre reinforced aluminium 6061 matrix composites. The initial numerical models presented are for composite plates with [90/0] and [+55/-55] fibre. The materials used for the finite element models are elastic-plastic and temperature-dependent. To test the numerical models' ability to predict the curvature, composite laminates were manufactured to the same specifications using the same manufacturing process conditions to validate with the numerical models. The curvatures between the models and manufactured plates showed a good correlation, giving confidence in the models' ability to predict the residual stresses for more complex components. The validated methods developed were then applied to a pressure vessel (burst tube) to demonstrate what residual stresses would be developed within a real component due to the manufacturing process.

ACKNOWLEDGEMENT

This project would not have been possible without the support of many people. First and foremost, I would like to express my deep and sincere gratitude to my research supervisor Dr Neil Fellows, who read my numerous revisions and helped to provide project focus. He provided invaluable advice, continuous support, and patience during my PhD study. He has taught me the methodology to carry out the research and to present the research work as clearly as possible. It was a great privilege and honour to work and study under his guidance. I am extremely grateful for what he has offered me. I would also like to thank him for his friendship, empathy, and great sense of humour.

I am extremely grateful to Professor John Durodola for giving me the opportunity to do this research and provide invaluable guidance throughout. His vision, sincerity, and motivation have deeply inspired me. Many thanks to Dr Shpend Gerguri and the University research support team, Professor Khaled Hayatleh, Jill Organ, and Catherine Joyejob, who were always on hand to offer guidance. I would like to thank all the members of the Department of Mechanical Engineering and Mathematical Sciences. It is their kind help and support that have made my study and life in Oxford a wonderful time. My thanks go to all the people who have supported me to complete the research work directly or indirectly.

Thanks to Oxford Brookes University for awarding me the 150th Anniversary fully funded Scholarship, providing me with the financial means to complete this project. Many thanks to Tisics Ltd for providing support for the project.

I am extremely grateful to my parents for their love, caring and sacrifices for educating and preparing me for my future. I love you very much! Thanks to my late grandfathers, Mr Huang and Mr Ran, for your great wisdom and for always encouraging me to keep going and achieve my goals.

Finally, to my caring, loving, and supportive fiancée, Scott and the extended family. Your encouragement when the times got rough is much appreciated and noted. It was a great comfort and relief to know that you were willing to provide management of our household activities while I completed my work. My heartfelt thanks.

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List of Symbols

MMC	Metal matrix composite
CFMMC	Continuous fibre reinforced composite
MPa	Megapascal
GPa	Gigapascal
°C	Celsius temperature
K	Kelvin temperature
V_f	Volume fraction
F	Force
A	Area
L	Length
L_0	Original length
ΔL	Elongation
σ	Stress
σ_y	Yield stress
σ_u	Ultimate stress
σ_r	Radial stress
σ_θ	Hoop stress
γ	Engineering shear strain
ε	Strain
ε_e	Elastic strain
ε_p	Plastic strain
ε_{ss}	Creep rate
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	Principal strain
c	Elastic stiffness tensor
S	Compliance tensor
H	Material constant
n	Hardening exponent
Q	Activation energy
T	Temperature
R	Universal gas constant
E	Young's modulus
E_f	Modulus of elasticity of fibre material
E_m	Modulus of elasticity of matrix material
ν	Poisson's ratio
V_f	Volume fraction of the fibre reinforcements
ν_m	Poisson's ratio of matrix material
α	Coefficient of thermal expansion
α_f	Coefficient of thermal expansion of the fibre
α_m	Coefficient of thermal expansion of the matrix
α_v	Volumetric thermal expansion coefficient
V	Volume
G	Shear modulus
G_f	Shear modulus of elasticity of fibre material
G_m	Shear modulus of elasticity of matrix material

CHAPTER 1

INTRODUCTION

CHAPTER 1 INTRODUCTION

1.1 Introduction to metal matrix composites (MMCs)

Metal matrix composites (MMCs) combine metals with one or more reinforcing materials. The reinforcing material is usually fibres, with continuous fibres enabling directional optimisation of robustness and stiffness. In applications such as aerospace (Figure 1.1), metal matrix composites (MMCs) have better performance than conventional monolithic structural materials (Arsenault, 1996).



Figure 1.1 Hubble in orbit (NASA, 2005)

MMCs have several advantages, which include a combination of the following properties such as high strength and stiffness, high elastic modulus, low weight, high wear resistance and toughness, improved elastic modulus and fatigue properties, elevated temperature stability, high thermal conductivity, control of thermal expansion coefficient (Rohatgi, 1993a; Suresh, 2013).

The individual material properties within advanced composites materials can be combined to enhance desirable properties whilst reducing the effect of less desirable properties. The UK

based manufacturer TISICS Ltd is a pioneering company in the production of continuous fibre reinforced composites. The monofilament ceramic fibre reinforced composites manufactured by TISICS Ltd has been used in spacecraft and aircraft components. Due to the lower melting point of the aluminium alloy compared with titanium alloy, aerospace components made of silicon carbide monofilament fibre reinforced aluminium alloy composite have a unique safety advantage during re-entry to the Earth's atmosphere during decommissioning. However, the high cost of metal composite products is a still big challenge in terms of increasing the scale of commercial applications.

Aluminium metal reinforced with silicon carbide fibre have advantages such as improved thermal performance and fatigue performance. Therefore, this research focuses on optimising the performance of silicon carbide fibre reinforced aluminium matrix composites.

1.2 Aims and objectives

Residual stresses are introduced into the composites during processing due to the mismatch of the thermal and mechanical properties of the silicon carbide monofilament fibre and the aluminium matrix. Residual stresses can negatively influence the mechanical performance of composites. For long fibre reinforced titanium metal composites, for instance, residual stresses can decrease the static strength of the material and shorten the thermal fatigue life (Durodola and Derby, 1994). The temperature used to consolidate the aluminium alloy at the solid-state (Tejas, 2009), the layups of the composite material, and the composite structure have a significant effect on the residual stresses.

Extensive research has been done to predict the mechanical properties of carbon composites and MMCs (Durodola *et al.*, 2013; Tejas, 2009; Rosales-Iriarte, Fellows and Durodola, 2012; Harvey, 2009, etc.). However, there is limited literature on the prediction of mechanical properties of Al/SiC MMCs (Guo, 2016) and the change in mechanical properties due to post-manufacture heat treatment of Al/SiC MMCs.

This project aims to develop numerical models to predict the residual stresses in advanced continuous silicon carbide fibre reinforced aluminium matrix composite components subjected to manufacturing and post heat treatment processes.

The main objectives of the project are:

- Review literature relating to material properties, fibre layup, manufacturing processes and analytical and numerical modelling of continuous fibre reinforced metal matrix composite materials.
- Develop numerical models to predict the effect of manufacture, heat treatment, layup, and component shape on the residual stresses of silicon carbide fibre reinforced aluminium matrix composites.
- Validate numerical models using results of curvature measurement and tensile testing of material test coupons subjected to manufacture and post-manufacture heat treatment.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

CHAPTER 2 BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

The following literature review presented the technical background on fibre reinforced metal matrix composites, including classifications, residual stresses, and thermomechanical behaviours. To the best of the author's knowledge, there is presently no published work related to modelling the change in mechanical performance due to post-manufacture heat treatment as presented in this thesis on the subject of silicon carbide fibre reinforced aluminium alloy composite material. There is a great number of pieces of literature available on the analysis of composite materials. Chapter 2 only presents the published work directly relevant to the present research.

2.2 Metal matrix composites

Extensive research studies into metal matrix composites started over three decades ago (Taya and Arsenault, 2016). There are two components of MMCs: the metal matrix and the reinforced constitute. During the fabrication process, the two materials are mixed and then become permanently combined. In most cases, the metal matrix will be an alloy, and the reinforcement is usually a type of ceramic material (Taya and Arsenault, 2016). With the addition of ceramic fibres and particulates, the base metal matrix's mechanical properties are significantly improved (Taya and Arsenault, 2016).

2.2.1 Classifications of metal matrix composites

According to the reinforcement types, there are two groups of metal matrix composites (Hull and Clyne, 1996): continuous reinforcement composites and discontinuous reinforcement composites.

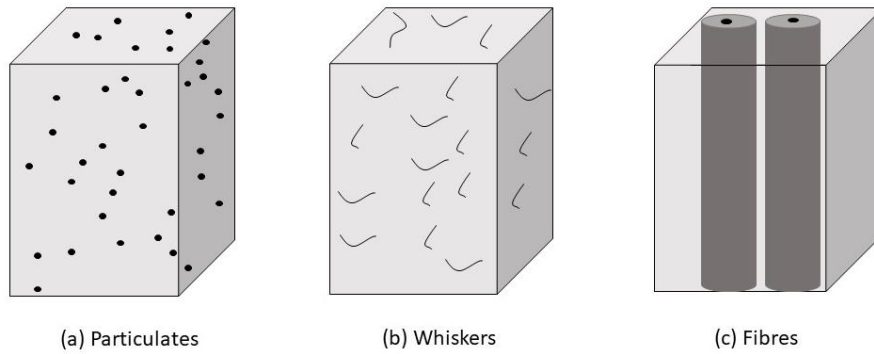


Figure 2.1 Schematic diagram for different types of metal matrix composites

2.2.2 Discontinuous reinforcement composites

In discontinuous reinforcement composites, the alloy is reinforced by either short fibres (whiskers) or particles (see Figure 2.1 (a) and (b)). The most common reinforcement materials for discontinuous reinforcements are silicon carbide, titanium carbide, boron carbide, etc. (Nichols, 1988). Materials such as alumina and silicon nitride are manufactured as either particulate or whiskers. The whiskers' price is 30+ times more than that of the particulates (Nichols, 1988).

Although the cost decreases with increasing production volume, particle reinforced composites still have a cost advantage compared with continuous fibre reinforced composites (Nichols, 1988).

In general, the longer the reinforcements are, the better the material performance is. Whiskers reinforcement composites can offer better mechanical properties than composites with particle reinforcement, such as higher strength and stiffness (Nichols, 1988). However, the manufacturing process for whiskers is much more complicated and more expensive. One of the reasons for this is that the brittle ceramic whiskers break easily into smaller pieces during the process.

2.2.3 Continuous reinforcement composites

In continuous reinforcement composites, long fibres are used as reinforcement to enhance the composite's mechanical properties. These reinforcing constituents can either be tow-based or monofilament fibres. Tow-based fibres are bundles made of hundreds or thousands of fibres; each fibre diameter varies from 5-15 μm (Durodola *et al.*, 2017). These fibres are woven into a mat or spread to form a thin structure (Durodola *et al.*, 2017).

The other type of continuous reinforcement is monofilament fibres (see Figure 2.1 (c)). The diameter of monofilament fibres is much larger than tow-based fibres. Like silicon carbide fibres, monofilaments have a diameter of approximately 80 to 150 μm (Durodola *et al.*, 2017).

The initial interest in continuous fibre reinforced metal matrix composites (CFMMCs) started in the 1960s. The composite was developed initially for aerospace applications (Taya and Arsenault, 2016). The base materials used were aluminium and copper matrices, then reinforced with tungsten and boron. In the 1970s, the research decayed mainly due to the high costs and the limitations in the production process (Clyne and Withers, 1995; Durodola, et al 1994 and 2017). However, the demand for high-performance materials kept increasing in non-aerospace and non-military fields. The development of new low-cost fibres created new opportunities to develop continuous fibre reinforced MMCs (Clyne and Withers, 1995; Durodola, 2017).

Continuous fibre reinforced composites are anisotropic materials. The degree of anisotropy depends on the orientation of the reinforced fibres (Taya and Arsenault, 2016)). When the composite is under load, especially axially, the long fibres become the primary carrier of the force. The role of the matrix is to transfer and distribute the load to the fibres through the matrix-fibre interface (Clyne and Withers, 1995). The highest strength of the composite usually follows the direction of the fibre reinforcement. A key benefit of CFMMCs is the ability to tailor the component behaviour via fibre orientation or through variation of fibre dispersion within the component. The direction of this research will focus on continuous fibre reinforced composites (CFMMCs).

2.3 Advantages of continuous fibre reinforced MMCs (CFMMCs)

2.3.1 Structural properties

Aluminium matrix composites have significantly better structural properties than those without fibre reinforcements. For example, aluminium 6061 alloy with fibre reinforcement has much higher strength and stiffness than aluminium 6061 alloy (Table 2.1). Table 2.1 compares the structural properties of unreinforced, particle reinforced, and fibre reinforced 6061 aluminium alloy composites.

Table 2.1 Comparison of structure properties of unreinforced, particle reinforced and fibre reinforced 6061 aluminium alloy composite (Nichols, 1988)

Properties	Matrix metal ⁽¹⁾	Particle reinforced MMC ⁽²⁾	Fibre reinforced MMC ⁽³⁾
Strength (MPa)	~290	290-480	620-1,240
Stiffness (GPa)	~70	80-140	130-450
Specific strength ⁽⁴⁾	~100	100-170	250-390
Specific gravity ⁽⁵⁾	2.5-2.8	~2.8	2.5-3.2
Transverse strength (MPa)	~290	290-480	30-70
Transverse stiffness (GPa)	~70	80-140	34-173
Maximum use temperature (°C)	180	300	300
Plane strain fracture toughness (MPa-m ^{1/2})	18-35	12-35	—

2.3.2 Thermal and physical properties

The maximum service temperature of reinforced 6061 aluminium MMCs is significantly higher than the base metal. When the fibre reinforcements have a volume fraction of 50%, the service temperature of the 6061 aluminium MMCs is 67% greater than that of the 6061-aluminium alloy (Table 2.1). The composite materials' application range under high-temperature environments has been dramatically extended with the reinforcements added.

Under high-temperature environments, MMCs also have better strength. However, because the matrix dominates the transverse properties in CFMMCs, the composite's strength in the transverse direction is similar to the unreinforced metal (Nichols, 1988).

MMCs have extraordinary wear resistance compare with conventional metals (Nichols, 1988). A piston used in Toyota vehicles, utilising alumina-silica fibre reinforcement, provided an 85% improved wear resistance than the original cast iron with nickel insert.

MMCs can provide a combination of mechanical properties that can be designed based on the requirement. This ability does not exist in conventional monolithic materials. For instance, graphite fibre reinforced composite with copper, aluminium and titanium matrix has a high heat transfer coefficient. It is desirable for applications such as radiators working in a space environment (Nichols, 1988).

Another example is a fibre reinforced composite used in a Toyota truck diesel engine piston. An assemblage containing a ring-shaped ceramic fibre was designed to be a critical area in the final product. It can be combined with selective matrix materials to form the composite component (Nichols, 1988). This design improved the engine's performance and reduced the manufacturing cost (Nichols, 1988).

2.4 Applications

MMCs were initially developed for aerospace industries; however, the material applications have now widened to military, advanced structural and ground transportation (auto and rail) industries (Miracle, 2005). In 1999, the worldwide MMCs market accounted for 2500 metric tons valued at over \$100M (Miracle, 2005). Despite the COVID-19 crisis, the global market for MMCs was estimated at \$458.1M in 2020, reaching the revised size of \$620.4M within the following seven years (Global Industry Analysts, 2020).

Steel is the standard material for manufacturing pressure vessels; it provides good strength and impact resistance. A low weight combined with high strength is particularly attractive for pressure vessel components. Carbon fibre reinforcement is applied to pressure vessels. It allows the pressure vessel to be stronger and lighter, improving fuel economy when used in aerospace applications (Figure 2.2).



Figure 2.2 A carbon nanotube composite overwrap pressure vessel (NASA, 2017)

The composite plate with fibre layups is cut into the designed length and width for the vessel shells during the manufacturing process. The plate is then rolled into cylinders of the required diameter. The pressure vessel head is fabricated using various techniques, and it is welded onto the vessel shell to achieve an airtight component. Quality checks and inspections are

applied to the pressure vessels, such as high impact testing, to ensure the final product meets the standard.

2.5 Silicon-carbide fibre reinforced aluminium matrix MMC

The most commonly used fibre reinforcements for an aluminium matrix are silicon carbide and carbon fibres (Suryanarayanan, Praveen and Raghuraman, 2013). The composite material, made of aluminium matrix and silicon carbide reinforcements, can provide excellent strength to weight ratio (three times higher than mild steel) (Ballmes *et al.*, 2010; Thostenson, Ren and Chou, 2001). The composite also has many other attractive mechanical properties such as high stiffness, high strength, good wear resistance, longer fatigue life and better corrosion resistance (Ballmes *et al.*, 2010; Thostenson, Ren and Chou, 2001).

Aluminium also has a lower melting temperature than titanium. Aerospace components made from titanium may not wholly burn when re-entering the earth's atmosphere, causing threats to life on earth. The solution is to use aluminium matrix composites for the applications instead of titanium alloy based composites. Thanks to improved manufacturing processes, silicon carbide reinforced aluminium metal matrix composites (Al/SiC MMCs) are cost-effective. They have massive potential in applications such as transportation, machine building and plant engineering (Ballmes *et al.*, 2010; Baumli *et al.*, 2013).

Even though Al/SiC MMCs have many extraordinary properties, there is still a big drive to improve their mechanical properties and make them more economical and commercially attractive. The current challenges are outlined in the next section.

2.6 Thermomechanical behaviour analysis

It is helpful to have the ability to predict the thermomechanical behaviour of composite materials by using analytical and numerical models. This provides the possibility to tailor the desired material performance before embarking on physical processing. The essential thermomechanical properties are creep behaviour and thermal expansion.

Thermomechanical properties which are generally anisotropic are closely related to the microstructure morphology of the materials (Taya and Arsenault, 2016).

2.6.1 Creep law

Creep is the time-dependent deformation of a material under load at a given temperature. When the service temperature is higher than 30% of the homologous melting temperature, the creep will become significant in metals. There are typically three creep deformation stages: primary/transient stage, secondary/steady stage, and tertiary creep stage, as illustrated in Figure 2.3.

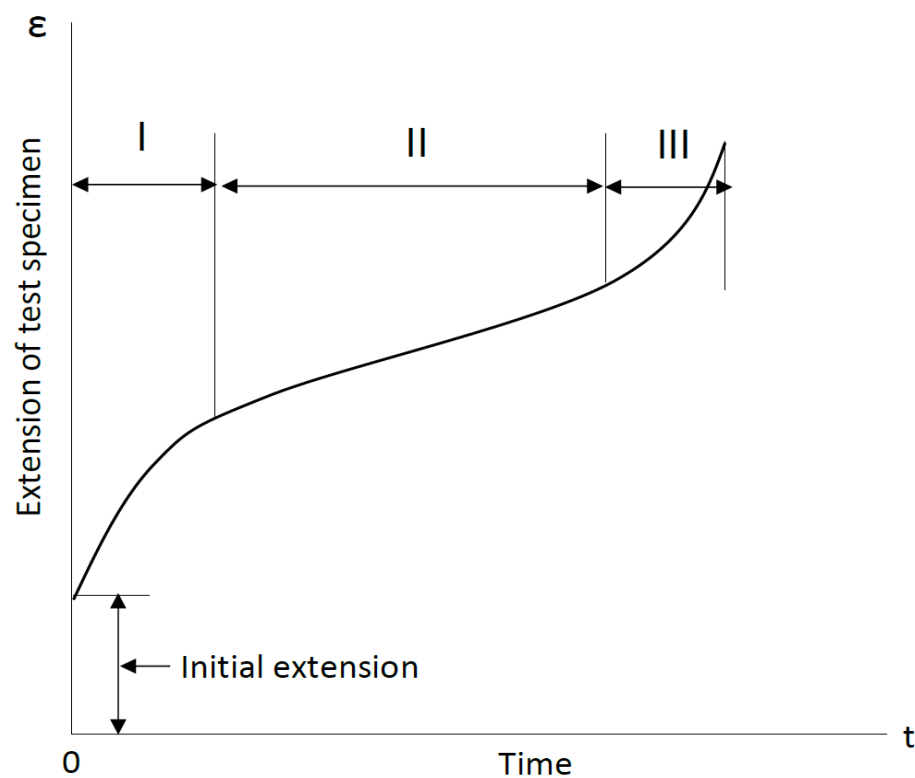


Figure 2.3 Three stages of a creep deformation (David N. French, 1991)

The three stages of a creep deformation:

- The primary/transient stage I: the creep rate is very high and slows down as work hardening sets in, counteracting the creep flow processes.
- The secondary/steady stage II: this is the longest stage in a creep deformation, and the creep rate ($\dot{\epsilon}_{ss}$) is almost constant at this stage (see Figure 2.4).
- The tertiary creep stage III: the strain rate increases rapidly due to necking or internal void formation.

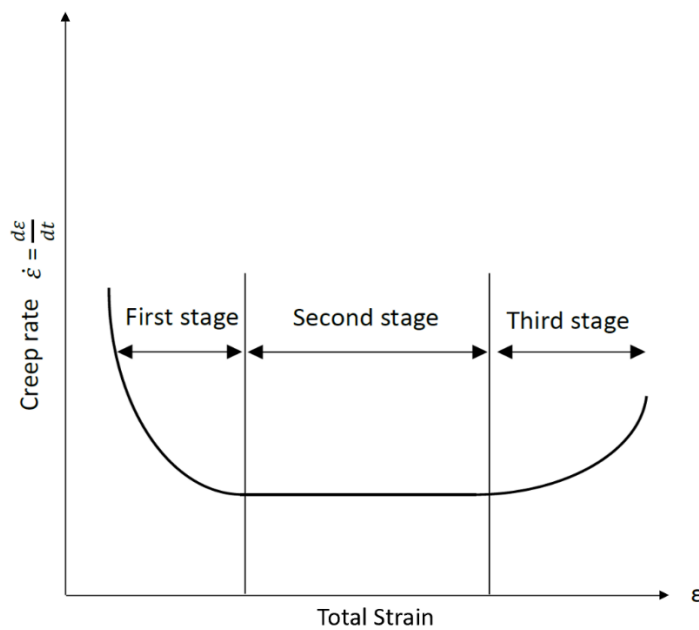


Figure 2.4 Creep rate as a function of total strain

Creep is a thermally activated process; the steady-state creep law is given by Equation 2.1,

$$\dot{\epsilon}_{ss} = A\sigma^n \exp\left(-\frac{Q}{RT}\right) \quad \text{Equation 2.1}$$

A is the pre-exponential constant, Q is the activation energy for the rate-controlling process, T is the absolute temperature, and R is the universal gas constant.

2.6.2 Coefficient of thermal expansion (CTE)

Thermal expansion is a phenomenon of a material dimension changes when heated at a constant temperature (Huber *et al.*, 2006). Most materials expand under heat, and in some special circumstances, some materials contract when they are heated (Huber *et al.*, 2006).

The coefficient of thermal expansion measures the dimension change of a material with a temperature change. The volumetric thermal expansion coefficient is given by Equation 2.2.

$$\alpha_v = \frac{1}{V} \frac{dV}{dT} \quad \text{Equation 2.2}$$

where V is the volume, and T is the temperature.

There are two types of thermal stresses in a metal matrix composite, temperature gradient induced thermal stress and CTE mismatch-induced thermal stress. Temperature gradient induced thermal stress is caused by the temperature gradient in the composite material. CTE mismatch induced thermal stress of a composite is because of the mismatch in the CTE between the metal matrix and the reinforced fibres. The research in this project focuses on the CTE mismatch-induced thermal stress.

2.7 Rule of mixtures

Rule of mixtures is an analytical method for predicting various properties of composite materials. It is based on several simplifying assumptions using weighted contributions from the fibre reinforcements and the metal matrix. For unidirectional (direction along the fibre direction) fibre reinforced matrix, we can make assumptions as follows:

- Reinforced fibres are uniform, parallel distributed and continuous
- Bonding between fibres and matrix are perfect
- Fibre and matrix experience equal strain under longitudinal load
- Fibre and matrix experience equal stress under transverse load

The thermal-mechanical properties of the composite can be estimated using the rules of mixtures. Detailed derivations in the rules of mixtures expressions are presented in Appendix A. Equation 2.3 to Equation 2.8 show the rule of mixture equations for predicting elastic and thermal properties of composites.

Modulus of Elasticity in longitudinal direction:

$$E_1 = E_f V_f + E_m (1 - V_f) \quad \text{Equation 2.3}$$

Modulus of Elasticity in transverse direction:

$$E_2 = \frac{E_f E_m}{E_f V_f + E_m (1 - V_f)} \quad \text{Equation 2.4}$$

Shear modulus of the composites:

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{(1 - V_f)}{G_m} \quad \text{Equation 2.5}$$

Poisson's ratio of the composites:

$$\nu_{12} = \nu_f V_f + \nu_m (1 - V_f) \quad \text{Equation 2.6}$$

Coefficient of Thermal Expansion (CTE) in the longitudinal direction (along the fibres):

$$\alpha_1 = \frac{1}{E_1} (\alpha_f E_f V_f + \alpha_m E_m [1 - V_f]) \quad \text{Equation 2.7}$$

Coefficient of Thermal Expansion (CTE) in transverse direction:

$$\alpha_2 = \alpha_f V_f (1 + \nu_f) + \alpha_m (1 - V_f) (1 + \nu_m) - \alpha_1 \nu_{12} \quad \text{Equation 2.8}$$

Where,

E_f - modulus of elasticity of fibre material

E_m - modulus of elasticity of matrix material

V_f - volume fraction of the fibre reinforcements

G_f - shear modulus of elasticity of fibre material

G_m - shear modulus of elasticity of matrix material

ν_f - Poisson's ratio of fibre material

ν_m - Poisson's ratio of matrix material

α_f - coefficient of thermal expansion of the fibre

α_m - coefficient of thermal expansion of the matrix

2.8 Residual stresses

Residual stress was mentioned for the first time by Heyn E. in 1941 as an internal strain (Tejas, 2009). In 1981, Ruud C. O. and Mordfin L. defined the residual stress as unknown stresses induced in a component through manufacture or assembly procedure which exists after all applied forces have been removed (Tejas, 2009). When a metal matrix composite is cooled down to room temperature from the fabrication or annealing temperature, residual stresses remain in the composite because of the mismatch of the thermal expansion coefficients between the matrix and reinforcement fibre (Arsenault and Taya, 1987).

Residual stresses are vital as they have both positive and negative effects on components depending on the application. Tensile residual stresses can lead to early failures, such as fatigue and fracture; In contrast, compressive residual stresses can be helpful, such as strengthening brittle materials, extending fatigue life and increasing the time of surface crack initiation (Tejas, 2009).

2.8.1 Classifications

Residual stresses are divided into macro-scale and micro-scale residual stresses based on the distance (Tejas, 2009).

The macro residual stress, also called type-I stress, covers many grains and is measured by destructive methods such as layer removal or hole drilling techniques (Tejas, 2009). Type-I residual stress is generated by the mismatch of properties of different areas, sharp thermal gradients or non-uniform plastic deformation (Tejas, 2009). Changes in Type-I residual stress will result in a macroscopic deformation.

Micro residual stresses are classified into type-II and type-III stress. Type-II stress generates in polycrystalline materials; they have different sizes in different grains (Tejas, 2009). Type-III stresses have various atomic scales, and they may be produced due to the interface difference and the dislocation stress fields (Tejas, 2009). In reality, all manufacturing processes will generate residual stresses to some degree.

Residual stresses can also be classified into three types based on their origin (Chason *et al.*, 2002; Tejas, 2009):

- a) Chemical origin - Residual stresses from chemical origins are due to the volume changes from chemical reactions or phase transformation, such as the deposition of atoms from the vapour. It can increase the driving force in the surface chemical potential, which leads to compressive stress generation during thin film growth.
- b) Mechanical origin – This type of residual stress is generated due to non-uniform plastic deformation.
- c) Thermal origin – residual stress generated from the thermal origin are due to temperature change and expansion coefficient misfits in the materials (see Figure 2.5). During heating up, the expansion of the matrix is significantly higher than the reinforced fibre due to its larger coefficient of thermal expansion (see Figure 2.5 (b)). As a result, residual stresses developed in the composite materials (see Figure 2.5 (c)).

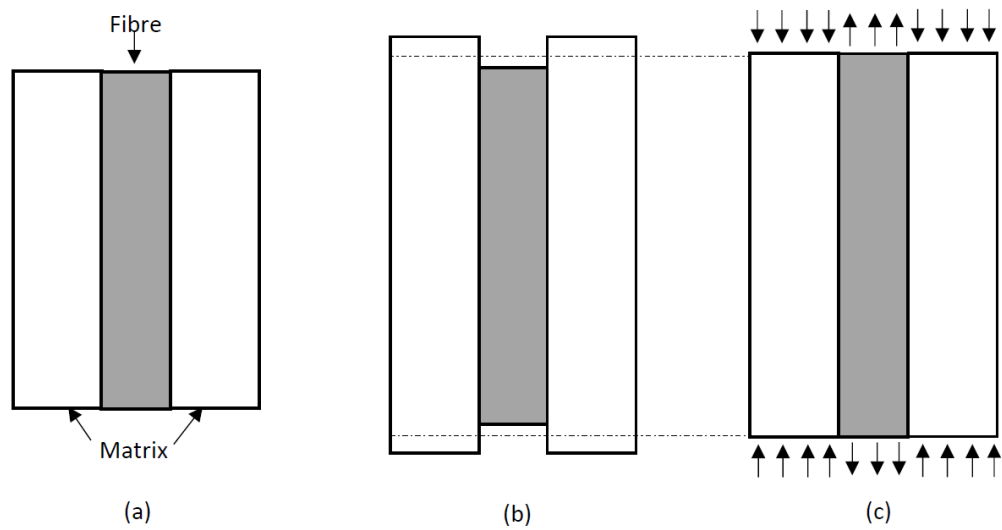


Figure 2.5 One-dimensional model of a continuous fibre reinforced metal matrix composite

2.8.2 Development of residual stresses during manufacturing processes

Manufacturing processes required for composite components can cause residual stresses. Some of the common techniques are machining, forming, heat treatment, joining and coating processes (Sosa, 2005). The machining processes are operations to cut or remove materials to fabricate a component, such as turning, drilling, or milling. The mechanical operations often cause compressive stresses in the cut zones (Sosa, 2005). The residual stresses generated are often related to the material properties, the cutting force and the sharpness of the tool (Sosa, 2005).

When an aluminium alloy is cooled down rapidly in a heat treatment process, temperature gradient will be generated at the surface and the inner structure. As the temperature at the surface is lower than the interior, this causes a difference in the contraction rate, resulting in the generation of residual stresses.

2.8.3 Effects of residual stresses

Residual stresses have multiple effects on composite materials, such as altering dimensions, mechanical properties, plastic deformation, and internal friction (Sosa, 2005).

The induction of residual stresses can change the response of a composite component to static loading (Sosa, 2005). When the part is under external load, stresses generated in the material add up with the pre-existing thermal stresses. These stresses will reduce the load required to reach the yield point, and therefore plastic deformation can happen earlier, leading to component failure.

The generated residual stresses caused by thermal operations can cause alteration of the dimensions of unidirectional fibre reinforced composite plate components. Uneven stress distribution within the composite plates can cause distortion. During the cooling down process from the high curing temperatures, asymmetric composite laminates will generate curvatures. The reason is the differences between the CTE and the elastic properties in each layer (Moore, Ziaei-Rad and Firouzian-Nejad, 2014).

2.8.4 Methods of determine residual stresses

2.8.4.1. Non-destructive methods

Non-destructive methods are based on measuring the variation of the material properties (Sosa, 2005). These methods measure parameters like crystallographic lattice (Tejas, 2009) distance compared to that in the stress-free material. However, the relations of the measured parameters are complicated, and the results obtained are challenging to interpret (Sosa, 2005). Many measurements attempt using diffractive, acoustic, ultrasonic, magnetic techniques (Tejas, 2009). The most commonly used methods are X-ray diffraction and neutron diffraction techniques (Sosa, 2005).

a) X-ray diffraction technique

Diffraction techniques are based on the fact that when the material is stressed below the yield point, the elastic strain within a material causes the atomic planes in the metallic crystal structure to change their distance (Tejas, 2009). The interatomic planes in the metallic crystal structure are used as internal strain gauges, and the difference within the lattice spacing is recorded as angular shifts in diffraction peak positions (Tejas, 2009). The measured lattice spacing will be related to the elastic strain, determining the residual stresses within the materials (Sosa, 2005).

The x-ray diffraction technique uses x-ray beams with a penetration of around 0.1 mm in steel (Sosa, 2005). The poor penetration depth hugely limits the capability of the method when a thick material needs to be measured. The X-rays are directly applied to the component surface, which requires a perfectly cleaned surface (Sosa, 2005). Also, only a tiny area of the component can be measured each time, and it can therefore be very time-consuming and expensive (Sosa, 2005). Overall, the x-ray diffraction technique is an expensive but reliable method for residual stresses measurements. It is suitable for components with small regions with high-stress gradients.

b) Neutron diffraction method

The neutron diffraction method is similar to the x-ray diffraction technique, which determines the strains from measuring the changes in lattice spacing inside the material (Sosa, 2005). It can determine the stresses up to several centimetres deep within the components (Sosa, 2005). However, the method is costly and time-consuming (Sosa, 2005).

2.8.4.2. Semi-destructive methods

Semi-destructive methods are based on the principle that by removing a small region of material at the position of interest, the residual stresses that existed there will be partially or fully released, measured by strain gauges using stress-strain relations (Sosa, 2005). When a small piece of material is removed, it will break the stress equilibrium established inside the

component. A set of strain gauges put on the spot of interest can measure the change in strain, and the results are used to calculate the residual stresses. The damage using these methods is relatively small, and the tested specimens may still be usable for further application (Tejas, 2009). However, these methods have low penetration, can be expensive to use and can lack accuracy (Sosa, 2005). Examples of these methods are expressed below.

a) Centre-hole drilling method

The principle of this method is to release residual stresses inside the material through the hole drilling process. By drilling a small circular hole on the specimen, the initial residual stress at the hole's edge reduces to zero (Sosa, 2005). The state of residual stresses can be analysed by measuring the strain in three different directions (see Figure 2.6).

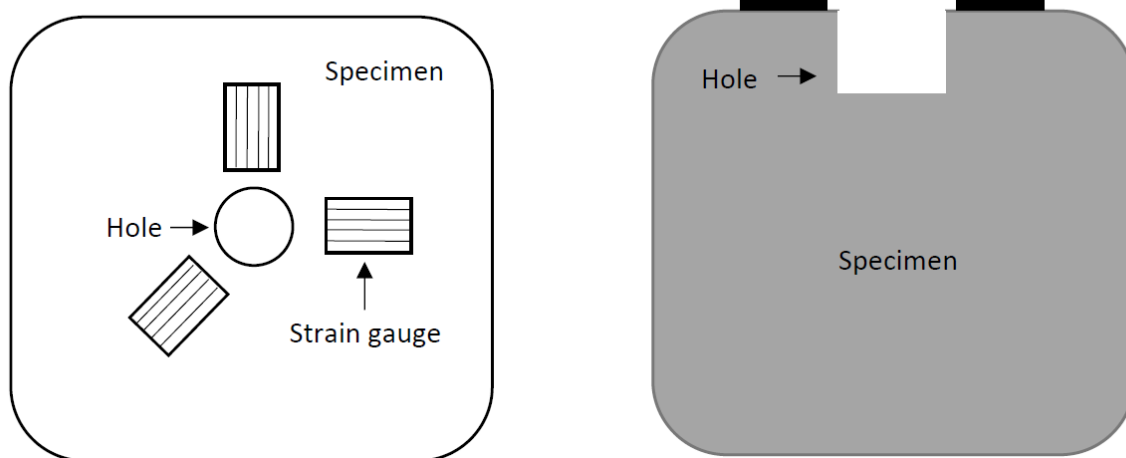


Figure 2.6 Schematic diagram for the centre-hole drilling residual stresses measurement

The residual stress field of the specimen can be calculated using strain data and other material properties, like Young's modulus and Poisson's ratio (Tejas, 2009).

b) Deep-hole drilling method

The deep-hole drilling method is a stress relief method that involves drilling a hole through the specimen (see Figure 2.7). This technique assumes the stress relief behaviour of the

material is uniform and that the stress-strain behaviour is linear (Sosa, 2005). The reference hole, which is through the component, is measured carefully before and after trepanning. The difference between the diameter and column height determines the residual stresses (Sosa, 2005).

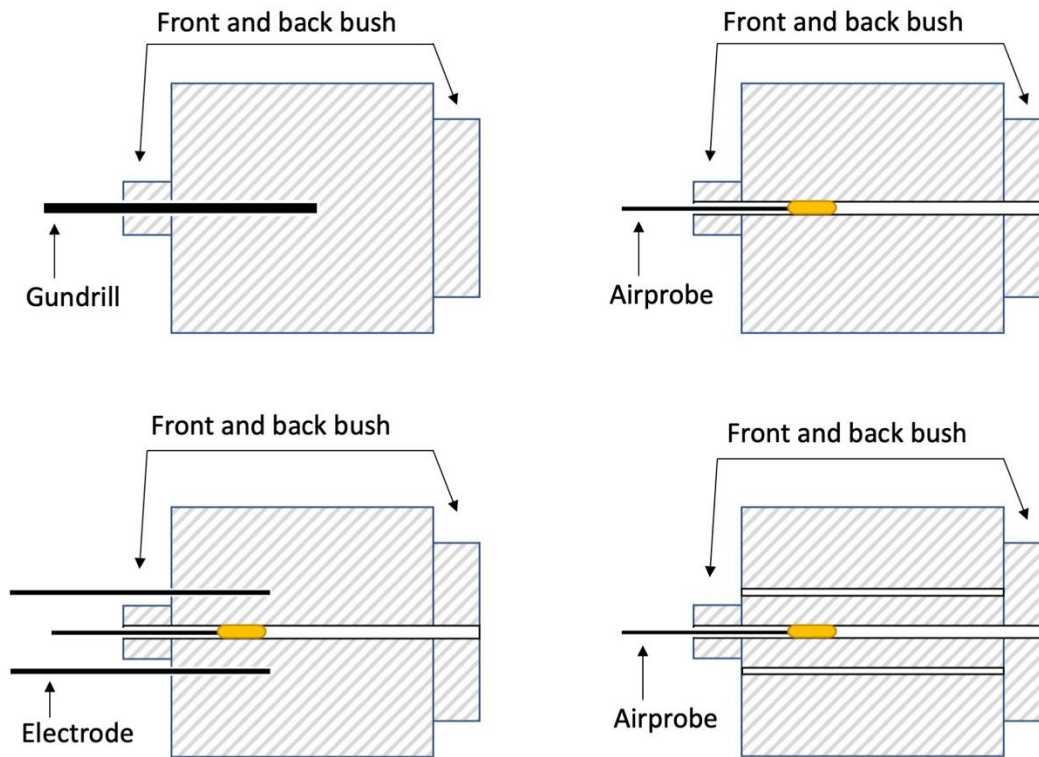


Figure 2.7 Stages of deep-hole drilling method

This method provides good residual stress data with reasonable accuracy; however, it causes more damage than the centre-hole drilling and ring-coring methods (Sosa, 2005).

2.8.4.3. Destructive methods

Destructive methods measure the residual stresses through cutting or slicing parts of a component, which will cause permanent damage to the material (Sosa, 2005). These methods are entirely destructive, and they cannot be used to analyse the residual stresses of materials in service (Tejas, 2009). However, the methods measure the average strain in the material, which can be very accurate (Sosa, 2005). The most commonly used methods are explained below.

a) Crack compliance method

This method measures the residual stresses by introducing a crack into the specimen. The damage creates stress relief, and the strain change is measured by strain gauges placed on both sides of the material (Figure 2.8).

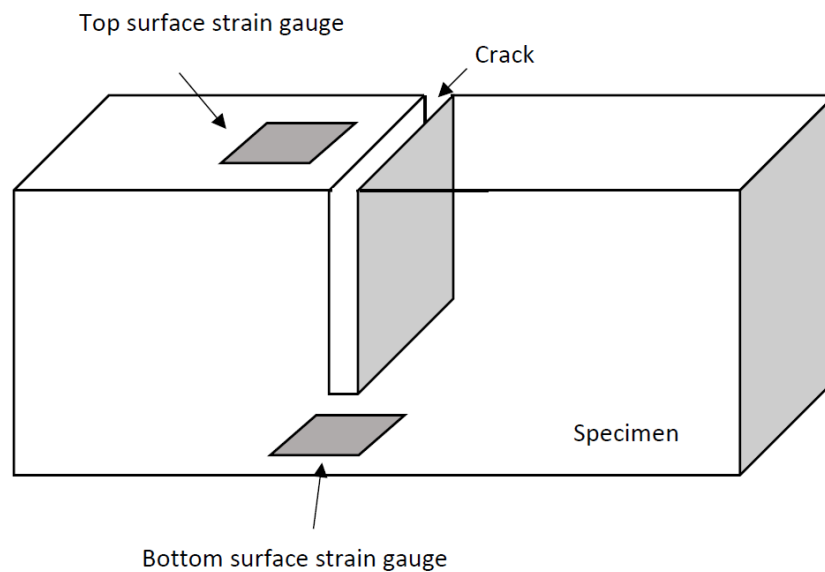


Figure 2.8 Schematic of the crack compliance method

The crack compliance method can measure various materials such as metals, composites and polymers (Sosa, 2005). It is relatively easy to apply, with commonly used equipment like strain gauges and conventional machining (Sosa, 2005).

b) Contour method

In this method, the specimen is cut into two pieces which will release the pre-existing stresses. The residual stress relief causes uneven surfaces (contours), and the outline of the two new surfaces can be measured.

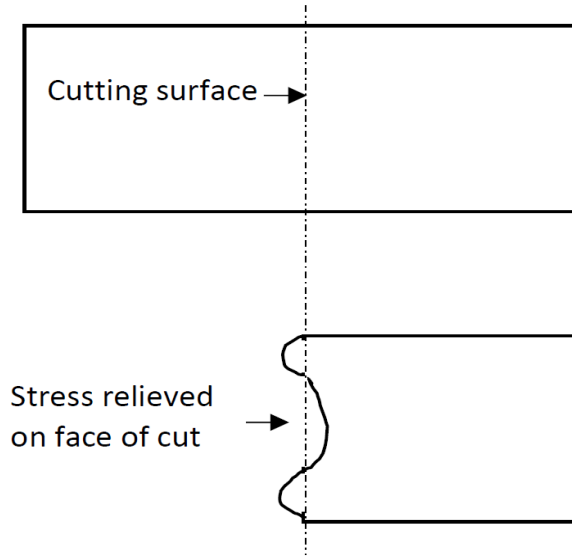


Figure 2.9 Schematic of the contour method

The coordinated measuring machine is used for the measurements, and the results are used for the stress calculation. The contour method can determine the residual stresses normal to the face of the cut, and the results have reasonable accuracy.

2.8.5 Analytical models

2.8.5.1. One-dimensional model (Voigt model)

During the manufacturing process, aluminium matrix and reinforced fibres shrink with the drop in temperature back to room temperature. A one-dimensional model can estimate the thermal stresses generated in the composite (see Figure 2.10). The composite model is assumed to experience a uniform temperature drop ($\Delta T > 0$) and is initially stress-free. Young's modulus and CTEs of the matrix and fibre are denoted by E_m , E_f , and α_m , α_f ($\alpha_m > \alpha_f$), respectively. The CTE of the composite is represented with α_c .

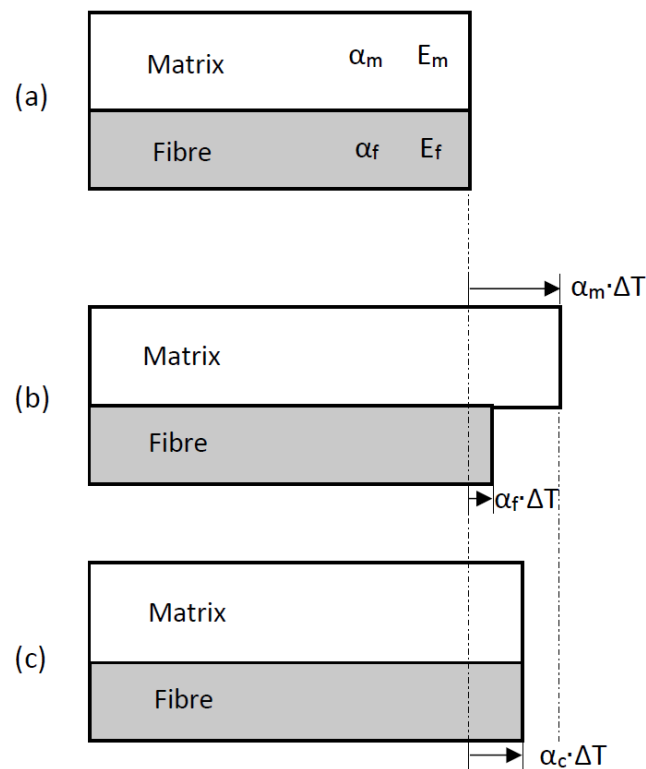


Figure 2.10 One-dimensional model for the analysis of residual stresses in a continuous fibre reinforced metal matrix composite (a) initial state, (b) constraint-free state, (c) equilibrium established state.

2.8.5.2. Cylinder model

The cylinder model can be applied to predict the internal thermal residual stresses for a composite with monofilament reinforcements. The model was originally studied to predict the mechanical behaviour of fibres with different coating materials. It can estimate the internal stresses with the combination of externally applied loads (Choy *et al.*, 1995). The model contains a long cylinder fibre with perfectly bonded surrounding matrix material (see Figure 2.11).

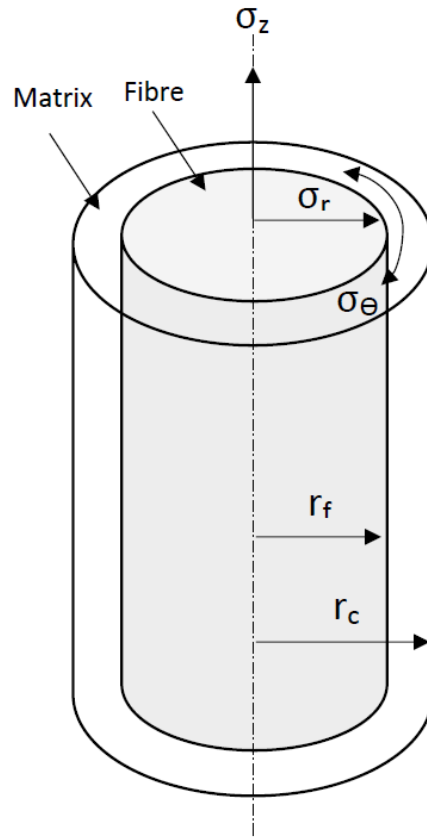


Figure 2.11 Cylinder model with external radius r_c and fibre radius r_f

2.8.6 Factors that affect the magnitude of residual stresses

In the manufacturing procedure, thermoplastic composites are heated to a designed temperature and cooled down to solidify. During the cooling process, the volumetric shrinkage of the matrix is much faster than that of the fibres. The reason is the misfit of the thermal expansion coefficient between the matrix and the fibres. Consequently, compressive residual stresses are developed in the fibre whilst tensile residual stresses are developed in the matrix (Tejas, 2009).

The four parameters in order of magnitude of residual thermal stresses in the fibre reinforced composites are:

1. temperature difference
2. coefficients of the thermal expansion
3. elastic coefficients of the layer
4. fibre volume fraction

These parameters are dependent on the fibre types, fibre arrangement, crystal structure of the matrix, interface properties between the fibre and matrix and the composite material processing conditions (Tejas, 2009). Apart from the parameters stated, plasticity should also be taken into consideration when cooling.

2.9 The manufacturing process of MMCs

There are various fabrication methods for metal matrix composites. Those methods are classified into five types based on the temperature of the matrix during the manufacturing process (Bhaskar *et al.*, 2014), they are liquid state processes, solid-liquid processes, deposition techniques, in situ processes and two-phase (solid-liquid) processes. The research work uses hot isostatic pressing (hipping) process, which is a two-phase process.

The hot isostatic pressing process was originally developed for porosity removals and diffusional bonding of nuclear reactor components. Nowadays, the technology has been commercially applied in the industries such as aerospace, marine, power generation, automotive. The basics of the hipping process have been reviewed by several authors (Atkinson and Davies, 2000; Bahei-El-Din and Dvorak, 1995).

The hipping process is used to remove pores to improve the density of pre-sintered components. It involves applying high isostatic pressure and high temperature in a closed vessel simultaneously. The high isostatic pressure is generated by a mechanical compressor

and a heated gas in the container. When the gas (usually inert) is heated, the gas molecules collide with all the surfaces of the hipping component consistently at a relatively high speed. Under certain circumstances, there are 10^{30} collisions happening in one square meter per second. Since the gas is evenly spread in the hipping vessel, there are on average the same number of molecules in a unit space moving at an identical velocity in all directions. Thus, the object is experiencing an isostatic pressure perpendicular to the surface.

Isostatic pressing is different from conventional unidirectional pressing. Unidirectional pressing applies normal pressure on an object in a die using a ram. The pressure is transmitted directed by the surface of the ram and the die to the object. This process involves a movement between the object and the die walls, which leads to friction, thus causing the pressure to be non-uniform in all directions.

The hipping process does not have the same unidirectional pressing problem because the hipping does not have the ram or die to perform physical contact between the pressure and the treated object. It can achieve a uniform press in all directions (photographic shape reduction as shown in Figure 2.12. The hipping method can manufacture components with complex shapes, such as turbine discs in the aerospace industry.

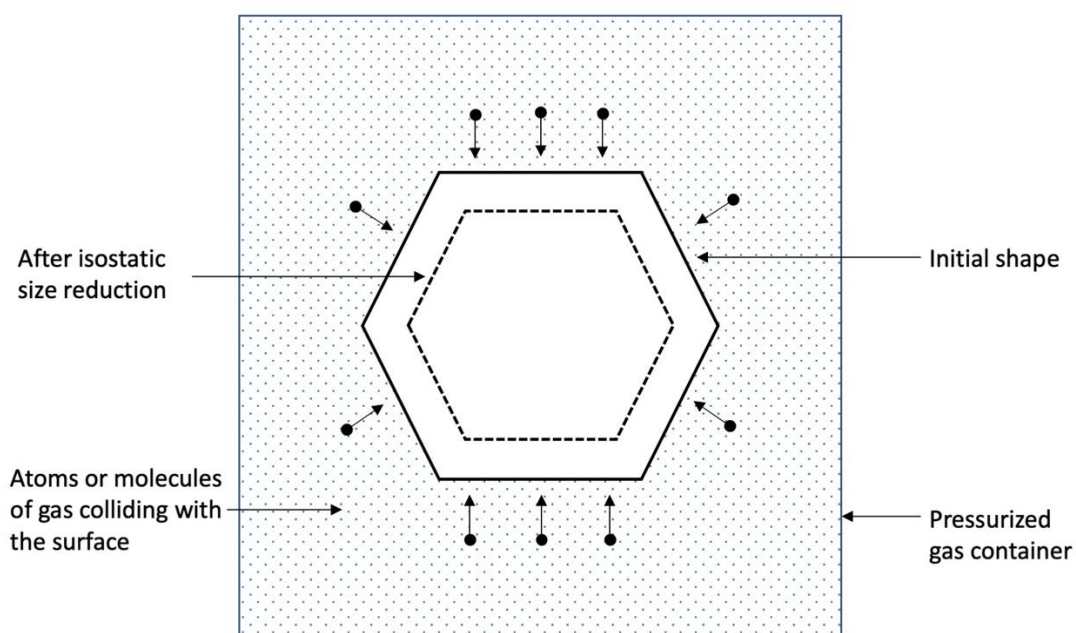


Figure 2.12 Illustration of the photographic reduction during hot isographic pressing

Table 2.2 Applications of Hipping (Atkinson and Davies, 2000)

Application	Primary Aim	Materials
Densification of casting	To remove macro-porosity and micro-porosity	Ni- and Co-based superalloys Ti alloys Al alloys Steels Cu alloys
Densification of pre-sintered powders	To achieve full theoretical density and to avoid excessive grain growth (difficult by conventional methods)	WC- based hard metals Si_3N_4 , Al_2O_3 , ZrO_2 And other advanced ceramics Be alloys
Consolidation of encapsulated powders	To achieve full theoretical density and to avoid segregation and excessive grain growth	High-speed tool steel Ceramics Metal matrix composites (e.g., Al-SiC) Ceramic-ceramic composites Magnetic materials (e.g., ferrites) Ceramic superconductors Fe-TiC
Interface bonding	To diffusion bond similar and dissimilar materials (overcoming problems of conventional joining and uniaxial diffusion bonding techniques) To bond and densify coating	Bronze and steel Ni alloys and steel Ceramic-metal bonding Ta, Ti, Al, and W in Al housing for sputter targets
Specialised applications	To remove pores and gaseous impurities from optical materials	ZnS Sn-In-O lanthanates and aluminates
Reactive hipping	To synthesise compounds by exothermic reaction during heating at hot isostatic press temperature	Intermetallics, e.g., TiB_2 and Nb_3Al

Hipping is an effective treatment for metal components to remove both macro and micro-porosity. Micro-porosity is casting pores developed during the metal solidification and shrinkage effect. Conventional hot working such as forging can remove these cast pores, but

this will add extra cost, and the original shape of the components will be sacrificed. Hipping can be used to eliminate micro-porosity without changing the designed shape. Also, the microstructure of the metal, such as grain structure, phases present and the precipitate arrangement, will not be altered due to the hipping process. Table 2.2 lists the applications of hipping, the primary aims, and examples of materials processed.

Benefits of hipping process also include:

- Removal of casting pores
- Improvement in mechanical properties (increased fatigue resistance, resistance to wear and abrasion, better service temperature)
- Densification of powdered metal parts to theoretical density
- Precise shape forming
- Cost-effective diffusion bonding
- Reduced property scatter

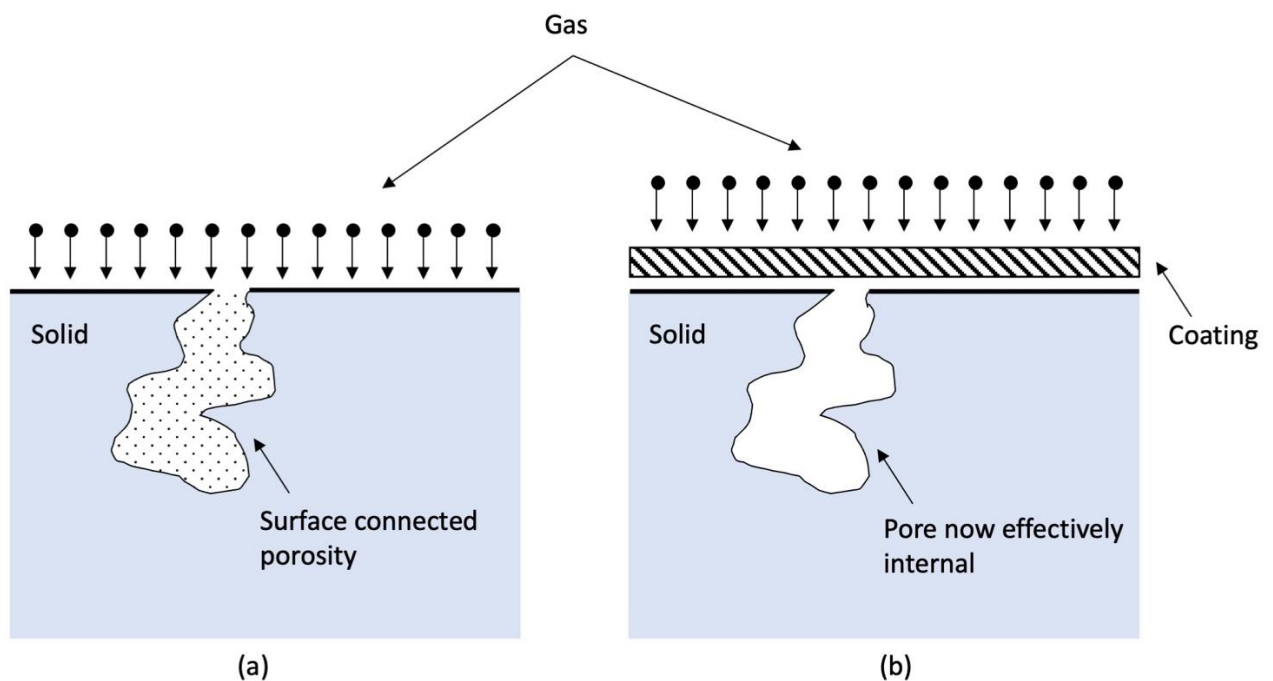


Figure 2.13 Pores connected with the surface (Atkinson and Davies, 2000)

The pore removal of the HIPping process depends on various parameters, such as temperature, pressure, size, and position of the pores. When the pores are connected with the surface, they cannot be removed by HIPping directly. The solution to this problem is to coat the surface before the HIPping treatment, refers in Figure 2.13. The pores are connected with the surface, as shown in Figure 2.13 (a). After coating, as shown in Figure 2.13 (b), the connected surface pores become internal, and they can be evacuated using the HIPping process.

2.10 EDM Machining of Metal Matrix Composites

Metal matrix composites are considered difficult-to-machine materials due to the ceramic reinforcement. Conventional methods such as turning, milling, drilling, and grinding have been applied to MMCs. However, the machining performance is not ideal because of the severe cutting tool degradation, increased cutting force and uneven cut surfaces (Chen, Gu and He, 2020). A diagram is shown in Figure 2.14, listing the classification of traditional and nontraditional machining processes. Unconventional cutting methods have been employed to improve the machinability of MMCs, such as electrical discharge machining (EDM), wire EDM, electrochemical machining (ECM) and arc discharge machining (ADM) (Zhao *et al.*, 2013). EDM is the method employed for machining samples for this research project, and it will be mainly discussed in this section.

Electrical discharge machining (EDM) is also known as spark machining, spark eroding or wire erosion. It is a machining process to remove desired sections from the workpiece by applying rapid discharges on the electrodes (Figure 2.15). The workpiece is placed in a dielectric liquid such as deionised water. The purpose of the dielectric liquid is to act as the conductor between the electrodes and the workpieces. Also, it is used as a flushing agent to remove the eroded debris from the workpiece. It is important to choose the right fluid that depends on the type of materials being machined, the electric voltage, the fluid type, and the EDM fluid filters' efficiency.

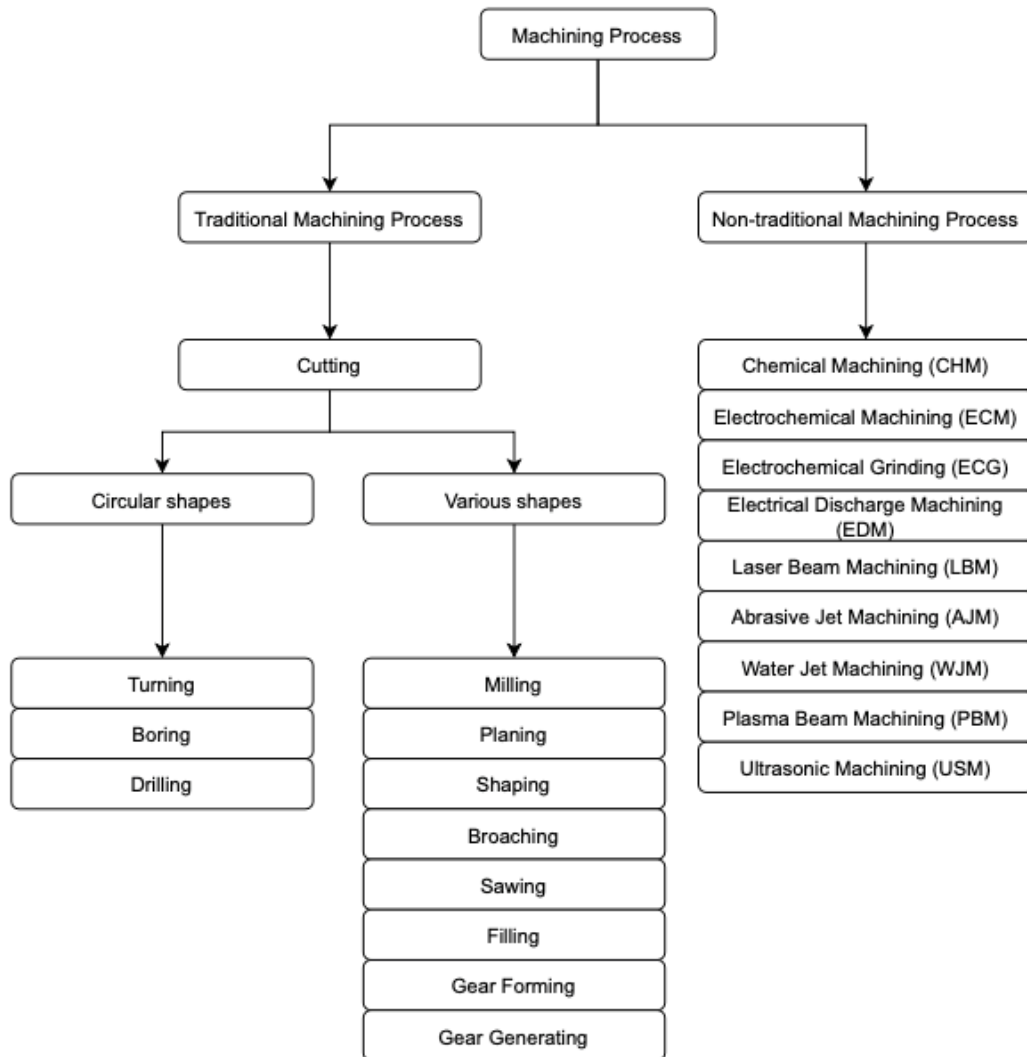


Figure 2.14 Diagram of machining processes (Nukman *et al.*, 2017)

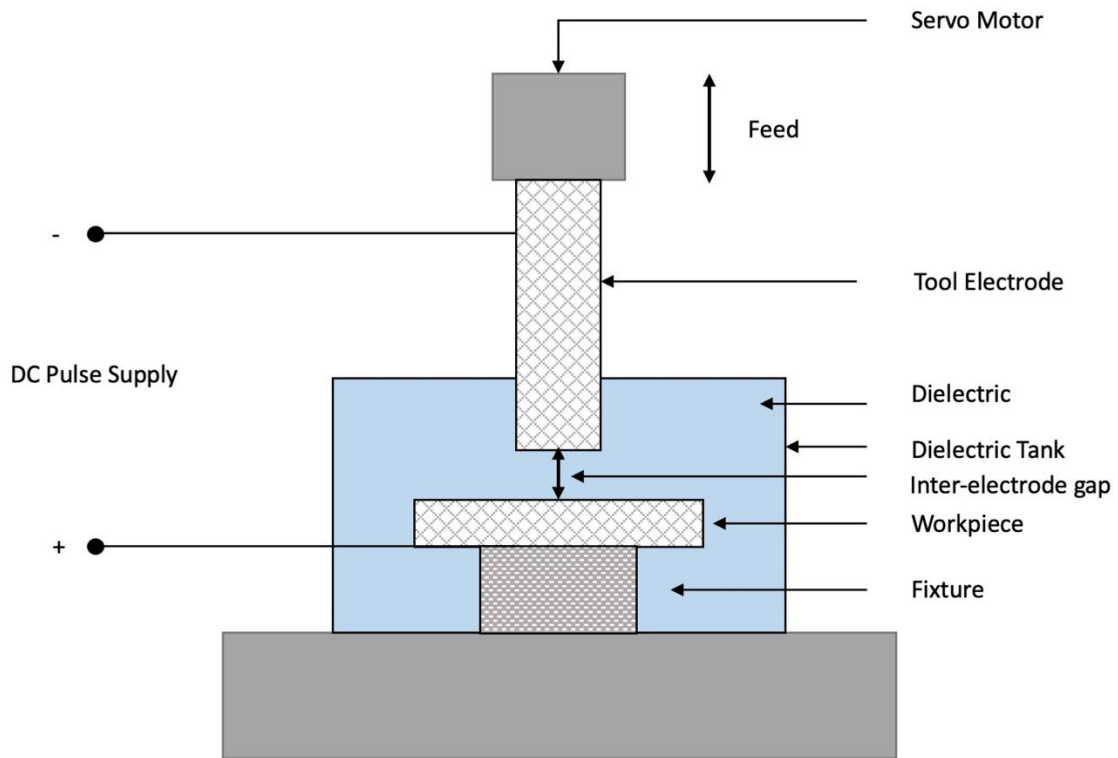


Figure 2.15 A schematic diagram of electric discharge machining (EDM) machining (Saha, 2008)

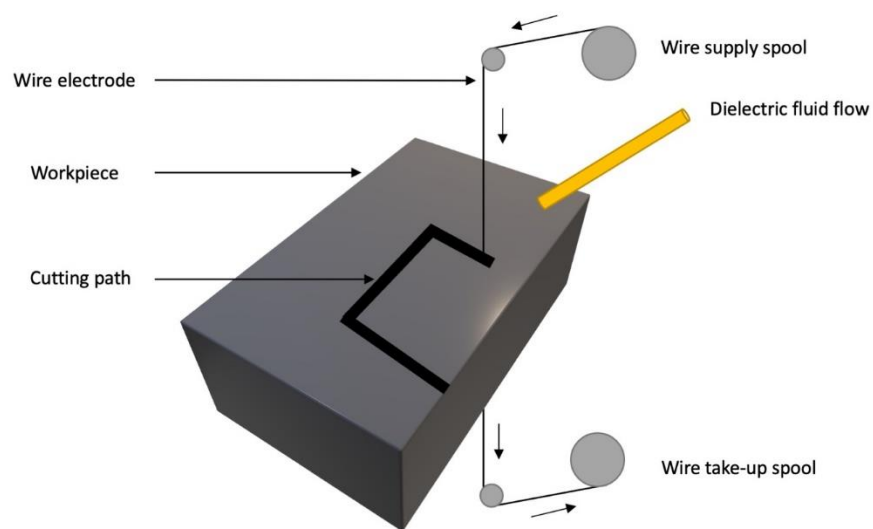


Figure 2.16 Schematic diagram of wire EDM process (Bisaria and Shandilya, 2015)



Figure 2.17 Wire EDM cutting process

Wire EDM machining, also known as wire cutting, uses thin metal wires as the electrodes. The cutting wire, usually brass, is fed continuously through the workpiece from a spool, as shown in Figure 2.16. The wire will pass through the workpiece submerged in the EMD fluid to generate the programmed cutting, shown in Figure 2.17. The wired EDM machine usually has upper and lower guides, which are CNC controlled. It allows the user to set the precise cutting programme in multiple axes and monitor the cutting process in real-time. Thanks to the multi-axis cutting guide and the thin wire (0.05mm – 0.3mm), wire EDM can be used to make tools and dies difficult to cut by other methods and achieve fine and delicate shapes (Chen, Gu and He, 2020).

2.11 SiC fibre reinforced aluminium alloy composite process technology

During the consolidation process, some fibre and matrix reactions could severely degrade the composite material properties (Durodola *et al.*, 2017). Because the solid-state processing method is a relatively low-temperature manufacturing technique, it is widely used for fibre reinforced MMC fabrication. (Durodola *et al.*, 2017). Foil-fibre-foil (FFF) manufacturing process is used for the fabrication of SiC fibre reinforced aluminium alloy composite. Figure 2.18 presents the manufacturing process.

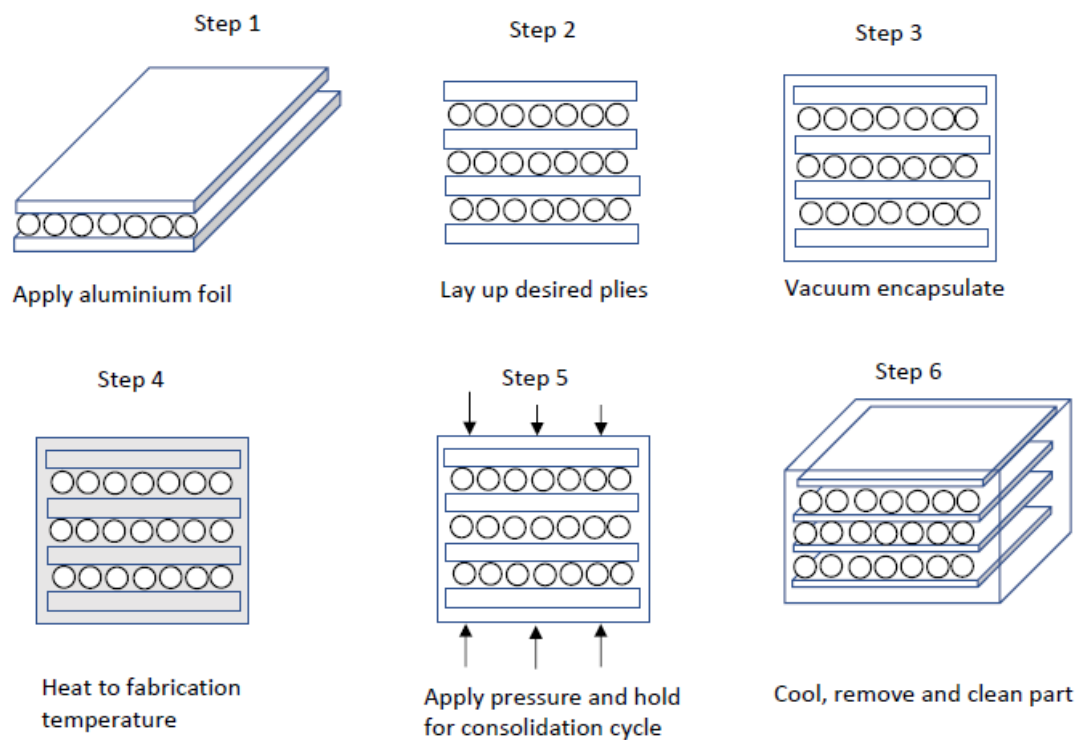


Figure 2.18 Schematic of continuous fibre reinforced aluminium composite fabrication by diffusion bonding (Rohatgi, 1993b)

Silicon carbide fibres are aligned and glued together to form the fibre ply. The fibre plies are then cut into the designed shape and stacked with aluminium alloy foils with custom angles. The layups, made of multiple metal foils and fibre layers, will go through the diffusion process in a vacuum environment. The aluminium metal foils are hot-pressed around the fibre layer until the fibres are surrounded by the aluminium material (John. V, 1983). The compressed layers, with unidirectional fibres, will become a composite unit layer. They can be stacked as layup plies with tailored fibre angles. After the designed pressure and thermal processes, the composite material will be formed, cooled down and removed from the mould.

Foil-fibre-foil (FFF) manufacturing process is a well-developed technique. It has reasonable control of the composite microstructure and the matrix composition (Durodola *et al.*, 2017). However, it is difficult to place the fibre precisely, and it is hard to use the method to fabricate thin composite sheets (Durodola *et al.*, 2017). It increases the manufacturing cost and makes it an overall expensive technique.

2.12 Silicon carbide (SiC) fibre manufacturing process

Three main approaches to manufacture the SiC fibre are carbothermal reduction, chemical vapour deposition (CVD), and polymer pyrolysis (Wei Xie, 2012). Silicon carbide before the manufacturing process is shown in Figure 2.19. The carbothermal reduction method is an easy operating technique. However, the SiO_2 preparation requires expensive chemical reaction materials. The fabrication process also requires high temperatures, from 1400°C to 1650°C (Wei Xie, 2012). Part of the carbon material will be consumed during the manufacturing process, damaging the final fibre structure (Wei Xie, 2012).

Compared to the carbothermal reduction method, the polymer pyrolysis method requires a lower temperature, around 1200°C (Wei Xie, 2012). The operation steps are simple, but the technique requires costly reaction material to achieve the desired fibre coating (Wei Xie, 2012). Also, the operation steps often need to be repeated multiple times, which can be inefficient (Wei Xie, 2012).

The chemical vapour deposition (CVD) method requires a low fabrication temperature (800°C to 1250°C) (Wei Xie, 2012). However, the technique's equipment is expensive and specially made (Wei Xie, 2012). This technique's main advantage is that the manufacturing procedure using this method can be robust and rapid. The primary diameter of the coated fibre ranges from 80 to 140 microns (Wei Xie, 2012; Andrulonis *et al.*, 2017). Figure 2.19 presented the schematic of SiC fibre.



Figure 2.19 Silicon carbide crystal

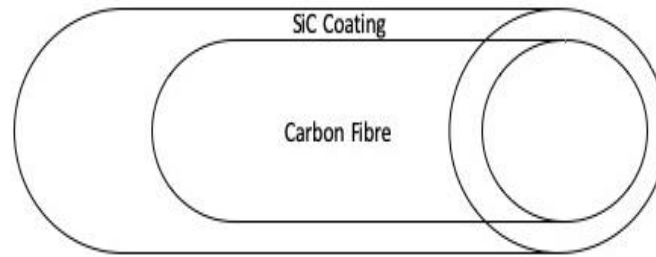


Figure 2.20 Schematic of a SiC Fibre

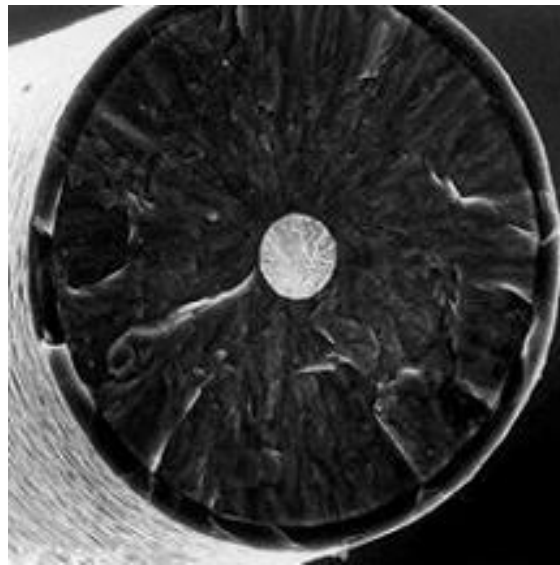


Figure 2.21 Cross-sectional picture of a SiC Fibre

2.13 Aluminium alloys

A wide range of aluminium alloys is classified into two groups, heat treatable alloys and non-heat-treatable alloys. Heat treatable alloys can be heated and cooled to vary their performance characteristics. The cooling process is normally rapid such as quenching the materials in water, oil or sand. Natural or artificial ageing can be applied for further treatment. Heat treatable alloys are denoted by the letter T in the aluminium material descriptor. Non-heat treatable alloys are also called work hardening alloys, denoted by the letter H in the aluminium material descriptor. These alloys developed characteristics through cold work, such as rolling and extruding.

Aluminium alloys are categorised based on the material characteristics and the elements added to the alloy. Aluminium is the primary metal, and the main added components include copper, silicon, zinc, magnesium. Aluminium 6061 is the metal matrix material used for this research.

2.14 MMC mechanics and deformation behaviour

For simple microstructure models, SiC fibre and the aluminium matrix are considered elastic materials. In reality, the SiC fibre will behave as elastic-brittle, and the aluminium 6061 matrix will be elastic-plastic. In FEA, the creep, yield and plasticity of the aluminium related to stress and temperature can be modelled (see chapter 3). Failure in the SiC/Al composite is governed by the SiC fibre strength and the yielding point of the aluminium 6061 alloy matrix (Zhou *et al.*, 2006). Fibres are assumed to be elastic up to the fracture point, about 3.24GPa in tension and 9GPa in compression (Mahmoodi and Aghdam, 2011).

The behaviour of elastic-plastic materials is displayed in Figure 2.22. The realistic material behaviour is shown in Figure 2.22 (a) and simplified behaviour with hardening modulus in Figure 2.22 (b). Figure 2.22 (c) shows that the simplest model assumes elastic perfectly plastic material. This type of material does not exist in real life and will only be used in this research to develop analytical models for comparative purposes.

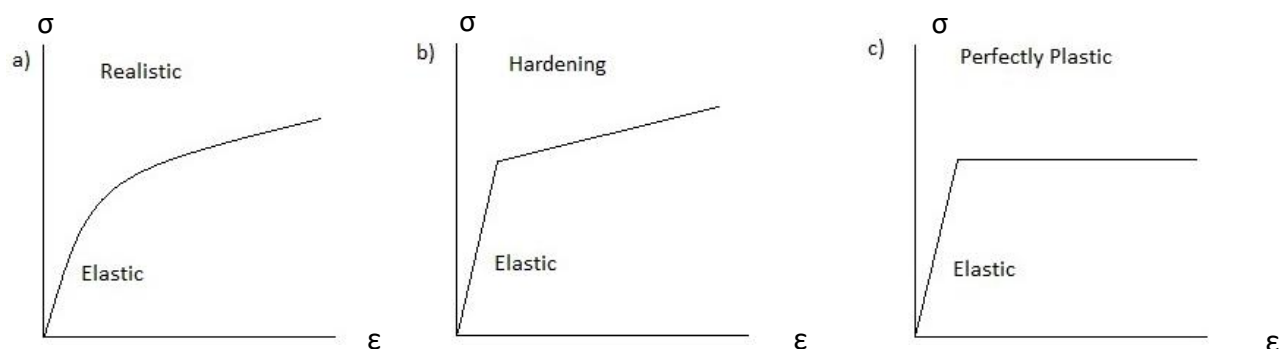


Figure 2.22 Modelling of Elastic-Plastic Materials

Elasticity, plasticity and failure models are expressed as below, using rules of mixtures. These are simplified analytical models with the ability to predict the yield and failure points of composite materials. The SiC fibre is elastic-brittle, and the metal base is considered as elastic-perfect plastic.

Elasticity:

$$E_1 = E_f V_f + E_m (1 - V_f) \quad \text{Equation 2.9}$$

$$\nu_1 = \nu_f V_f + \nu_m (1 - V_f) \quad \text{Equation 2.10}$$

$$\varepsilon_1 = \varepsilon_m = \varepsilon_f = \frac{\sigma_1}{E_1} \quad \text{Equation 2.11}$$

Plasticity (Zhou *et al.*, 2006):

$$\sigma_y = \left(\frac{E_f}{E_m} V_f + 1 - V_f \right) \sigma_y^m \quad \text{Equation 2.12}$$

$$E_p = E_f V_f \quad \text{Equation 2.13}$$

Failure (Zhou *et al.*, 2006):

$$\sigma_s = \sigma_s^f V_f + (1 - V_f) \sigma_y^m \quad \text{Equation 2.14}$$

The stress the MMC would yield is much lower than obtained from material testing aluminium with an equivalent cross-section. The reason is that the aluminium matrix is pre-tensioned by the residual thermal stress during the cooling down process, which causes the composite to have an earlier yield during tensile loading (Zhou *et al.*, 2006).

2.15 Classical laminate theory

Classical laminate theory (CLT) is a prediction method for analysing mechanical and thermal loading of composite laminate mechanical behaviour. It is based on assumptions as below (Hyer, 1981):

- 1) The laminate thickness is small compared with the in-plane length
- 2) The displacements are continuous throughout the laminate, and the strain-displacement has a linear relationship
- 3) The laminate has perfect bonding
 - The bond line is extremely thin
 - The bond is stronger than the material around it
 - The bond does not shear
- 4) The Kirchhoff hypothesis is valid regarding undeformed normals
 - Straight lines normal to the mid-plane remain straight (they do not bend)
 - Straight lines normal to the midplane remain unstretched (thickness is constant)
 - Straight lines normal to the mid-plane remain normal (always at a right angle to the neutral plane)

The classical laminate theory integrates the constitutive equations through the laminate thickness, with defined force and moment resultants. Resultant load, moment, midplane stress and curvature can be calculated with the classical laminate theory.

2.16 Room temperature shapes of unsymmetric composite laminate

Classic laminate theory predicted that the asymmetric laminates would deform into a saddle-like shape after being cooled to room temperature (Hyer, 1981). However, laminates develop into two stable cylindrical forms in most cases (Moore, Ziaei-Rad and Firouzian-Nejad, 2014). The deformation behaviour of thin unsymmetrical laminates attracted attention for

researchers to study the bistable structure. The bi-stability structure can settle at either cylindrical position without external aid compared with the unstable saddle-like shape.

Unsymmetrical composite laminates develop curvatures when cooled to room temperature. The curvatures are a great indicator for deformation shapes due to the material properties differences. Thus, it is important to understand the curvatures of composite laminates fully. Hyer (Hyer, 1981b) was the first to research the deformation behaviour of thin laminates. For the past three decades, comprehensive efforts were put into understanding bistable composite laminates, regarding the thermal load analysis (Giddings *et al.*, 2010), cured shapes modelling (Betts *et al.*, 2010), temperature-curvature relationships (Moore, Ziaei-Rad and Firouzian-Nejad, 2014), bistable structures snap processes (Zhang *et al.*, 2014) and morphing and energy harvesting (Emam and Inman, 2015). A load is added to the component in Moore's experiment, and the two cylindrical shapes were switched at room temperature (Moore, Ziaei-Rad and Firouzian-Nejad, 2014).

As discussed in the previous section, curvatures can be used as an indicator for deformations of composite laminates. Section 2.16.1 and 2.16.2 present the equations for curvature calculations on a plane and a surface.

2.16.1 Curvature on a plane

For a twice differentiable curve $Y(t) = x(t), y(t)$, the curvature is

$$k = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{\frac{3}{2}}} \quad \text{Equation 2.15}$$

When the function of a curve is $y = f(x)$, the curvature formula is simplified to

$$k = \frac{y''}{(1 + y'^2)^{\frac{3}{2}}} \quad \text{Equation 2.16}$$

2.16.2 Curvature on surfaces

For a surface map Y , Y_x and Y_y are derivatives respect to x and y , respectively.

The normal unit vector of the surface n can be defined as:

$$n = \frac{Y_x \times Y_y}{|Y_x \times Y_y|} \quad \text{Equation 2.17}$$

Gaussian curvature is the product of the principal curvatures, K_1 and K_2 . It is the intrinsic property of the surface. The Gaussian curvature is positive for spheres, negative for saddle-shaped surfaces, and it equals zero for planes.

$$K = \frac{l_{11} \cdot l_{22} - l_{12}^2}{g_{11} \cdot g_{22} - g_{12}^2} \quad \text{Equation 2.18}$$

Where g_{11} , g_{12} , g_{22} , l_{11} , l_{12} , l_{22} are,

$$g_{11} = ||Y_x||^2 \quad \text{Equation 2.19}$$

$$g_{12} = Y_x Y_y \quad \text{Equation 2.20}$$

$$g_{22} = ||Y_y||^2 \quad \text{Equation 2.21}$$

$$l_{11} = Y_{xx} \cdot n \quad \text{Equation 2.22}$$

$$l_{12} = Y_{xy} \cdot n \quad \text{Equation 2.23}$$

$$l_{22} = Y_{yy} \cdot n \quad \text{Equation 2.24}$$

The mean curvature is equal to the half sum of the principal curvatures, K_1 and K_2 .

$$H = \frac{l_{11} \cdot g_{22} - 2 \cdot g_{12} \cdot l_{12} + l_{22} \cdot g_{11}}{2(g_{11} \cdot g_{22} - g_{12}^2)} \quad \text{Equation 2.25}$$

Thus, the principal curvatures of the surface are

$$K_1 = H + \sqrt{H^2 - K} \quad \text{Equation 2.26}$$

$$K_2 = H - \sqrt{H^2 - K} \quad \text{Equation 2.27}$$

2.17 Finite element analysis (FEA)

Extensive research has been carried out to predict the mechanical properties of MMCs using finite element analysis. Researchers developed various numerical models on composites with continuous fibre reinforcements (Giugliano, 2016; Karimi, Sedighi and Afshari, 2015; Tejas, 2009; Harvey, 2009). For example, Giugliano investigated the thermal-mechanical behaviour of aluminium composites with continuous alumina and silicon carbide fibres. The work focused on the micromechanical FE model with different fibre volume fractions up to 63.61%. Durodola presented a finite element model for thermal residual stresses prediction. The model was also calibrated with published experimental data (Durodola *et al.*, 2013). Efforts were also put into numerical models to predict aluminium alloy behaviours during the welding process (Karimi, Sedighi and Afshari, 2015). There has been a lot of interest in composite laminates shapes at room temperature. Four decades ago, Hyer discovered that unsymmetric laminates form cylindrical or bi-stable shapes at room temperature, which differs from the prediction from classic laminate theory (Hyer, 1981a). This early research presented a theory to explain why this occurred. Moore investigated the bi-stable deformation on the asymmetric composite laminates using analytical and finite element simulations (Moore, Ziaei-Rad and Firouzian-Nejad, 2014). The work focused on the effect of material temperature dependency on specimens with different reinforcement layouts.

2.18 Thermal residual stresses modelling using FEA analysis

Finite element models have been published to evaluate thermal residual stresses within MMCs. Aghdam (Aghdam, 2001) presented two-dimensional and three-dimensional micro-structural finite element models to predict the unidirectional fibre reinforced composite. In the models, plane strain mesh elements in ABAQUS were used to mesh the geometry. Boundary conditions applied include axial and transverse loading, the effects of interface debonding and thermal residual stress. The results were validated with experimental data of SiC/Ti MMCs, and a good agreement was found. It was found that friction must be introduced at the interface to allow the model to perform correctly.

Teixeira-Dias (Teixeira-Dias, 2001) studied thermal residual stress prediction using a three-dimensional finite element modelling method. The SiC/Al FE models applied the thermal elastic behaviour for the reinforcement and thermo-elasto-visco-plastically for the matrix material. The unit cell continuous MMC model had 240 elements in total. The symmetric plane boundary conditions applied for the unit cell continuous MMC model are X, Y and Z planes; the other three planes are free moving. Several numerical parameters within the model needed to be controlled properly for the accurate numerical results. The FE model was validated with some numerical tests. Balokhonov (Balokhonov, 2020) demonstrated 3D numerical methods for boron carbide particle-reinforced composite at the micro level, using ABAQUS package. However, this research only focused on the cooling down process.

More recent works have been published (Wang, 2022; ZEMLIANOV, 2022; Metehri, 2021); however, an obvious limitation is that people have only studied the cooling process, not the entire residual stress variation during the manufacturing process.

2.19 Summary

Metal matrix composite with continuous fibre reinforcements provides excellent mechanical and thermal properties with aerospace, military, ground transportation and many more industries. The most commonly used reinforcements for aluminium alloy based composites are silicon carbide and carbon fibres. This composite has an excellent strength to weight ratio

thanks to the aluminium base. Aluminium composite also has unique thermal advantages compared with titanium in aerospace applications. Even though Al/SiC MMCs have many extraordinary properties, there is still a big challenge to improving mechanical performance and productivity. It is important to fully understand the thermal and mechanical behaviour of the composite as these effect the final mechanical and thermal performance of the component.

The strength properties of MMC's are known to be heavily influenced by the manufacturing process. When the aluminium alloy is cooled down during heat treatment, a temperature gradient within the component generates residual thermal stress. Various methods have been applied for residual stress measurement, both non-destructive and destructive. Nevertheless, it is still challenging to have an accurate and cost-effective measurement method. Classical laminate theory provides a method to analyse composite laminate behaviour. With known external loading, residual stresses and curvatures of the composite midplane can be quantified. Classical laminate theory predicted a saddle-like deformation for unsymmetric laminates. However, various studies proved that the saddle-like shape is unstable. It can bifurcate into two stable cylindrical structures with curvatures generated. This observable effect shows great potential for determining residual stresses.

Researchers developed various methods on modelling composites (2.17), it is very difficult to determine the residual stresses within MMCs. An indirect method is to predict the residual stresses through matching numerical and experimental curvature to overcome these issues. The main objective of this project is to determine the residual stress and optimise the performance of continuous silicon carbide fibre reinforced aluminium matrix composites. This new method for determining the residual stresses is developed through chapters 3 to 9. Chapter 5 will present the experimental measurement data of composite plate specimens. An analytical tensile test model will be developed in Chapter 6. In chapter 10, FE models of burst tube components will be developed to predict the residual stresses states. Different fibre layups, manufacturing processes and the effect of the fibre layups on the residual stress profiles will be investigated and discussed.

CHAPTER 3

**MATERIALS PROPERTIES AND
EXPERIMENTAL TECHNIQUES**

CHAPTER 3 MATERIALS PROPERTIES AND EXPERIMENTAL TECHNIQUES

This chapter investigates the thermal and mechanical behaviour of the silicon carbide reinforcement fibre and aluminium 6061 matrix material and presents the experimental methods used to validate the numerical models. The mechanical behaviour of composite material is highly dependent on the reinforced fibre and the aluminium matrix material properties, so the use of correct material properties is vital for accurate modelling of composite behaviour.

This chapter is divided into three sections. The first section presents the mechanical behaviour of the fibre and the matrix material. The second and the third section detail the test sample preparation and the experimental test procedures.

The material properties presented in this chapter have been used in the finite element analyses. With representative material models, a good predictive behaviour of composite response can be obtained. To this end; thermal loading and the creep effect were taken into considerations within the 3D component models developed. The results and verification of the finite element analysis are presented in Chapters 7, 8 and 9.

3.1 Material Properties

External thermal and mechanical loading were applied during the composite manufacturing process. To properly predict the composite thermal and mechanical characteristics, it is vital to fully understand the fibre and aluminium behaviours within a whole range of temperatures. In this section, aluminium 6061 alloy and silicon carbide fibre material properties with elevated temperatures are investigated and presented. Material properties such as Young's modulus, Poisson's ratio, thermal area reduction, coefficient of thermal expansion, stress-strain relations are studied. Yield stress, failure strain, ultimate tensile stress, and creep model are also investigated to predict composite behaviours.

3.1.1 Aluminium 6061 alloy

Aluminium 6061 alloy material data were obtained from the paper by P, Summers (Summers et al., 2015) unless otherwise stated. Each data point was extracted from the experimental curves using curve fitting software Web Plot Digitizer (Rohatgi, 2019). Curves were then fitted to the points to gain continuous data.

3.1.1.1 Design values of material constants

Based on the European Standard (Mazzolani, 2001), the material constants of aluminium 6061 alloys are presented in Table 3.1.

Table 3.1 Design values of material constants

Unit mass	$\rho = 2\,700\text{ kg/m}^3$
Modulus of elasticity	$E = 70\,000\text{ N/mm}^2$
Shear modulus	$G = 27\,000\text{ N/mm}^2$
Poisson's ratio	$\nu = 0.3$
Coefficient of linear thermal expansion	$\alpha = 23 \times 10^{-6}\text{ }^\circ\text{C}^{-1}$

3.1.1.2 Modulus of elasticity

Data of Young's modulus values at elevated temperature can be found in the paper by P, Summers (Summers et al., 2015). Figure 3.1 shows the Young's modulus of aluminium 6061-T6 with respect to temperature.

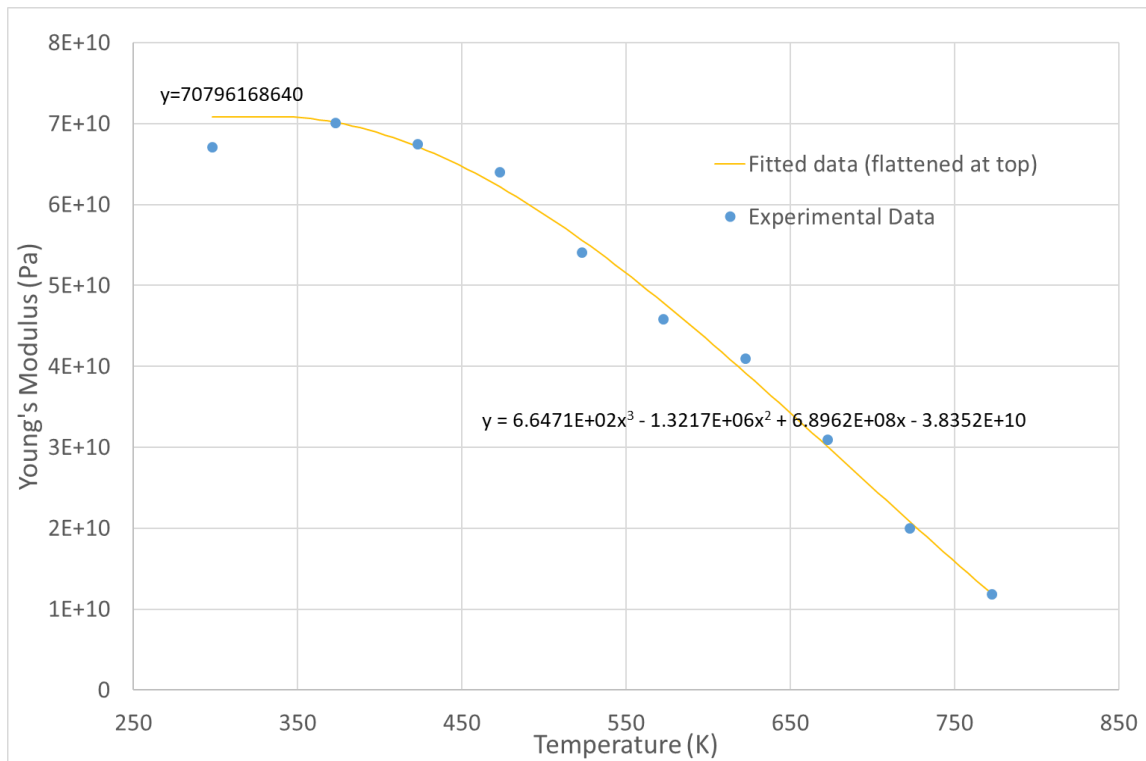


Figure 3.1 Young's modulus of aluminium 6061-T6 at elevated temperatures

3.1.1.3 Poisson's ratio

The values of Poisson's ratio with temperature for aluminium alloy 6061 could not be found in the literature. After considering the similarity, the data for aluminium 2024 was used instead (Yu, 2017). Figure 3.2 presents the relation between Poisson's ratio and temperature.

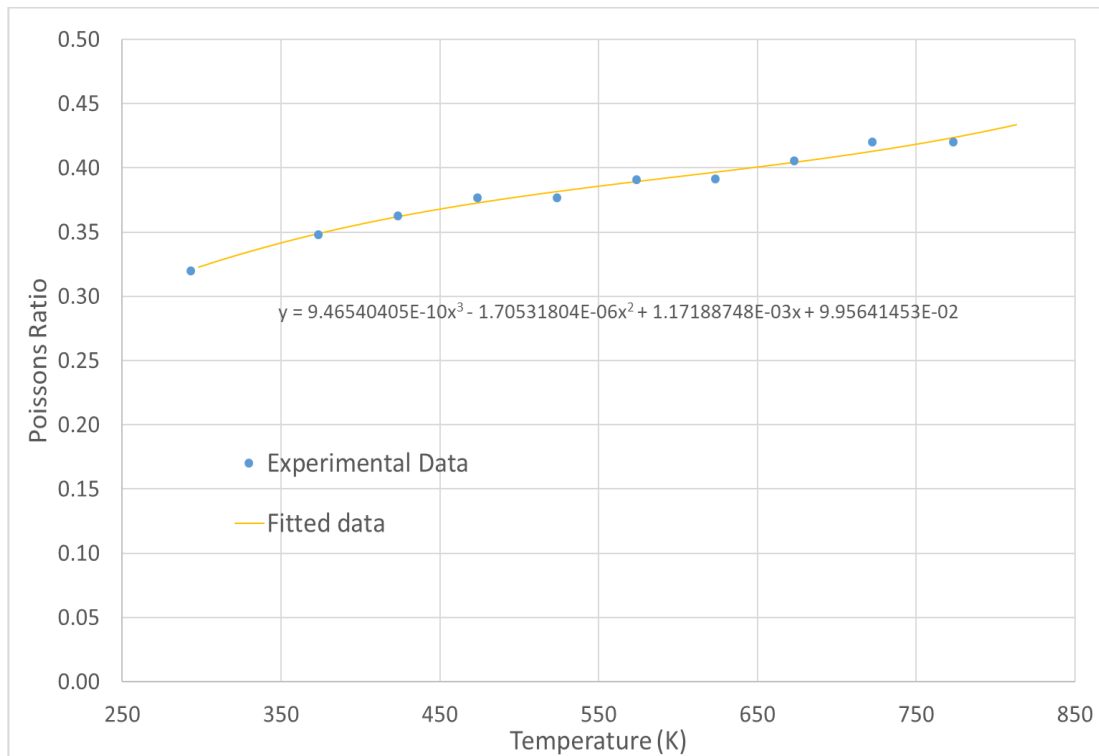


Figure 3.2 Poisson's ratio of aluminium 2024 at elevated temperatures

3.1.1.4 Area reduction

Area reduction is the reduction from a test sample's original pre-test cross-sectional area after tensile failure. It is usually presented as a percentage. Due to the area percentage reduction versus temperature curves' complexity, the experimental data set was split into four parts. Each part was then separately fitted, as shown in Figure 3.3.

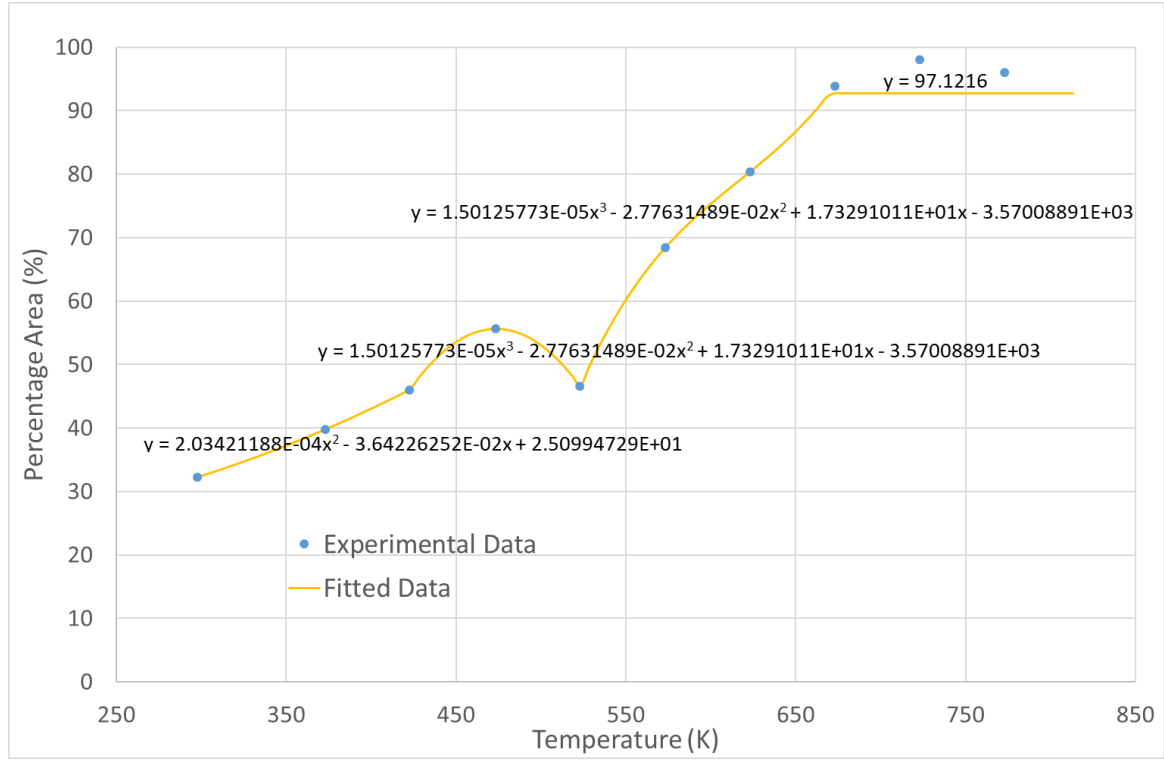


Figure 3.3 Percentage area reduction at different temperatures for Al 6061

3.1.1.5 Coefficient of linear thermal expansion

According to Eurocode 9 (Mazzolani, 2001), the relative thermal elongation (strain) of aluminium alloys $\Delta l/l$, can be determined using Equation 3.1 for θ between 0 °C and 500 °C.

$$\frac{\Delta l}{l} = 0.1 \times 10^{-7} \theta^2 + 22.5 \times 10^{-6} \theta - 4.5 \times 10^{-4} \quad \text{Equation 3.1}$$

Where

l is the length at 20 °C

Δl is the temperature-induced elongation

The mean coefficient of thermal expansion of aluminium alloy 6061-T6 at various temperatures is given in Table 3.2 (Karimi, Sedighi and Afshari, 2015).

Table 3.2 The coefficient of thermal expansion of aluminium alloy 6061-T6

Temperature (°C)	21	93	204	316	427	571
Aluminium 6061-T6($\times 10^{-6} \text{°C}^{-1}$)	22.4	24.41	26.6	27.6	29.3	34.2

The coefficient of thermal expansion (CTE) is calculated by dividing the change in expansion by the temperature change. The data presented in Figure 3.4 is the ratio of the CTE (CTE at different temperatures divided by the CTE at the initial temperature). The initial temperature is set to 25°C (298.15K).

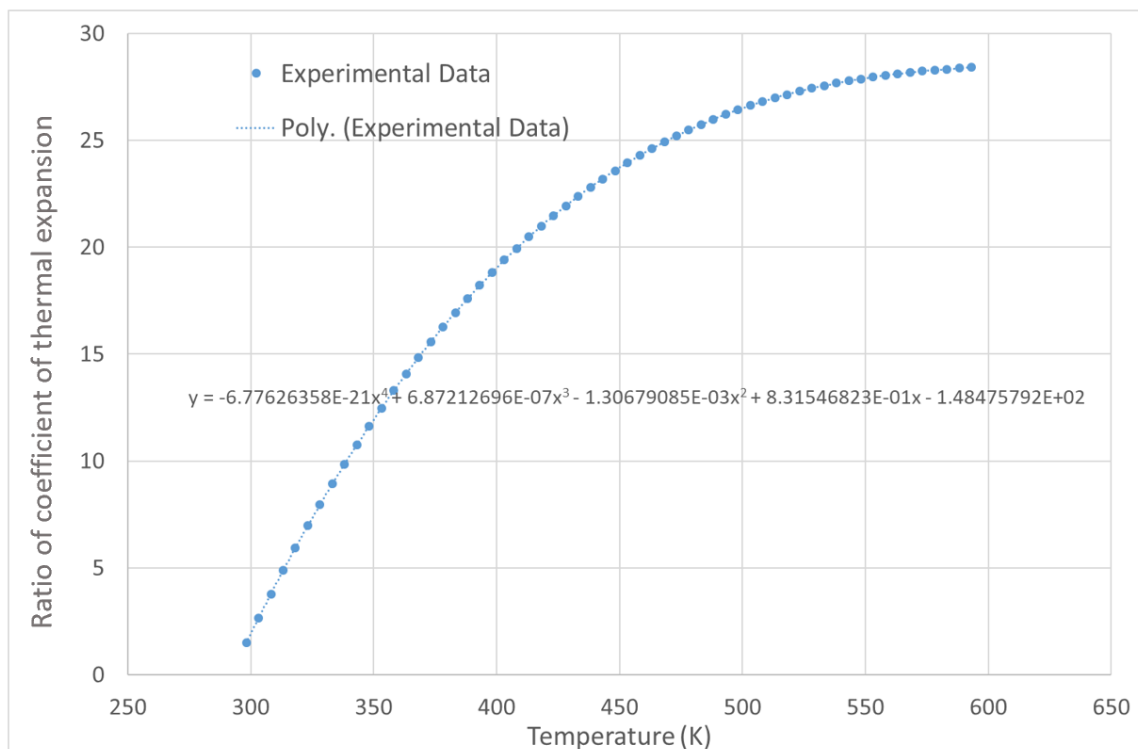


Figure 3.4 Ratio of coefficient of thermal expansion versus temperature for Al 6061

3.1.1.6 Ultimate tensile stress

Figure 3.5 shows the ultimate tensile stress curve of aluminium 6061 at different temperatures. The stress data points were converted into real stress before the curve fitting process. True stress is calculated by dividing the engineering stress by the true area percentage (calculated using the area reduction data from Figure 3.3). The ultimate tensile stress data were then divided into three sections and fitted separately.

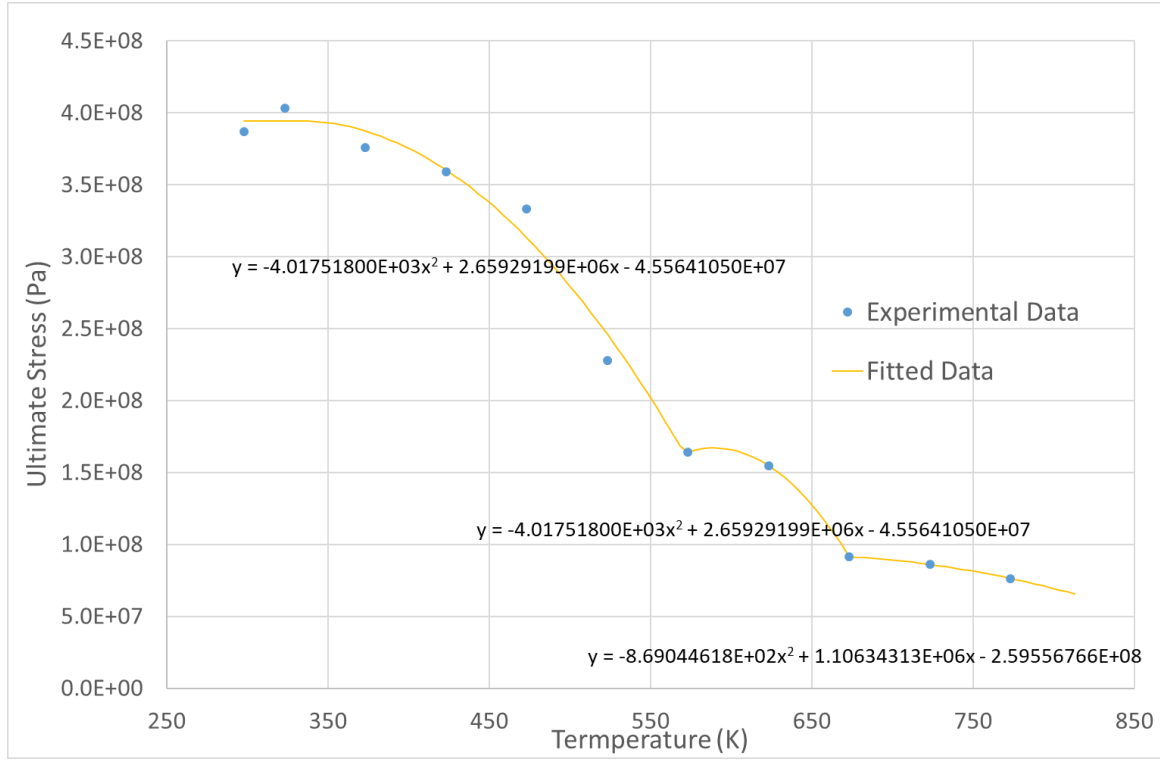


Figure 3.5 Ultimate stress versus temperature for Aluminium 6061

3.1.1.7 Failure strain

The effective plastic failure strain curve is presented in Figure 3.6. The curve was fitted in two sections. True strain, true stress and effective plastic strain were calculated using Equation 3.2, Equation 3.3 and Equation 3.4. Data of true stress and Young's modulus were obtained from Figure 3.1 and Figure 3.5.

$$\text{True strain} = \ln (1 + \text{engineering strain}) \quad \text{Equation 3.2}$$

$$\begin{aligned} \text{True stress} &= \text{engineering stress} * \exp(\text{true strain}) \\ &= \text{engineering stress} * (1 + \text{engineering strain}) \end{aligned} \quad \text{Equation 3.3}$$

$$\text{Effective plastic strain} = \text{engineering strain} - \frac{\text{true stress}}{\text{Young's modulus}} \quad \text{Equation 3.4}$$

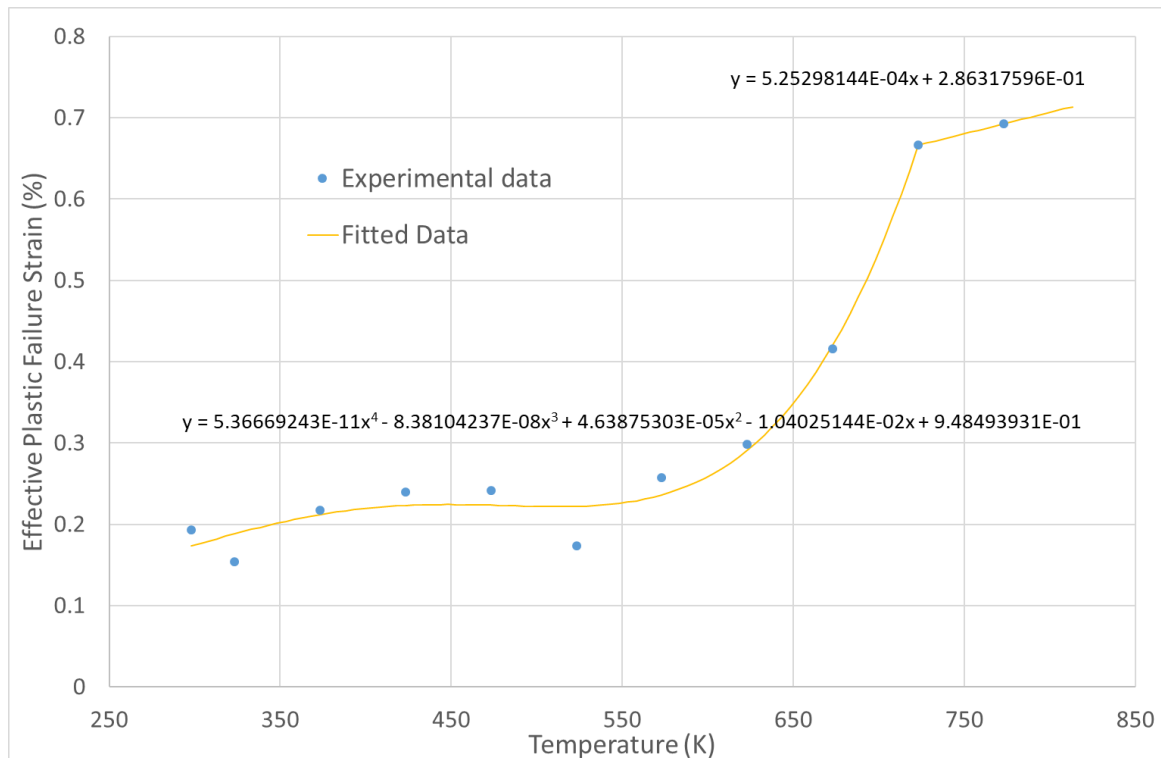


Figure 3.6 Effective plastic failure strain versus temperature for Aluminium 6061

3.1.1.8 Yield strength

Figure 3.7 showed the yield strength curve for aluminium 6061-T6 versus temperature. Two different approaches were used to obtain the 0.2% proof stress. The proof stress curve was fitted using data points after the engineering stress was converted into true stress. The second approach uses the 0.2% line on the individual stress-strain (Yu, 2017). In theory, we should obtain the same results using the two different methods. However, it was noticed that there was a slight difference in the numbers.

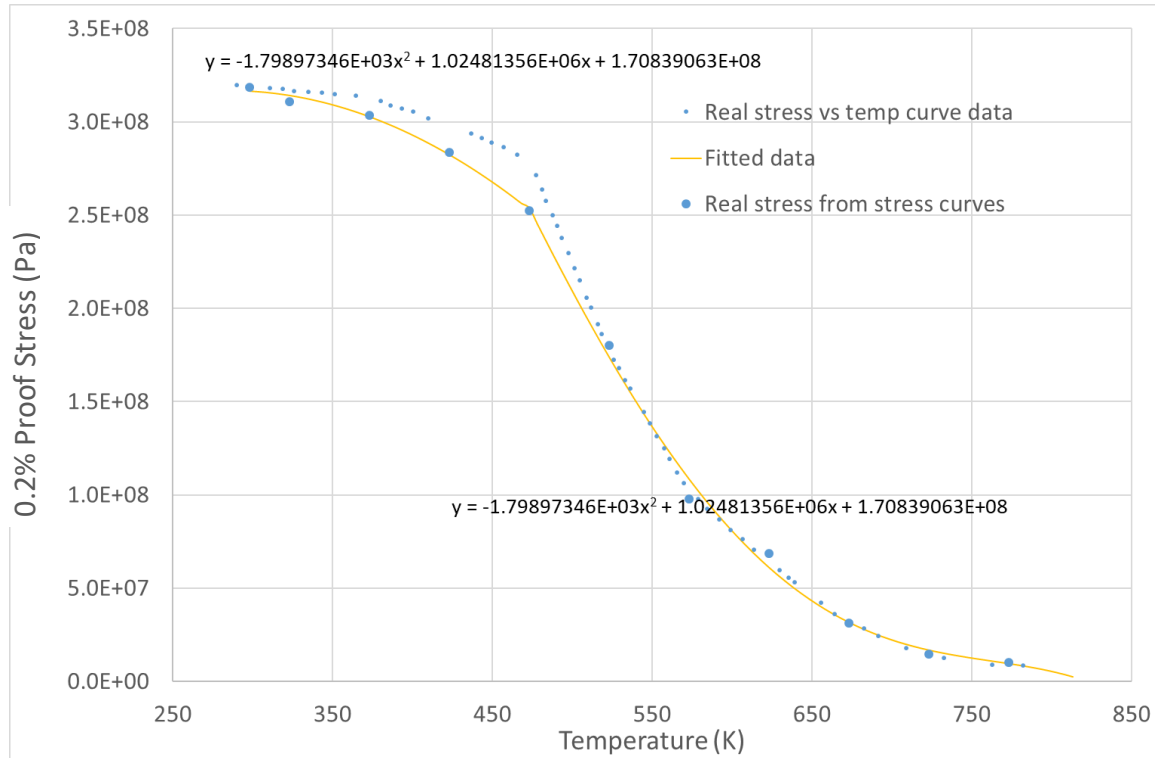


Figure 3.7 0.2% proof stress vs temperature for Aluminium 6061

3.1.1.9 stress-strain relations

Data points of true stress versus effective plastic strain at 50K temperature intervals were generated using the curve fits of the engineering stress-strain curves (Yu, 2017; Albak, 2021), Young's modulus curve (Figure 3.1), the ultimate tensile stress curve (Figure 3.5) and the effective plastic failure strain curve (Figure 3.6). The true stress versus effective plastic strain generated through this process is given in Figure 3.8, which shows the relation curves for both T6 and annealed conditions. The annealed condition of the matrix is reached shortly after reaching high temperature during the manufacturing process. The annealed data properties could only be found in the literature at room temperature (298.15K).

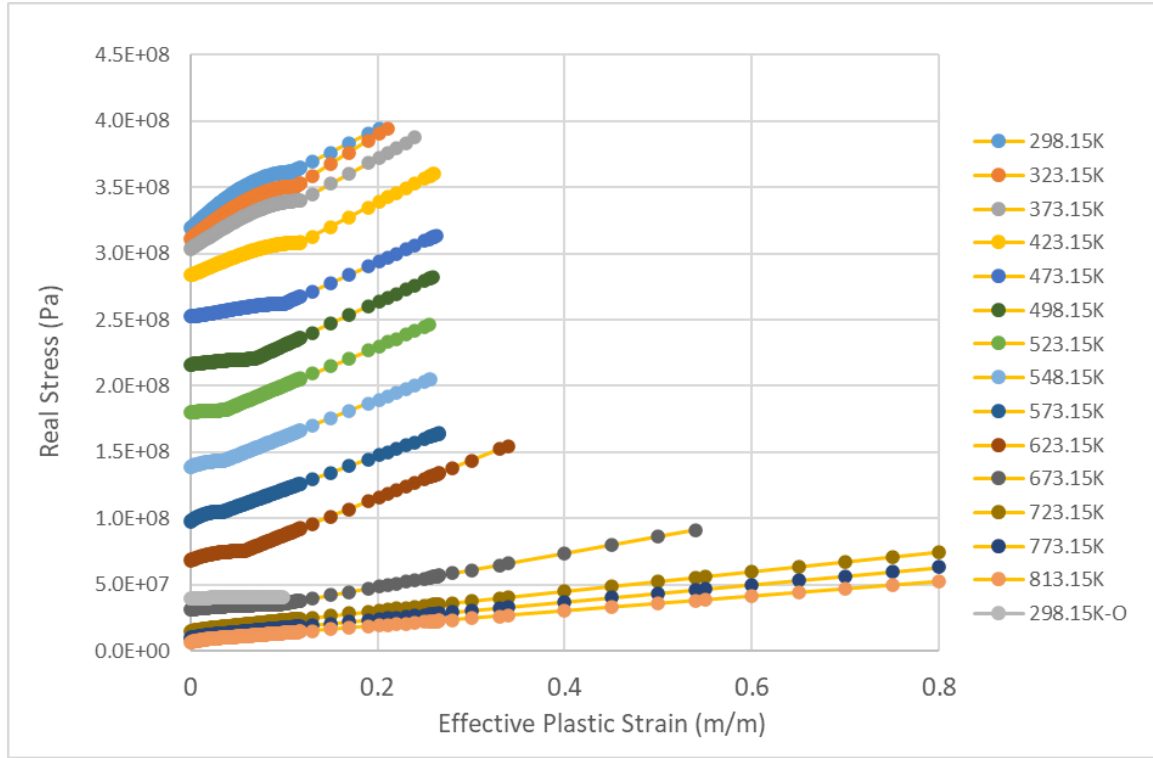


Figure 3.8 Real stress with effective plastic strain for Aluminium 6061

3.1.1.10 Strain hardening modulus

The strain hardening models for aluminium 6061 is given by Equation 3.5 (Summers *et al.*, 2015).

$$\frac{d\sigma}{d\varepsilon} = \theta - \beta (\sigma - \sigma_y) \quad \text{Equation 3.5}$$

where at the heating rate of 5 °C/min,

$$\theta \approx 3600 \text{ Mpa}, \beta \approx 34$$

and at the heating rate of 25 °C/min,

$$\theta \approx 3350 \text{ Mpa}, \beta \approx 31$$

3.1.1.11 Creep

Summers (Summers *et al.*, 2015) presented a creep model suitable for materials that exhibit insignificant primary creep regions. Their secondary creep rate model is given by

$$\dot{\epsilon}_{II} = 2.91 \times 10^{11} \times [\sinh (0.0404 \times \sigma)]^{1.74} \exp \left(-\frac{204}{8.314 \times 10^{-3} \times T} \right) \quad \text{Equation 3.6}$$

where σ and T are the current stress (MPa) and temperature (K), respectively.

The creep activation energy measured by Maljaars (Maljaars, Soetens and Katgerman, 2008) for aluminium 6061-T6 is 195KJ/mol.

3.1.2 Silicon carbide fibres

The mechanical properties of the SiC fibres used in this project are presented in Table 3.3.

Table 3.3 Material properties of TISICS SM3240 Silicon Carbide Monofilament (Rix, 2018)

Diameter	103±3 (µm)
Unit Mass	$\rho = 3\,570 \text{ kg/m}^3$
Modulus of elasticity	$E = 380\,000 \text{ N/mm}^2$
Ultimate Tensile Strength	3.8±0.2 GPa
Modulus of Rupture	7.0±1.0 GPa
Coefficient of linear thermal expansion	$\alpha = 4.5 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
Poisson's ratio (Huber et al., 2006)	$\nu = 0.18$
Yield Strength	3.6 GPa

3.1.2.1 Coefficient of linear thermal expansion

The fitted curve of the coefficient of thermal expansion, using data from Li (2013), is shown in Figure 3.9.

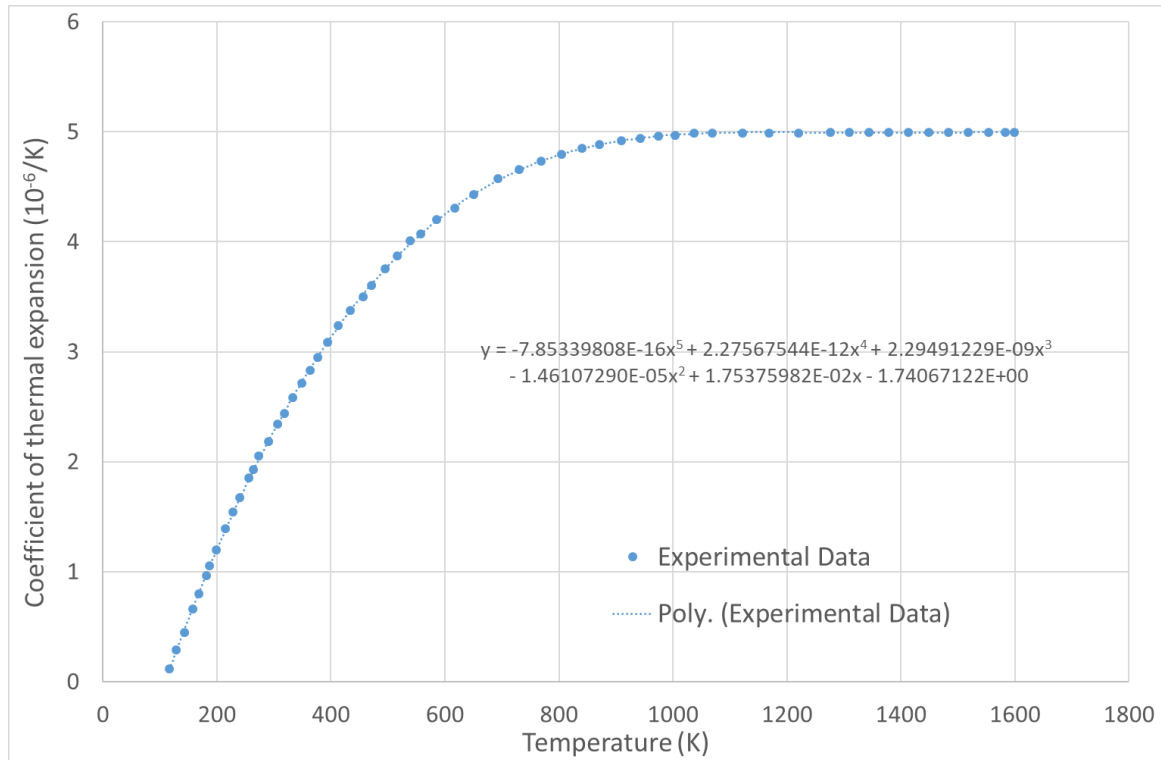


Figure 3.9 Coefficient of thermal expansion versus temperature for SiC

3.1.2.2 Young's modulus

The Young's modulus graph was obtained using the same curve fitting process, using data from (Leisen *et al.*, 2015) and is shown in Figure 3.10. The initial value was offset to match the data in (Li, 2013).

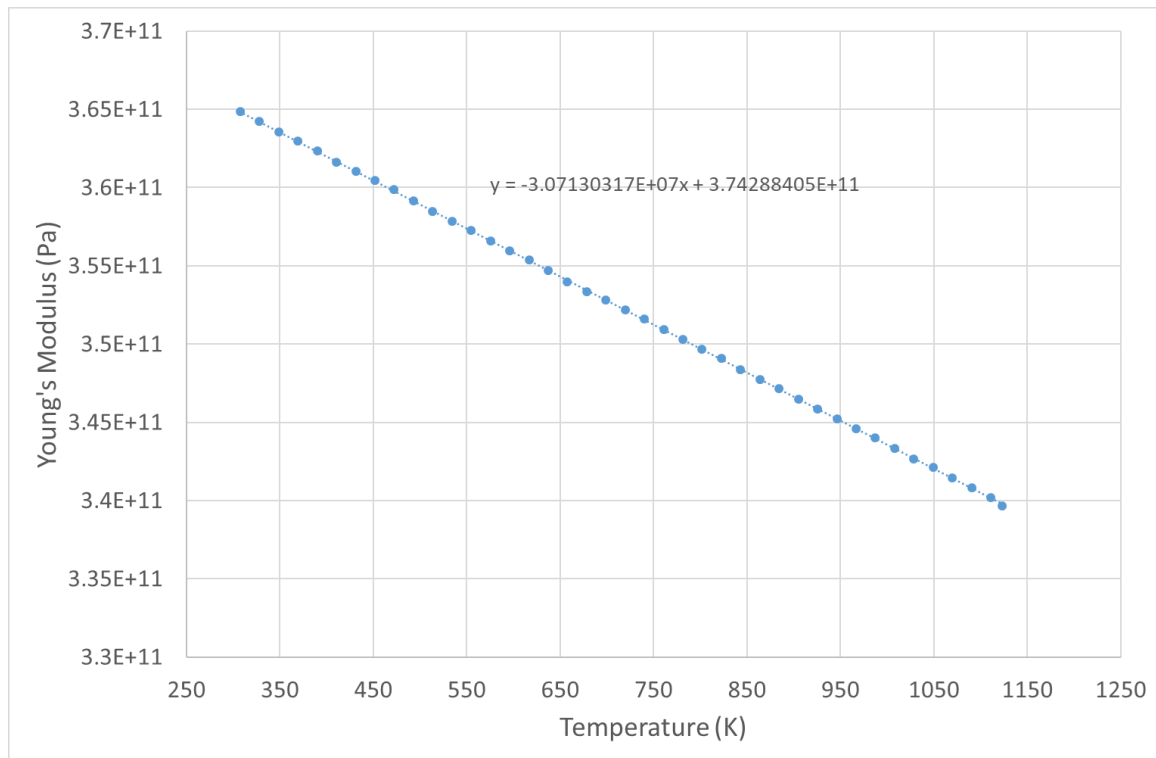


Figure 3.10 Young's modulus versus temperature for SiC

3.2 Test sample preparation

The test samples are made by laying up aluminium matrix foils and reinforcement fibres. The fibres have a diameter of 100 μm , and the thickness of the aluminium alloy foil is 127 μm . The specimen plates are then subject to a temperature and pressure heat treatment process (HIP process). Detailed information about the manufacturing heat treatment process cannot be given due to commercially confidentiality. Some specimens had a post manufacture heat treatment applied, which is discussed in detail in Chapter 4.

3.2.1 Sample preparation for curvature measurement of composite plates

SiC fibre reinforced aluminium 6061 MMC samples were cut from four plates of dimensions 200mm x 150mm. Table 3.4 presented the fibre layups. Two of the plates have a non-symmetrical layup, and two have a symmetrical layup. The symmetrical plates were used to create test specimens to test the effect of heat treatment. The non-symmetrical plates were designed and fabricated to determine the residual stresses through curvature measurement. $[90/0]_s$ and $[90/0]$ were chosen as they should produce large residual stresses due to the

perpendicular layup. [+55/-55] are commonly used fibre layups in the manufacture of the MMC burst tubes.

Table 3.4 Dimensions of SiC/Al MMC sample plates

Sample	MCO 360.1	MCO360.2	MCO360.3	MCO360.4
Fibre Layup	[90/0]	[90/0] _s	[90/0] _s	[+55/-55]
Total Layup	AL/AL/FI/AL/FI/AL/ AL	[AL/AL/FI/AL/FI/AL/ AL] _s	[AL/AL/FI/AL/FI/AL/ AL] _s	AL/AL/FI/AL/FI/AL/ AL
Total Thickness (mm)	0.73	1.46	1.46	0.73

3.2.2 Sample preparation for test specimens

Test specimens were cut from plates MCO360.2 and MCO360.3 to create four different aspect ratios with dimensions 15 x 15 mm, 15 x 30 mm, 15 x 45 mm and 15 x 75 mm, as shown in Figure 3.11. The plates MCO360.2 and MCO360.3 were designed to have a symmetrical layup (see Table 3.4) to remain straight during manufacture to aid the cutting process. The specimens for mechanical characterisation tests were cut from as-received plates followed the British standard ISO 6892-1, as shown in Figure 3.12. The test pieces had grip ends of 15 mm in width. The parallel length is connected to the ends with transition curves of a 20 mm radius. The length and width of the parallel side are 60 mm and 12.5 mm, respectively. The gauge length was 50 mm with a total length of 160 mm.

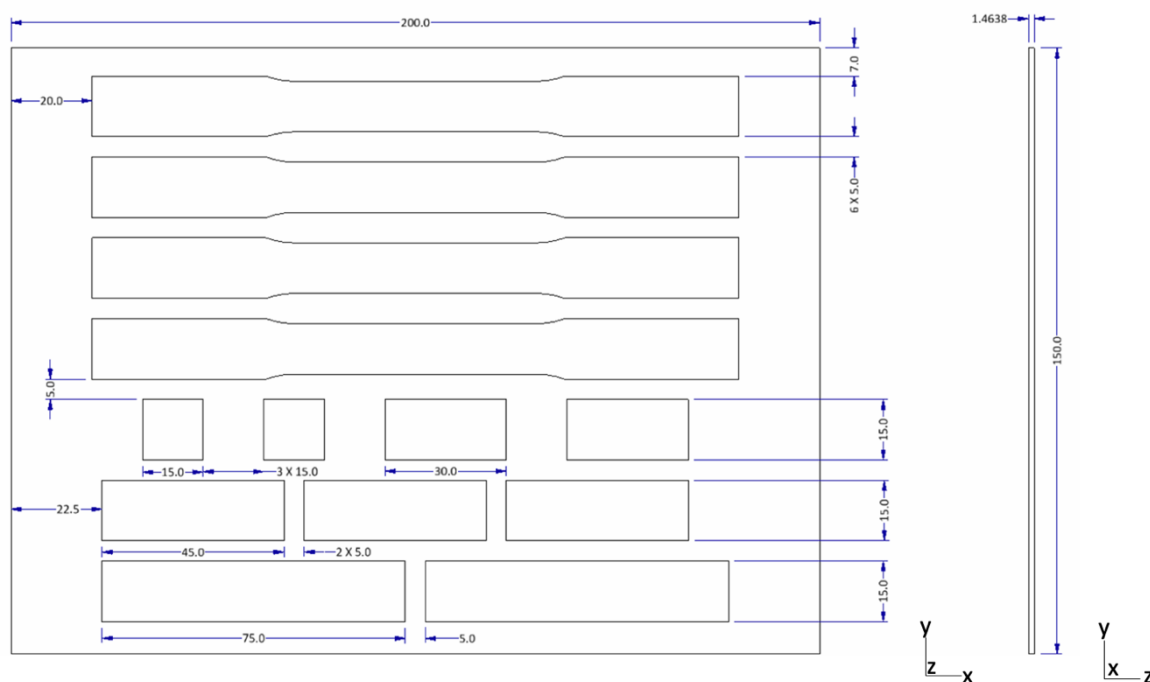


Figure 3.11 Cutting of the test specimens

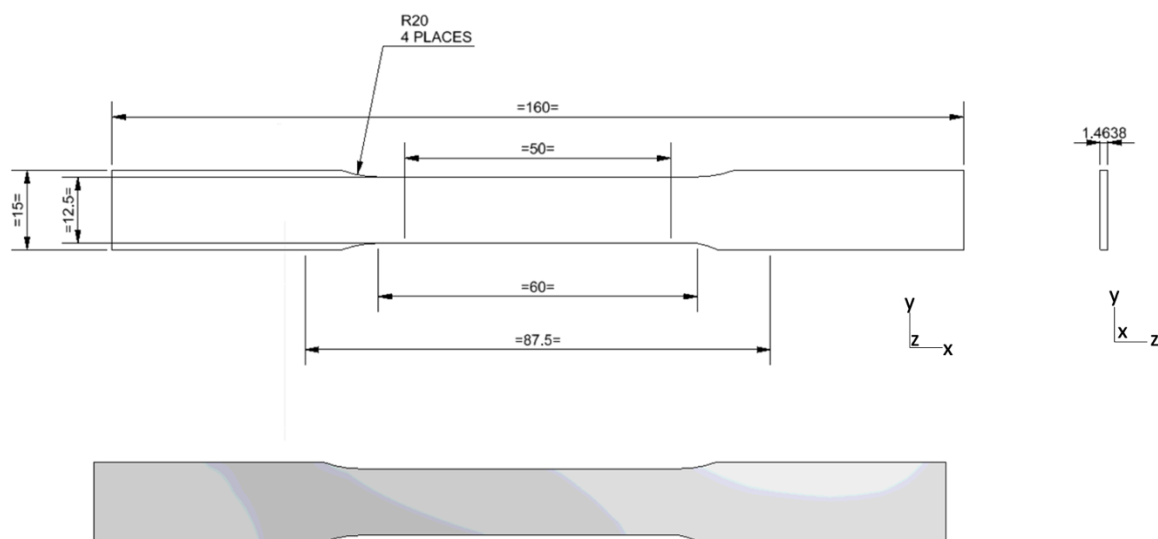


Figure 3.12 Machined test pieces of dog bone tensile test specimen

All the specimens machined from plates MCO360.2 and MCO360.3 are listed in Table 3.5. As can be seen from the table, there are at least four samples for each aspect ratio. Two of each ratio were given post-manufacture heat treatment (see chapter 4), and the others were used as references (no post-manufacture heat treatment). The samples in each ratio were selected from different plates (MCO360.2 and MCO360.3) to minimise manufacturing variations.

Table 3.5 Dimensions of test specimens (Dimensions in millimetres)

Aspect ratios	Plate MCO360.2	Plate MCO360.3
1:1	2-off (15 x 15)	2-off (15 x 15)
1:2	2-off (15 x 30)	2-off (15 x 30)
1:3	3-off (15 x 45)	3-off (15 x 45)
1:5	2-off (15 x 75)	2-off (15 x 75)
Tensile test specimen	4-off	4-off

3.3 Curvature analysis techniques

3.3.1 Determination of curvature

3.3.1.1 Determinate of surface coordinates

Surfaces of each specimen were measured using the DEA Swift manual coordinate measuring machine (CMM), with Inspect 3D software and Renishaw MH20i probe at Oxford Brookes University, as shown in Figure 3.13. Curvatures of the sample plates were then calculated using the surface displacement measurements. The details of the curvature analysis process are explained in section 3.3.2.

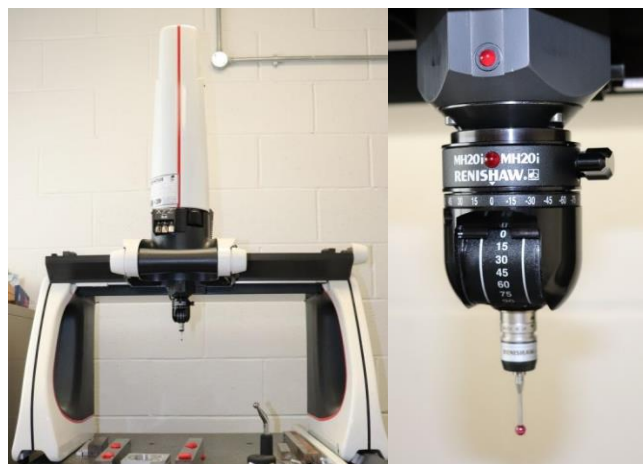


Figure 3.13 DEA Swift Manual CMM and Renishaw MH20i probe (pictures taken by the author)

Table 3.6 listed the Renishaw Mh20i probe specifications.

Table 3.6 MH20i Specifications

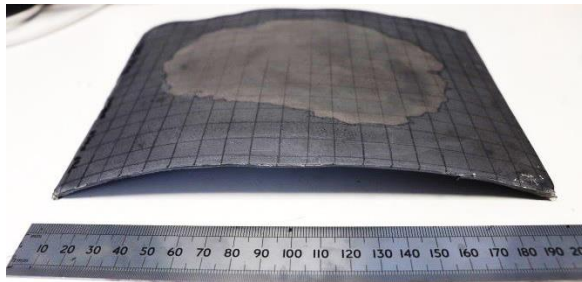
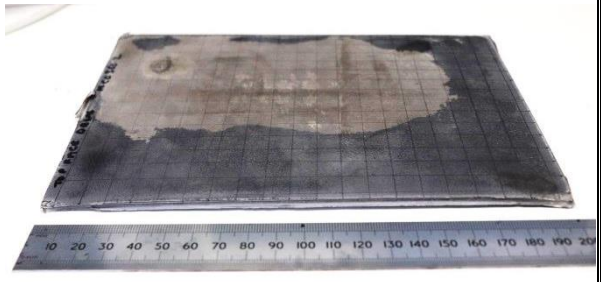
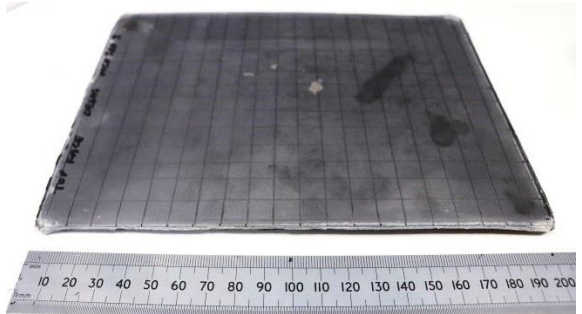
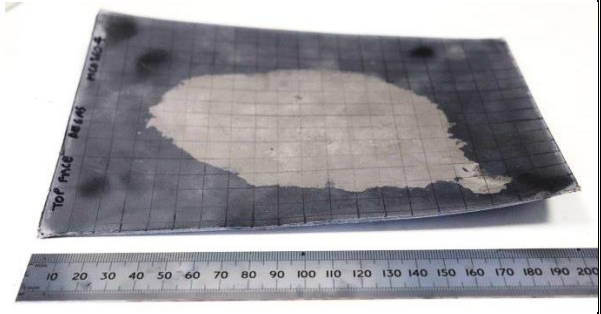
Probe criteria	Specification
Positional repeatability (2σ)	1.5 μm (0.00006 in) (At stylus tip with TP20 standard force module & 10 mm styli)
Range of articulation	An axis 0° to 90° in 15° increments; B axis $\pm 180^\circ$ in 15° increments
Maximum load	EM2 extended module 94.5 mm (3.72 in)
Operating temperature range	+10° to +40° C (+50 to +104°F)
Storage temperature ... -10° to +70° C range	-10° to +70° C (+14 to +158°F)
Probe mount	TP20 kinematic
Weight - without shank	210 g

Pictures of the 4 sample plates are shown in Table 3.7. X-Y-Z coordinates were collected for each point where the drawn grid pattern intersected. Each point's interval was 10mm x 10mm; 300 locations were collected in total per plate. The location measurement processes on each plate was repeated three times. In total, there were 900 data points collected for each plate. The MCO 360.2 and MCO 360.3 composite laminates were relatively flat because of the symmetrical fibre layup. As can be seen, the plates MCO 360.1 and MCO 360.4 exhibit significant curvature due to manufacturing, as was expected due to their non-symmetrical layups.

The MCO 360.1 laminate had the cross-ply architecture of [90/0] fibre layups. After the laminate was cooled to room temperature, residual stresses were generated in both fibre layers. Using classical laminate theory, a saddle-like deformation was predicted at room temperature. As discussed in Chapter 2.16, the unstable deformation will bifurcate into two stable cylindrical shapes. The observed result for the MCO 360.1 laminate agrees with the published papers (Hyer, 1981).

As seen in Table 3.7, a cylindrical structure can be observed. A more diagonal cylindrical deformation is observed for the laminate plate of MCO 360.4 due to the [+55/-55] fibre orientations.

Table 3.7 Pictures of composite plates with different layups (pictures taken by the author)

	
MCO 360.1	MCO 360.2
	
MCO 360.3	MCO 360.4

3.3.1.2 Shadow machine

A shadow machine was used to measure the curvature of specimens before and after the post-manufacture heat treatment (Figure 3.14). Displacement points of each specimen were measured by the machine and then the data was used for curvature calculation. The reference groups with various aspect ratios were sent to EDM precision technologies Ltd for EDM wire cutting. The samples were cut through the symmetric plane using the dimensions shown in Figure 3.15. The testing group samples were heat-treated first, then cut open using the same EDM wire cutting method. All the specimens had symmetric fibre layups. They were placed sideways during the EDM wire machining.



Figure 3.14 Shadow machine for curvature measurement (picture taken by the author)

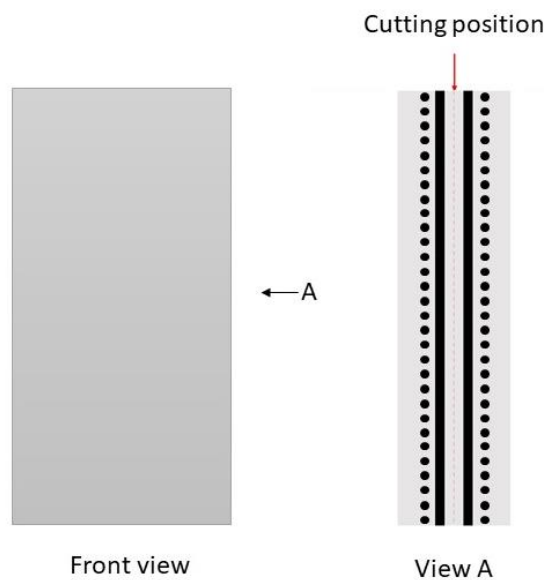


Figure 3.15 Side view diagram of cutting position on the specimen

3.3.2 3D Curvature analysis procedures

The coordinates measured by the CMM machine were imported into an Excel file and processed using Matlab software developed in this work, see Appendix F for the Matlab Code. The surface images were generated by fitting the X-axis, Y-axis, and Z-axis coordinates with curve fitting equations. Then the curvatures of each plate were calculated using Equations

2.88-2.100 in chapter 2. The maximum value of the curvature was obtained by curvature comparison on each surface.

The X-Y data points measured by the shadow machine were imported into Excel files, then analysed by Matlab. For each specimen, two sets of curvature data (left and right side) were obtained. These data points were curve fitted by Matlab using the embedded curve fitting function. Curvatures were then compared for the different aspect ratios.

For calibration purposes, the surface data collected were also imported into Catia for curvature analysis. The curvature results analysed were compared with the Matlab calculations.

3.3.3 Hardness test equipment

Vickers hardness tests were carried on the samples that were sectioned into the different aspect ratios (Figure 3.11) before and after heat treatment. The Vickers method is based on an optical measurement system. A range of light loads using a diamond indenter are applied to create indentations on the testing pieces which are then measured and converted into hardness values. Chapter 4 discusses the heat treatment process in detail.

3.4 Application of the chapter outputs

The material properties presented in section 3.1 are used in the finite element models presented in chapters 7 to 10. The post heat treatment, of some of the samples, is discussed in chapter 4. The results for the hardness tests are presented in chapter 5 and the results for experimental curvature and FE model curvature are given in chapters 5 and 8 respectively.

CHAPTER 4

POST MANUFACTURE HEAT TREATMENT

OF SIC/AL METAL MATRIX COMPOSITE

CHAPTER 4 POST MANUFACTURE HEAT TREATMENT OF SIC/AL METAL MATRIX COMPOSITE

4.1 Introduction

In the context of metals, heat treatment is the process of heating a metal component to a suitable temperature over a period of time, followed by cooling. Normally it is desirable to apply the heating and cooling as evenly as possible. For stress relief heat treatment, the heating temperature should be set to an appropriate value below the metal transformation stage to relieve any stresses remaining in the metal component due to the manufacturing processes. Heat treatment can also be applied to increase the yield strength and ultimate tensile strength of the metal.

This research investigated the heat treatment of silicon carbide fibre filament reinforced aluminium 6061 alloy composite manufactured by TISICS. The manufacturing process to produce aluminium reinforced silicon carbide fibre components requires a heat treatment process that focuses not primarily on obtaining good aluminium properties or reducing component residual stresses but on creating a well-bonded component.

Specimens were prepared, as detailed in chapter 3, to investigate of the effect of the manufacturing process. To provide additional information about the condition of the aluminium after the initial manufacture heat treatment and to see if the material strength properties could be improved a post manufacture heat treatment was applied to some of the specimens. This chapter gives details on the equipment and procedure used to carry out the post manufacture heat treatment.

4.2 Heat treatment procedures

There are three steps to increase the strength of aluminium alloys (ASM, 1991), they are solution heat treatment, quenching and ageing hardening. The first step is solution heat treatment, which is to soak the material at a relatively high temperature for enough time. It

is to achieve a nearly homogeneous solid solution (ASM, 1991). In this step, it is vital to avoid overheating. Overheating can cause degradation of tensile strength, ductility and many other material properties. Apart from preventing material melting, the lower limit of solution heating temperature should remain above the temperature at which solution happens. Solution heating, also called soak time, is defined based on the material types, thickness and shapes.

Quenching is the process of cooling down the metal rapidly from the heat-treatment temperature. Conventional quenching media includes water, oil, brine, salts. Quenching media selection should consider the material hardness, surface and centre hardness requirements, and the acceptable sample distortion (ASM, 1991). The purpose of quenching is to preserve the solid solution generated during the solution heat treatment. It is the most crucial step of the heat treatment process. The material must be transferred into the quenching media at a rapid speed without any interruption. The transfer of specimens from solution heat treatment must be less than the maximum quench delay time, as stated in Table 4.1.

Table 4.1 Maximum quench delay for aluminium products with difference thickness (ASM, 1991)

Thickness		Maximum quench delay (s)	
mm	in	Alclad	Non clad
0.41	0.016	6.4	4.4
0.51	0.020	8.0	5.5
0.64	0.025	10.0	6.8
0.81	0.032	12.8	8.8
1.02	0.040	20.0	11.0

Age hardening is the process to harden the material either at room temperature (natural ageing) or at an elevated temperature (artificial ageing). It is the procedure after the solution heat treating and quenching process. The selection between natural ageing or artificial ageing depends on the types of alloys and whether the materials will achieve a stable mechanical property at room temperature. Aluminium 2xxx series alloys will be able to reach a steady

state in a few days. However, 6xxx or 7xxx aluminium alloys are less stable at room temperature, and thus artificial ageing is required.

Precipitation heat treatment is a long-time heating process with the heating temperature range from 115 to 190°C. The selection of time and temperature should consider the material types, particle size, distributions.

4.3 Heating equipment

Generally, there are three types of heat treatment equipment for aluminium alloy: air chamber furnaces, induction heaters and molten salt baths (ASM, 1991). The choice of heating equipment depends on the types of aluminium alloy and the application of the material.

Air chamber furnaces and molten salt baths can be applied for the solution heat treatment of aluminium alloys. Molten salt baths can heat the materials faster than the air chamber furnace under the condition that the bath temperature is controlled and kept within the desired range. However, molten salt baths are hazardous. Extra safety measurements and special equipment should always be provided for the operators. Because of the residual salt on the heating parts, parts should be cleaned and free from a large amount of salt before the quenching process. Thus, constant fresh water should always be available for rinsing parts after quenching.

Air chamber furnaces are more comprehensive than salt baths due to the flexibility in temperature control. Before each heating process, the furnaces are heated beforehand to achieve a uniform temperature within the chamber. It is to prevent uneven heat treatment and prevent the heating influences from both cold and hot walls.

Induction heating has the advantage of a rapid heating rate in comparison with the other two methods. However, a proper temperature range for solution treatment of the heating material needs to be analysed before conducting induction heat treatment.

For this research, an air chamber furnace with an electronic controller from Nabertherm was used. The furnace has the function of allowing three heating temperatures, heating speed and the holding temperature, to be predefined by the operator.

4.4 Post Manufacture Heat treatment specimen preparation

The specimens for the heat treatment are listed in Table 4.2. Each batch is made of one sample only, and this is to ensure that the specimen is removed from the solution heating temperature at the fastest speed possible for quenching.

Table 4.2 Heat treatment schedules for test specimens

Batch number	Aspect ratios	Quantity	Solution heat treatment	Precipitation heat treatment (Age hardening)
1-2	1:1	2	530°C (30 mins)	175°C (30 hours)
3-4	1:2	2		
5-7	1:3	3		
8-9	1:5	2		

4.5 Post manufacture Heat treatment process

Aluminium alloys in T6 tempers typically have high strengths without decreasing other desired properties. To achieve the temper designation of T6 for aluminium 6061, the solution and precipitation heat treatment was applied (Figure 4.1). For the first phase (solution heat treatment), the SiC fibre reinforced aluminium 6061 composite samples were heat-treated to 530°C for 30 mins.

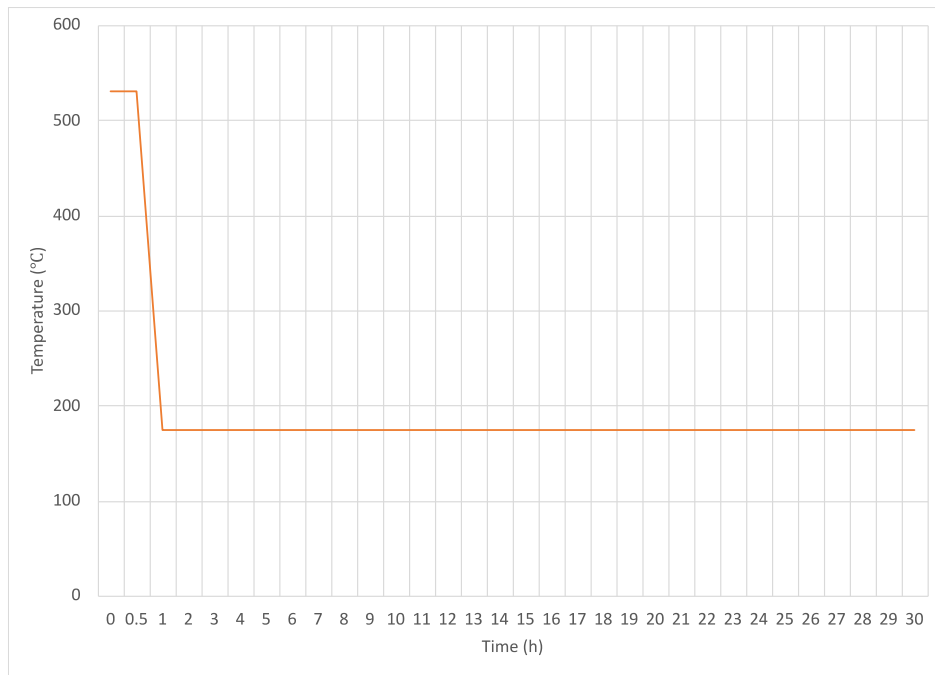


Figure 4.1 Solution and precipitation heat treatment for SiC/Al 6061 composites

During the solution heating process, dimension changes of parts can happen because of creep within the heated materials. A simple rack was prepared and used for all the samples during the heat treatment process to prevent any distortion. A 'U' shaped steel wire was used to attach the specimens to the rack, keeping them horizontal (in terms of longest length) within the heating chamber (see Figure 4.2).



Figure 4.2 Heat treatment of experimental specimen

Water was used during the second phase to quench the specimens after the heat-treatment process. The heat-treated material was removed from the furnace at a fast speed, followed by water quenching from the solution heat treatment temperature. The average quench delay is less than 4 seconds. The most challenging part of quenching is the dimension change due to the uneven cooling. The contact time difference between quenching parts and quenching media can cause warpage or distortion.

The water quench is maintained at room temperature (around 20°-30°C) and agitated. To minimise the contaminates and to control the temperature, freshwater was added periodically by the author. All the samples were quenched by complete immersion in water. For the third phase (participant heat treatment), the samples are moved back into the furnace, after a couple of seconds in the water, with the temperature set to 175°C for 30 hours. Whilst quenching the samples, the furnace doors were open so that the temperature in the furnace drops to 175°C, ready for the specimens to go back into the furnace. This step is to achieve the T6 temper designation for the material. The three phases were repeated for each sample.

4.6 Determination of the residual stresses state of the components

When the composite specimens are solution heat treated and quenched immediately, the aluminium alloy is ductile similar to the annealed condition. The aluminium alloy within the specimens is assumed to be stress-free before the age hardening step. However, because of the difference in coefficient of thermal expansion between the fibre reinforcement and the matrix, residual stresses will be generated between the two constituents.

The residual stresses generated due to the quenching process can be relieved by precipitation heat treatment. The residual stress reduction depends on the temperature, ageing time, and the alloy's thermal properties. The creep of the aluminium alloy is the key to relieving residual stresses. The creep rate is related to the temperature, stress levels, time and other properties, as discussed in the previous chapters. It is impossible to estimate the level of residual stress relieved after heat treatment. Thus, the curvature measurements obtained from cut symmetric plate samples are required to determine the residual stress levels (Chapter 5.2.3).

4.7 Application of the chapter outputs

The presented post manufacture heat treated samples, that were cut down the mid plane (section 3.3.1), have their curvature measured and are assessed for their residual stresses in section 5.2.4. The test results for the presented dog bone specimens are given in section 5.2.4. These results combined with hardness tests carried on in section 5.2.5 are used to inform the material models used in the analytical and FE models presented in chapters 6, 7 and 10.

CHAPTER 5

EXPERIMENTAL DATA

CHAPTER 5 EXPERIMENTAL MEASUREMENT DATA

5.1 Introduction

Composite silicon carbide monofilament fibre reinforced aluminium 6061 matrix composite plates with various fibre layups were specially manufactured to validate FE models. Symmetric and asymmetric groups of composite plates were made, as discussed in Chapter 3. The three layups considered were an $[90/0]_s$ symmetric layup, an $[90/0]$ asymmetric layup and an $[+55/-55]$ asymmetric layup.

In this chapter the curvature analysis, using Catia and Matlab, of the SiC/Al composite plate specimens with $[90/0]$ and $[+55/-55]$ layups and the palm tree specimens with $[0/90]_s$ layups with our different aspect ratios are presented. The tensile tests and hardness tests conducted on the composite plate samples are also presented, which help indicate the hardness state of the aluminium.

5.2 Curvature analysis of SiC/Al composite specimens

The symmetric laminate plates have a balanced structure and, therefore, theoretically should have no curvature after cooling down, from the manufacturing process temperature down to room temperature. The symmetry of the plate keeps a balance between the residual stresses generated about the composite mid-plane. Due to manufacturing and furnace temperature variations, there will be slight differences in cooling speed within the composite, which will give rise to some undulation on the surface of the composite. Regardless, residual thermal stresses will also be within the laminates due to the thermal mismatch between fibres and matrix (even if the surface is completely flat). The composite plates with the fibre directions of $[90/0]$ and $[+55/-55]$ have noticeable out-of-plane deflections. It is because of the asymmetric fibre layups and the differences in the coefficient of thermal expansion between the fibre and matrix components. The 3D surface positions of the $[90/0]$ and $[+55/-55]$ composite plates were measured using a coordinate measuring machine (CMM), as discussed in Chapter 3. The data was then imported into Catia for curvature analysis.

5.2.1 [90/0] and [+55/-55] composite laminates

A comparison of the test plate curvature and the simulation curvature, using Catia, for the [90/0] composite plate (MCO 360.1), is displayed in Figure 5.1. Using the curvature analysis tool embedded in Catia, we can obtain the maximum curvature of the plate. The maximum curvature of the [90/0] composite plate after the cooling down process was 48.5m^{-1} . The simulated curvature seen from different views is presented in Figure 5.2.

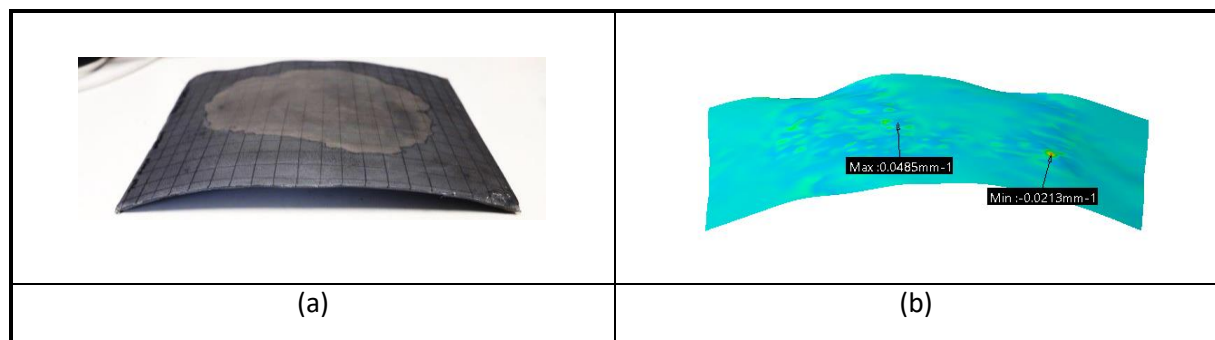


Figure 5.1 Curvature measurement of [90/0] composite laminate

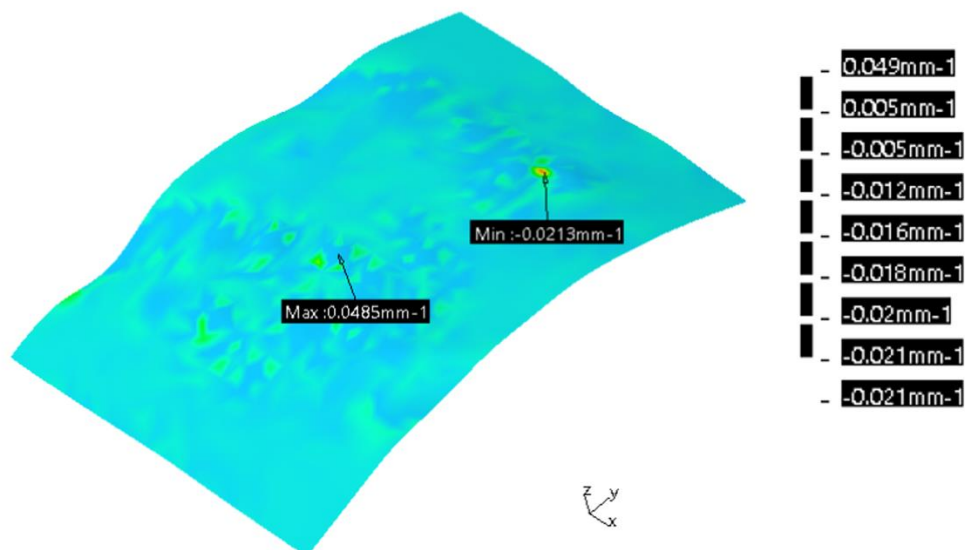


Figure 5.2 Curvature analysis of [90/0] composite laminate

A comparison of the test plate curvature and the simulation curvature, using Catia, for the [+55/-55] composite plate (MCO 360.4), is displayed in Figure 5.3. The plate has a maximum

curvature of 49.2m^{-1} . The simulated curvature seen from different views is presented in Figure 5.4.

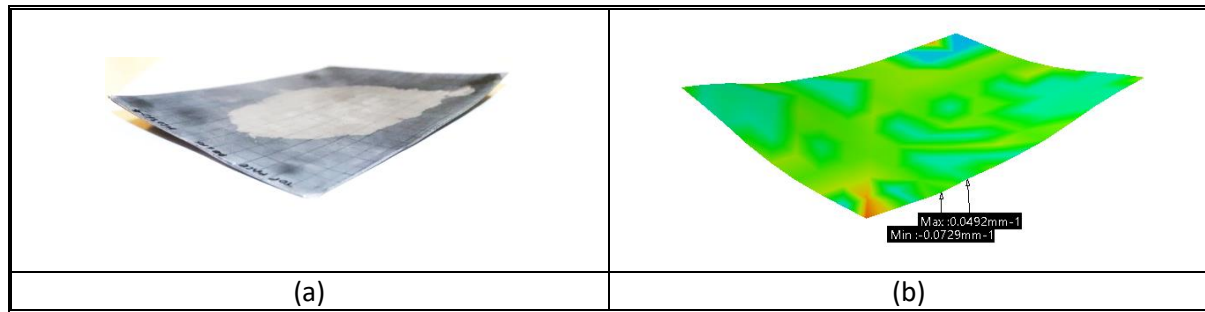


Figure 5.3 Curvature measurement of [+55/-55] composite laminate

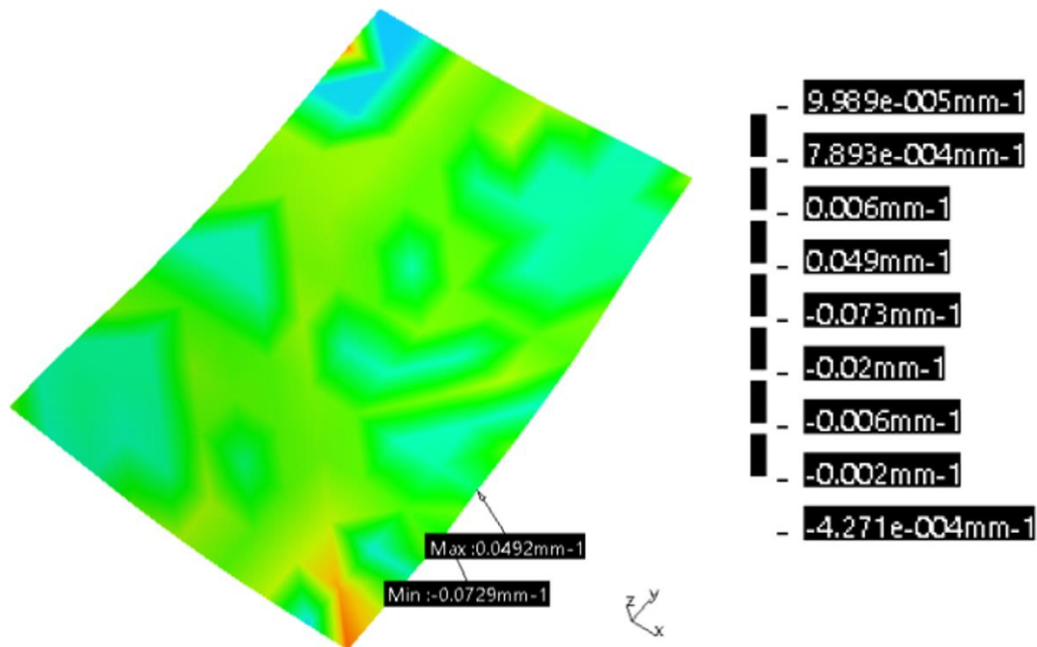


Figure 5.4 Curvature analysis of [+55/-55] composite laminate

5.2.2 SiC/Al composite with an aspect ratio

As discussed in Chapter 3, the composite plates MCO360.2 and MCO360.3 were cut into specimens with four different aspect ratios, as presented in Table 5.1 and then cut partially down the midplane to produce curvature in the two separated halves of the composite (fronds, like on a palm tree). Different aspect ratios were examined to see if the boundary conditions regarding length to width and distance from the freed edge made a difference in

the curvatures generated. The samples had 15 x 15 mm, 15 x 30 mm, 15 x 45 mm and 15 x 75 mm dimensions. The first 3 aspect ratios were selected to have an evenly distributed sample sizes. The large ratio of 1:5 was chosen to see the effect on more extreme cases. Due to the limitation of the funding and the high machining cost, 2 specimens were made from each aspect ratio group. It was important to see whether there would be differences experimentally from the ideal conditions within the FE models. For each aspect ratio, both the left and right side curvatures of each specimen were measured from the midline position. In total, four groups of curvatures were measured by shadow machine and analysed with Matlab.

Table 5.1 Dimension of specimens with aspect ratios

Aspect ratio	Dimensions (mm)	Specimen quantity	Curvature quantity
1:1	15 x 15	2	4
1:2	15 x 30	2	4
1:3	15 x 45	2	4
1:5	15 x 75	2	4

Figure 5.5 shows the aspect ratios of the various specimens after initial cutting with the EDM. The cutting diagram for each composite plate was presented and discussed in Chapter 3. All the specimen pieces have a width of 15mm and lengths ranging from 15mm to 75mm.

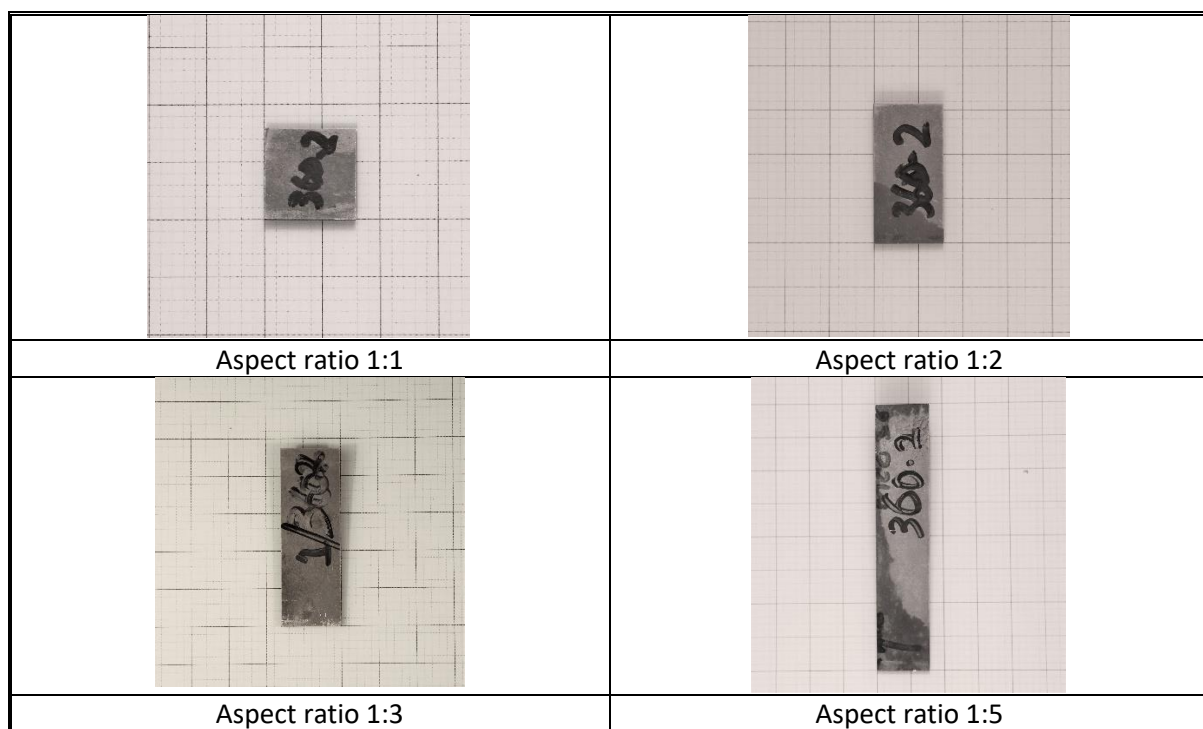


Figure 5.5 Experiment specimens with an aspect ratio

The side view of the composite specimens before secondary EDM cutting is shown in Figure 5.6. Figure 5.6(b) shows that the fibre layups, where the individual fibres can be observed from the zoomed-in view. The composite is made up of 4 fibre layups within an aluminium matrix. After manufacture, there is some undulation of the surface for the symmetric composites, but essentially there is no out of the plane movement. To release the residual stresses to assess their magnitude, each specimen was partially cut down the midplane using an EDM. The cutting requires exceptionally high precision and due to the long cut, pinching of the wire and hardness and non-conductivity of the fibres, a low cutting speed needed to be set. The process was also labour intensive as many trials were needed to be done, with observation, to find a fast but reliable speed at which to do the cut. Thus, it was costly to machine the specimens.

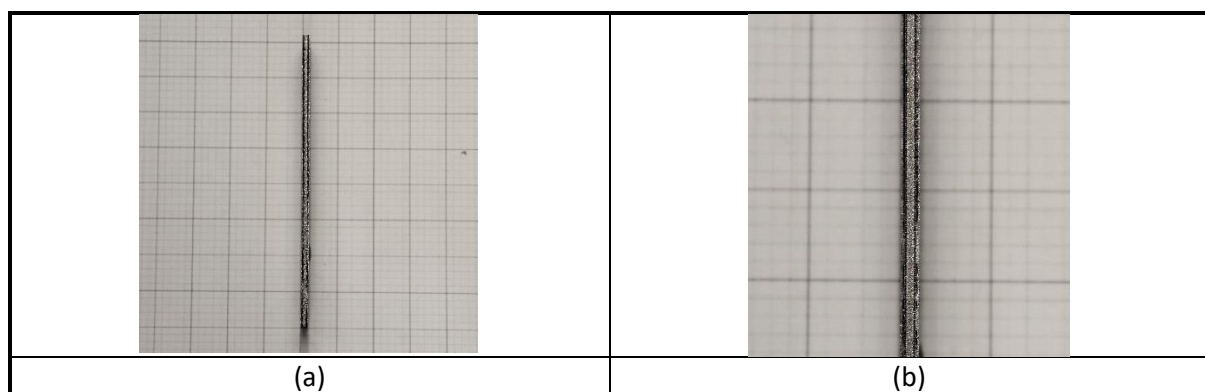


Figure 5.6 Composite specimens before EDM cutting

A zoomed-in view of a cut specimen is shown in Figure 5.7. Individual SiC fibres can be seen at 0 degrees (black edge) and 90 degrees (black circles). As the EDM cuts along the central symmetric plane (mid-plane), out-of-plane deflections occur on the component's released left and right sides. Theoretically, the curvatures (displacements) on each side should be the same, but there will be differences due to manufacturing variations and the cut down of the mid-plane being unlikely to be exactly in the middle. Therefore, both sides of the specimen are measured to obtain curvatures for each side with the average curvature used for analysis. The curvatures should be representative of the residual stresses.

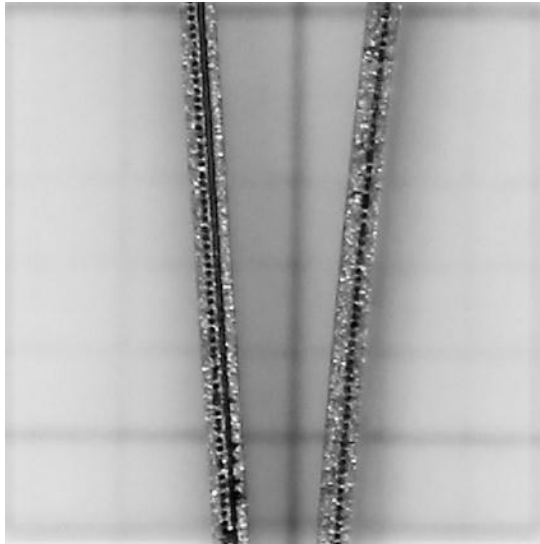
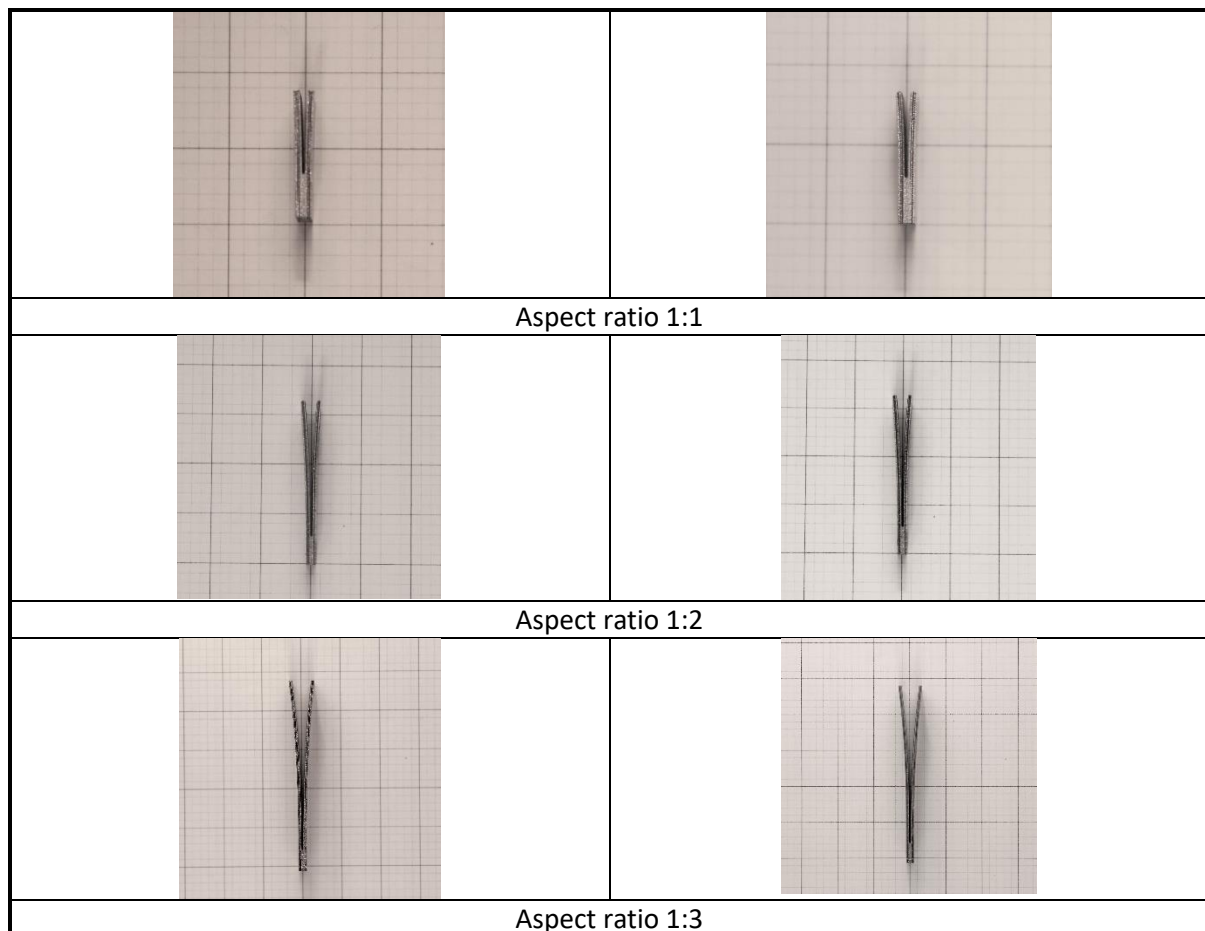


Figure 5.7 Composite specimens machined vertically at the centre position

The EDM cutting at the symmetric plane was repeated for all the experimental specimens. Figures of the palm curvatures of the different aspect ratio samples are illustrated in Figure 5.8. Deflection positions along the length of the outside edge cut relative to the midplane for both the left and right sides were collected and analysed.



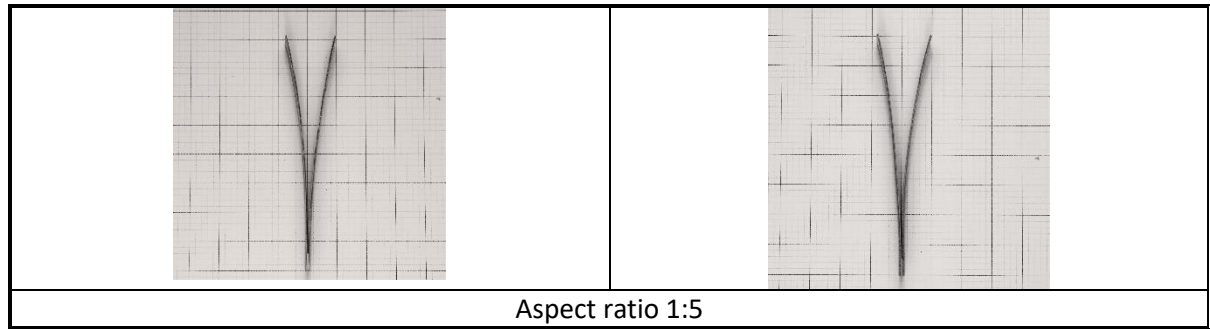


Figure 5.8 Palm curvature of machined specimens with an aspect ratio

The position data were imported into Matlab and fitted to determine the curvature. The fitted curves for all the specimens are presented in Figure 5.9. Fitted curves for both left and right sides are shown. The fitted curves were analysed, and the maximum curvature for each fitting is calculated within Matlab. More results of the calculation process are presented in APPENDIX B.

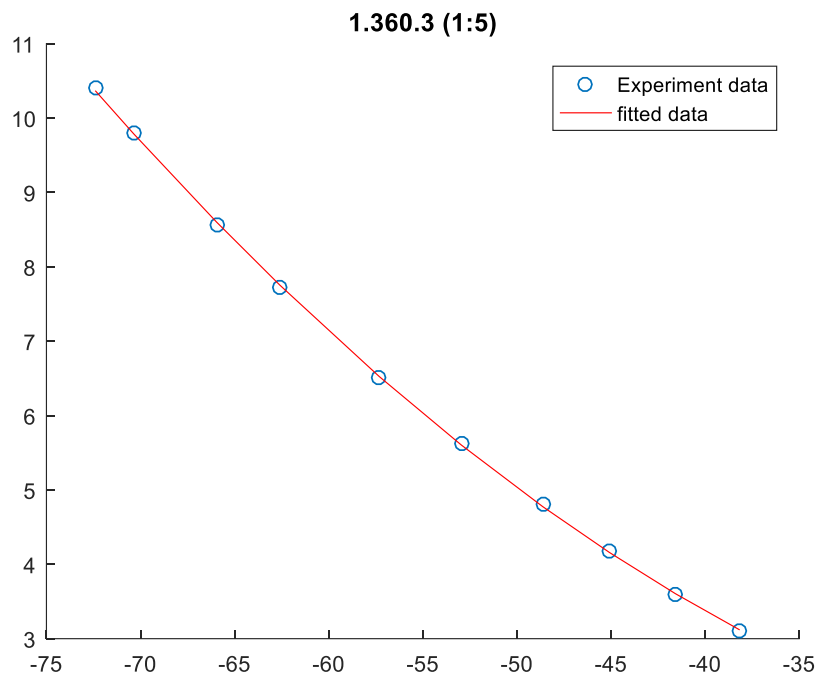


Figure 5.9 Matlab curvature analysis of composite with 1:5 aspect ratios

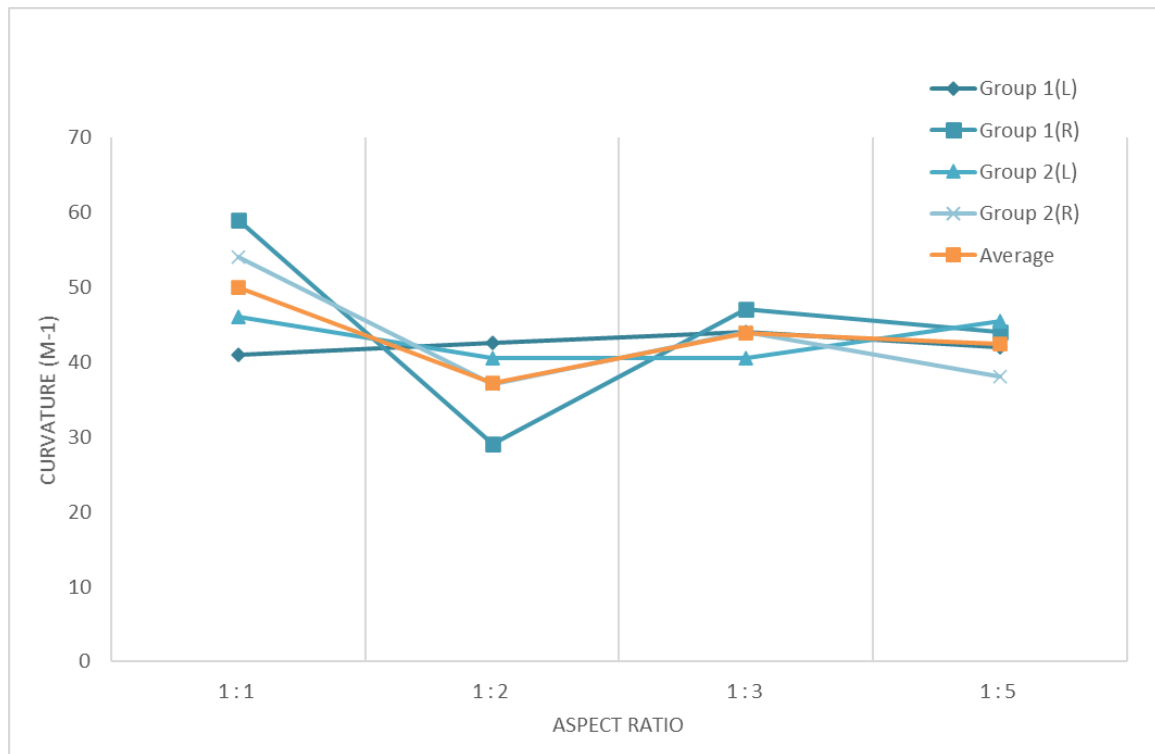


Figure 5.10 Curvature analysis results of specimens with aspect ratios

Figure 5.10 presented the results of curvature analysis for each sample. For each range of aspect ratio, the curvatures from each specimen are slightly different from each other. It may be down to machine tolerances and manufacturing variations. An average curvature value was obtained for each ratio which is shown by the orange curve. We can see the average results have a slight downward trend with an increase in ratio value. Overall though, the ratio differences do not significantly influence the value of the specimen curvatures.

5.2.3 Composite specimens with post-manufacturing heat treatment

As discussed in Chapter 3, the composite plates MCO360.2 and MCO360.3 were cut into specimens with four different aspect ratios and then cut partially down the midplane to produce curvature in the two separated halves of the composite.

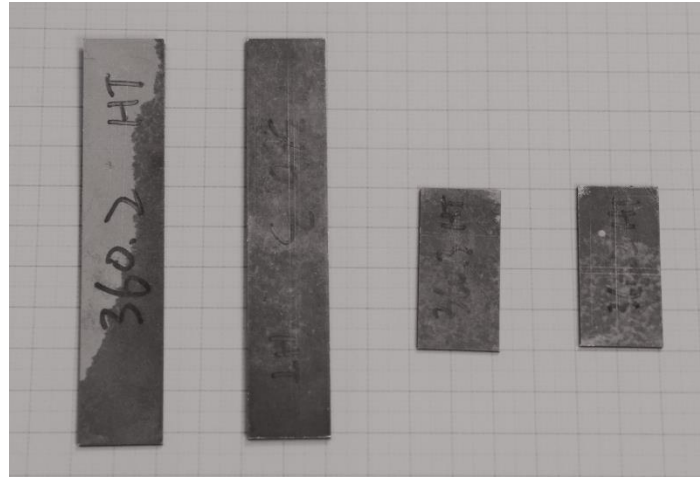


Figure 5.11 Specimens with post-manufacturing heat treatment with 1:5 and 1:2 aspect ratios (left to right)

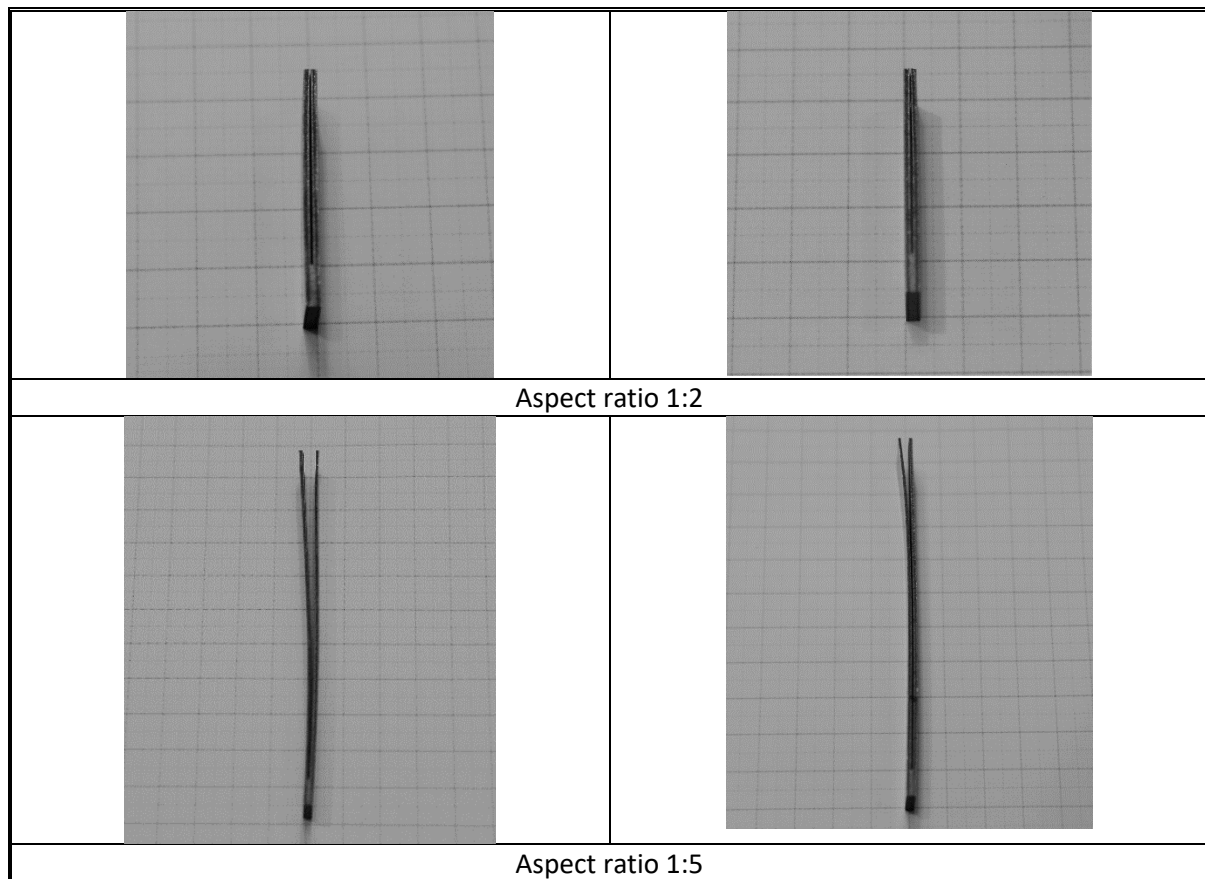


Figure 5.12 Machined specimens with an aspect ratio

Figure 5.11 and Figure 5.12 presents the specimens with post-manufacturing heat treatment. The samples with an aspect ratio of 1:2 demonstrated no curvature after machining, and for aspect ratio 1:5 specimens, the gaps were minimal, with fronds moving in the same direction. The reason for the small gap may be due to manufacturing variations and how well the cut

goes down the middle of the specimen. From the author's observation, the cut was not on the midplane, particularly as the cut got longer. The specimens not opening up has possibly made the EDM cutting more difficult to control. The results indicate that the post-manufacturing heat treatment seems to significantly (or maybe completely) remove the residual thermal stress, thus having no curvature after EDM.

5.2.4 Tensile test analysis of SiC/Al composite

In this section, the tensile test results of the composite specimens are presented. The experimental test specimens were machined from the plates MCO360.2 and MCO360.3, as listed in Table 5.2. The dimensions of the test pieces are detailed in Chapter 3. The specimens were made from the two symmetric layup composite plates (two from each plate) to examine any manufacturing variations.

Table 5.2 Test pieces for experimental tensile analysis

Groups	Sample	Fibre Layup	Heat-treated	Test number
Group 1	MCO360.2	[90/0] _s	No	Test 1
			Yes	Test 2
Group 2	MCO360.3		No	Test 3
			Yes	Test 4

Test 1 and 2 used the specimens made from plate MCO360.2, and test 3 and 4 used the specimens made from plate MCO360.3. The test 2 and 4 specimens were heat-treated, whilst the test 1 and 3 specimens were not.

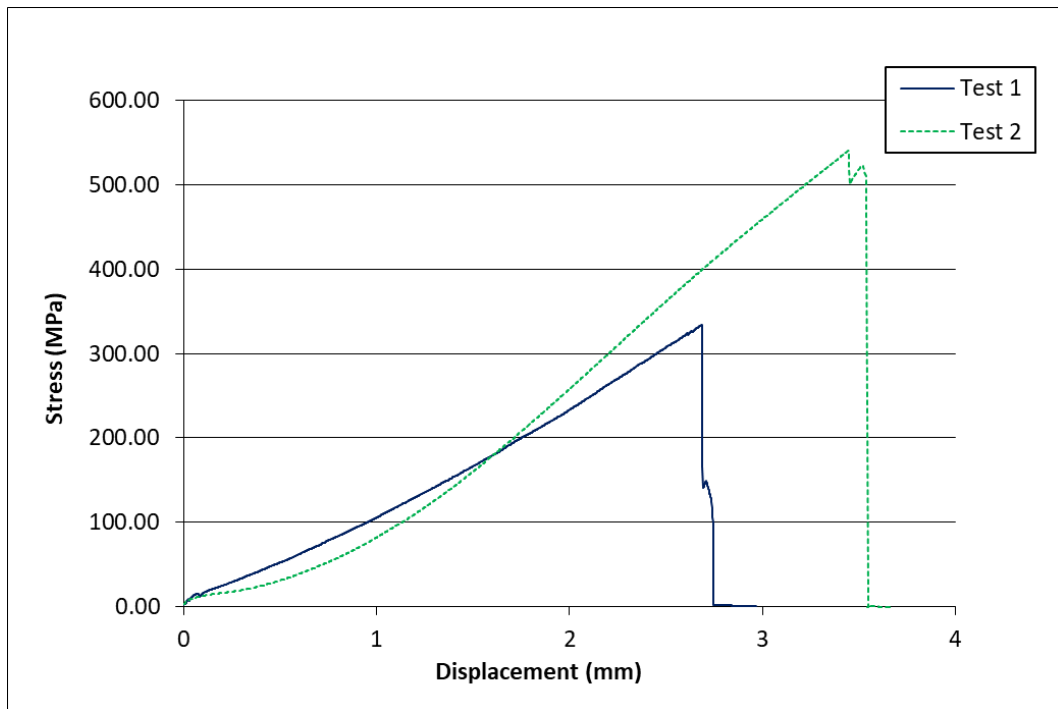


Figure 5.13 Experimental tensile test of group 1

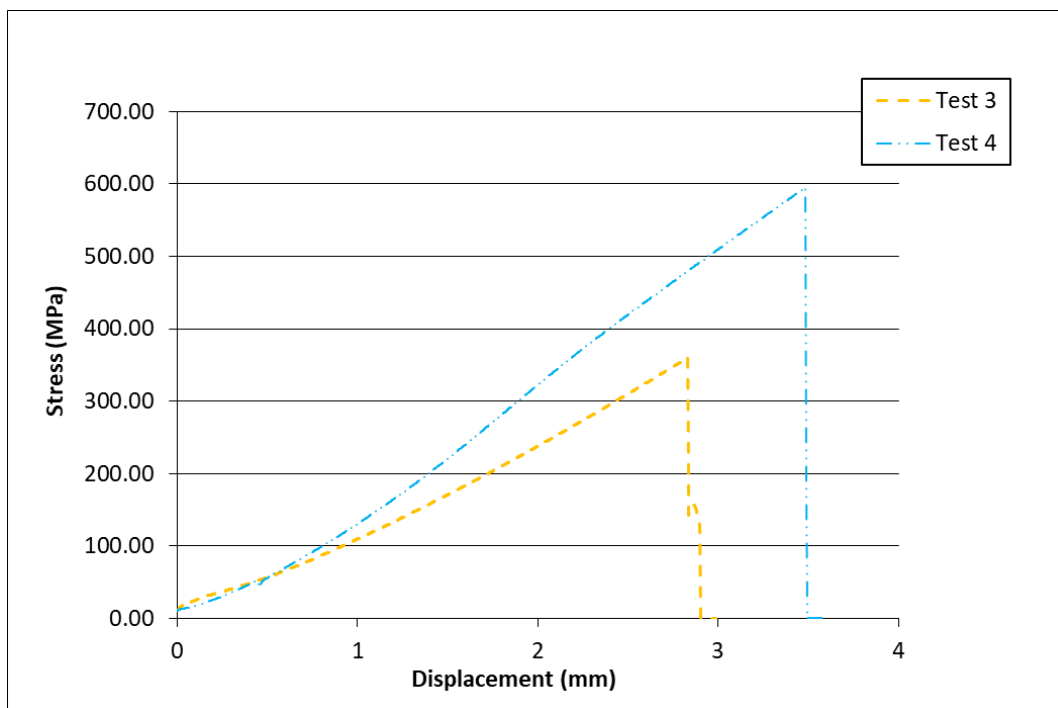


Figure 5.14 Experimental tensile test of group 2

The two groups' experimental tensile test graphs are presented in Figure 5.13 and Figure 5.14. Test specimens before heat treatment have maximum stresses of 334MPa and 359Mpa. The maximum stresses of test pieces after heat treatment are dramatically higher, at 540Mpa and

595Mpa. The displacement at maximum stress for all the test specimens are listed in Table 5.3.

Table 5.3 Maximum tensile stresses and displacements of experimental analysis

Test number	Heat-treated	Maximum tensile stress (MPa)	Displacement at peak stress (mm)
Test 1	No	334	2.68
Test 2	Yes	540	3.44
Test 3	No	359	2.82
Test 4	Yes	595	3.48

5.2.5 Hardness analysis of SiC/Al composite

Vickers hardness tests were carried out for specimens before and after heat treatment. The equipment used is discussed in Chapter 3. By applying a range of light loads using a diamond indenter on the testing pieces, indentations are measured and converted into hardness values. The author tested two sets of specimens before and after heat treatment. Each set contains three groups of samples with ten locations picked on each sample for the hardness test. The testing locations were selected evenly across each piece. There are six groups of specimens tested in total using 60 testing points. The average hardness value was then calculated for each sample group. The average Vickers hardness values obtained are shown in Figure 5.15.

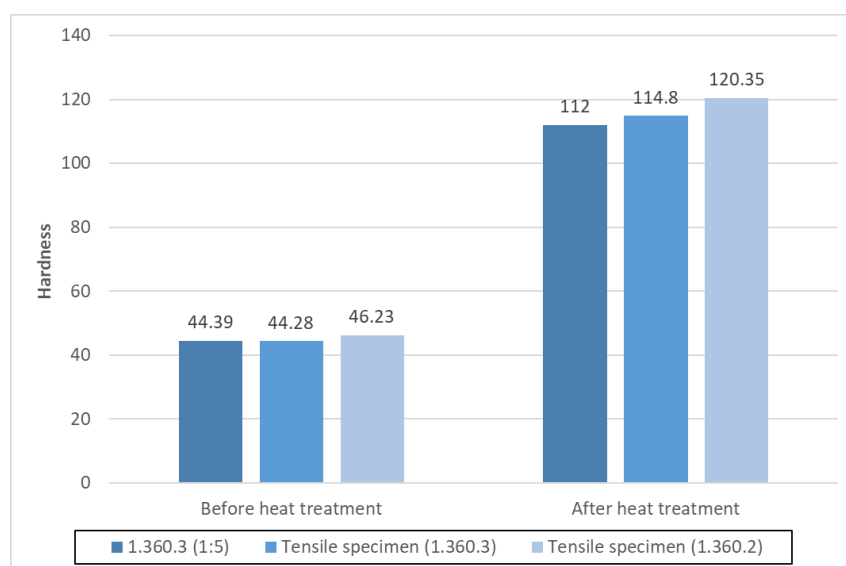


Figure 5.15 Vickers hardness test results

The sample groups have an average Vickers hardness of around 45 before heat treatment. From the hardness values for tests before heat treatment, the state of the aluminium matrix would appear to be in the O (annealed) condition (voestalpine, 2018). After the heat treatment process, the composite samples' average Vickers hardness dramatically increases to around 116. This indicates that the state of the aluminium matrix is in the T6 condition (Voestalpine, 2018; Aluminum, 2014).

5.3 Application of the chapter outputs

The curvature results and hardness results presented in this chapter are used to justify the choices made in terms of the set aluminium temper conditions in the analytical model presented in Chapter 6 and the FE models presented in chapters 7-10. The curvature results are also used in chapter 9 to validate the FE models by comparing them with the FE curvature model results presented in chapter 8.

CHAPTER 6

ANALYTICAL TENSILE TEST MODEL

CHAPTER 6 ANALYTICAL TENSILE TEST MODEL

6.1 Introduction

To compare with results obtained experimentally and to better understand the state of the aluminium during heat treatment and due to post heat treatment, analytical tensile test models have been created in this chapter based on the unidirectional fibre reinforced matrix slab model to determine the initial residual stresses due to manufacture and then force balance calculations to determine failure when subject to tensile loading. The slab model method is detailed in APPENDIX E. The results from the experiments (section 5.2.4) and the analytical models are compared and analysed in detail below.

6.2 Analytical tensile test model

It is assumed that the aluminium is elastic perfectly plastic and on the failure of the fibres, both fibre and matrix stresses go to zero. The tensile test dog bone specimens' cross-sectional dimensions were applied for the analytical model. The input parameters and results for the analytical model can be found in APPENDIX G. The model does not include a creep model, and so an assumption needs to be made about the temperature at which the aluminium and SiC fibre form a bond during the cooling down process to generate the residual stresses due to mismatch in the CTEs of the materials. In the model, it was found that setting the bond temperatures above 498.15°C caused yield to occur in the aluminium during cooldown. This resulted in similar residuals stresses for all models when the bond temperature was set above 498.15°C. Given there was not much variation in residual stresses it was decided to set the bond temp to 500°C (798.15K), which is lower than temperatures used by other authors (Durodola and Derby, 1994).

The initial residual stresses after heat treatment cause the aluminium to have yielded. It is assumed that the aluminium is in the O condition based on the hardness tests (see chapter 5), after initial manufacture and is in the T6 condition after the post-manufacture heat treatment. As the tensile loading increases in the analytical model, the stress values of both

fibre and the matrix component rise accordingly. As the load increases, the stresses increase until the fibres break.

Figure 6.1 and Figure 6.2 show the analytical model simulations of loading force and stress versus displacement for the specimens with post-manufacturing heat treatment. Initial residual stresses of 55MPa and -576MPa were applied for the analytical model. The residual stress values were calculated from the slab model. Aluminium matrix temper O has a yield stress of 55MPa, and the failure stress of silicon carbide continuous fibres is 3.6GPa. The total fibre volume fraction is 16.1%. The [0/90] cross-ply fibre orientation has an effective volume fraction of 8.05% in the tensile direction. The detailed calculation is given in APPENDIX E.

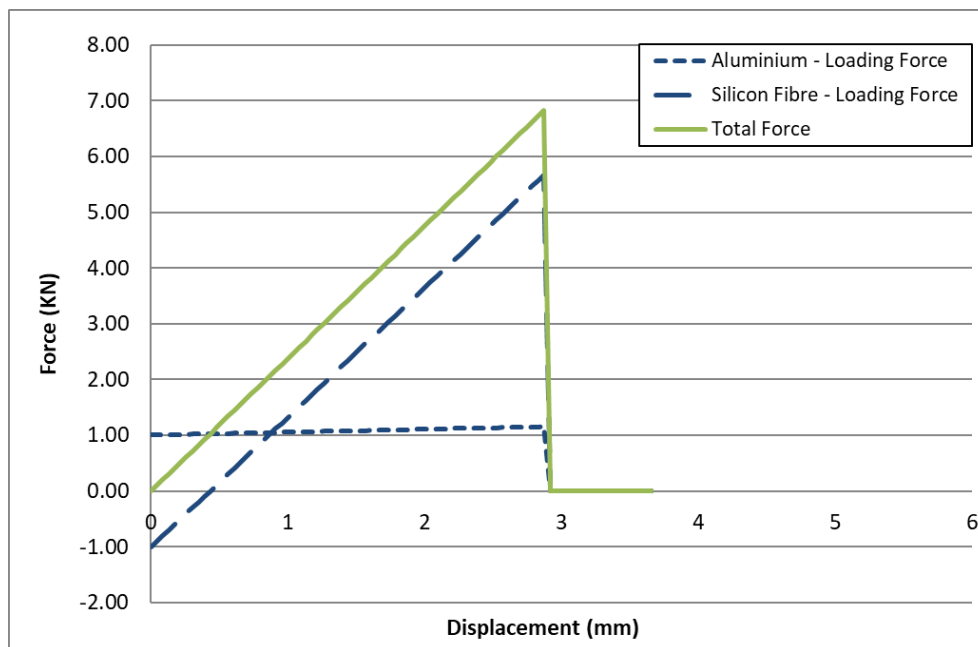


Figure 6.1 The analytical model of loading force versus displacement for specimens before heat treatment

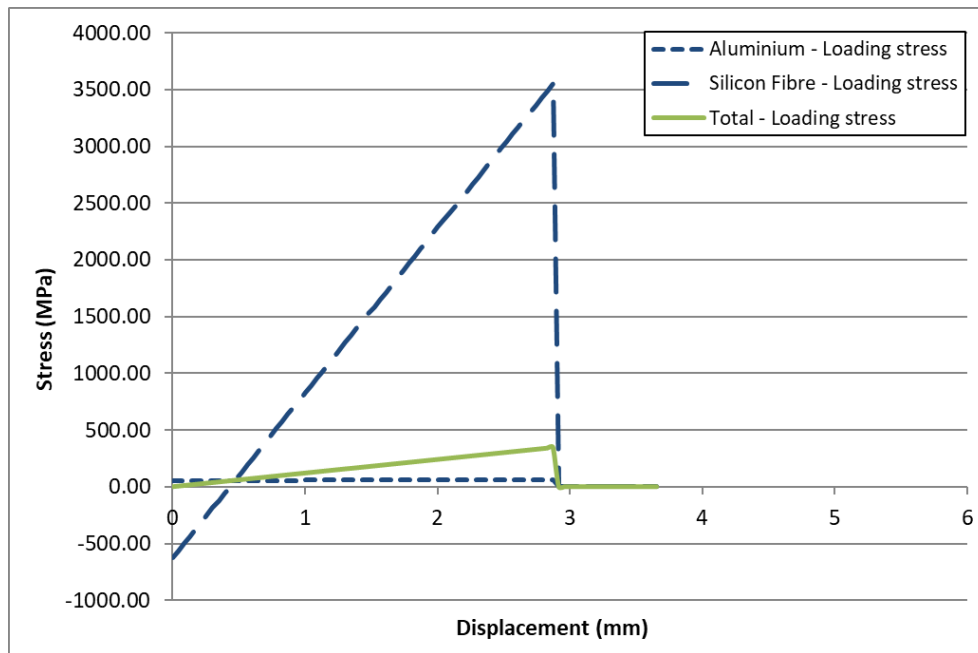


Figure 6.2 The analytical model of loading stress versus displacement for specimens before heat treatment

It can be seen from the results that the silicon carbide fibres are much stiffer (as defined by the material properties) than the aluminium matrix, which means most of the applied load is transferred onto the fibres. Due to the stiffness of the aluminium in comparison to the silicon carbide, the final force is almost completely defined by the strength and stiffness of the silicon fibres, as shown in Figure 6.1.

Figure 6.2 shows the tensile stress loading model with increasing displacement. The silicon carbide fibres have much higher failure stress in comparison with the aluminium matrix yield stress. As shown in Figure 6.2, the total loading stress lies between the stress on the aluminium and the stress on the fibre. The stresses are related to the volume fraction due to the overall forces needing to balance out the residual applied load. As the loading stress increases, the total stress of the component reaches the peak value, defined by the fibres breaking. In Figure 6.2, we can see a slight increase in the total stress just after the fibres break, but this is caused by the incrementation within the model and is not real. As discussed earlier, fibre reinforcements carry most of the loading until the fibre failure stress is reached. The aluminium matrix acts as a loading transfer media before the fibre fracture. At the second the fibre fracture occurs; all of the loadings are transferred immediately to the matrix.

However, because of the relatively low strength of the aluminium, the maximum stress will be achieved almost instantaneously, leading to the complete failure of the composite.

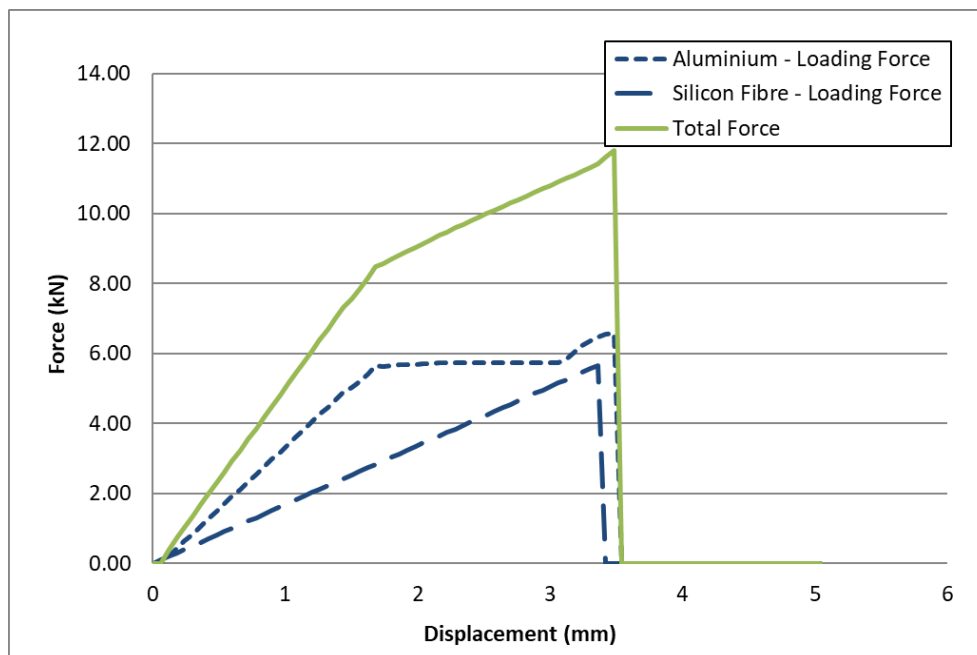


Figure 6.3 The analytical model of loading force versus displacement for specimens after heat treatment

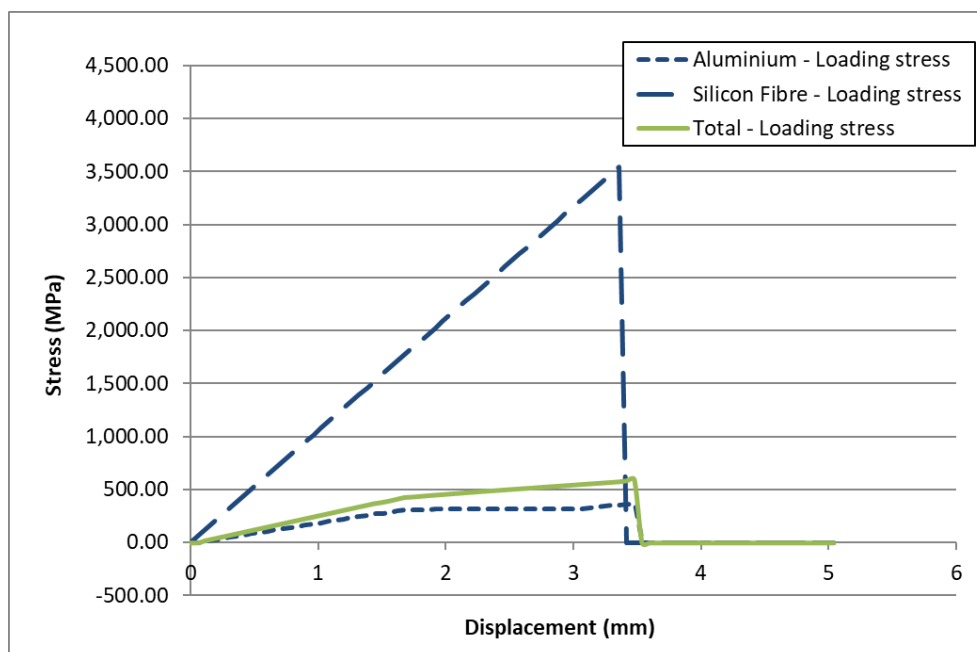


Figure 6.4 The analytical model of loading stress versus displacement for specimens after heat treatment

Figure 6.3 and Figure 6.4 show the analytical model simulation of loading force and stress with displacement for specimens after heat treatment. Again, the aluminium is treated as behaving as an elastic perfectly plastic material. The aluminium matrix is heat treated to the T6 condition with a yield stress of 275MPa. The failure stress of the fibres is 3.6GPa, and the volume fraction in the pulling direction is effectively 8.05%. Residual thermal stresses were removed thanks to the post-manufacturing heat treatment (see section 5.2.3). The total force combines the aluminium and silicon carbide fibre loads, as displayed in Figure 6.3. After the heat treatment, the yield strength is predicted to increase dramatically. The total force that can be applied before failure is significantly increased due to this.

Figure 6.4 shows the tensile stresses during loading for the heat-treated specimens. The total loading stress combines the effect of the stresses on the fibre and the matrix components. Again, the stresses increase with load until the fibres break. The aluminium matrix reaches its yield point at around the 1.8 mm displacement position. At the moment the fibre fractures, all of the loading is transferred immediately to the matrix, thus generating a slight increase in total stress due to the way the model is incremented. In reality, the composite would fail as soon as the fibres fail due to the sudden overload this would cause on the aluminium matrix. The results of this analytical model are compared with the experimental test results in the next section.

6.3 Tensile test analysis of SiC/Al composite

In this section the tensile test results from the experiments (section 5.2.4) are presented and compared with the analytical results. The experimental data is based on the average test values from the two tests carried out for the two different post-manufacture heat treatment conditions (treated and non-treated). The experimental and analytical tensile analysis results are listed in Table 6.1.

Table 6.1 Experimental and analytical tensile analysis results

	Heat-treated	Average maximum tensile stress (MPa)	Displacement at peak stress (mm)
Experiment Test	No	346	2.75
	Yes	567	3.46
Analytical model	No	338	2.82
	Yes	594	3.48

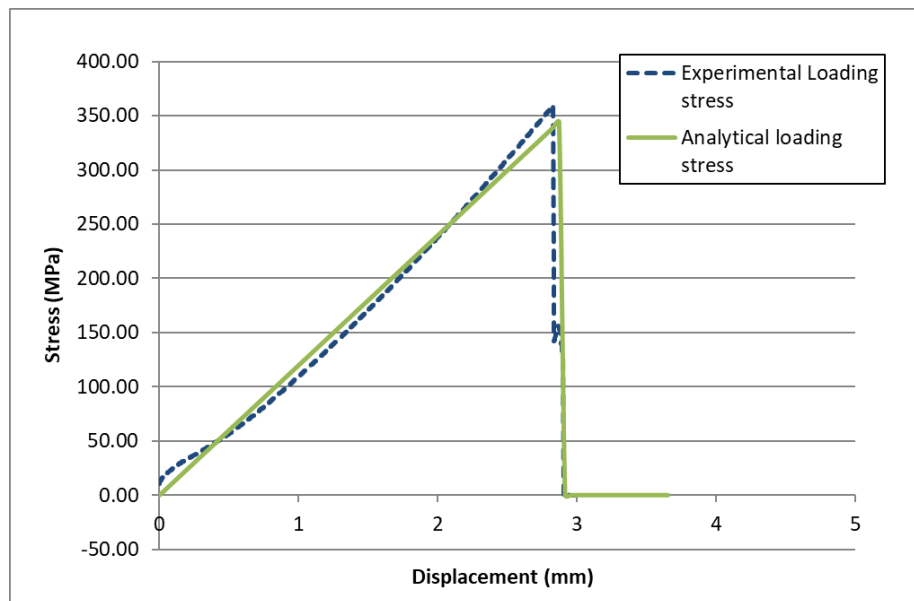


Figure 6.5 Result comparison of experimental tensile test and analytical model before heat treatment

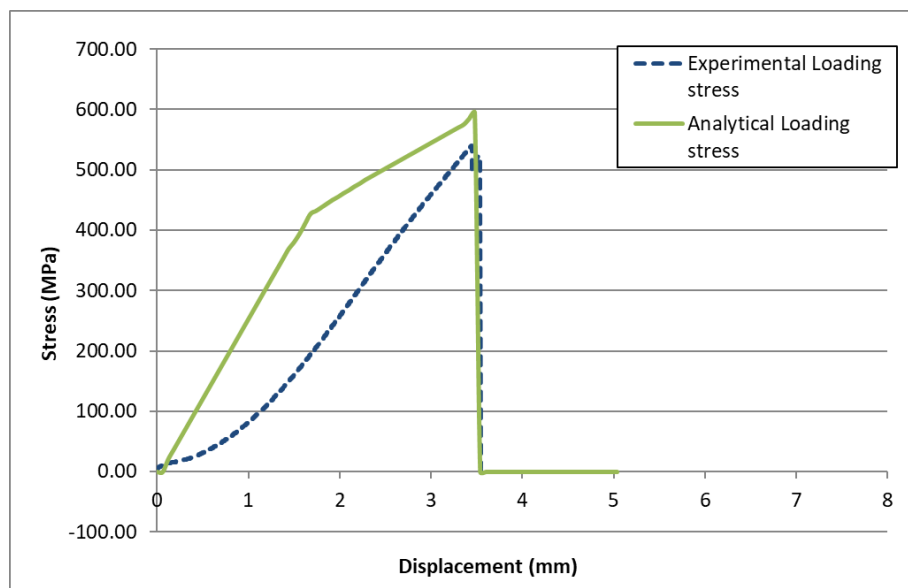


Figure 6.6 Result comparison of experimental tensile test and analytical model before heat treatment with adjustments

Figure 6.5 shows the tensile results for the specimens before heat treatment, it presents a great match in terms of peak stresses and displacements with a maximum stress of 346MPa and 338Mpa and a displacement of 2.75mm and 2.82mm, for the experiment and model, respectively. The shapes of the experiment and model curves show a very good correlation. Figure 6.6 displays the experimental and numerical tensile test results for specimens after heat treatment. As discussed earlier, the residual thermal stress was removed due to the post-manufacturing heat treatment. The analytical model predicted the maximum stress of 594MPa, which over predicted the tensile stress. The difference could be due to various reasons, such as the aluminium alloy strength and SiC fibre strength having not performed as theoretically predicted, or the fibre volume fraction is lower than the design value.

The shapes of the experimental and analytical curves do not match well. The shape of the experimental curve would seem to indicate that the aluminium yields earlier than predicted in the numerical model, but that would go against the increase in hardness measured after heat treatment (section 5.2.5) and the rise in tensile strength, as seen when comparing Figure 6.4 and Figure 5.13.

The results between the analytical tensile test method and the experimental tests are very close in terms of pre-heat-treated samples and give good results in terms of failure strength but do not follow the loading curve well. This demonstrates that the assumptions in terms of initial residual stresses and the effect of heat treatment on strength of the aluminium are reasonable but there is underlying material behaviour that is not fully being accounted for within the model.

6.4 Application of the chapter outputs

This chapter has used an analytical slab model to predict the initial residual stresses in composite laminate plates due to manufacture (see APPENDIX E) and uses force balance to predict the tensile test results assuming elastic perfectly plastic behaviour for the aluminium alloy. A temperature point at which the Aluminium and Silicon Carbide fibres become bonded

on cooldown was set based on there being little variation in stresses when set above $\sim 500^{\circ}\text{C}$ and this value being lower than values found in literature. The temper conditions prior to the post manufacture heat treatment and after were assumed to be O and T6 based on the hardness test results (see section 5.2.5). After the heat treatment the residual stresses were assumed to be zero based on the curvature results of the heat treated samples (see section 5.2.3). The tensile test results show some differences when comparing the post heat treatment analytical and experimental results but do give good confidence that some of the assumptions that have been made are substantially correct. This means that this analytical method could be used to give reasonable answers to the strength of composite laminate plates due to manufacture and post manufacture heat treatment. This also give confidence that the post manufacture residual stresses obtained in this chapter can be used as a rough check on the residual stress results when utilising a more fully defined aluminium alloy material model within the FE models developed in chapter 7 and discussed in chapter 8.

CHAPTER 7

FINITE ELEMENT ANALYSIS PROCEDURE

CHAPTER 7 FINITE ELEMENT ANALYSIS PROCEDURE

7.1 Introduction

This chapter presents the procedure used to develop Finite Element models to predict the deformation and stresses in Al/SiC MMC plates and pressure vessels.

Initial work was carried out for elastic slab, single fibre and multiple fibre models. In addition elastic two-dimensional (2D) and three-dimensional (3D) models were developed and compared with classical laminate calculations. A comparison of results between Ansys and LS-Dyna was also carried out to see if the packages gave consistent results. This built confidence in the initial procedures and software. LS-Dyna was chosen as the main software to use as it had an easy to implement appropriate creep material model and Oxford Brookes had access to unlimited licences which helped speed up the model development time. This initial work is presented in Appendix E and F.

In section 7.2 elastic FEA models are developed to compare the bistable curvature deformation of composite laminate plates against experimental data from literature. The curvature is generated by unbalanced residual stresses created by the manufacturing heat treatment. Classic laminate theory predicts a saddle-like deformation of plate laminates. Hyer's study (Hyer, 1981) indicates two stable cylindrical shapes will occur at room temperature, as discussed in Chapter 2. The saddle-like shape predicted by the CLT theory (See APPENDIX E) is unstable as it can snap to either of the stable conditions.

Due to creep and yielding within the aluminium during thermal cooling of Al/SiC MMC composites, purely elastic models will generate errors. Section 7.3 presents the creep model that was added, in addition to plasticity and thermal effects on material properties, to the models presented in section 7.2 and used in the models presented in section 7.4.

In section 7.4 the models developed to predict the residual stresses in burst tubes, manufactured by TISICS Ltd, are presented.

7.2 FEA modelling of bistable composite laminates

7.2.1 FEA simulations

Material properties applied in the FEA models are from Moore's study (Moore, Ziaei-Rad and Firouzian-Nejad, 2014). The study applied graphite-epoxy composite material with temperature dependency. FE models used shell elements utilising the constant stress element within LS-Dyna. Each node possesses three degrees of freedom, two translations and one rotation. The layup of the composite was carried out in LS-Dyna using the part composite method with beta angle specified to orientate the layers in the appropriate experimental directions given in the literature. The mesh size was $1\text{ mm} \times 1\text{ mm}$ for modelling a $150\text{ mm} \times 150\text{ mm}$ laminate. A total of 22,500 shell elements with 68101 nodes were used for building the model. All the models have square dimensions of $150\text{ mm} \times 150\text{ mm}$ with a total thickness of 0.73 mm . The middle point of the plates are fixed in all degrees of freedom in the FEA models. Thermal loading goes from 180°C to 25°C (room temperature). A small vertical force of 0.1 N was applied on the four corners of the models to snap the models from the first stable shape to the second stable shape. Mesh dependency and time step convergence were investigated first, which is presented in sections 7.2.2 and 7.2.3.

Table 7.1 Material properties and model layups

Elastic material properties	$E_1(\text{GPa})=182-0.272T+0.518 \times 10^{-3}T^2$ $E_2(\text{GPa})=147-0.803T+1.16 \times 10^{-3}T^2$ $G_{12}(\text{GPa})=7.46+6.52 \times 10^{-3}T-2.73 \times 10^{-5}T^2$ $G_{23}(\text{MPa})=22.2$ $\nu_{12}=0.629-1.22 \times 10^{-3}T+3.94 \times 10^{-3}T^2$ $\nu_{23}=0.01$ $\rho(\text{kg/m}^3)=1072$
Layup	Upper layer of composite laminate (thickness= 0.365 mm) Bottom layer of composite laminate (thickness= 0.365 mm)

Table 7.1 presents the graphite-epoxy composite material properties, the values are from Moore's study (Moore, Ziaei-Rad and Firouzian-Nejad, 2014). Longitudinal, transverse and shear elastic modulus are in GPa unit and the temperature is in Kelvin (K) that can transform to the $^\circ\text{C}$. FE models have two composite layers, both layers have thickness of 0.365 mm .

7.2.2 Mesh dependency analysis

A mesh dependency study is performed using a 4 x 5 mesh, 8 x 10 mesh, 16 x 20 mesh, 32 x 40 mesh, ... 192 x 960 mesh (Figure 7.1).

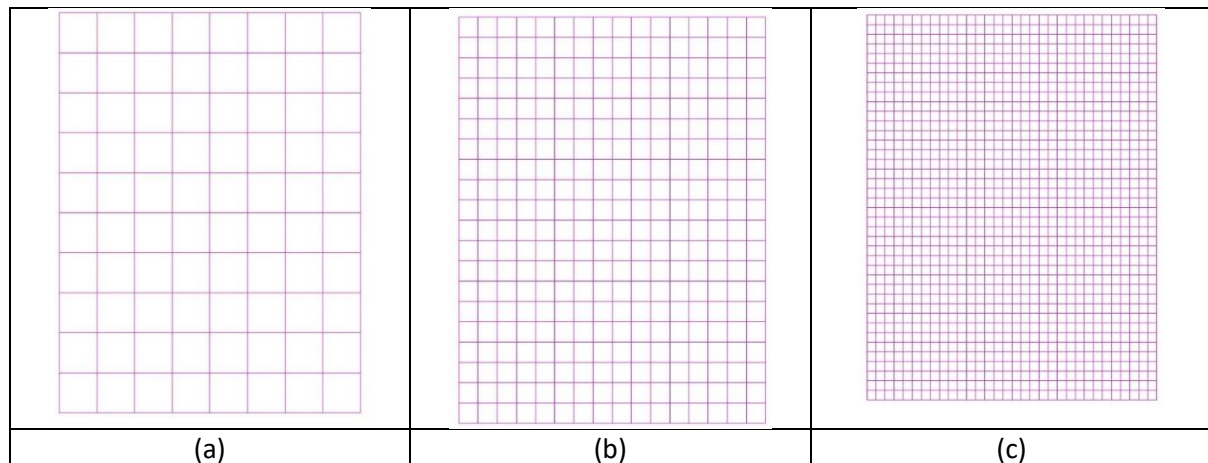


Figure 7.1 Mesh structure of FEA models with (a) 8 x 10 mesh, (b) 16 x 20 mesh and (c) 32 x 40 mesh

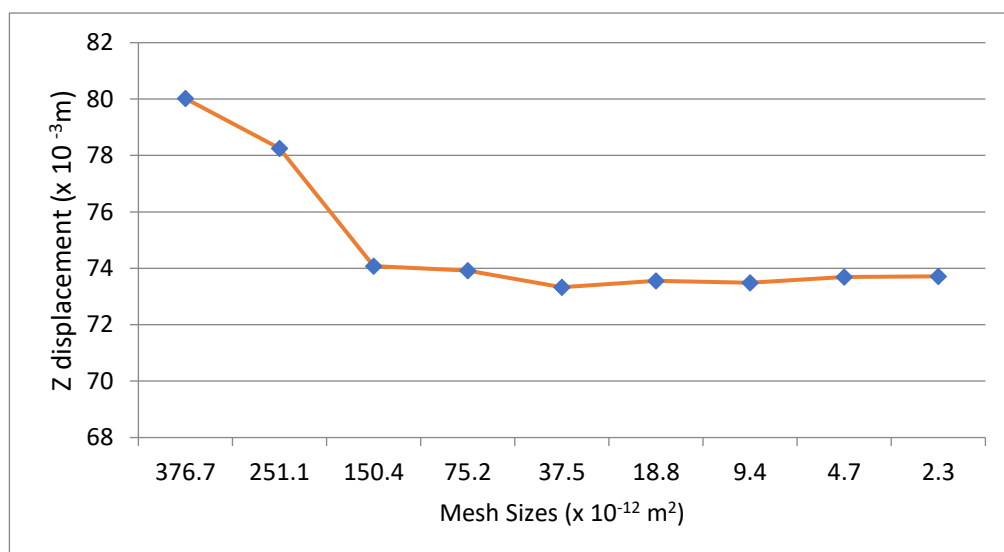


Figure 7.2 FEA results of mesh dependency study

As shown in Figure 7.2, the displacement results converged as the mesh density increased. It is important for finite element models to reach a stable state. Mesh sizes at the converged region are used for FEA modelling.

7.2.3 Time step dependency analysis

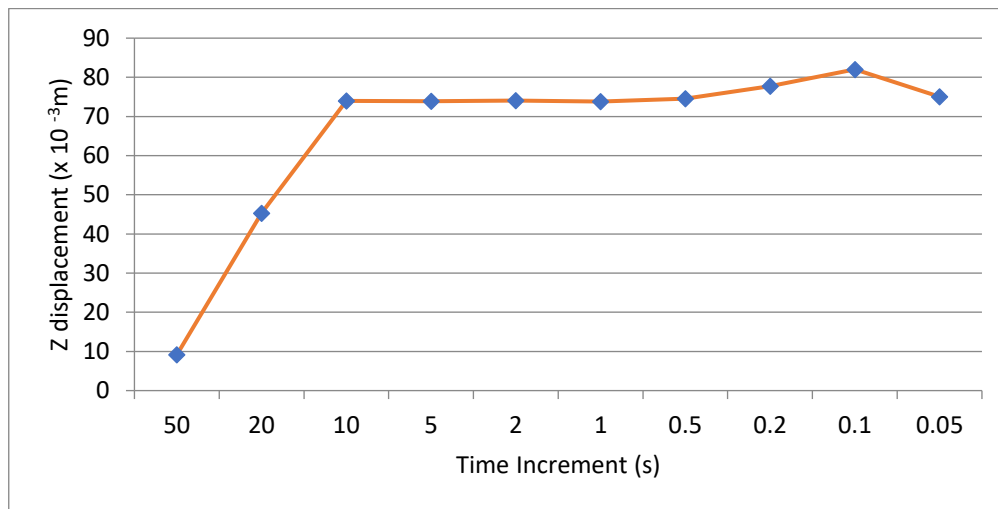


Figure 7.3 FEA results of time dependency study

Time step dependency is investigated for the FEA model convergence study. Time increment n is used where n varies from 50 to 0.05. As presented in Figure 7.3, the displacement results converged as the time step increased. For the composite laminate analysis with $[90_4/0_4]$, $[-30_4/30_4]$ $[60_4/30_4]$ fibre layups, a time step within the converged region was employed. The results of the models developed in this section are presented in section 8.1.

7.3 Checking the implementation of the creep rate model

The models up to this point have not included creep (or plasticity). In this section, the results for calculating the creep analytically will be compared with the results using the same model parameters within the same creep model implemented within LS-Dyna. This is to show a correct implementation of the creep model within the FEA.

The analytical secondary creep rate using hyperbolic-sin law was discussed in Chapter 3. When an aluminium bar is under a 20MPa axial load, and the environmental temperature is 400°C , the theoretical secondary creep rate is given as:

$$\dot{\epsilon}_{II} = A[(\sinh(B \sigma))^n \exp\left(-\frac{Q}{RT}\right)] \quad \text{Equation 7.1}$$

With A equals to 2.91×10^{11} and B equals to 0.0404, the analytical secondary creep rate of aluminium 6061 under 20MPa pressure in the 400 °C environments is therefore $3.54E - 05 \text{ s}^{-1}$. The secondary creep strain rate can be estimated when the external load changes from 20MPa to 35MPa, as demonstrated in Figure 7.4.

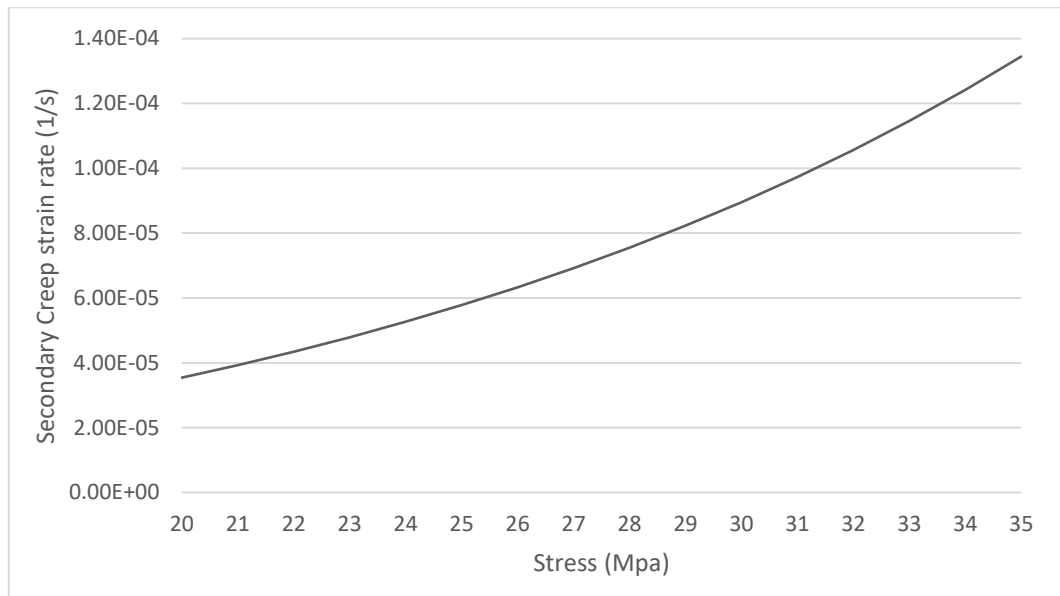


Figure 7.4 Analytical model results of secondary creep strain rate curve under 400°C thermal load

An FEA creep model was developed using LS-Dyna to validate with the analytical model. The Young's modulus, Poisson's ratio, thermal expansion coefficient and material parameters of plasticity are all assumed to be temperature-dependent. Chapter 3 presents the temperature-dependent material properties in detail. In the LS-Dyna finite element model, the material model 188 (Thermo-elasto-viscoplastic creep) was employed as it incorporates thermal and creep effects that are required to fully model the behaviour of the Aluminium 6061 during the manufacturing process. In the material setting card, the 'CRPLAW' parameter is set to zero. Constant A, B, Q and m values inputs are $2.9E11 \text{ (s}^{-1}\text{)}$, $4.04E-8 \text{ (Pa}^{-1}\text{)}$, 24536.93 (K) and 1.74 (-). The stress unit of the FE creep model is Pa, which is different from MPa as used in the analytical model. Thus, constant B of $4.04E-8 \text{ (Pa}^{-1}\text{)}$ is applied instead of 0.0404 (MPa⁻¹) for the FE model. Constant Q is the ratio of activation energy 204 (kJ/mol) and universal gas

constant 8.314×10^{-3} (kJ/mol-K). The meshed FE model with applied boundary conditions is presented in Figure 7.5. The representative deformation results using the FEA method is illustrated in Figure 7.6.

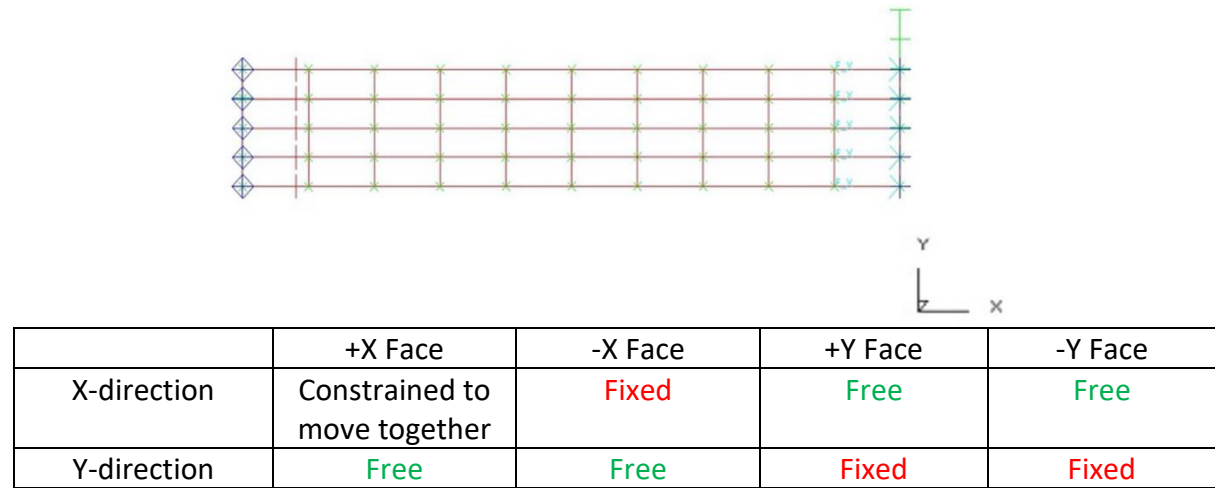


Figure 7.5 Boundary conditions of creep model

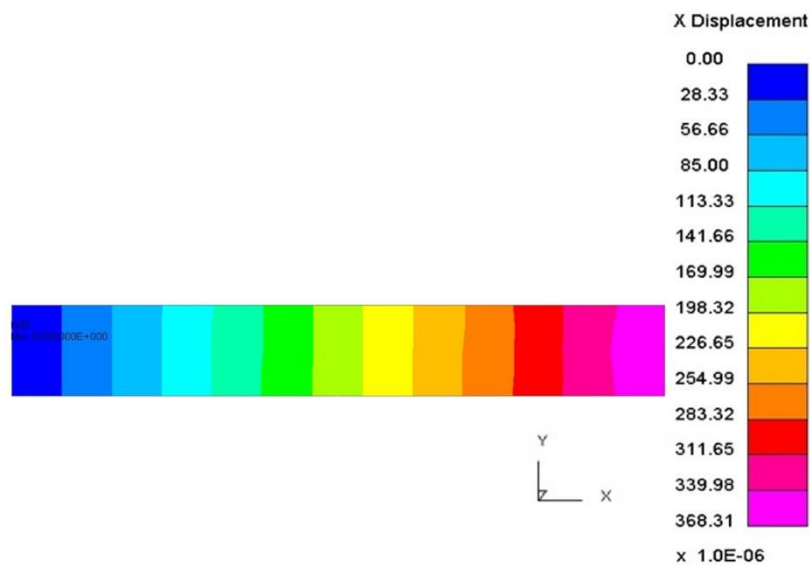


Figure 7.6 FEA deformation of creep model

When the FEA model is under the uniformed thermal load, with the external axial stress of $\sigma_1, \sigma_2, \sigma_3 \dots \sigma_n$, the corresponding deformation is $l_1, l_2, l_3 \dots l_n$. Thus, the secondary creep strain rate is expressed as in Equation 7.2.

$$\dot{\varepsilon}_{II} = \frac{\varepsilon_2 - \varepsilon_1}{\Delta t} = \frac{\frac{l_3 - l_2}{l_2} - \frac{l_2 - l_1}{l_1}}{\Delta t} \quad \text{Equation 7.2}$$

Figure 7.7 presents the secondary creep strain rate curve (using displacements obtained from the FEA model) when the external load varies from 20MPa to 35MPa.

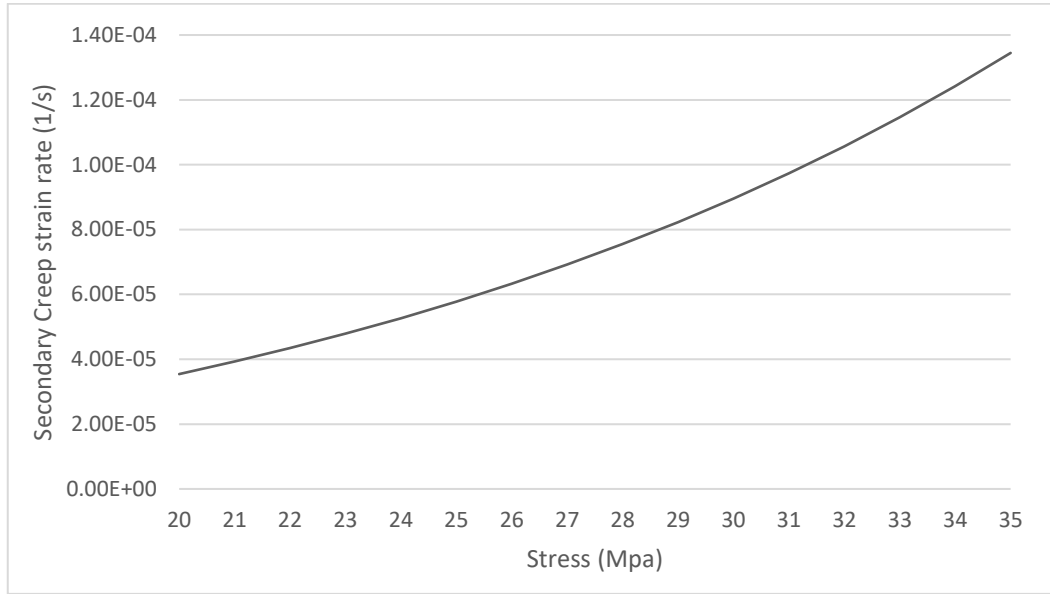


Figure 7.7 FEA model comparison of secondary creep strain rate curve under 400°C thermal load

A range of secondary creep strain rates were calculated with an analytical model under different pressure and thermal loading conditions. The finite element model correlates reasonably well with the analytical model. Figure 7.8 demonstrates the comparison of the analytical and FEA creep strain rates for a range of stress values.

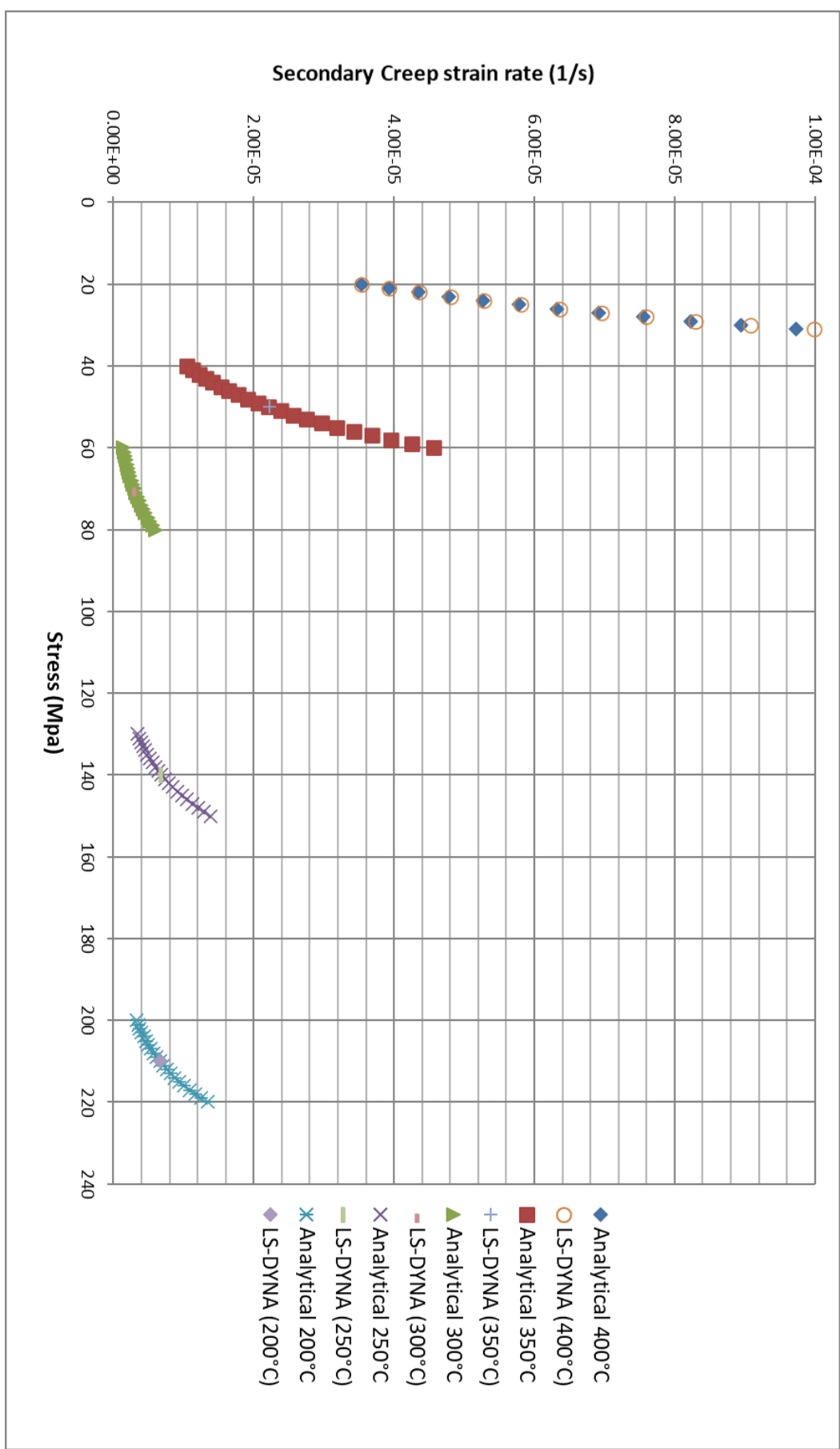


Figure 7.8 Comparison of the analytical and FEA creep strain rates

The FEA creep model results have a very good correlation with the theoretical secondary creep rate model. When the thermal load ranges from 200°C to 400°C , the FEA values match the values from the analytical estimation.

The material properties presented in this section were implemented into the material models presented in Chapter 3 and implemented into the FE models in the following sections.

7.4 2D Modelling of the experimental laminate plates

Models were developed with the same dimensions as the test specimens presented in chapter 3. The models use the material properties for the SiC fibre and the aluminium 6061 matrix presented in section 3.1. The total fibre volume fraction was 16.1%. The materials used for the simulation are elastic-plastic and temperature-dependent. The creep effect of the aluminium matrix is also considered in the FEA simulations. The FE models were given identical manufacturing thermal and pressure loadings, as the experimental plates. The exact cooling rate and pressure reduction rate at the end of the heat treatment process was not specified, only the total cooldown time. What is known is that the components are left in a switched off furnace with the door shut until they reach room temperature (based on instruction supplied by TISICS). The cooling was modelled linearly rather than considering the exponential decay that would normally be seen.

The dimensions of the full plate models were 200mm x 150mm, with a total thickness of 0.7319mm. The architecture detail is given in section 3.2. FE models used shell elements with quadratic shape function for displacement. Each node possesses six degrees of freedom, three translations and three rotations. The mesh size was 10 mm x 10 mm for modelling a 200 mm x 150 mm laminate. The models used a mesh size of 10mm with 300 elements for each model (Figure 7.9).

Models with different aspect ratios were also modelled with a width of 15mm and length of 15mm, 30mm, 45mm, and 75mm for aspect ratios of 1, 2, 3 and 5 respectively. The mesh size was 1 mm x 1 mm. For a 15 mm x 15 mm laminate, there was 225 elements in total.

The middle points of all the plates were restricted in all degrees of freedom in the FEA models.

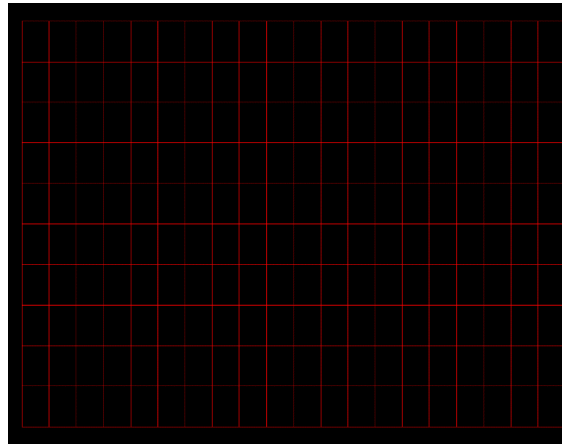


Figure 7.9 Mesh structure for finite element analysis

7.5 3D modelling of the experimental laminate plates

To investigate the localised stresses around the fibres a 3D cell model of the composite plates was developed. Both models applied solid element with tetrahedron mesh. There 12869 and 21820 elements for $[90/0]$ and $[+55/-55]$ composite model, respectively. The material properties are the same as used for the 2D plates in the previous section and the boundary conditions are discussed in APPENDIX E. The meshed component for the finite element analysis is shown in Figure 7.10.

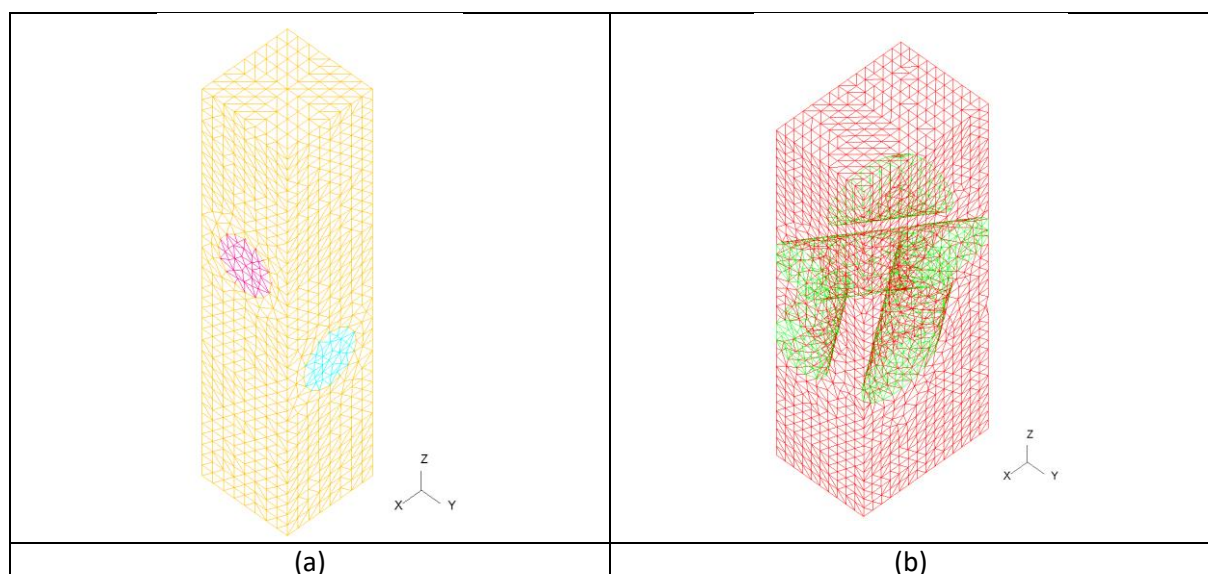


Figure 7.10 Mesh structure for FEA model (a) $[90/0]$ composite model (b) $[+55/-55]$ composite model

7.6 Silicon carbide fibre reinforced aluminium alloy matrix components

TISICS use burst tubes to analyse the manufacture and performance of metal matrix composites for use in pressure vessels. They were made with different wall thicknesses and various fibre layup architectures. The composite, made of silicon carbide fibre reinforced aluminium alloy matrix, is placed within the aluminium cylindrical body to strengthen the overall performance of the burst tube (Figure 7.11).

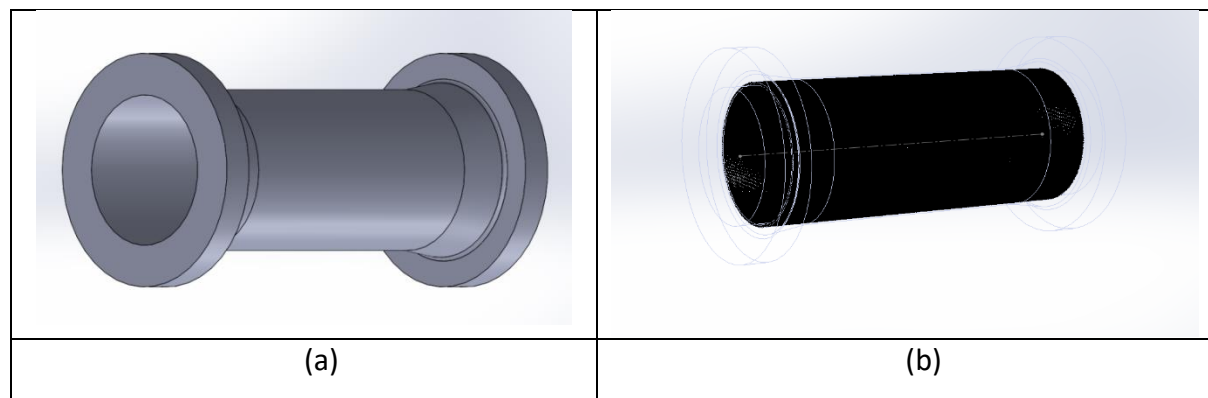


Figure 7.11 (a) 3D model of burst tube component (b) metal matrix composite reinforced region

The architecture of the reinforcement composite, such as fibre layups and the number of plies, varies for different pressure vessel specifications. As a result, the internal and external radius will have slight differences. However, the burst pressure is the main parameter to investigate when looking into the effect of various composite architectures.

7.6.1 Manufacturing process

The component is made by laying up aluminium matrix foils and reinforcement fibres outside a thin steel (circular tube) mandrel. A thick outer steel mould is placed over the laid-up component, and the mandrel and mould are welded onto two end flanges. A low temperature and vacuum are then applied to the component to de-gas it, and the vacuum is then sealed in by welding the outlet closed. The evacuated component is then subject to a temperature and pressure heat treatment process (HIP process). After the heat treatment process, the inner steel sheet and outer mandrel are removed by machining and acid etching and

dependant on the final dimensions, the outer and inner surfaces of the component are skimmed.

7.6.2 Numerical models of composite components

It can be costly and time-consuming to test composite components. The cost of SiC/Al composite components makes destructive testing particularly expensive. The diameter of the fibres and the structure of the components also makes it very difficult to measure the internal stress levels. An excellent way to tackle this problem is to analyse the components using finite element models. The FE models allow the stress levels to be studied within the component if the exact properties of the component and the manufacturing process can be created. In addition, the effect on stress levels of variations such as the material properties, volume fractions, component architecture, and heat treatment conditions can easily be implemented and studied.

Finite element models were developed to determine the residual stresses within the composite burst tubes after the manufacturing process. The models were built and analysed using the LS-Dyna package.

The aluminium foils and fibre plies were laid up in a cylindrical shape before the heat and pressure treatment. There are 158 fibres per inch within the fibre plies. A component with an average of 6 plies of fibre reinforcement could have over 3,700 fibres in total. It would be impossible to model the entire burst tube fibres individually due to the number of fibres involved, both in terms of setting the model up but also in terms of the computational resources required. Thus, simplified 3D models will be used in the finite element models that will attempt to capture the key features so as not to lose accuracy.

7.6.2.1 Dimensions of the Burst tube

The aluminium SiC fibre reinforced composites for the burst tube component investigated are made of six layers of fibre plies and nine aluminium foils. The fibres have a diameter of 100 μm , and the thickness of the aluminium alloy foil is 127 μm . The overall thickness of the tube

wall was measured at 3.542 mm, including outer cladding aluminium walls. The tube has a theoretical volume fraction of 8.45%. The calculation is presented in detail in APPENDIX H.

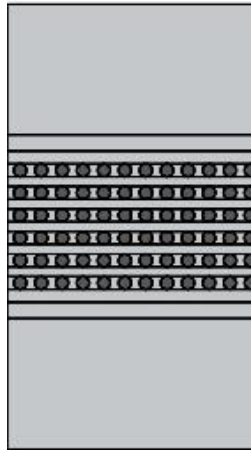


Figure 7.12 3D model of the composite reinforced region of burst tube component

Figure 7.12 shows a 3D model of a section of burst tube based on the actual component with the fibres having identical 0° layups.

7.6.2.2 Material properties

Table 7.2 presents the materials used in the models. The mechanical and thermal properties of both the matrix and fibre material were discussed in detail in Chapter 3. The aluminium matrix and outer wall are treated as a temperature dependant elastic-plastic material including creep, and modelled as discussed in section 7.3. Because of the properties of the silicon carbide monofilament fibres, the material is treated as elastic in the numeric model analysis. It is to be noted that the aluminium matrix will change to annealed condition during the final stage of the manufacturing process, as discussed in Chapter 10. Different models have been developed including models where the matrix material is treated as annealed during the manufacturing cooling period.

Table 7.2 Material selection of burst tube component

Location	Material
Aluminium foil matrix	Aluminium alloy 6061-T6
Outer cladding wall	
Fibre ply	Silicon carbide

7.6.2.3 Mesh selection and boundary conditions

The meshing of the models was difficult due to the geometry of the models and the requirement within LS-Dyna for the different parts of the component to be joined (an aspect of using LS-Dyna implicit models, as discussed later). The initial geometry was drawn in Solidworks and then exported as a step file to the appropriate meshing package. Meshing was performed using LS-Pre, Catia and Altair Hypermesh, but eventually, Catia was used as this was the easiest package to create the meshes required. A combination of hexahedron and tetrahedron meshes were utilised but it was found that hexahedron solid elements tended to be more reliable and efficient than tetrahedron elements. All the final models used hexahedron meshes. For a rectangle sized aluminium layer or cylindrical fibre section, a sweep 3D mesh was applied in Catia. Surface meshes were created, which are then used to sweep a solid mesh along the 3D model geometry to create a 3D solid mesh. The mesh was setup up with an attempt to keep to an element aspect ratio of 1:1 whilst trying not to exceed an element aspect ratio of 3:1.

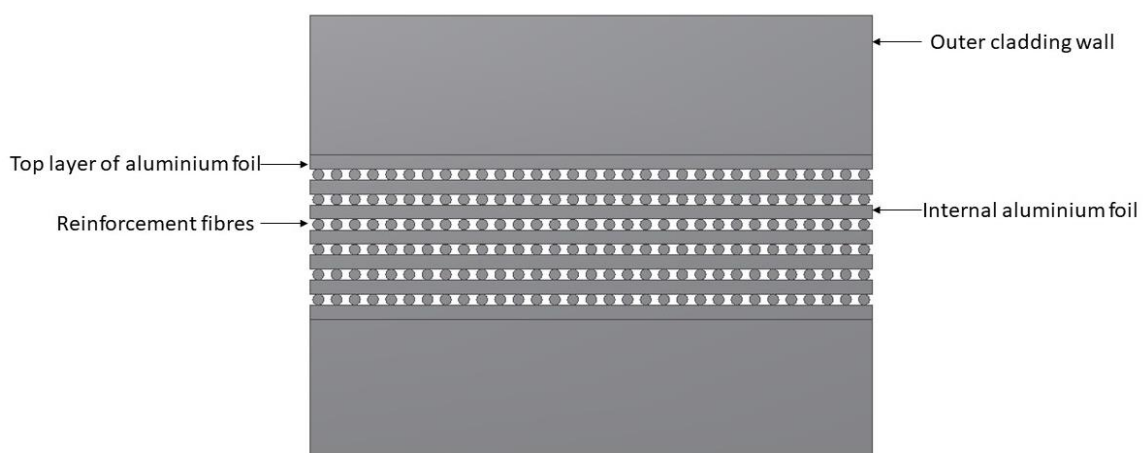


Figure 7.13 3D model geometry of axial reinforced fibre composite

Figure 7.13 shows a meshed section of the burst tube wall showing the outer cladding wall, the aluminium foils and the reinforcement fibres. Although the mesh is not visible, at this level of zoom, the elements are tetrahedrons and are evenly spaced through the through-thickness (3D Sweep). This model, containing many fibres, was too complex to run.

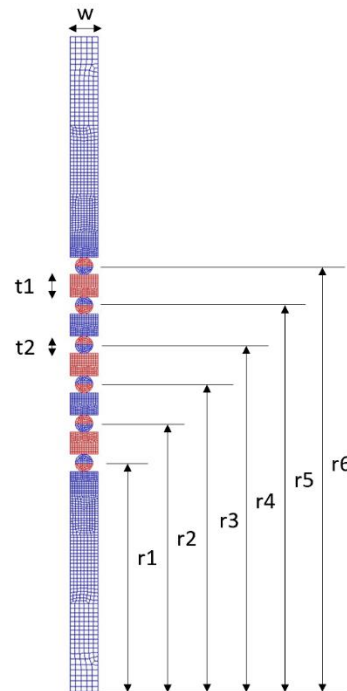


Figure 7.14 Dimensions of reinforced fibre composite unit model

Table 7.3 Dimensions of reinforced fibre composite unit model

Burst tube inside diameter (ID)	Burst tube outside diameter (OD)	Burst tube Internal radius (r)	Width of the model (w)	The thickness of aluminium foil (t1)	Diameter of fibre (t2)
94.421	101.7	47.21	0.161	0.127	0.1
r1	r2	r3	r4	r5	r6
1.29	1.52	1.75	1.98	2.2	2.43

All dimensions in mm

To reduce the complexity of the model; unit models (only one fibre per layer, placed at the centre of the section) were modelled and analysed to provide a more cost-effective solution, see Figure 7.14. Dimensions of the unit model are presented in Table 7.3. To do this,

symmetry boundary conditions due to the axisymmetric symmetry of the vessel, plane movement constraints and a fixed top surface are applied to the models as shown in Figure 7.15 and Table 7.4. Due to manufacturing variations, the fibres would not align perfectly in the different layers, even for $[0]_6$ and $[0/90]_3$ layups. For layups that are angled to the axial/circumferential direction such as $[+55/-55]_3$ the fibres in each layer will move position relative to other layers due to the difference in diameters of each layer level. An assumption that the fibre in a layer would align with the fibres in other layers would only be applicable for very limited points along the burst tube (probably never). The assumption that they do align will affect the matrix stress distribution but should not affect the overall average stresses in the matrix.

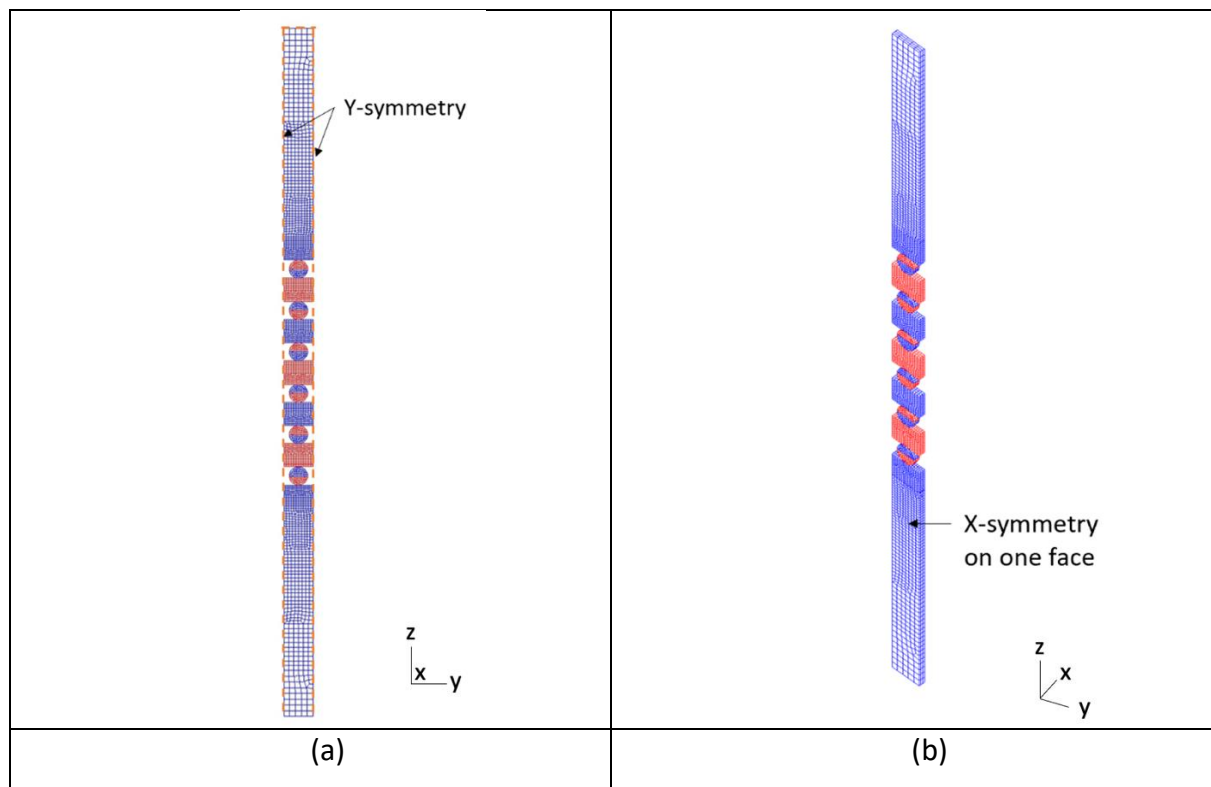


Figure 7.15 (a) Boundary condition (b) Constraint representation

Table 7.4 Boundary conditions of FE unit model

	+X Face	-X Face	+Y Face	-Y Face	+Z Face	-Z Face
X-direction	Fixed	Constrained to move together	Free	Free	Free	Free
Y-direction	Free	Free	Fixed	Fixed	Free	Free
Z Direction	Free	Free	Free	Free	Fixed	Constrained to move together
Rotation about X-axis	Free	Free	Fixed	Fixed	Fixed	Free
Rotation about Y-axis	Fixed	Free	Free	Free	Fixed	Free
Rotation about Z-axis	Fixed	Free	Fixed	Fixed	Free	Free

The unit model presented in Figure 7.15 has an axial fibre layup. One Z face and one X face have fixed boundary conditions (symmetry) and both Y faces have symmetry boundary conditions applied. The remaining Y and Z faces are constrained so that their nodes move in-plane along their respective axis. Equal pressure loads are applied to the moving Y and Z faces to simulate the pressure applied during manufacture.

Another simplification in the initial FE models was that the Y faces were assumed to be parallel with each other. This is not true as the curvature of the tubes will create a small taper angle on these faces. This assumption was tested by including the taper angle in later models. The taper angle was 0.37° which causes a variation in the y-direction thickness (going from 8.01mm at the bottom to 8.58mm at the top). All the other dimensions are identical to the parallel model.

To allow for the correct movement in the taper model two coordinate systems were set up to align with the taper angle. This caused some issues with the nodes fixed to move together in-plane due to this condition needing to be applied to the global coordinate system. This caused some face edges to have nodes with more than one coordinate system applied to them which prevented the model from running. To overcome this the outside edges and corners of the section had to be split into separate regions with coordinate systems set to

global if they interacted with the -X or -Z face and to the appropriate taper coordinate system otherwise.

7.6.2.4 Thermal loading of FE models

During the manufacturing process, the burst tube experiences two cycles of thermal loading. The thermal effect has a relatively small influence on silicon carbide fibres compared with aluminium matrix which will change properties due to annealing at the high temperatures. The heating stage for the finite element model follows the actual manufacturing process. When the parts are cooled down to room temperature residual stresses will be induced due to the differences between the properties of the materials, in particular the difference in thermal expansion. At an elevated temperature, the creep effect within the aluminium creates residual stress relaxation. Thus, the cooling stage plays a vital role in the final residual stress within the components. Teixeira-Dias and Menezes (Ballmes et al.), investigated the effect of cooling rates on Al/SiC MMCs. They found that stress and plastic strain influence are significant when cooling rates are below 50K/s and at lower temperatures. The effect on residual stress levels is small when room temperature is reached. The process for cooling the burst tube was the same as for the plates modelled in section 7.4. A correlation greater than 0.75 is considered to be a “strong” correlation between two variables. As good correlation (0.9) in results was obtained for the plates the same cooling and pressure reduction rate were used for these models as well.

7.6.2.5 Modelling changes in the Aluminium temper condition

For all the non-tapered models it is assumed that the aluminium temper condition remains in the T6 condition. From the hardness results presented in chapter 5, this is not true. To check the effect of the change in aluminium conditions during manufacture some of the tapered models included the change from the T6 to the O temper condition.

To achieve this the FE analysis of the manufacturing process was divided into heat-up and cool-down processes. Three analysis methods were used in total to determine which gives the most realistic results, as shown in Table 7.5.

Table 7.5 Finite element analysis methods for component with [0/90/0]_s fibre alignments

Stages	Method 1	Method 2	Method 3
Stage 1: Heat-up	Aluminium (T6) Mortar contact	Aluminium (T6) with Mortar surface contact	Aluminium (T6) with Mortar surface contact
Stage 2: Cool-down		Aluminium (T6) with Mortar tied surface contact	Aluminium (Annealed) with Mortar tied surface contact

Initially, the FE analysis used T6 condition aluminium for both heat-up and cool-down stages (Method 1) with a mortar contact. Various contact types were used but the only one to give realistic flow without run errors was the mortar or mortar-tied contact. The use of mortar tied during the heat-up stage also resulted in non-convergence of the models. During cooldown the component will expand but the materials, due to the manufacturing process, should be bonded. It is important in the model therefore that the surfaces do not come apart during the cooldown stage. The mortar contact does have an element of stickiness but to test whether different results would be obtained the mortar tied contact was used during cooldown (Method 2 and Method 3). Method 1 can be run as a single model but methods 2 and 3 need to have separate models for the heat-up and cool-down stage. Method 1 was run both as a single model covering both stages and as two models covering the two stages separately. To split the models into two stages the stresses and displacements were taken from the 1st stage and used as inputs for the 2nd stage (Using the spring back function within LS-Dyna).

Method 3 applied annealed aluminium properties for the cool-down stage. This was expected to give the most realistic results. From the literature (Ko, Y.G. and Hamad, K., 2017) the aluminium property change (T6 to annealed) will happen after quite a short time at high

temperatures. There was no information found about the annealed properties of aluminium at elevated temperatures so the T6 properties at the high temperature was retained but the final stress strain curve at room temperature were set to be annealed. Although the correct O condition properties were not applied at the high temperature the error is expected to be small as at high temperature, even in the T6 condition, the aluminium is very soft. All the intermediate T6 stress curves were removed so that as the component cooled the model would interpolate between the T6 condition high temperature values and the O condition (annealed) room temperature values.

The results of the three methods are presented for one of the fibre layups in chapter 10 and for subsequent layups the method that gave the most realistic results was used.

7.6.2.6 Applying Burst Pressure after Manufacture

To predict the burst pressure of the manufactured burst tubes a loading pressure was applied to the burst tube models (Stage 3) after the manufacturing stages (Stages 1 & 2). The key changes required to the existing models to mimic a burst tube test are to change the radial and axial pressure loads and adjust the boundary conditions. In particular, the axial pressure must be a function of the internal pressure (based on the relative internal area of the pressure tube to that of the wall thickness area) and the boundary conditions restricting the movement of the top face of the burst tube must be removed. LS-Dyna only allows the death of single point constraints (SPC) if the nodes affected only have one single point constraint applied. On the top face (+Z) edges more than one SPC is applied to the nodes, so the death time option was not feasible. To overcome this, two different methods were employed. In the first method the top boundary condition was replaced by creating a contact on the top face with an extra part that was modelled as a rigid body. The part was then removed by either introducing a death time in the contact or by running a restart file with the contact removed. In the second method fresh models were run where the stresses and displacements were taken from the previous model and used as inputs into the new model (using the spring back function and fresh boundary conditions were applied within LS-Dyna). Although this is a more complicated way to run the models it does give total flexibility to change the conditions for

the internal pressure run of the model. The internal pressure model run times (release of Z boundary constraint and apply pressure and axial load) were also very quick in comparison to the initial model run times (manufacturing process) and so it was much quicker to test changes quickly. Due to these considerations, the fresh model approach was used. As discussed in section 7.6.2.5 the initial models (manufacturing process) were split into two stages with the heat-up and cool-down run separately. This means the total process including internal pressure loading was carried out via three models for each fibre alignment.

The contact was set to be mortar-tied as was applied during the cool-down stage of the previous models. The maximum applied radial pressure was 20MPa and the maximum axial pressure was 6.245 times the applied pressure giving a total value of 124.9MPa.

To investigate the effect of the temper condition on the maximum burst pressure three methods were applied (labelled Method 4, Method 5 and Method 6, to distinguish them from the initial methods used in the manufacturing process, see section 10.2.2.5). Method 4 uses the T6 temper condition for the aluminium material properties during the entire process (T6 in Stages 1, 2 & 3). Method 5 applied annealed properties to the aluminium after the manufacturing phase (T6 in Stages 1 & 2) and immediately before releasing the contact (essentially a process of superposition). This method was done to see if the correct results could be obtained even if the annealed material properties were not applied before cooldown (saving time with the modelling by only doing two stages rather than 3 stages). Method 6 applied the annealed properties during the cooling and then during the burst stage (T6 only in stage 1).

7.6.3 Mesh dependency analysis

A mesh dependency study was performed on the non-tapered models using the mesh structures presented in Figure 7.16.

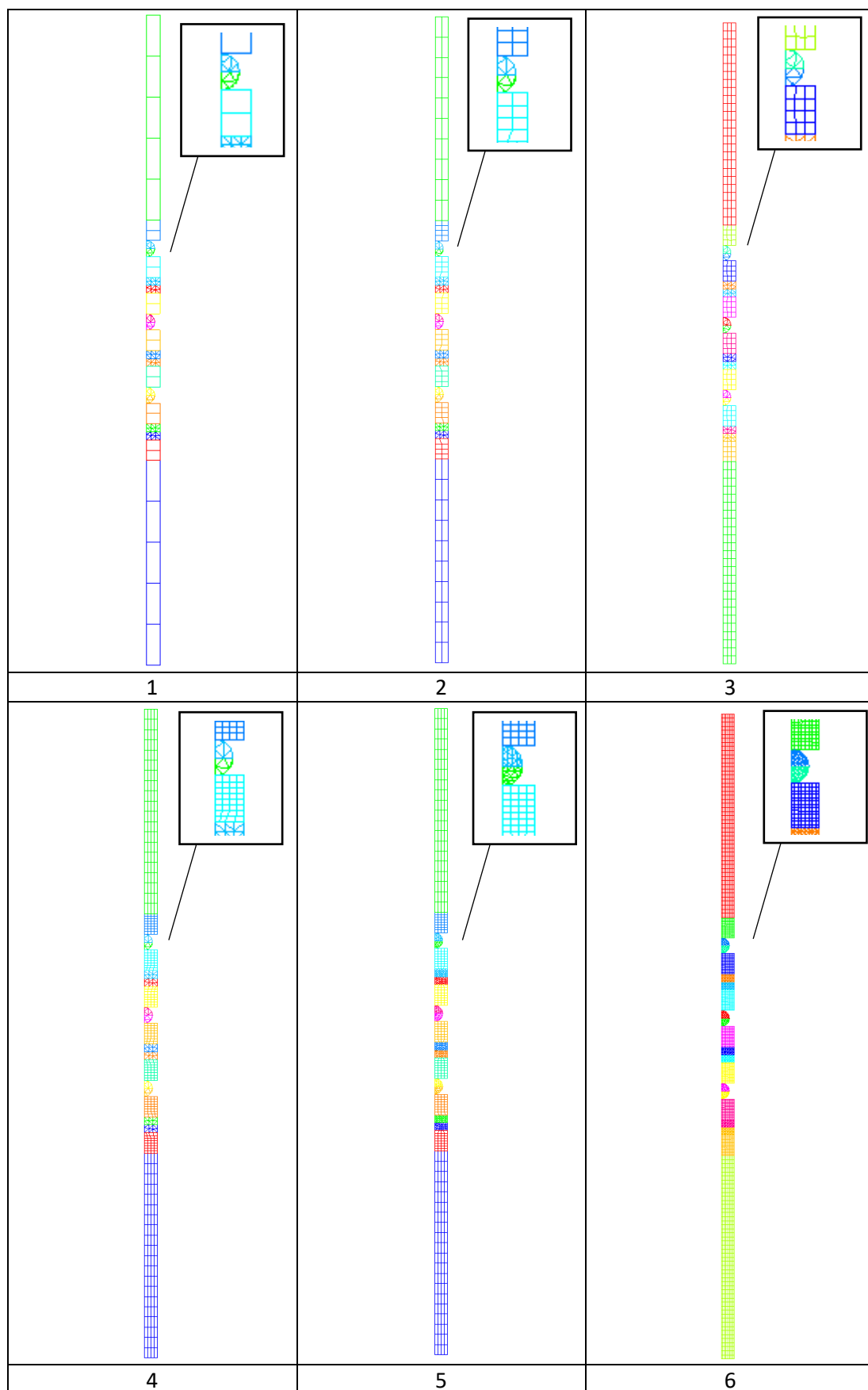


Figure 7.16 Mesh structure of the analysis models

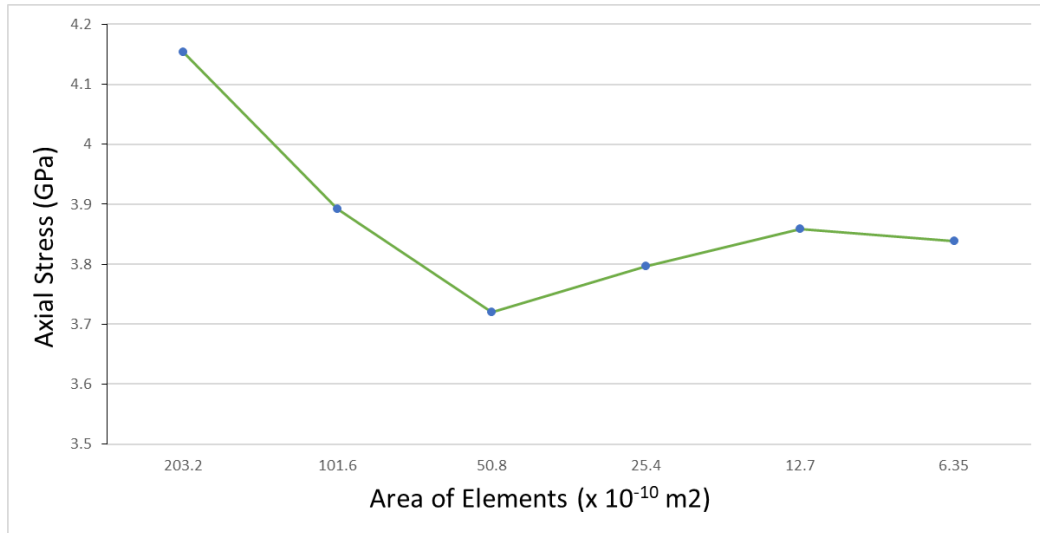


Figure 7.17 FEA results of mesh dependency study

As shown in Figure 7.17, the axial stress results converged as the mesh density increased. As the mesh density increased, the differences of axial stress results decrease from 6% to 0.7%. The last few changes are less than 1% of the final value. Finite element models need to reach a stable state and so meshes using the lower element size values were used for the non-tapered finite element models. A mesh dependency analysis was also carried out for the tapered models with the outcome presented with the overall burst tube model results in chapter 10.

For the tapered model a mesh convergence analysis was conducted using three levels of mesh densities, ranging from 2720 to 5392 elements. The mesh convergence analysis was only conducted for the heat-up stage (stage 1) to reduce runtimes. The results are presented in Figure 7.18. There seems to be good mesh convergence in terms of the X, Y and Von Mises stresses but significant variation in stress in the Z-axis. The reason seems to relate to the mesh density between the fibre and matrix. Visually when looking at the Z stresses around the fibres there are clear stress concentrations around nodal contact points. All other stress results are consistent and realistic; however, more mesh refinement will improve the Z-direction results. As the Von Mises stresses results show good consistency between the models, the results for failure pressure should be accurate. The medium mesh was used in the rest of the FE simulations.

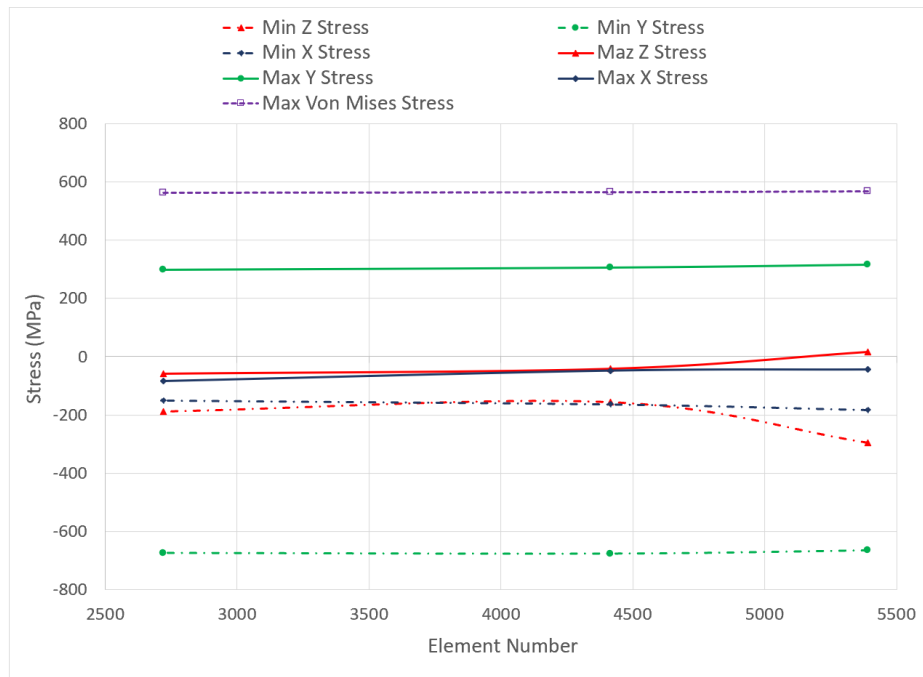


Figure 7.18 Mesh convergence of FE model with [0/90/0]_s fibre alignments

7.7 Application of the chapter outputs

FE models have been developed for three different cases. The first models were for elastic bilinear graphite-epoxy composite plates. These models were developed to see if the developed 2D FE/Curvature method would yield similar curvature results to those found in literature for identical layup, material properties, dimensions, and thermal loading. The second models introduced plasticity and thermal changes into the aluminium material model so that curvature comparisons with the experimental Aluminium Silicon Carbide plates could be made. Similar 2D models were developed but also 3D cell models were also developed so that variation within the aluminium matrix and fibres could be analysed. The third models developed were for burst tubes. These 3D models implemented the same material behaviour as used for the experimental plate models. The results of the models are presented, compared and analysed in chapters 8, 9 and 10.

CHAPTER 8

FEA LAMINATE CURVATURE RESULTS

CHAPTER 8 FEA LAMINATE CURVATURE RESULTS

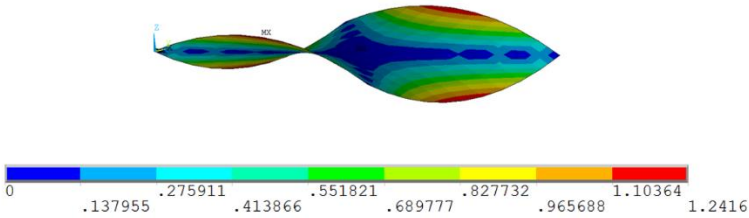
This chapter consists of two sections, the first section presents curvature FE model results for elastic conditions based on modelling thermal loading of graphite-epoxy composite MMC in the literature. The second section presents curvature FE model results, that incorporates creep and plasticity in the Aluminium model, based on modelling the manufacture of Al/SiC MMC experimental plates.

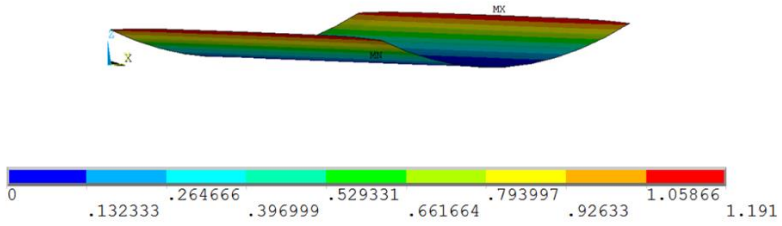
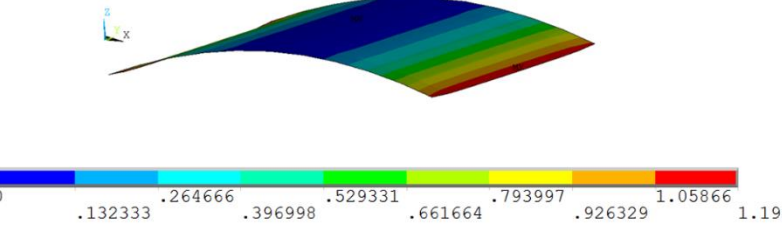
8.1 Elastic composite curvature results

8.1.1 $[90_4/0_4]$ laminate

The FEA curvature results of the $[90_4/0_4]$ laminate are shown in Table 8.1. In columns (b) and (c) of the table, we can see similar curvatures. The two stable cylindrical shapes are rotated 90° relative to each other, about an axis perpendicular to the laminate surface, and inverted, but with the same magnitudes. The figure in column (a) has a saddle shape which is predicted by classical laminate theory.

Table 8.1 Room temperature shapes of a square $[90_4/0_4]$ laminate in millimetre

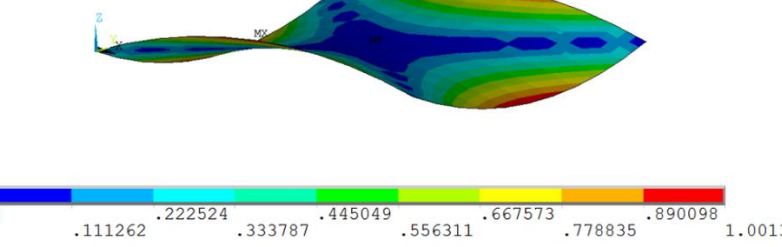
	$[90_4/0_4]$
(a) Unstable saddle shape	 The figure shows a 3D visualization of the curvature of a square $[90_4/0_4]$ laminate. The shape is a saddle, with a central point of zero curvature (blue) and curved edges. The color bar below the plot provides the following values: 0, .137955, .275911, .413866, .551821, .689777, .827732, .965688, 1.10364, 1.2416.

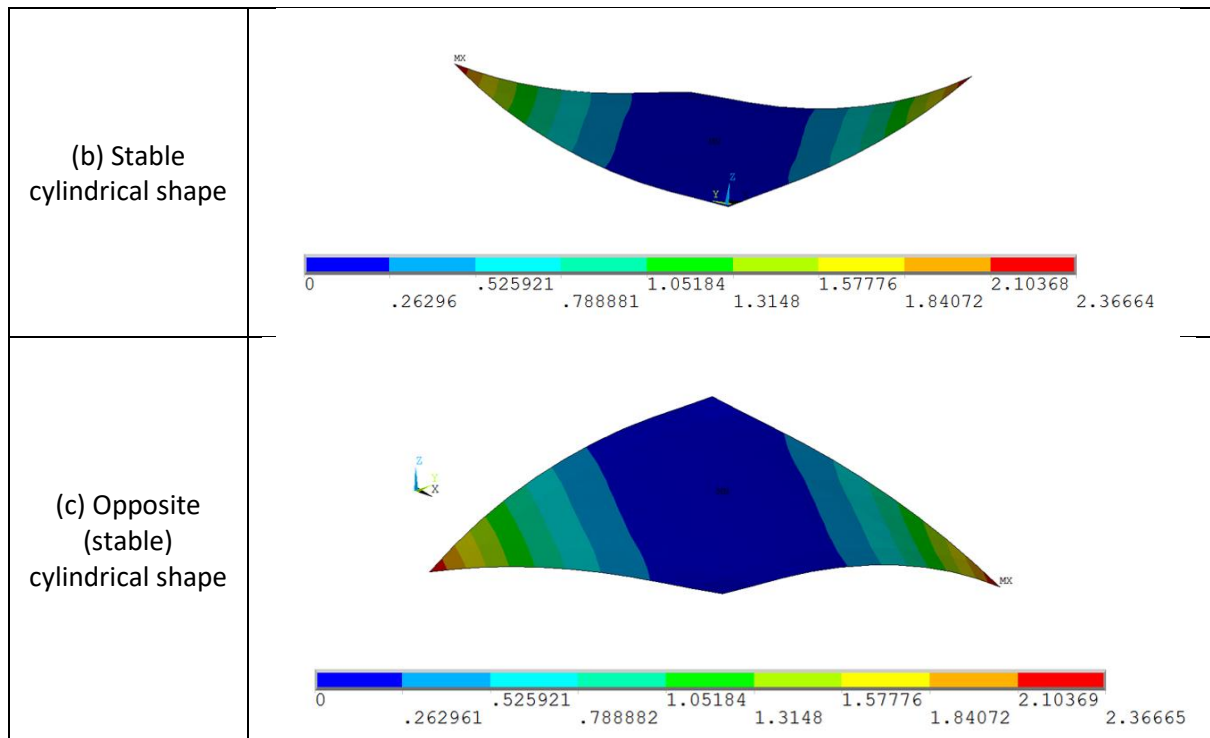
(b) Stable cylindrical shape	
(c) Opposite (stable) cylindrical shape	

8.1.2 $[-30_4/30_4]$ laminate

The curvature results of the $[-30_4/30_4]$ laminate are shown in Table 8.2. It can be seen that the laminate has a twisted curvature shape from the figure in column (a) of the table. The two stable cylindrical shapes are rotated 90° relative to each other, about an axis perpendicular to the laminate surface, and inverted, but with the same magnitudes.

Table 8.2 Room temperature shapes of a square $[-30_4/30_4]$ laminate in millimetre

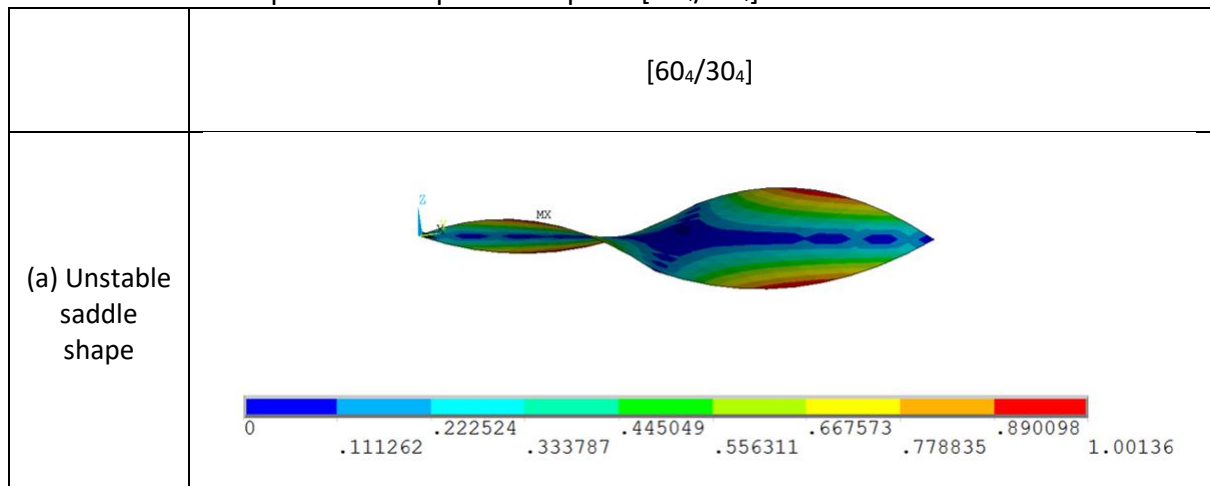
	$[-30_4/30_4]$
(a) Unstable saddle shape	

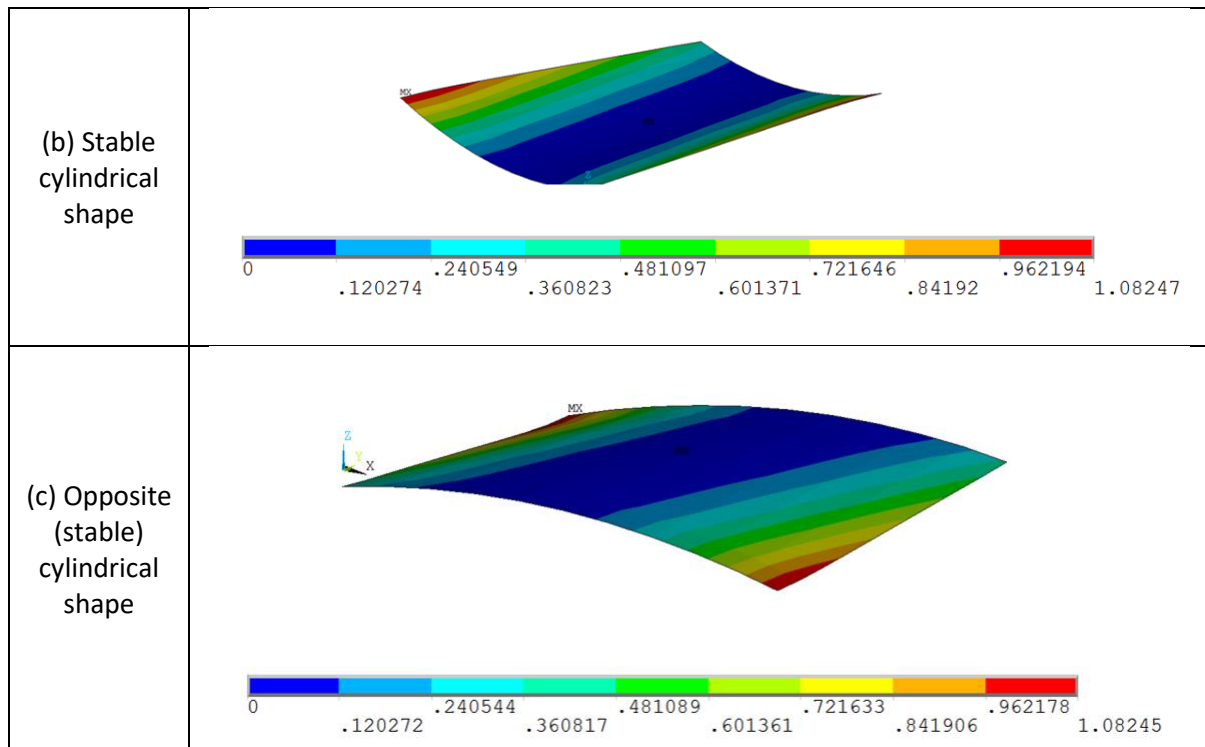


8.1.3 $[60_4/30_4]$ laminate

Table 8.3 presents the unstable and stable curvature shape of the $[60_4/30_4]$ laminate. The two stable cylindrical shapes are rotated 90° relative to each other, about an axis perpendicular to the laminate surface, and inverted, but with the same magnitudes. The unstable shape has a twisted curvature, as seen in the table's figure in column (a).

Table 8.3 Room temperature shapes of a square $[60_4/30_4]$ laminate in millimetre





8.1.4 Bi-stable curvature result analysis

The finite element results are compared with the paper's results (Moore, Ziaei-Rad and Firouzian-Nejad, 2014) in Table 8.4. The classical laminate theory assumes a constant value of curvature along any direction within the composite. However, in the FEA, the curvature is found to vary along with the directions. Thus, the averaged value was used to compare the FEA and paper results' curvature values.

Table 8.4 Matlab surface fitting of laminate FEA results in millimetre

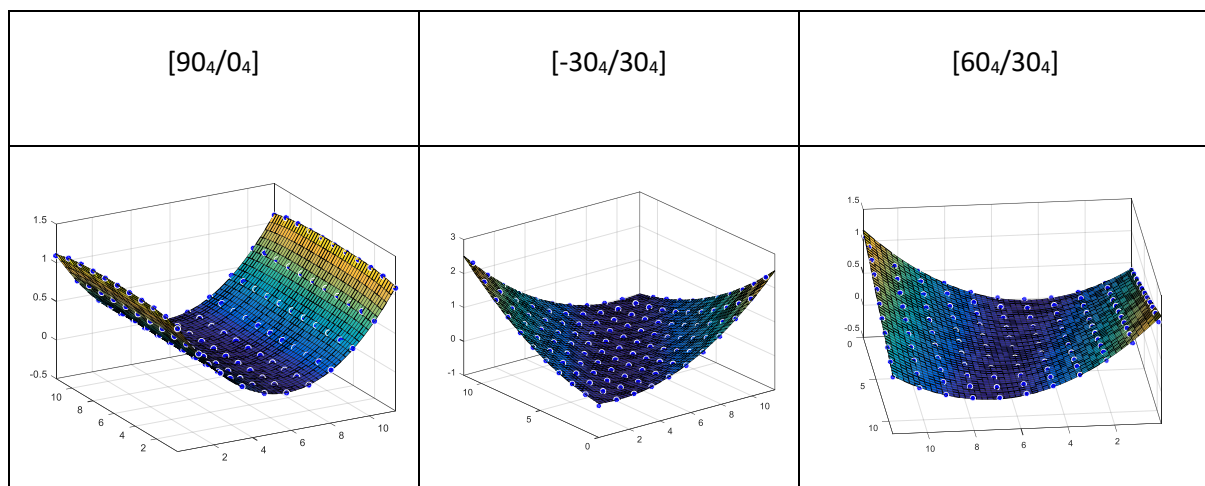


Figure 8.1 demonstrates the Matlab plots of the laminate surface based on FEA surface displacements. Matlab was used to calculate the curvature using the data shown. There is a good match between the Matlab results and the results obtained from the paper. The largest curvature was obtained for the $[-30_4/30_4]$ laminate. In comparison, the curvature values from the $[90_4/0_4]$ and $[60_4/30_4]$ laminates are around half of the magnitude of the $[-30_4/30_4]$ laminate.

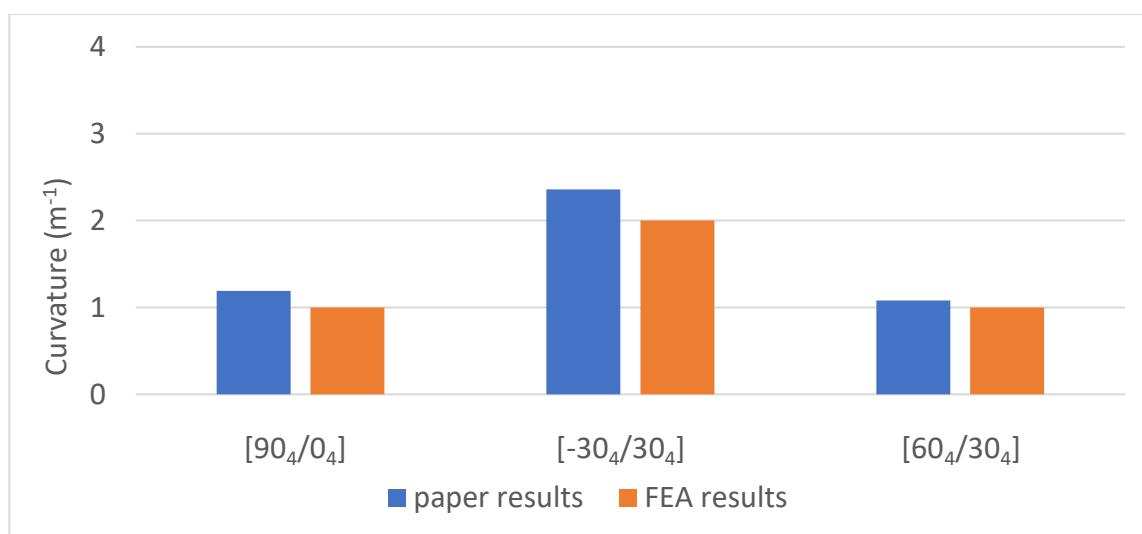


Figure 8.1 Curvature results comparison of laminate composite

8.2 Composite curvature results with creep and plasticity

8.2.1 FEA modelling of SiC/Al composite laminate

In this section results of FEA 2D (shell) and 3D plate cell models are presented. These results are compared with the results of the experimental plates in Chapter 9.

8.2.2 $[90/0]$ and $[+55/-55]$ composite shell models

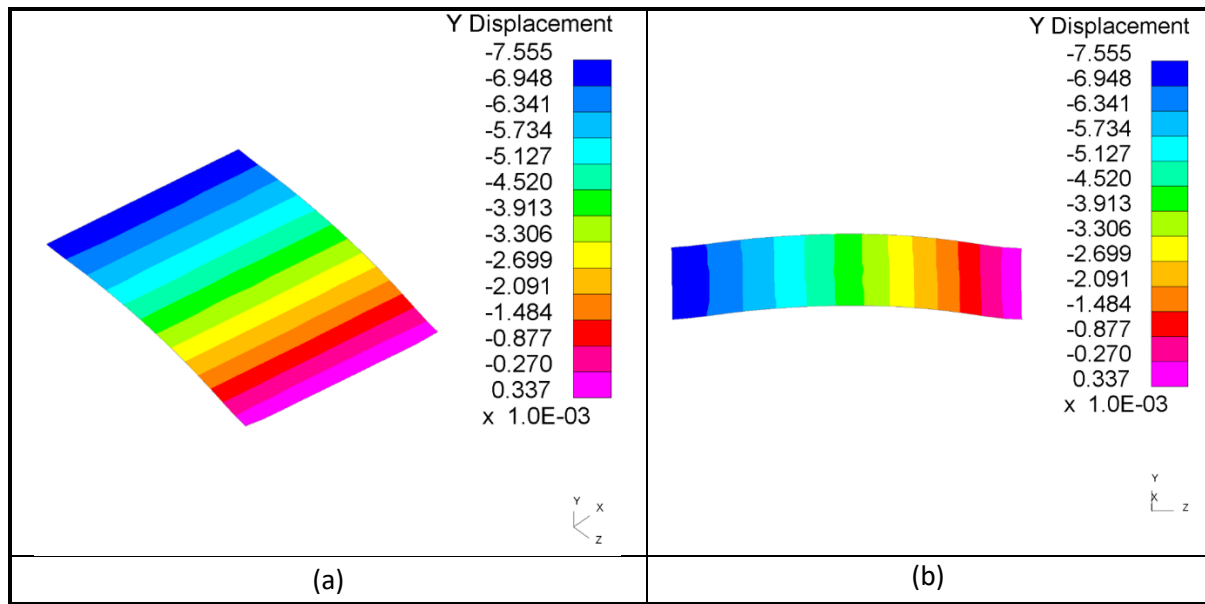


Figure 8.2 [90/0] composite laminate (a)(b) finite element simulation Y displacements

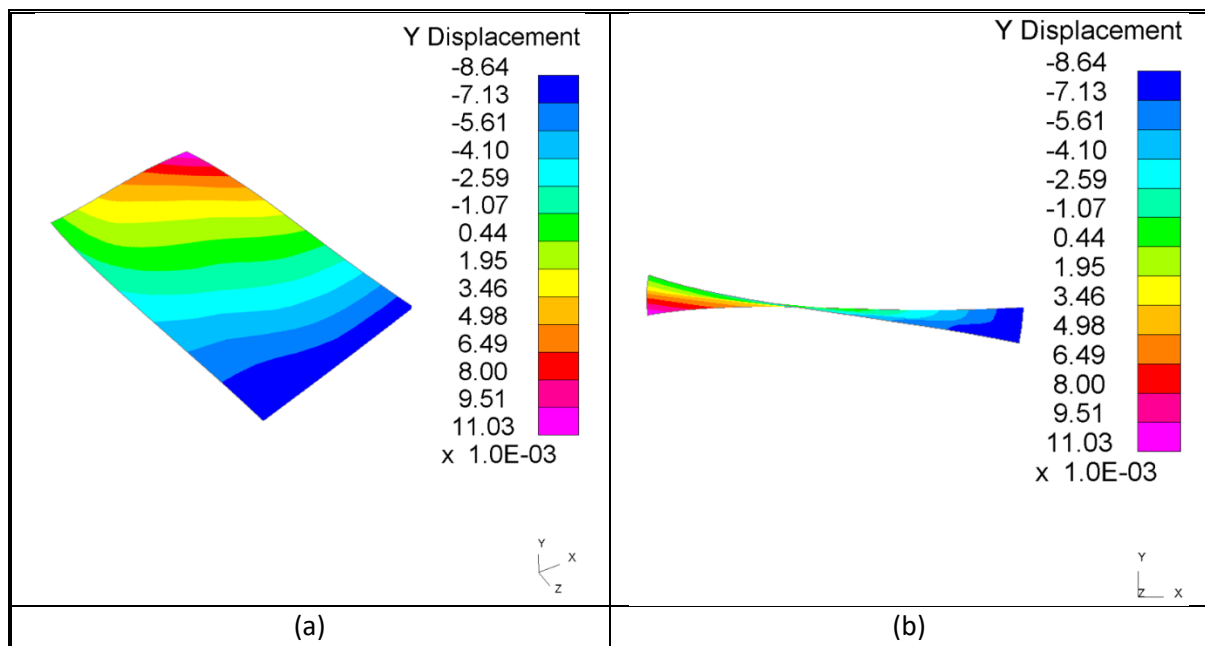


Figure 8.3 [+55/-55] composite laminate (a)(b) finite element simulation Y displacements

Figure 8.2 presents the Y displacement results for the [90/0] composite laminate finite element model. The model has a maximum out of plane displacement of 7.55mm. The displacement results for the [+55/-55] composite laminate finite element model are presented in Figure 8.3. The model has a maximum Y displacement of 11.03mm.

8.2.3 Composite shell models with an aspect ratio

FE models with aspect ratios are presented in this section.

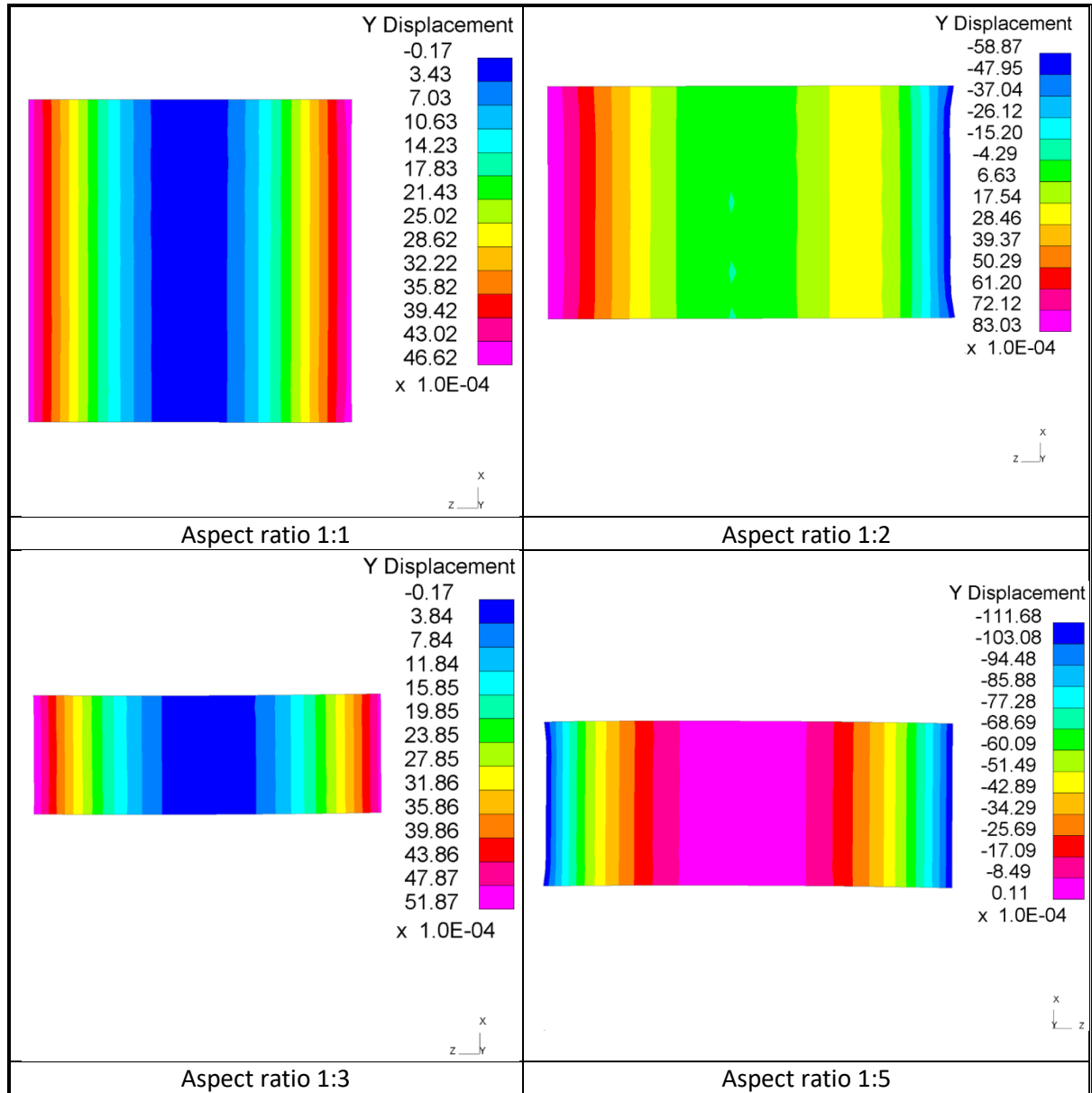


Figure 8.4 Out-of-plane deflections of FEA models with different aspect ratios

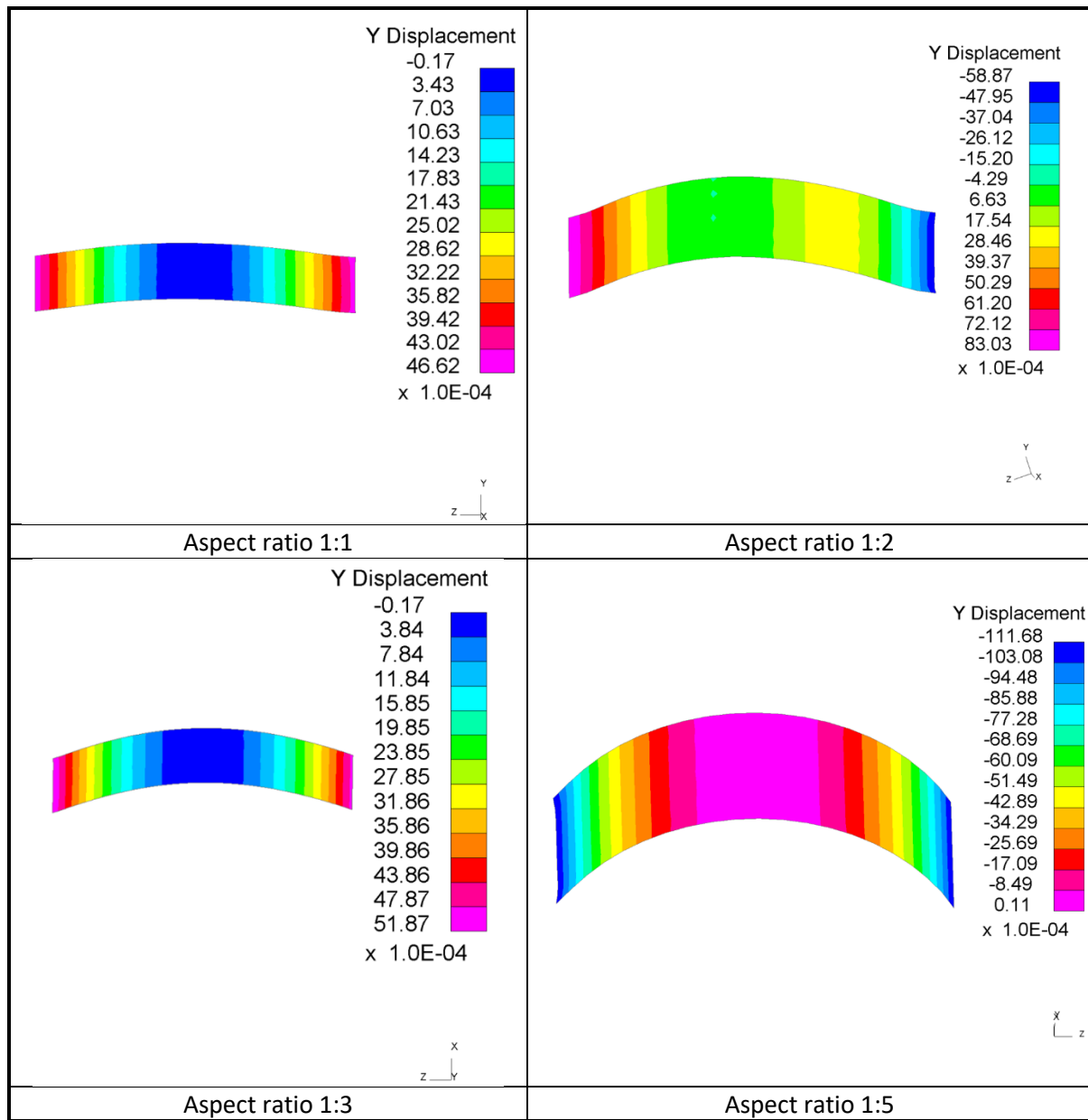
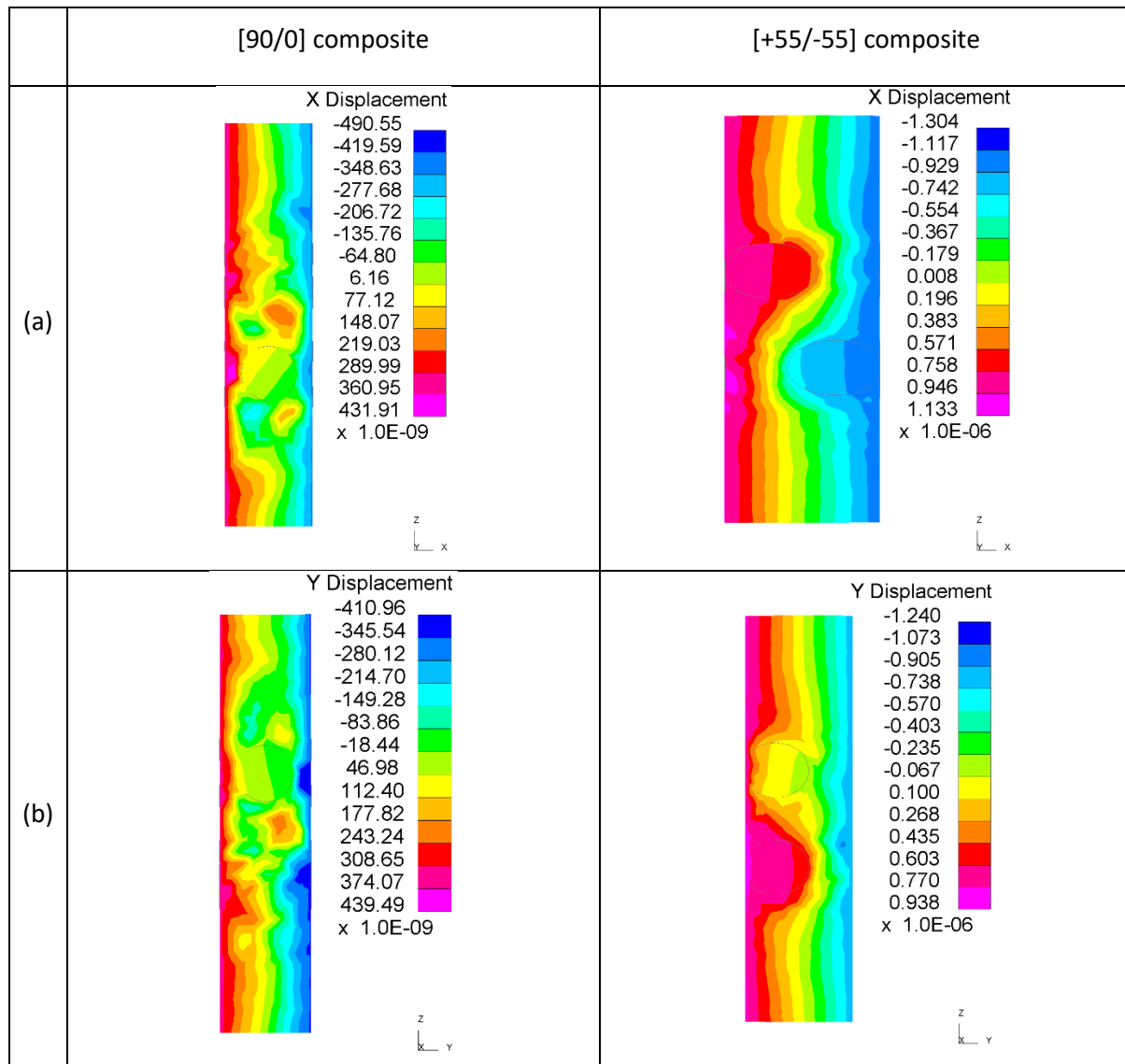


Figure 8.5 Side views of FEA models with different aspect ratios

The FEA simulation results of curvature models are shown in Figure 8.4. Large out-of-plane deflections (Y-displacement) were observed from Figure 8.5. FE model with an aspect ratio of 1:1 has a Y displacement of 0.99 mm. Models with an aspect ratio of 1:2 and 1:3 have out of plane deflections of 3.47mm and 5.18mm, respectively. The model with the largest aspect ratio of 1:5 also has the maximum Y-displacement, which is 8.759mm.

8.2.4 [90/0] and [+55/-55] composite 3D models

In this section the results of [90/0] and [+55/-55] composite unit cell models, comprising two layers of fibre reinforcements with an aluminium alloy matrix, are presented.



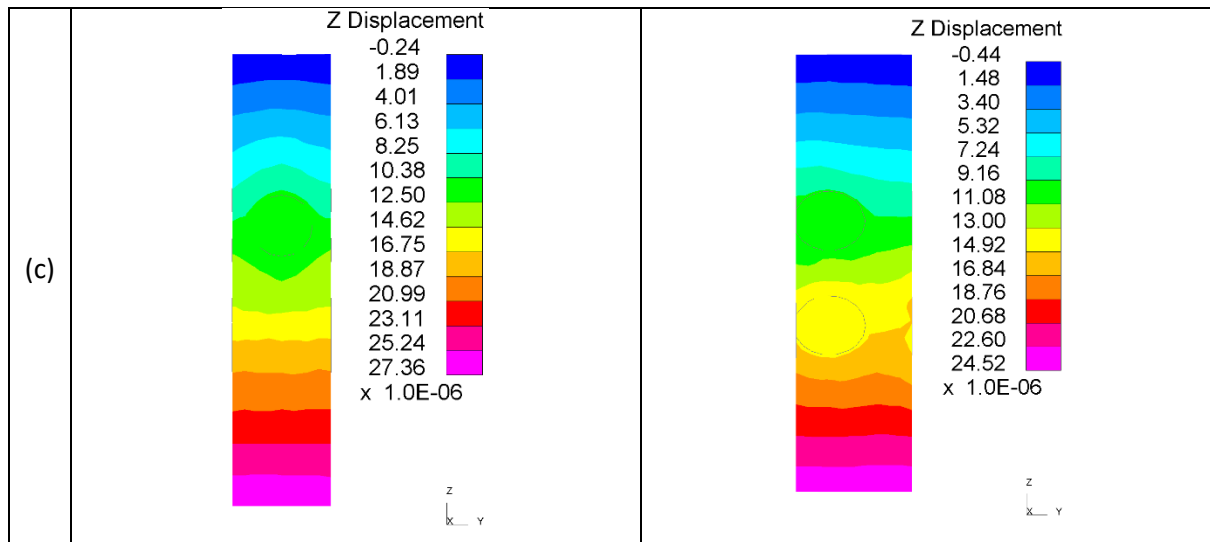
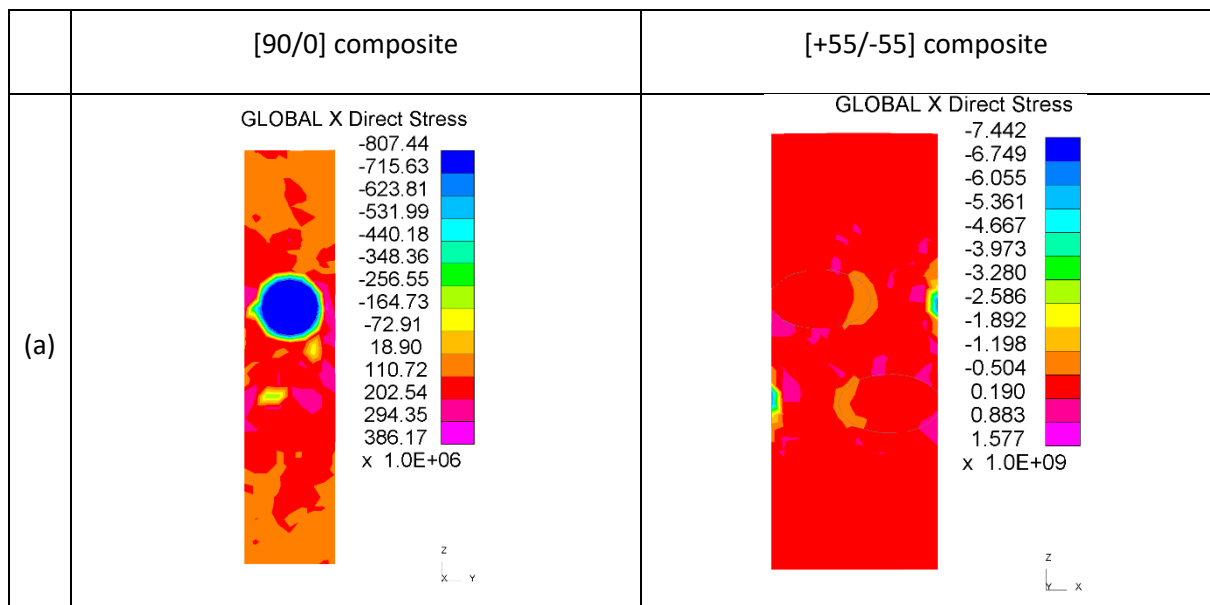


Figure 8.6 Displacement of [90/0] and [+55/-55] composite model at (a) X direction (b) Y direction (c) Z direction

Figure 8.6 shows that the composite models have uniform deformation at room temperature. The magnitude of the [90/0] composite model for X, Y and Z displacements is $0.922\mu\text{m}$, $0.850\mu\text{m}$ and $27.6\mu\text{m}$, respectively. For the [+55/-55] composite model, X, Y and Z displacements are $2.43\mu\text{m}$, $2.17\mu\text{m}$ and $24.96\mu\text{m}$, respectively.



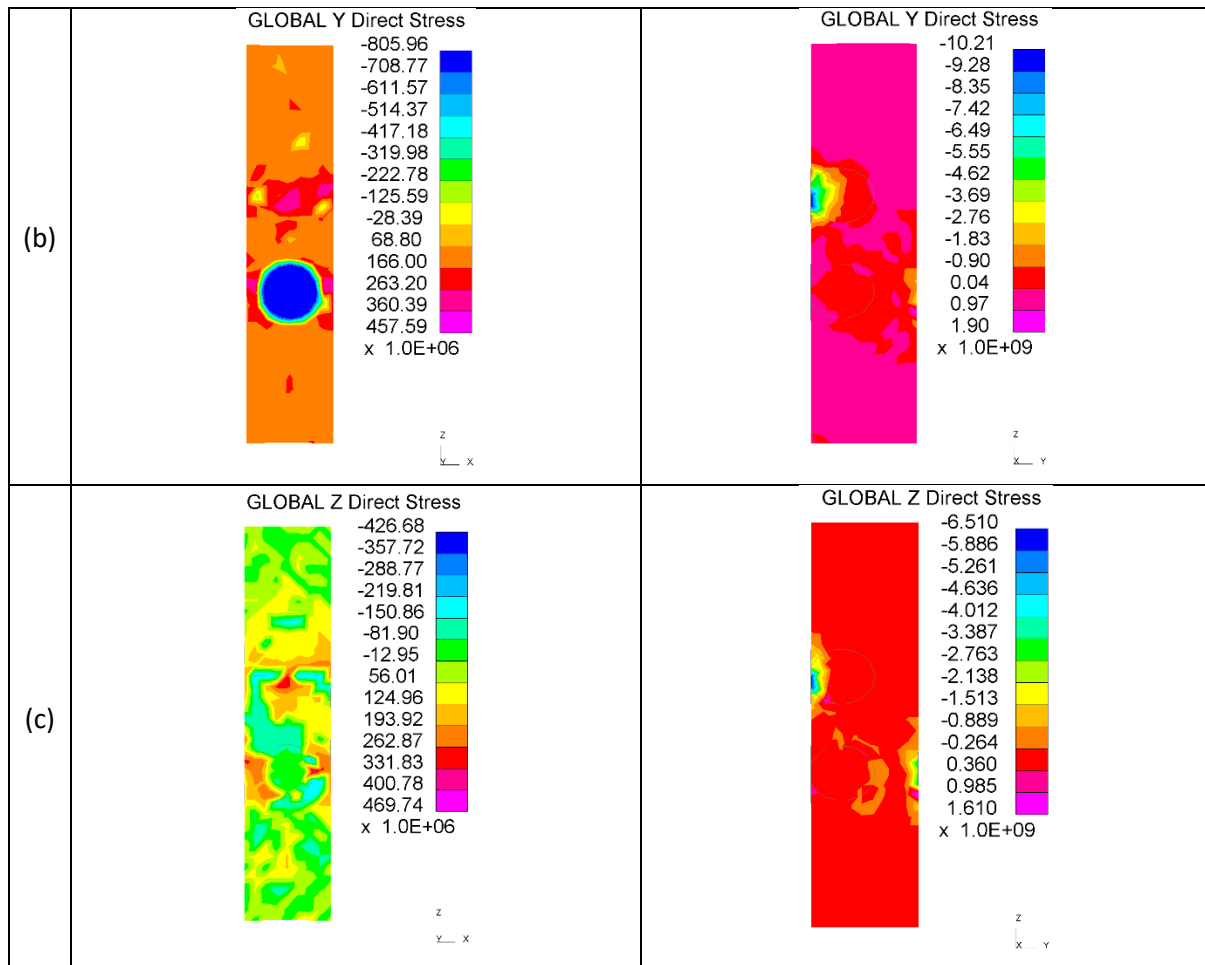


Figure 8.7 Thermal stress of [90/0] and [+55/-55] composite model at (a) X direction (b) Y direction (c) Z direction

Figure 8.7 illustrates the stress of the 3D composite unit cell models with [90/0] and [+55/-55] layups. The [90/0] composite model has a maximum value of 386MPa and a minimum value of -807MPa in direction X. The maximum and minimum stress values in the Y direction are 458MPa and -806MPa, respectively. In the Z direction, the stress has similar values for the maximum and minimum values of 470MPa and -427MPa. The fibre stresses can be seen to be around -800MPa. This value contrast strongly with the fibre stress value obtained from the analytical tensile stress model presented in Chapter 6, which was -3600MPa. The slab model has been shown to give comparable results with FE models that do not include creep, see Appendix E. This shows the importance of including creep in the models.

The [+55/-55] composite model has a maximum value of 1.58GPa and a minimum value of -7.44GPa in direction X. In the Y direction, the maximum and minimum stress values are

1.9GPa and -10.21GPa, respectively. In the Z direction, the stress has similar values for the maximum and minimum values of 1.61GPa and -6.51GPa. The high compression stress values within the [+55/-55] layup model appear to be caused by meshing errors around the corner edge of the unit cell at the intersection between fibre and matrix (as can be seen in the mesh quality when zooming in closely to this region). As these high stresses are highly localised around a couple of points, the model's ability to predict the residual stresses away from these points should not be compromised.

8.3 Application of the chapter outputs

This chapter gives details results of the models developed in chapter 7. Comparison with curvature results from literature with the 2D FE elastic composite plate models is encouraging with similar values and similar trends in terms of layup orientation. Detailed stress and displacement results were presented for the 2D FE and 3D cell FE models of the experimental Al/SiC MMC plates. The validity of the results is analysed in Chapter 9 by determining the plate curvature in the FE models and comparing this with the experimental curvature results.

CHAPTER 9

VALIDATION OF FEA MODELS WITH EXPERIMENTAL DATA

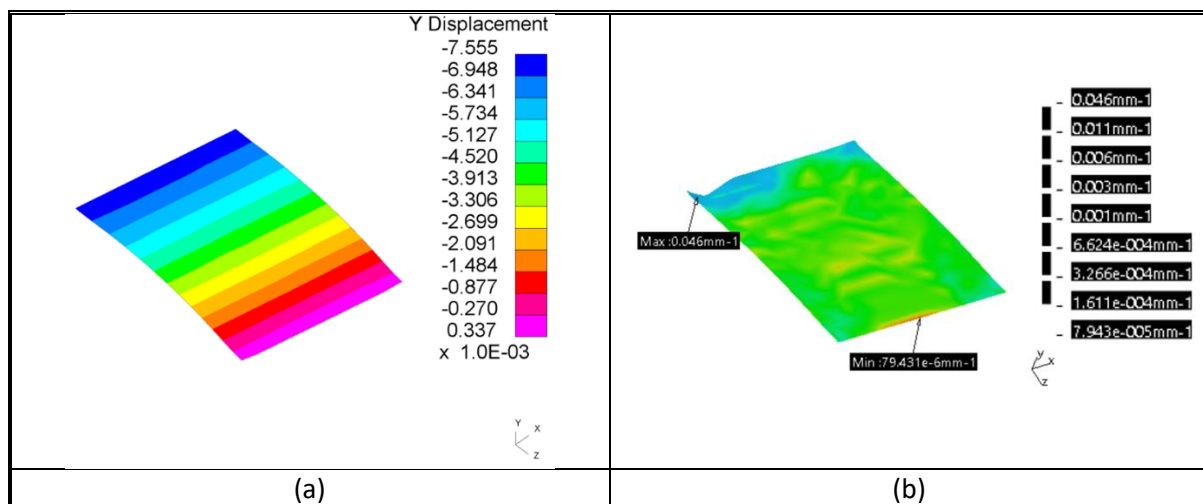
CHAPTER 9 VALIDATION OF FEA MODELS WITH EXPERIMENTAL DATA

9.1 Introduction

This chapter presents the curvature validation results of the finite element simulations and experimental measured data. Firstly, the comparison results of composite plates with [0/90] and [+55/-55] fibre layups were demonstrated. The effect of aspect ratio on curvature is also investigated using data from the split palm specimens (see chapter 3) and comparing with the FE models.

9.2 Curvature validation analysis of SiC/Al composite specimens

Results of FEA models with [0/90] and [+55/-55] fibre layups were presented in section 8.2.2. A method to determine the curvature was presented in section 5.2. In this section the curvature results obtained from the numerical results are compared with the experimental results obtained from section 5.2.1.



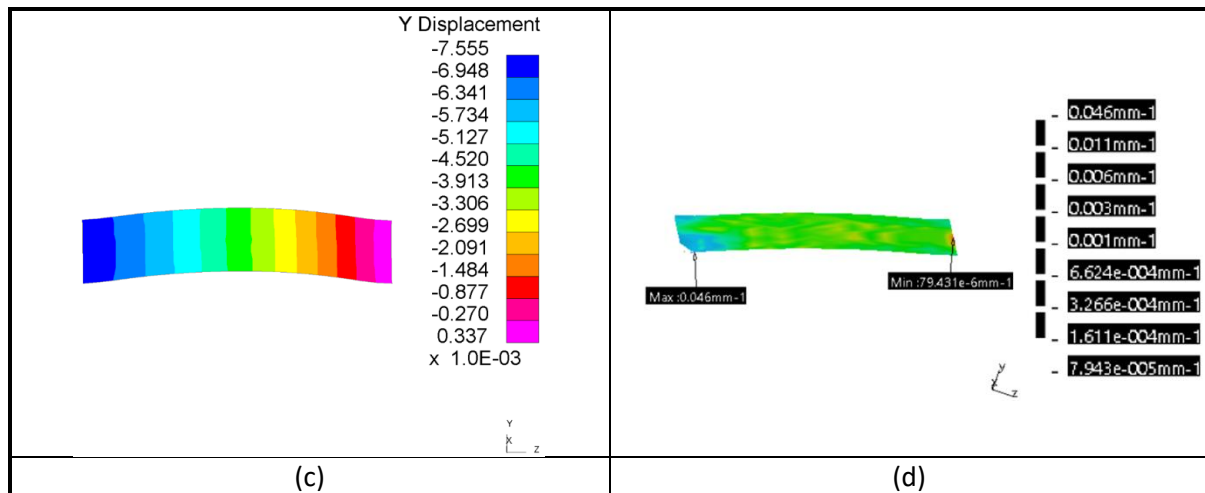


Figure 9.1 [90/0] composite laminate (a)(c) finite element simulation displacement results, (b)(d) curvature analysis of the FEA results

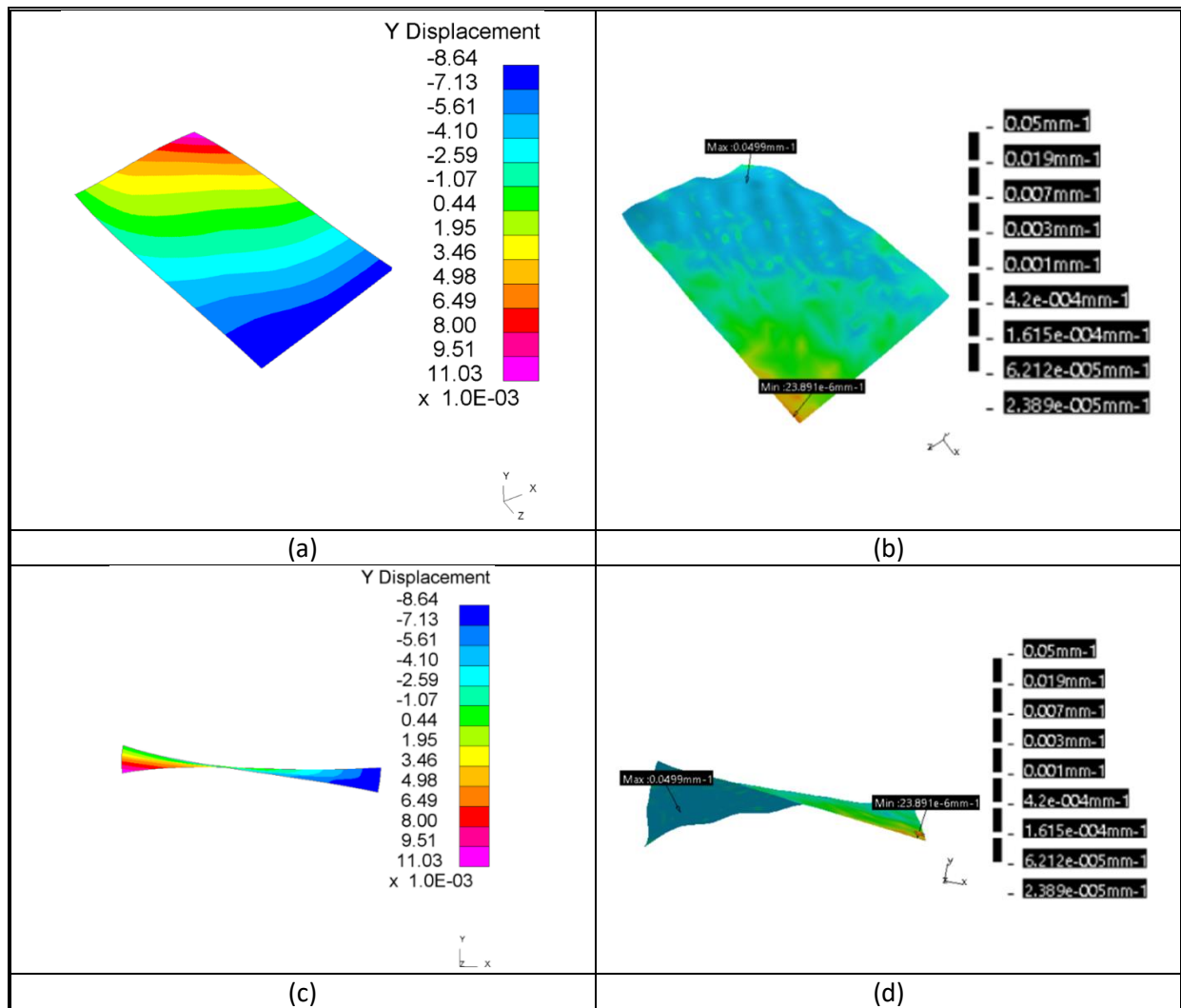
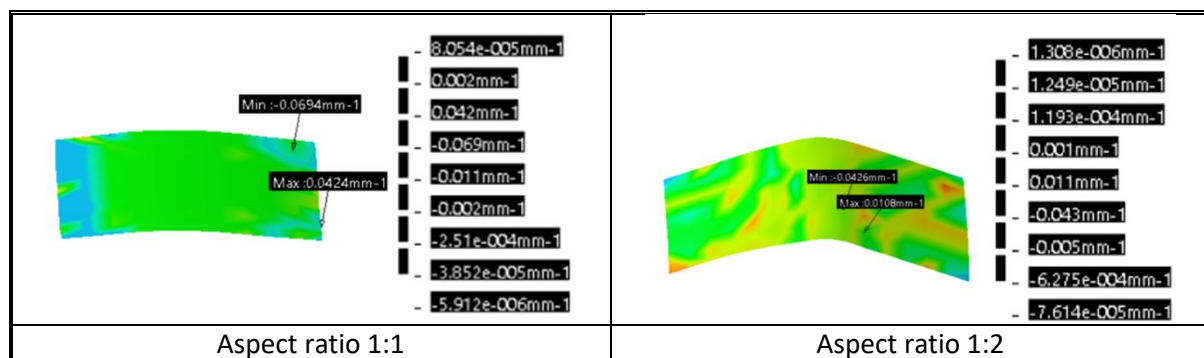


Figure 9.2 [+55/-55] composite laminate (a) (c) finite element simulation displacements results, (b)(d) curvature analysis of the FEA results

Figure 9.1 presents the displacement and curvature results for the [90/0] composite laminate finite element model. The model has a maximum curvature of 46m^{-1} , which is a good match with the experimental data (10% error). The displacement and curvature results for the [+55/-55] composite laminate finite element model are presented in Figure 9.2. The model has a maximum curvature of 49.9m^{-1} , which is a good match with the experiment results (11% error). The difference in the results could be down to manufacturing geometry variations and cooling process variations. Overall, the curvature results of the finite element simulations give good confidence in the ability of the finite element models to predict the correct residual stresses.

9.3 Aspect ratio analysis of SiC/Al composite

FEA models were created in section 7.4 to see if the curvature results concerning aspect ratio could be predicted correctly. The dimensions of the FEA models are based on the data from Table 5.1. The models have the same dimensions and the same aspect ratios as the specimens. Displacement results of the FEA models are shown in Figure 8.4 and Figure 8.5. Curvature analysis results of experimental specimens with aspect ratios were presented in Figure 5.10. Using the same curvature analysis method discussed in section 5.2, we can obtain the curvatures of the FEA models. Curvature analysis results from the FEA simulations are shown in Figure 9.3. The results are compared with the experimental analysis, as shown in Figure 9.4.



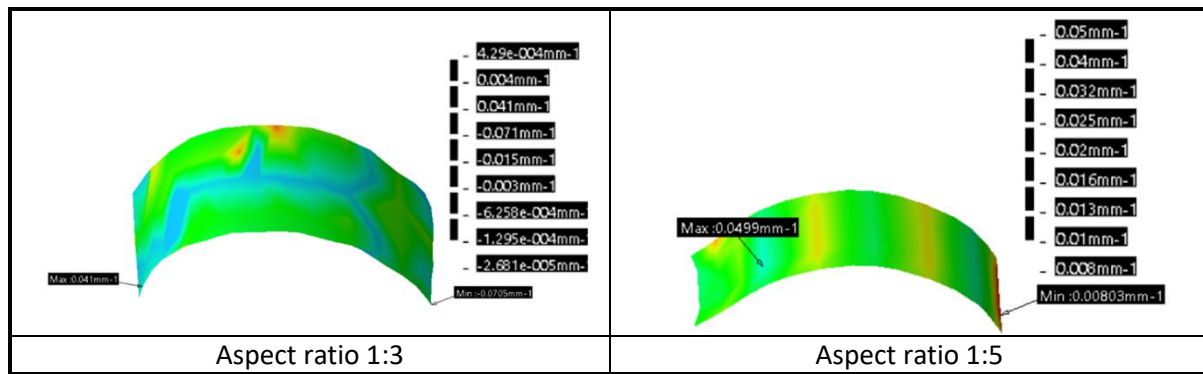


Figure 9.3 Curvature analysis of the FEA results with different aspect ratios

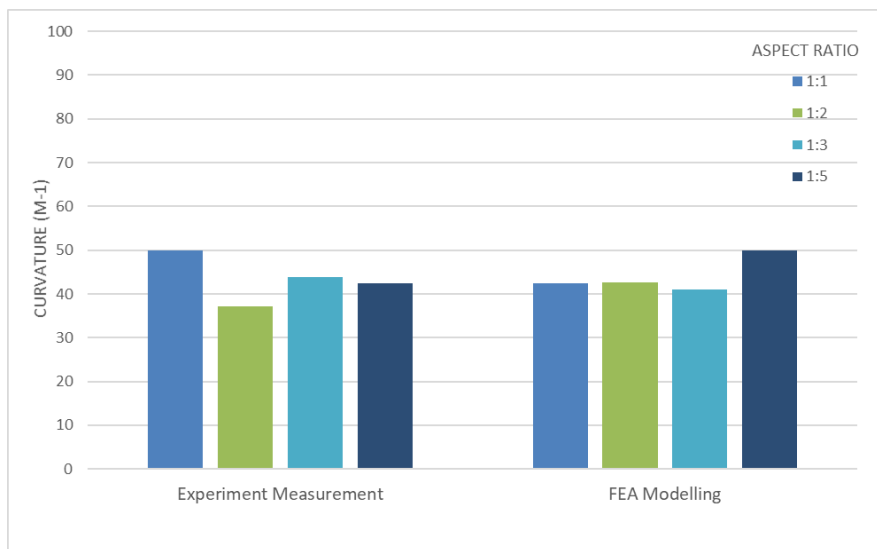


Figure 9.4 Curvature results comparison between experimental measurement and FEA models

As we can see from Figure 9.4, the curvature measurements of the FEA models had an average value of 44m^{-1} for all the aspect ratios. In comparison with the experiment results, the FEA modelling has a smaller average value of (43.4m^{-1}). We expect differences in the FEA simulations and experiment measurement results due to manufacturing variants and machine measurement tolerances. The FE material models are also likely not to model the material behaviour perfectly. Given all these uncertainties, the results demonstrate a surprisingly good match (98.6%) that gives good confidence that the FEA models performed realistically.

9.4 Validation of 2D and 3D models

3D models of [+55/-55] cross-ply layup composite and [90/0] layup composite were presented in section 8.2.4. FEA solid models with identical dimensions and material properties were created to validate the 2D modelling method (section 8.2.2). The FEA out-of-plane results are shown in Figure 9.5 and Figure 9.6.

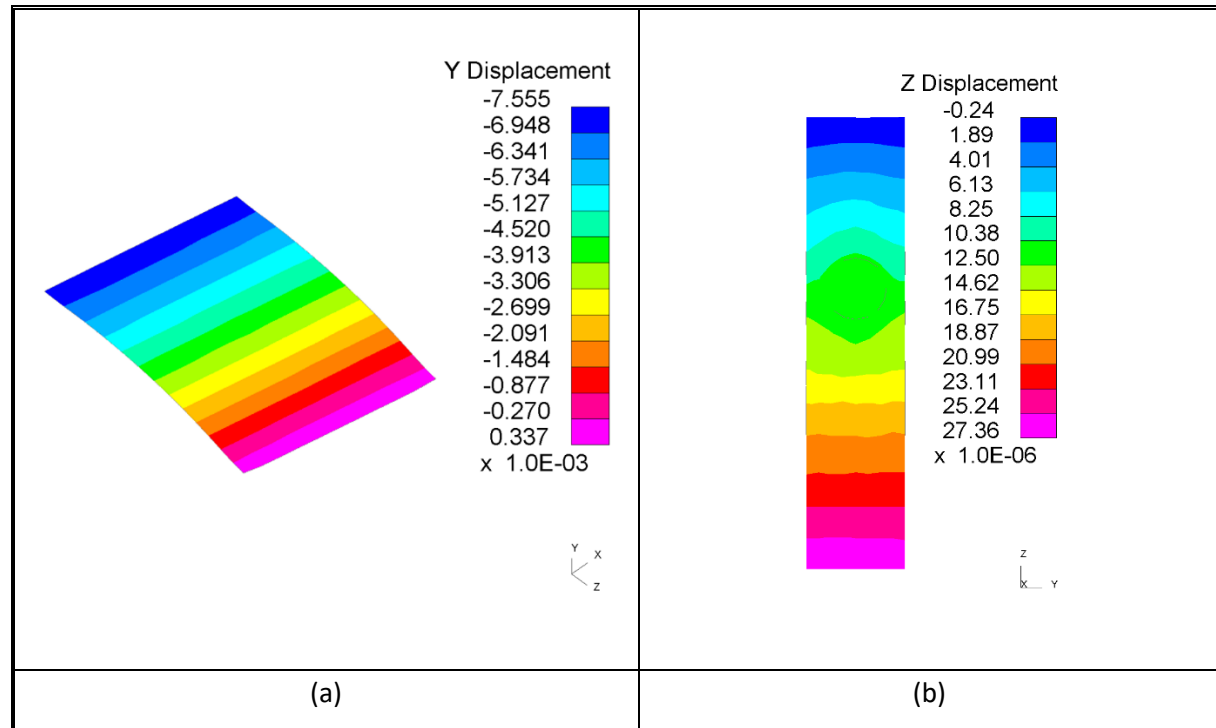


Figure 9.5 Out-of-plane displacement comparison of [90/0] cross ply lay-up composite model (a) two-dimensional model (b) three-dimensional model

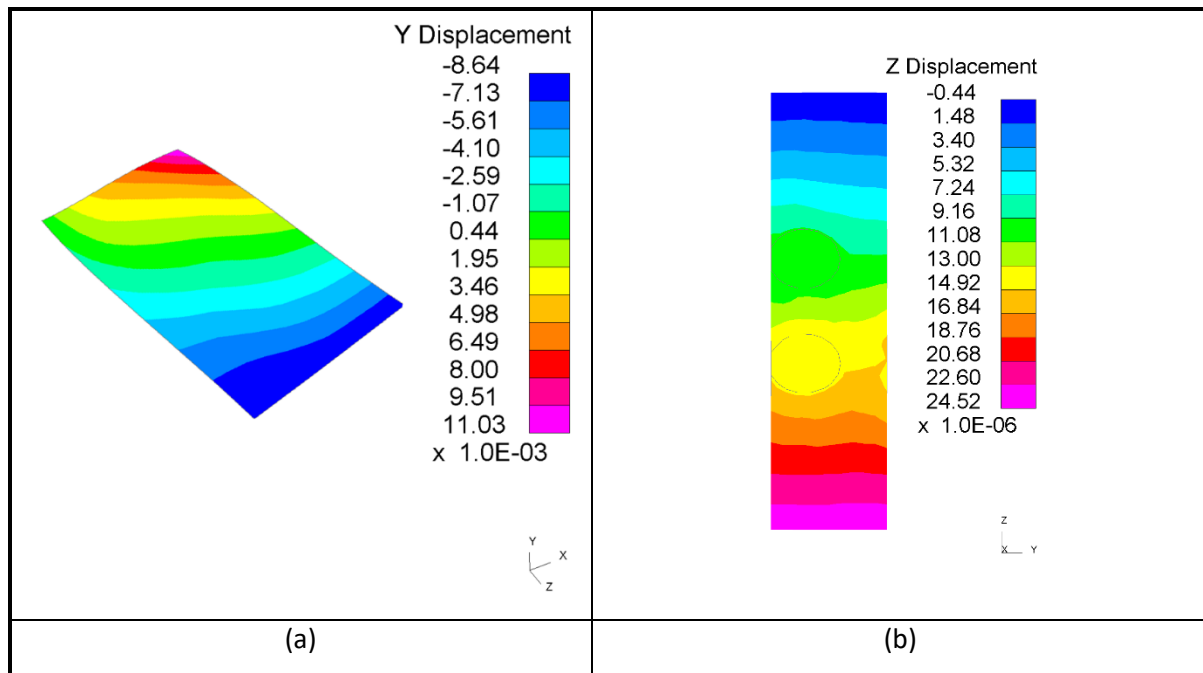


Figure 9.6 Out-of-plane displacement comparison of [+55/-55] lay-up composite model (a) two-dimensional model (b) three-dimensional model

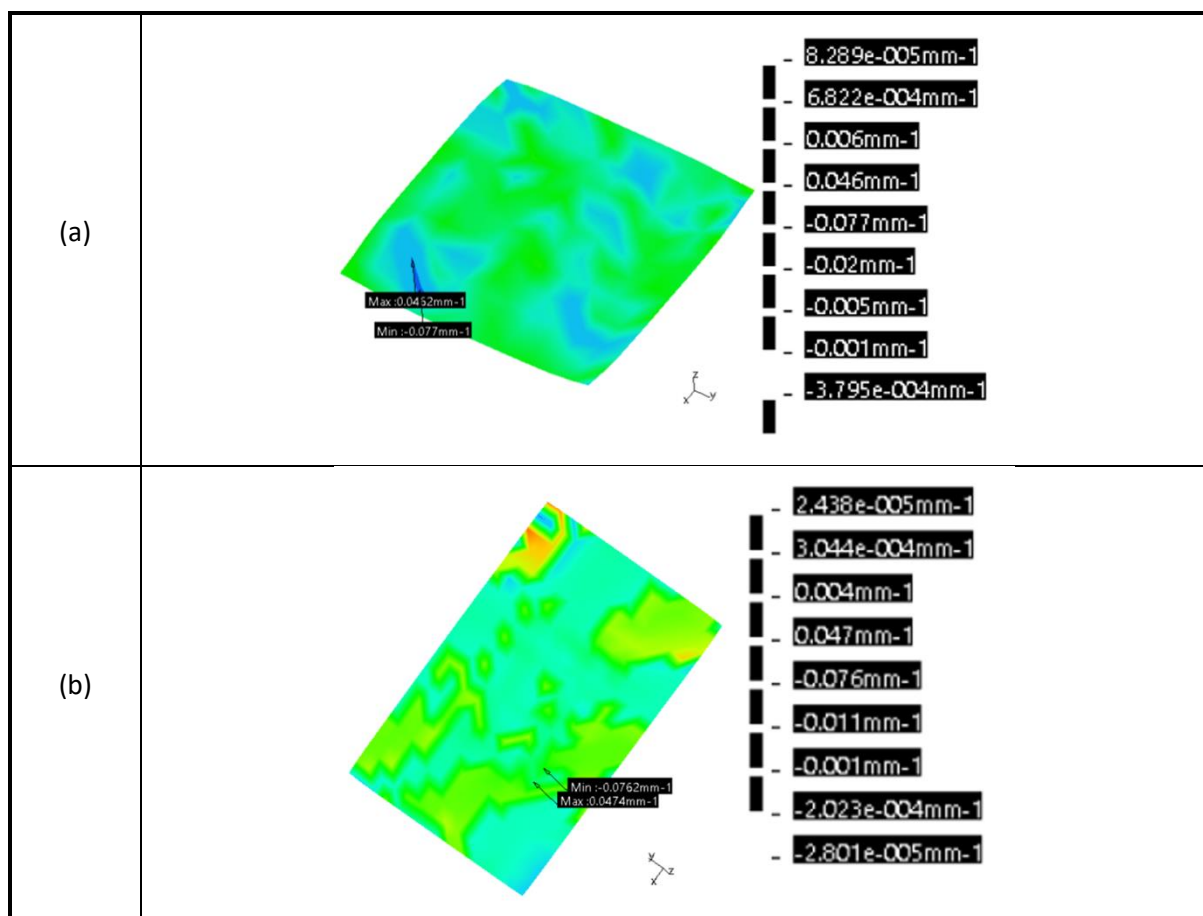


Figure 9.7 Curvature analysis of the FEA results (a) [90/0] composite laminate (b) [+55/-55] composite laminate

Table 9.1 Curvature comparison of out-of-plane curvature between two-dimensional and three-dimensional models

	[0/90]	[+55/-55]
Two-dimensional model	46m ⁻¹	49.9m ⁻¹
Three-dimensional model	46.2m ⁻¹	47.4m ⁻¹

The curvature results from the 3D models are shown in Figure 9.7. The results of curvature from the 2D and 3D models are compared in Table 9.1. The 2D models show a good match with the 3D FE simulations. It means the 3D FE models are performing well, as the 2D FE models showed a good correlation compared to the experimental data. The close results between the 2D and 3D models show that either method could determine the residual stresses. The 2D models are quicker to set up and run, but the 3D models give more information about the residual stress distribution in the matrix around the fibres.

9.5 Discussion of results

Non-symmetric composite plates were manufactured to see the effect that the manufacturing process would have on the curvature of the plates to identify the induced residual stresses. In chapter 7, the FE models developed saddle-like shapes or cylindrical structures depending on the composite aspect ratio. In experiments, cylindrical deformations were observed.

Classic laminate theory predicted that the plates would form saddle shapes after the cooling down process. The FE analysis predicted that both cylindrical and saddle shapes could be formed, as discussed in Chapter 7. As the laminate aspect ratio increased, the saddle shape got shallower, and the composite laminates bifurcate at a critical length. Experimentally, for the non-symmetric plates, cylindrical shapes were obtained after the cooling down process.

From the symmetrical layup composite plates, some samples were cut out with different aspect ratios. Some of these were heat-treated, and then all the samples were further cut

through the midplane by EDM. Although the samples were initially symmetric about the midplane, the cut halves are not symmetric. This causes curvature because of the coefficient of thermal expansion differences in the fibre and matrix.

Two-dimensional finite element shell models were developed and compared with the experimental curvature measurements. The experimental and numerical results showed a good correlation. Three-dimensional models were compared with the two-dimensional shell models, and very good results were obtained between the methods. The validation of the models gives good confidence that the numerical models can be used to predict the residual manufacturing stresses generated in these types of composites. The two-dimensional models can be used to find the overall average stresses in the fibre and matrix materials. Three-dimensional models can be used to find more detailed information about variations of the stress within the component but obviously at the cost of time (in both setting up and running).

CHAPTER 10

FINITE ELEMENT ANALYSIS OF SILICON CARBIDE FIBRE REINFORCED ALUMINIUM ALLOY MATRIX COMPONENT

CHAPTER 10 FINITE ELEMENT ANALYSIS OF SILICON CARBIDE FIBRE REINFORCED ALUMINIUM ALLOY MATRIX COMPONENT

10.1 Introduction

This chapter presents the results of the Burst Tube models developed in chapter 7. The effects of changing the fibre layup and including changes to the Aluminium condition state and taper angle in terms of the residual stress profiles are investigated.

10.2 Finite element analysis results

All the finite element results presented in this section simulated the entire manufacturing process, including hiping. The aluminium foils and fibre layers were placed as they would be before manufacturing (idealised positions), with voiding generated due to gaps within the fibre layers. The FE models nomenclature in terms of fibre layups is presented, such as $[0/90]_3$, where 0 represents a longitudinal fibre and 90 is a hoop fibre (a number after the bracket represents how many times the combination is repeated, and an 's' represents symmetry (mirrored)). An elevated temperature and high pressure were applied which makes the component compress and reduce the voiding. After the hiping process, all the voids should theoretically be removed.

10.2.1 Non-tapered FE models

Initially, as discussed in section 7.6.2, the FE models were created with non-tapered geometry (parallel vertical sides). Dimensions of the fibre reinforcements and aluminium matrix are presented in section 7.6.2.1. The models presented in this section assume the matrix was in the T6 condition through the entire process.

10.2.1.1 Component with $[0]_6$ fibre alignments

Figure 10.1 (a) and (b) display the displacement of the FE model before and after the manufacturing process. It is clear to see that the aluminium foil layers are well compressed around the fibres from the close-up views in Figure 10.2 (a) and (b).

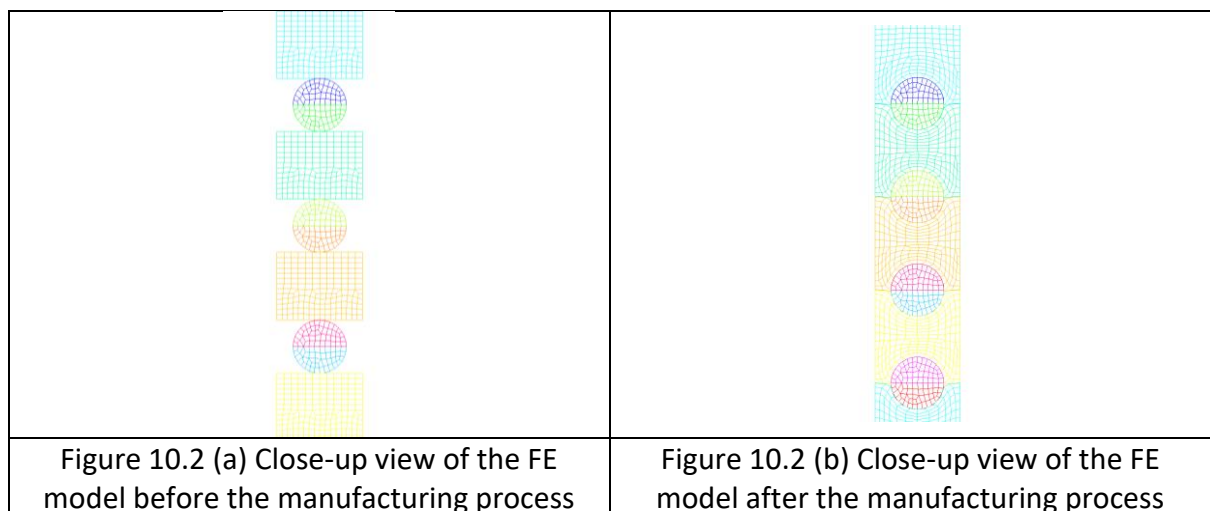
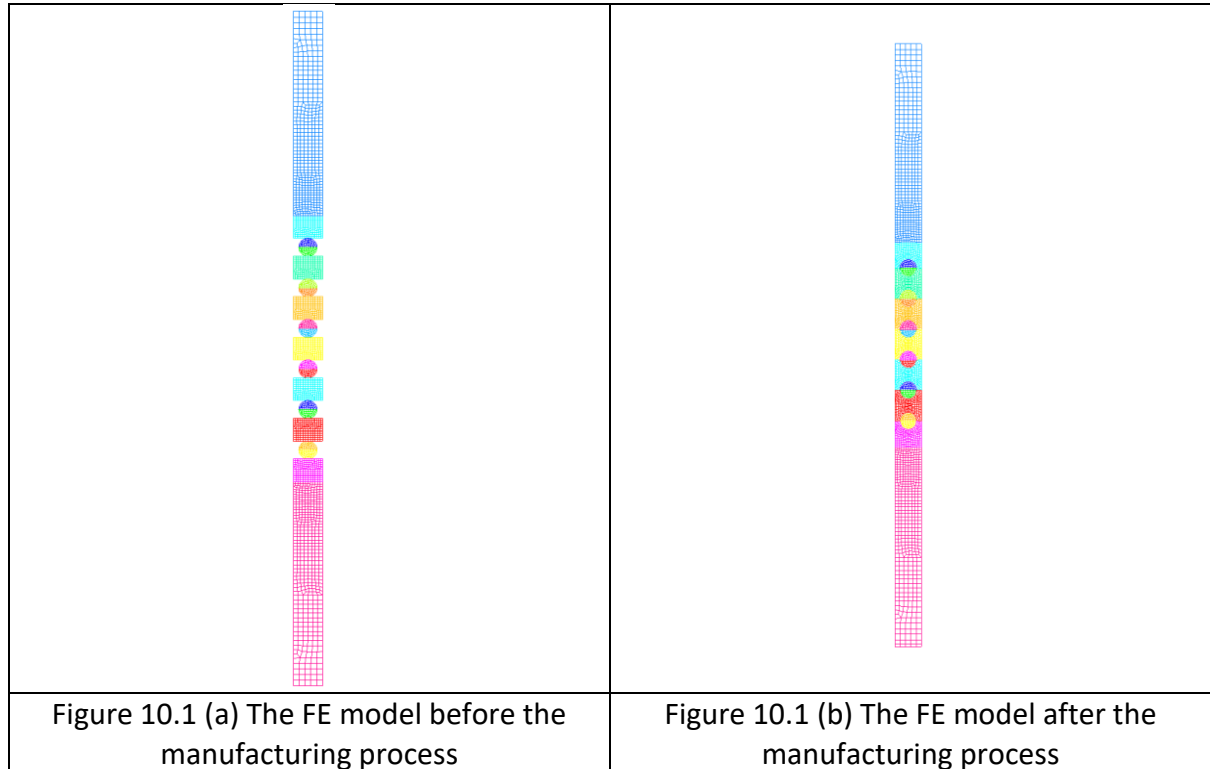


Figure 10.3 presents the axial stress and transverse stress of the composite burst tube model after the heat treatment. The residual stress results of fibre reinforcements are shown in Figure 10.4.

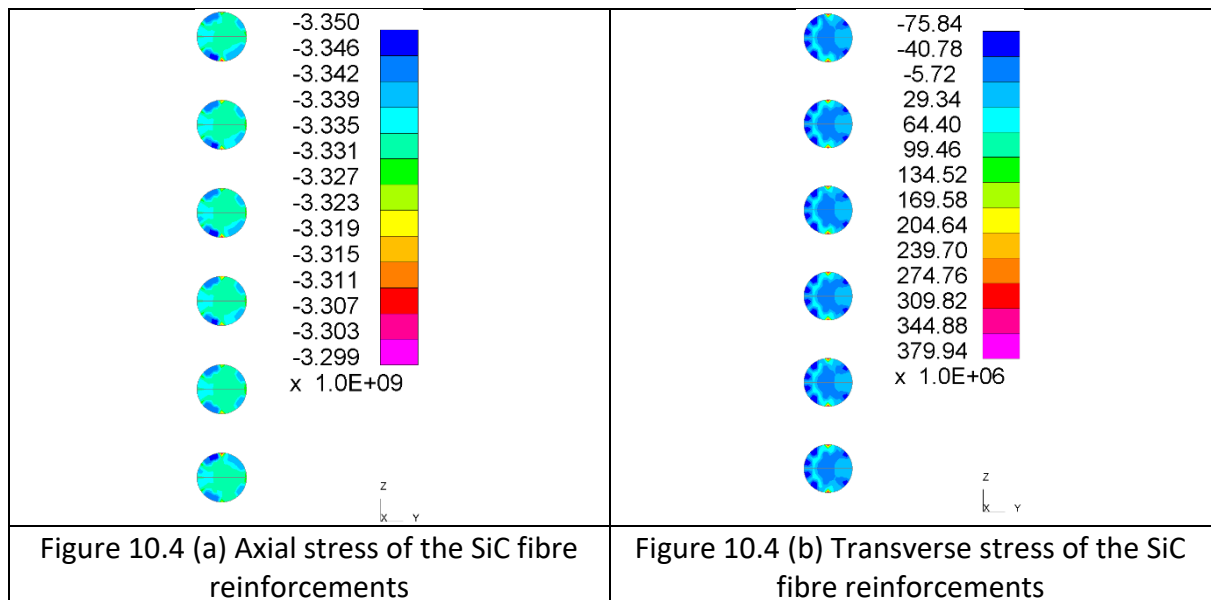
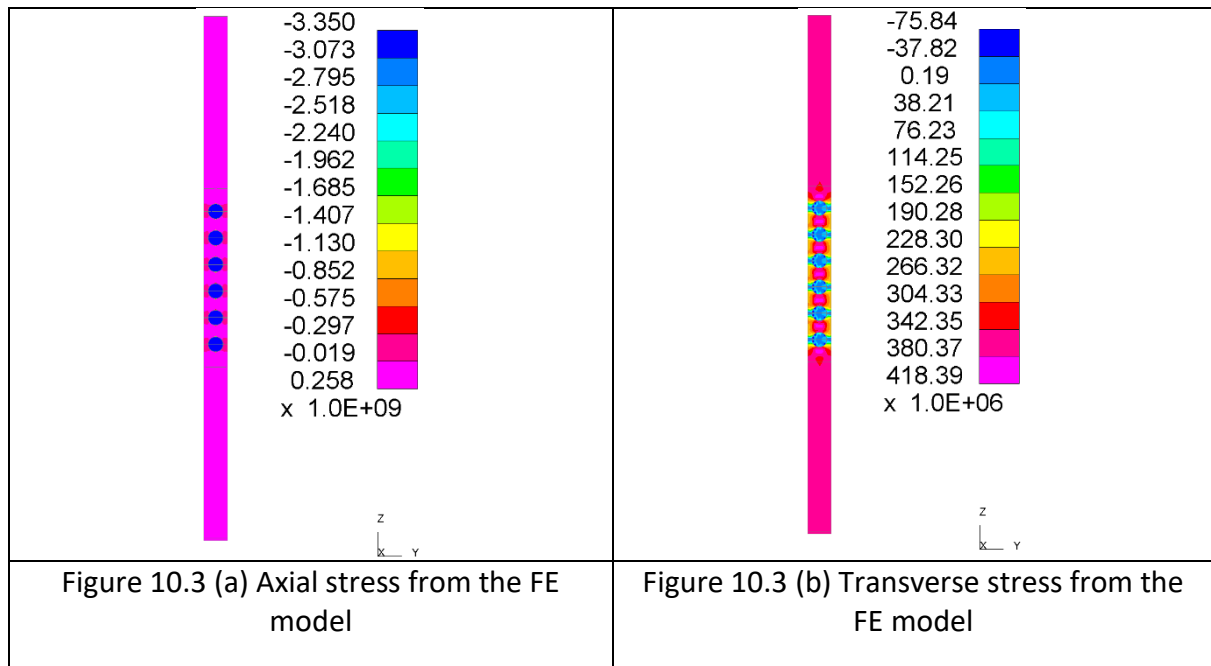


Figure 10.5 shows the axial plastic strain from the FE simulation after the heat treatment. A close-up view of the results is displayed in Figure 10.5 (b). Plastic strain within the aluminium, around the fibres, can be seen.

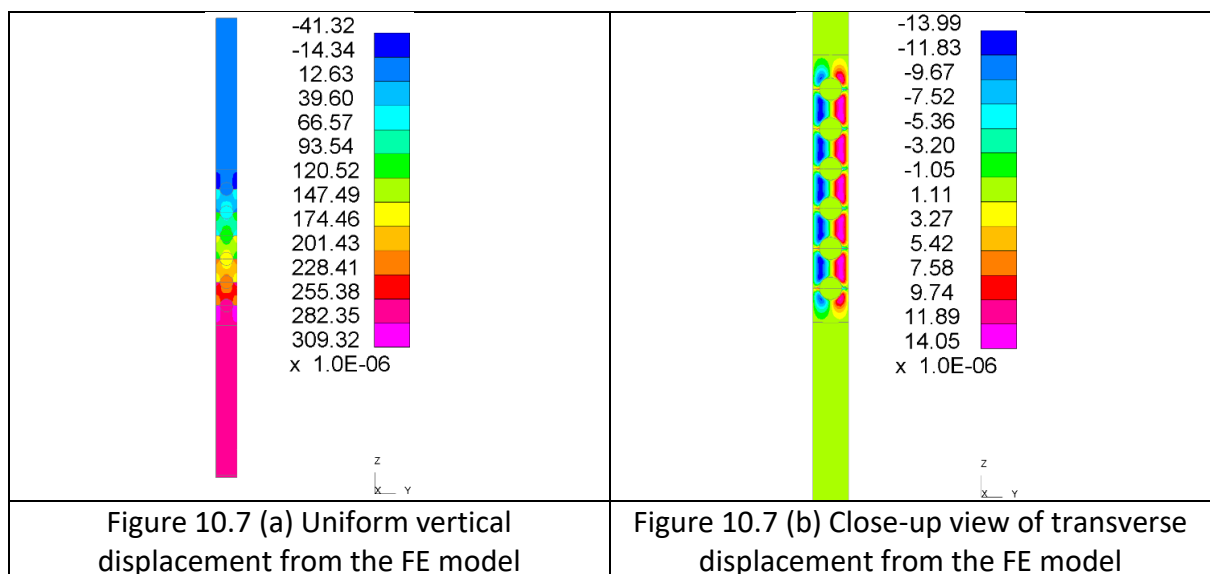
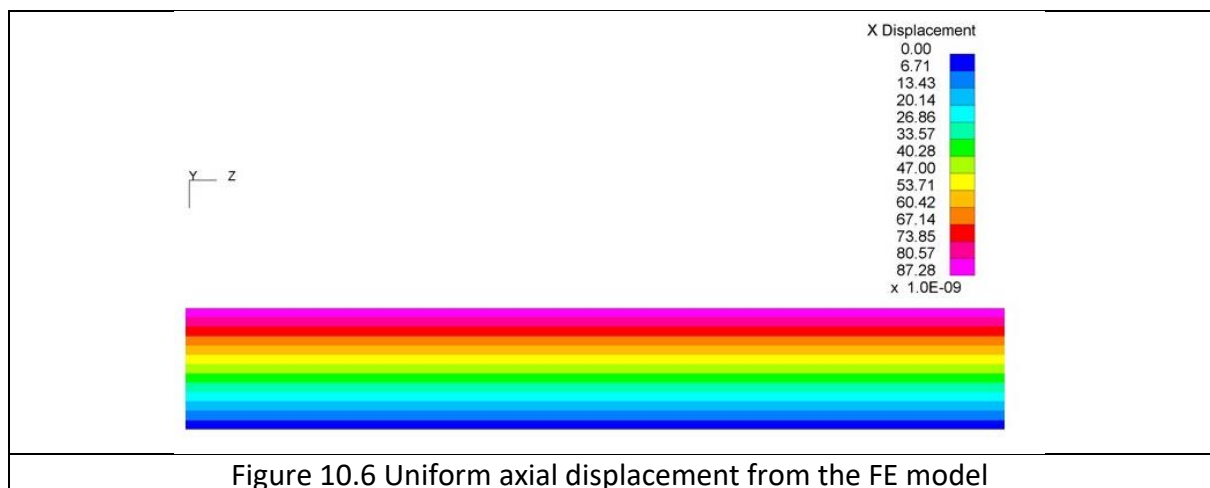
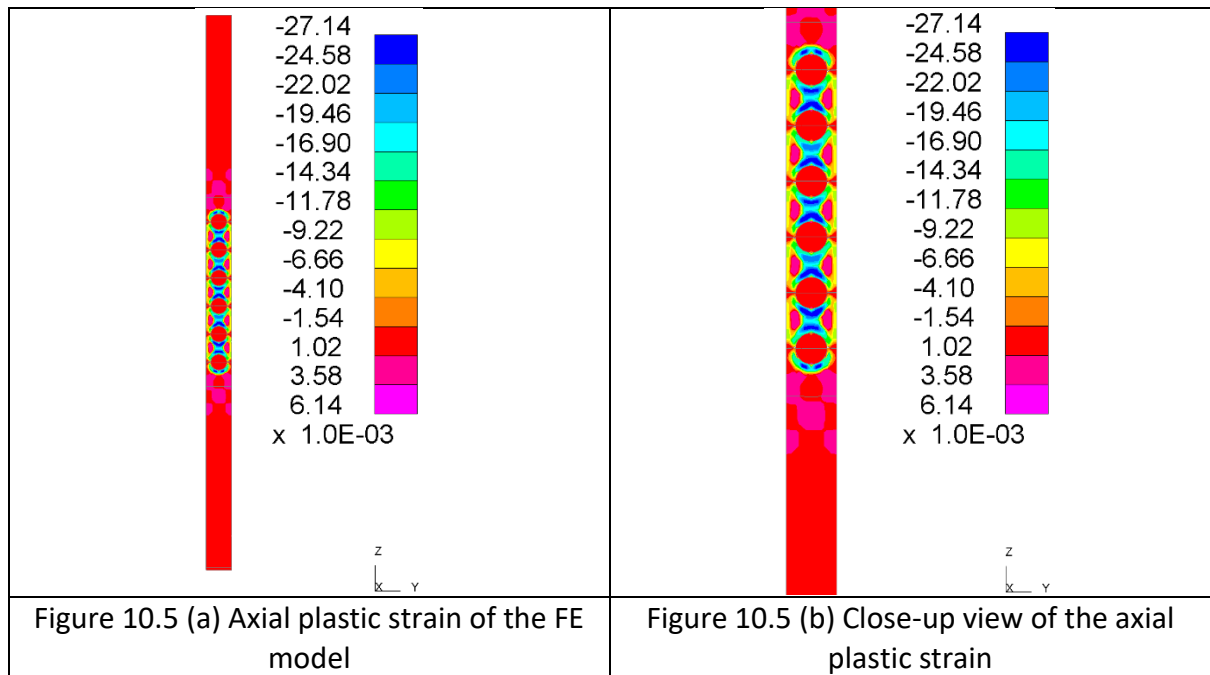
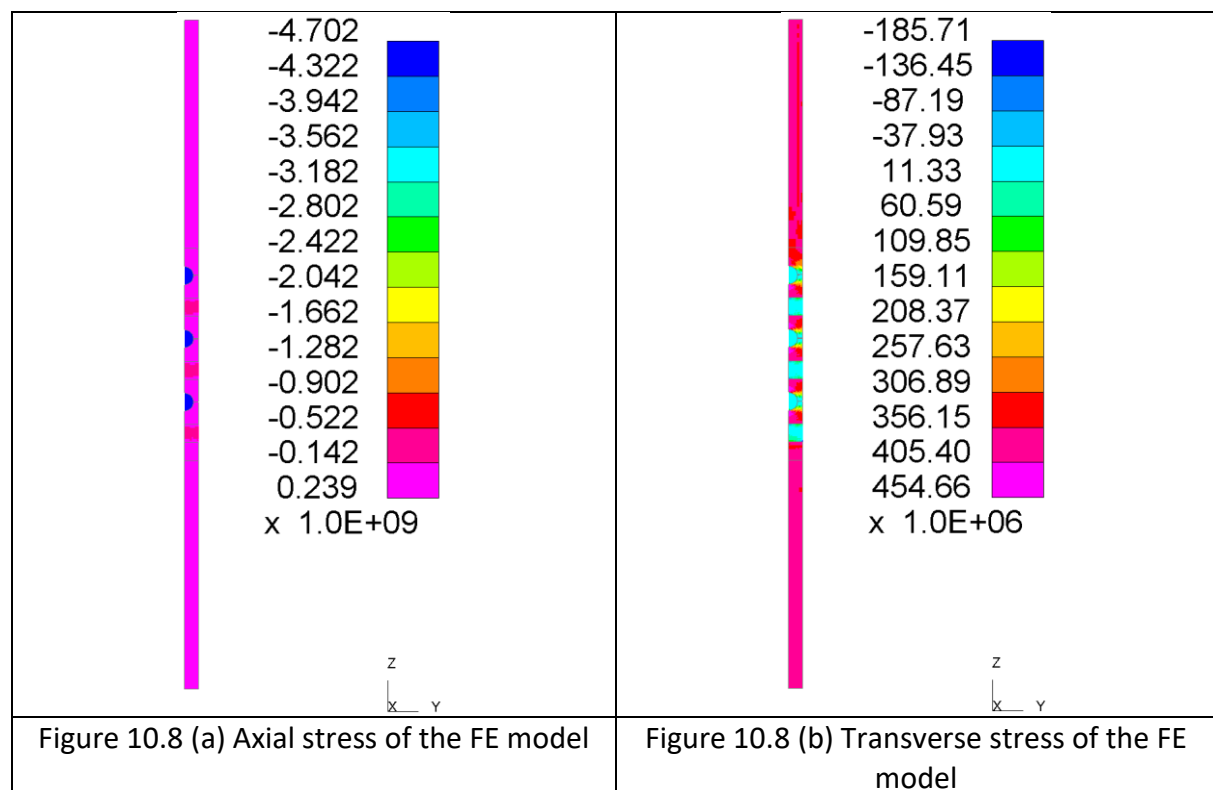


Figure 10.6 shows the axial displacement after the heat treatment, which shows an even displacement at different points along the x-direction. It shows the boundary conditions are working well with one edge (face) not moving and the other edge (face) moving in the plane. The radial and hoop displacements for the component are presented in Figure 10.7.

10.2.1.2 Component with $[0/90]_3$ fibre alignments

A composite component FE model with axial and transverse fibre orientations was developed. The model had six plies of fibre reinforcement with $[0/90]$ reinforcement layers repeated 3 times. The model showed good compression similar to $[0]_6$ model in section 10.2.1.1. Figure 10.8 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stresses results of fibre reinforcements are shown in Figure 10.9. Negative residual stress in the fibre in the axial direction is generated after the heat treatment. The matrix is under tension as shown by the positive stress values. The three layers of axial fibres have the same stress results, and the transverse fibres also have equal stress values.



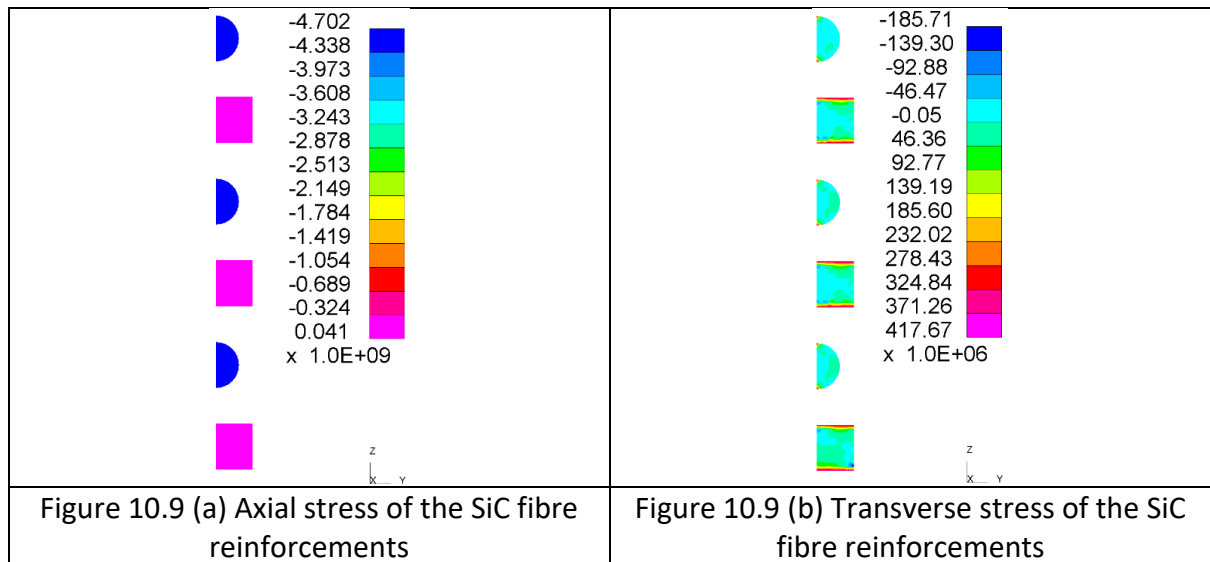


Figure 10.10 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.10 (b). Plastic strain within the aluminium, around the fibre, can be seen.

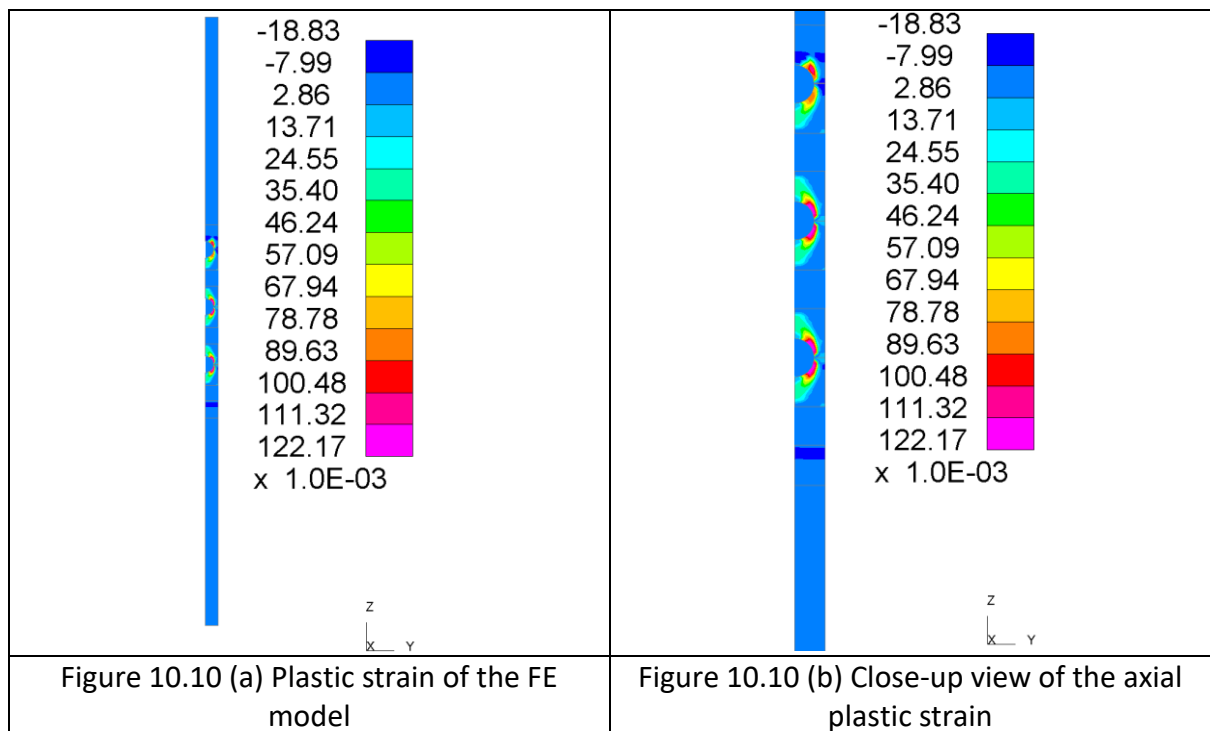
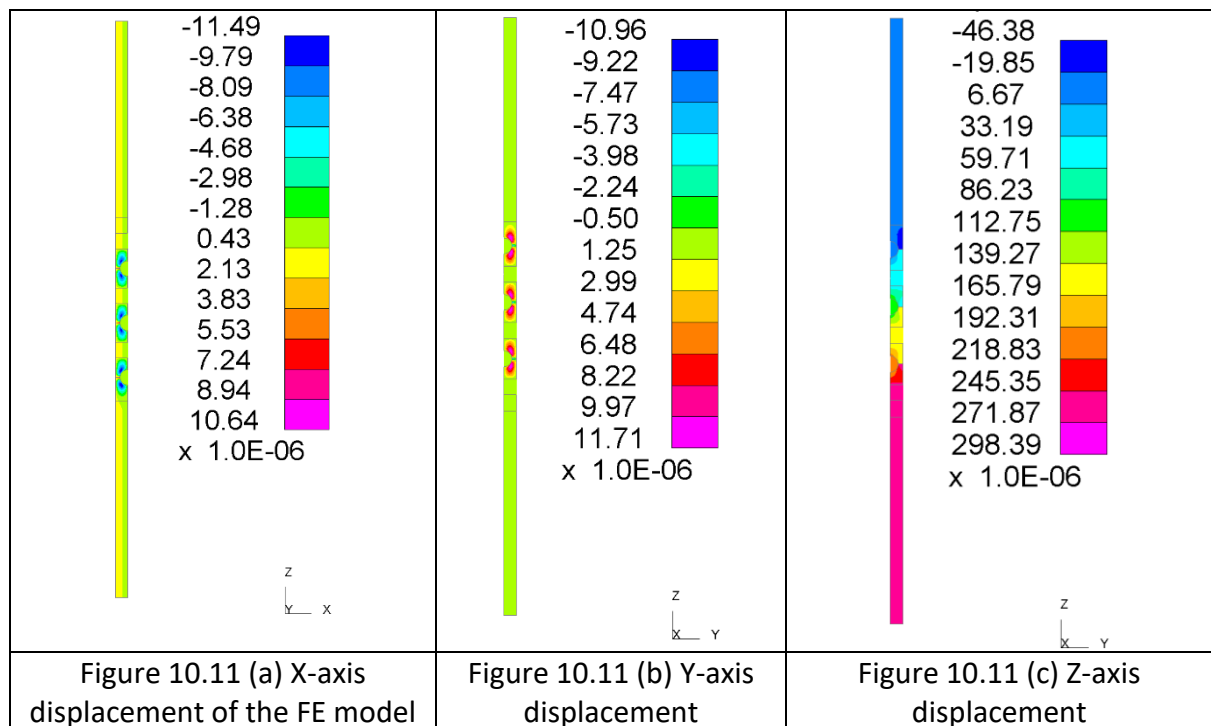


Figure 10.11 shows the displacements in the X, Y and Z directions which indicates that the boundary conditions look to be working correctly. Due to the cross-ply fibre orientation, the axial displacements are not the same as in the model with axially aligned fibres. The roughly

uniform deformation results (apart from around the fibres in the middle) in the radial direction (Z) shows that the component has been evenly compressed.



10.2.1.3 Component with $[90/0/90]_s$ fibre alignments

Figure 10.12 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stresses results of fibre reinforcements are shown in Figure 10.13 The model shows good compression similar to $[0]_6$ model in section 10.2.1.1.

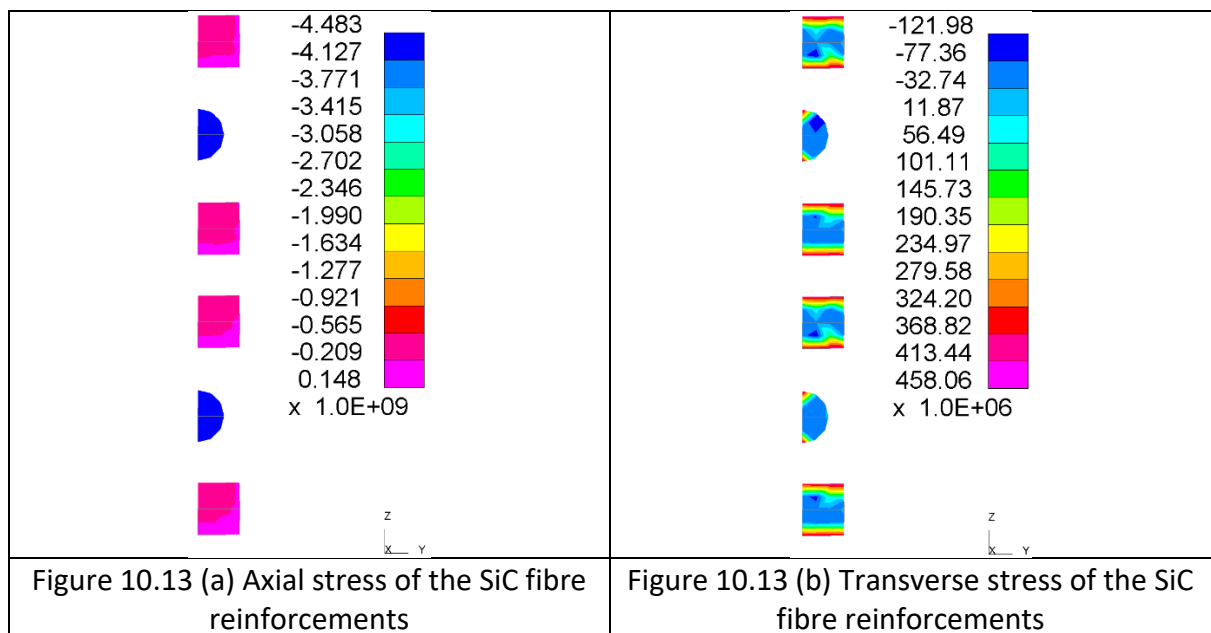
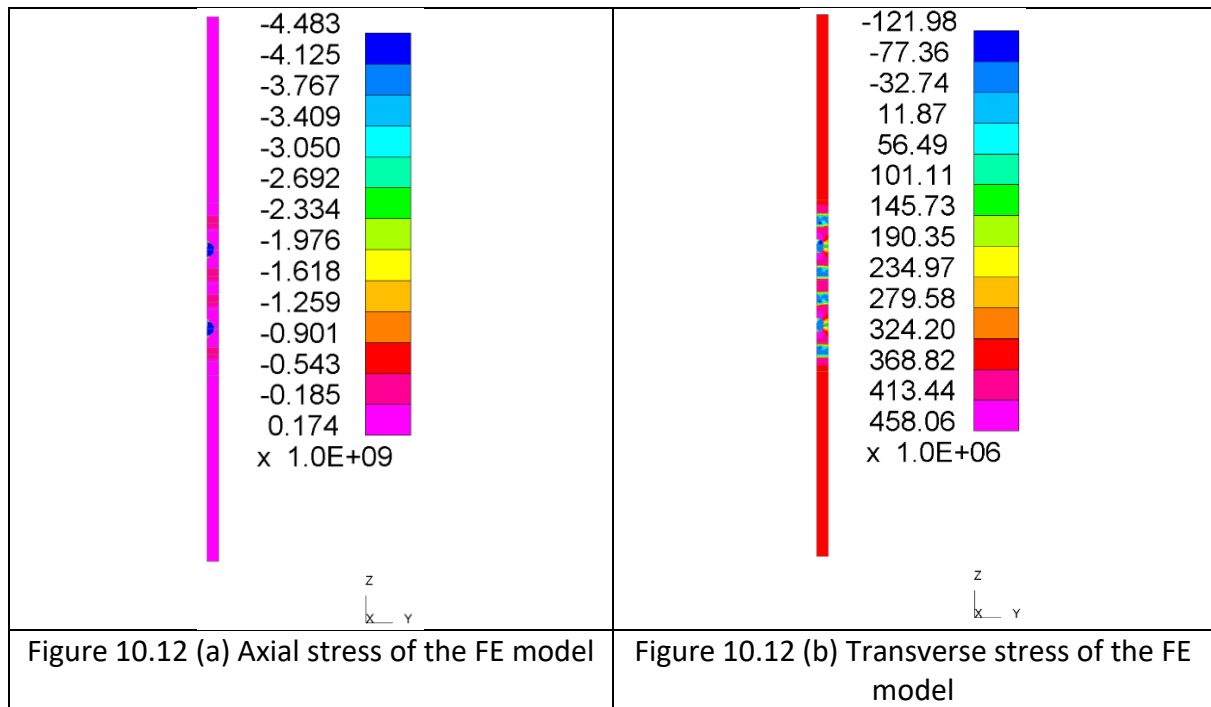


Figure 10.14 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.14 (b). Plastic strain within the aluminium, around the fibres, can be seen.

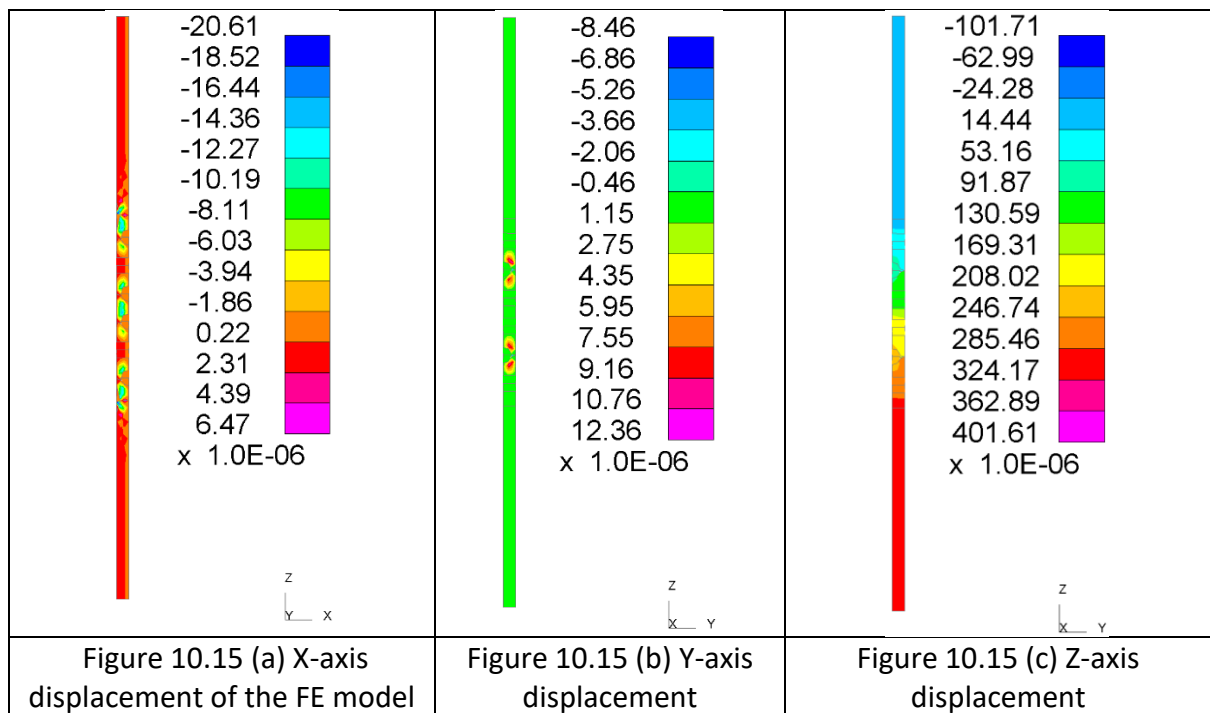
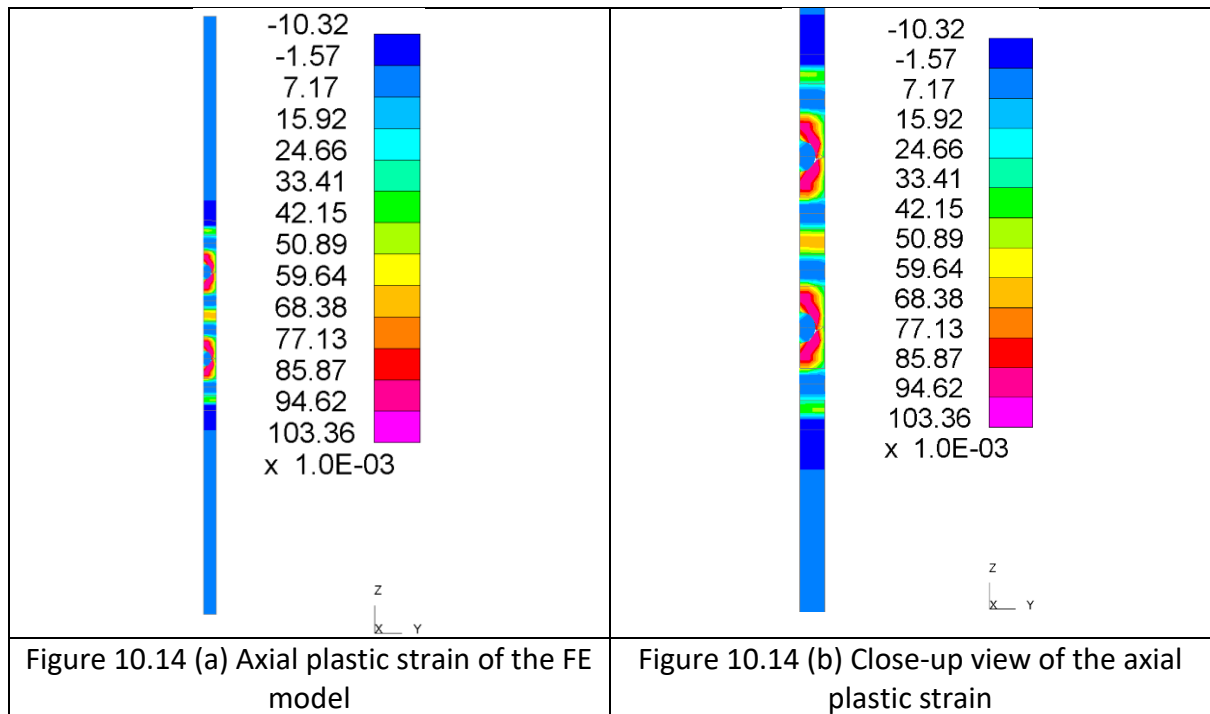


Figure 10.15 shows the axial displacements in the X, Y and Z directions which indicates that the boundary conditions look to be working correctly.

10.2.1.4 Comparison of Non-Tapered model results

The residual stress and strain values at the end of the manufacturing process for the three different fibre orientation models are presented in Table 10.1. Maximum values in magnitude were recorded in the table.

Table 10.1 Stress and strain results at room temperature

	Stress (MPa)				Plastic strain
	Axial	Hoop	Radial	Von Mises	
	[0] ₆ fibre alignments				
0° fibre	-3350	-76	146	3403	-
Matrix	258	418	131	319	-0.027
	[0/90] ₃ fibre alignments				
0° fibre	-4702	-25	149	4760	-
90° fibre	-569	102	312	529	-
Matrix	239	455	110	360	0.122
	[90/0/90] _s fibre alignments				
0° fibre	-4483	-99	173	4538	-
90° fibre	-402	109	196	374	-
Matrix	174	458	164	358	0.103

Stress and strain results of the three non-tapered models are listed in Table 10.1 and compared in Figure 10.16-Figure 10.19. Figure 10.16 shows the comparison of the axial residual stresses within the 0° fibres, the 90° fibres and the aluminium matrix. It is clear that the 0° fibres have the highest axial stress. The burst tubes with [0/90]₃ fibre alignments and [90/0/90]_s fibre alignments have similar results, but the model with [0/90]₃ fibre layups has a slightly bigger maximum axial stress at room temperature of -4702MPa. The axial matrix stresses are relatively low.

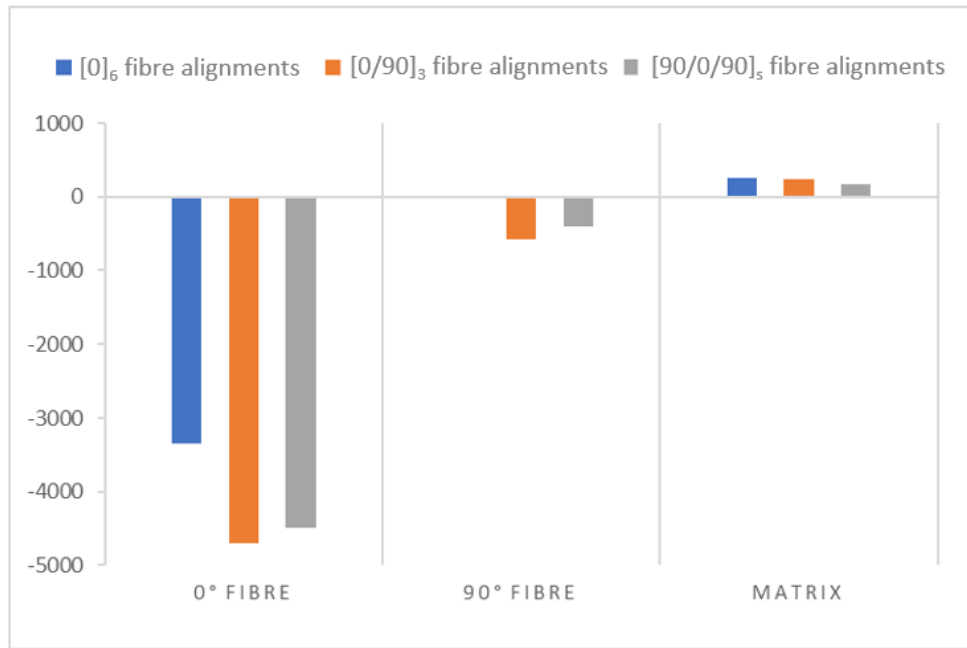


Figure 10.16 Axial stress results for the non-tapered models

In the hoop direction, the aluminium matrix has a similar value of residual stress for all models, around 443MPa, as shown in Figure 10.17. The fibre hoop stresses are all low.

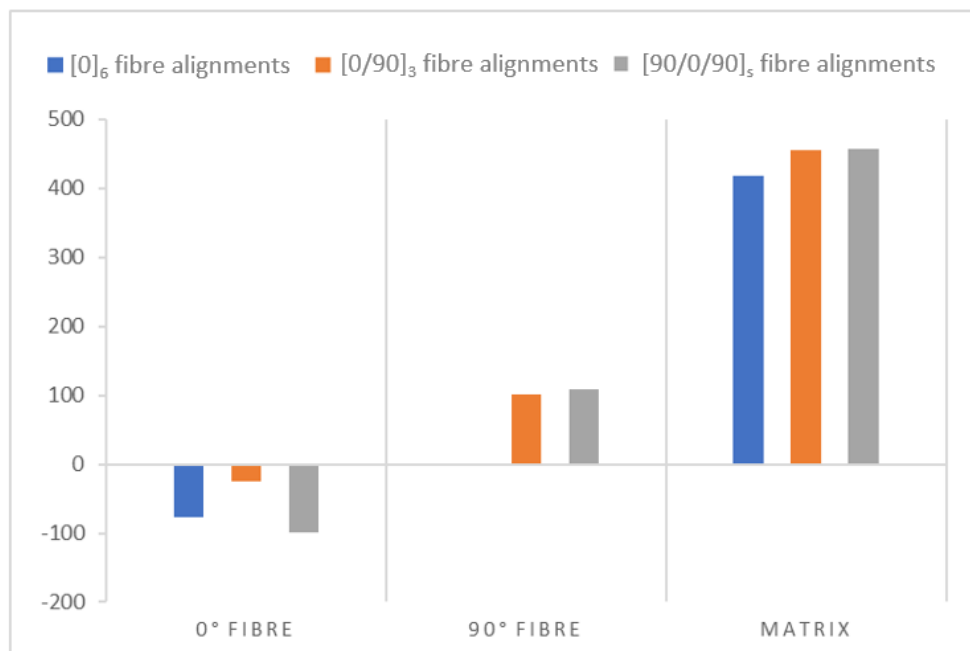


Figure 10.17 Hoop stress results for the non-tapered models

The model with the [0/90]₃ fibre layup has maximum radial stress in the 90° fibre direction (312MPa, Figure 10.18). However, the model with the [90/0/90]₅ fibre layup appears to have

the maximum radial stress in the 0° fibre direction (196MPa). The radial matrix stresses are relatively low and the maximum stress is in the axial [90/0/90]_s layup model (164MPa).

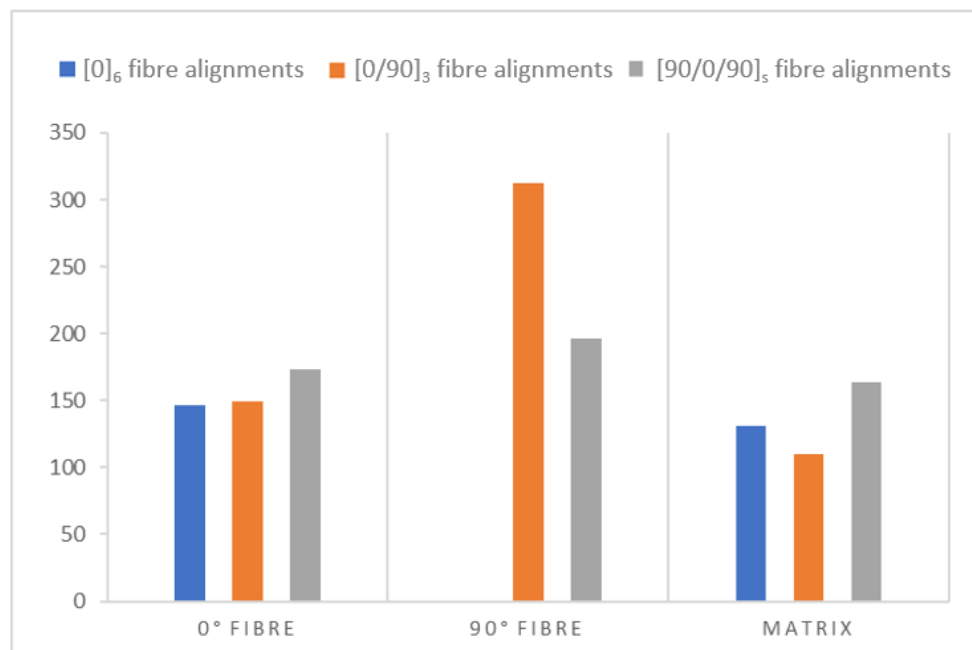


Figure 10.18 Radial stress results for the non-tapered models

Figure 10.19 shows the Von Mises stress of all three models. The Von Mises stress of the 0° fibre reinforcement is much bigger than the 90° fibres and the matrix. The maximum Von Mises stress, of 4760 MPa, appears in the models with [0/90]₃ fibre alignments.

So overall the models generate high fibre stress in the fibres orientated in the axial direction and high matrix stress in the hoop direction.

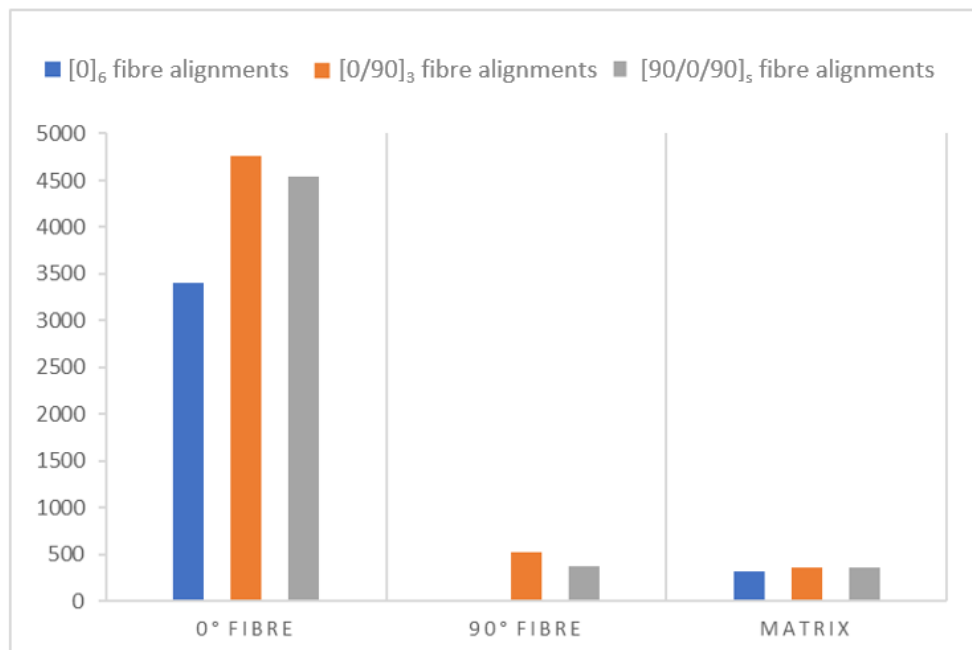


Figure 10.19 Von Mises stress results for the non-tapered models

An important point to note is that all the models have been run with parallel edges, elastic conditions for the fibres and assume the aluminium remains in T6 condition. Although the magnitude of the stress in the fibres are above the failure value of the fibres (3.8GPa) the fibres would not fail due to the stress being compressive. All the models have significant yield in the aluminium around the fibres.

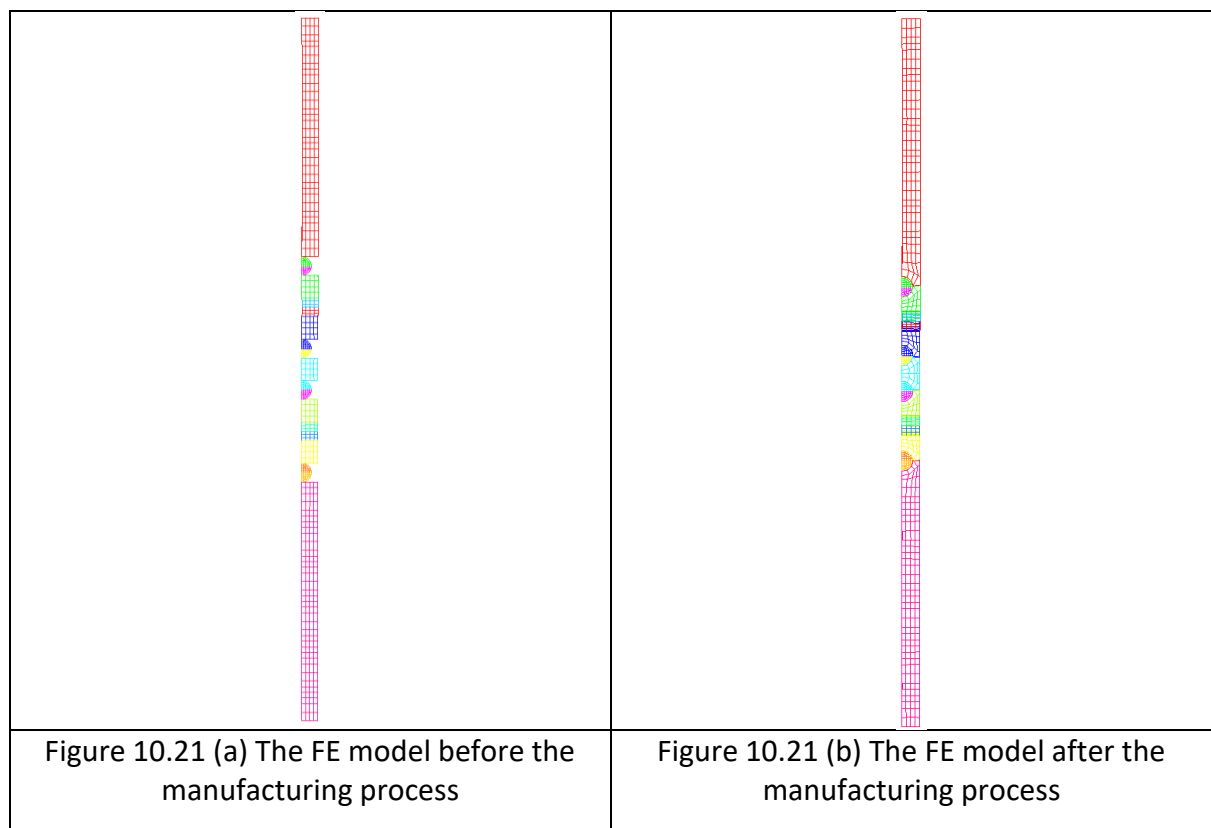
10.2.2 Tapered FE models

In this section, the FE models have been modelled using tapered geometry using the internal and external diameters of the experimentally measured burst tubes. Dimensions of the fibre reinforcements and aluminium matrix are presented in section 7.6.2.1. Both T6 and annealed conditions were used for the aluminium matrix properties dependant on the model and stage of manufacture. The entire manufacturing process has been broken down into stages (separate models) to see the effect of introducing and changing the T6 to O condition transition within the models and changing the contact from mortar to mortar tied, see section 7.6.2.5. The results of these methods will be presented in detail for the model with a [0/90/0]₅ layup and then for further models the results from the method that is deemed most accurate are presented.

10.2.2.1 Component with [0/90/0]_s fibre alignments

In this section the results of the tapered model with a [0/90/0]_s layup is presented using the three different methods that adjust the contact type and the temper condition as discussed in section 7.6.2.5.

In Figure 10.21 (a) and (b) the displacement of the FE model before and after the manufacturing process using method 1 are shown. It is clear to see the aluminium foil layers are well compressed around the fibres from the close-up views in Figure 10.22 (a) and (b), which was true for all the methods used.



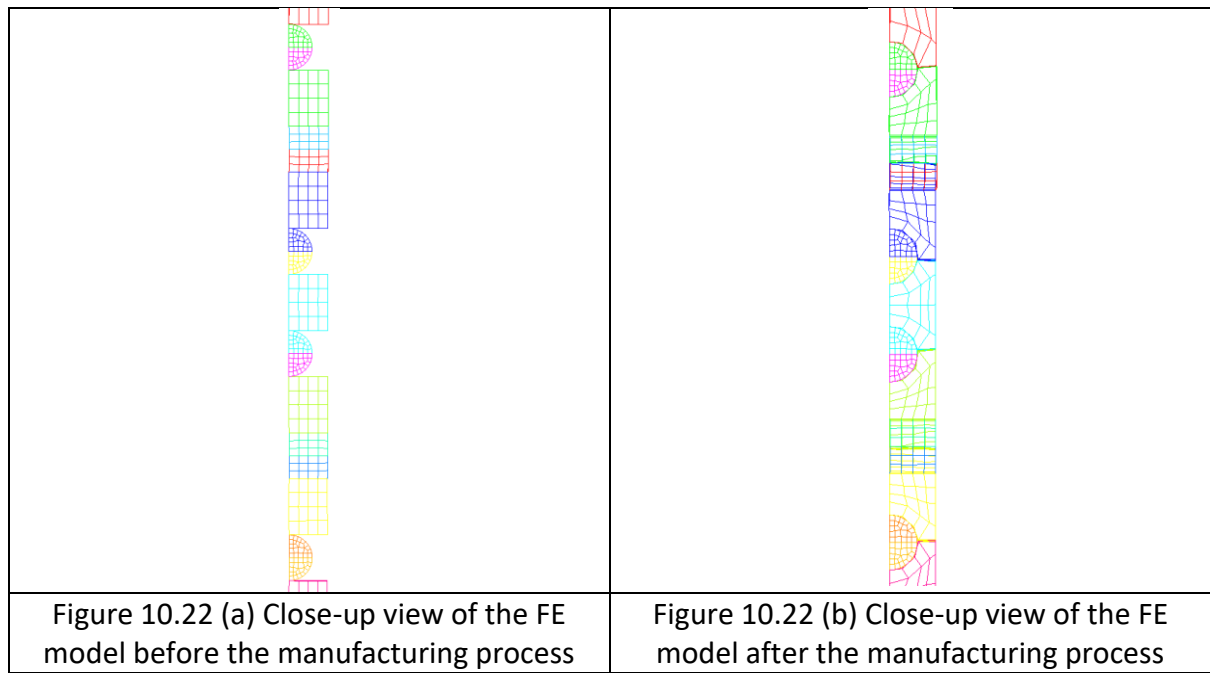
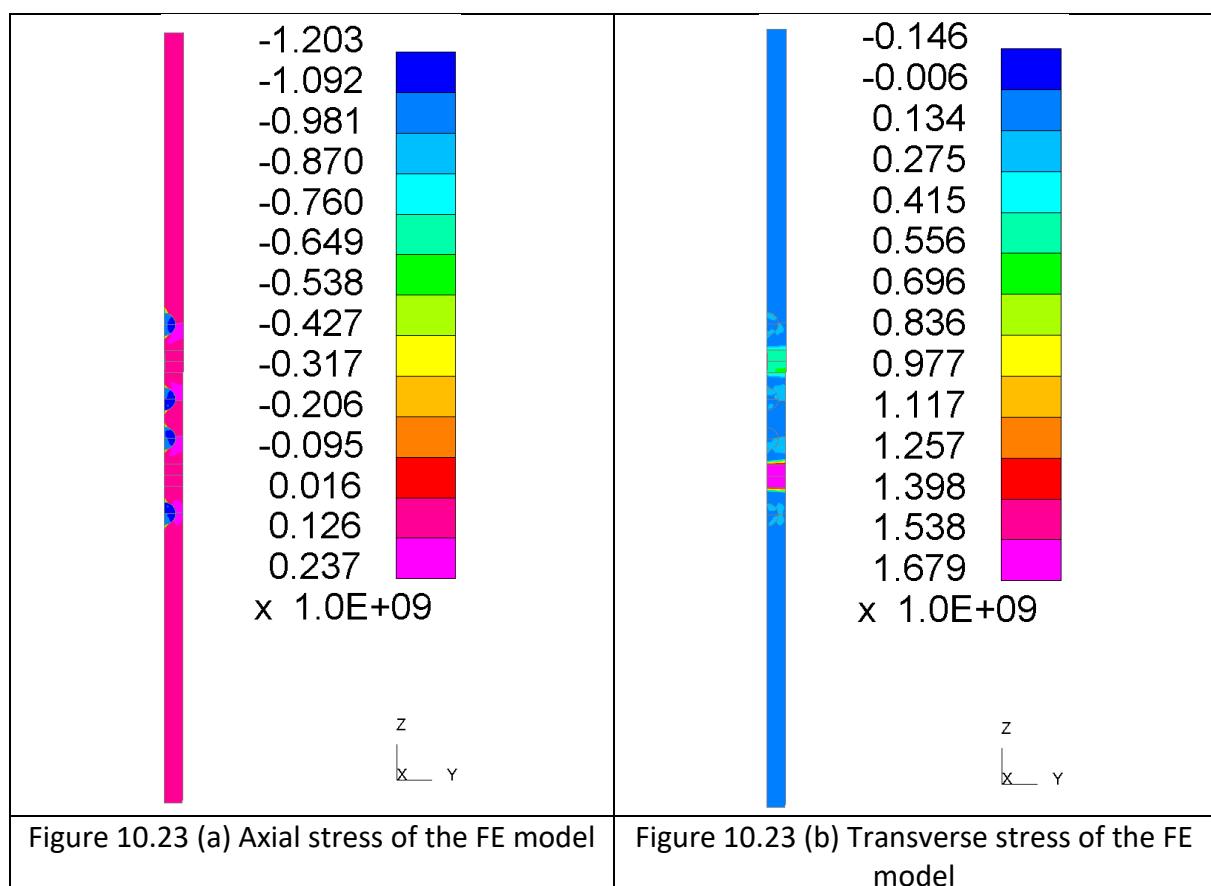


Table 10.2 Maximum magnitude (ignoring signs) of the residual stresses from the three analysis methods

Stresses after heat stage (MPa) – Separate 1 st Stage				
Methods 1, 2 and 3				
	Axial	Hoop	Radial	Von Mises
0° fibre	-185	-299	-295	389
90° fibre	-143	-666	-125	395
Matrix	-117	-117	-117	0.7
Stresses after cooldown stage (MPa) – Combined 2 nd Stage				
Method 1				
0° fibre	-3022	240	292	2962
90° fibre	-94	1598	121	1572
Matrix	300	500	183	368
Stresses after cooldown stage (MPa) – Separate 2 nd Stage				
Method 1				
0° fibre	-3018	238	284	2952
90° fibre	-96	1573	116	1549
Matrix	300	496	180	369
Method 2				
0° fibre	-3020	407	-322	2995
90° fibre	-148	1584	-157	1677
Matrix	293	490	205	365
Method 3				
0° fibre	-1203	303	-196	1224
90° fibre	128	1679	-99	1680
Matrix	237	239	127	152

Table 10.2 presents the maximum magnitude (ignoring signs) of the residual stresses from the different model methods, As the settings for the combined 2nd stage model are the same up to cooldown as for the separate 1st stage model it was expected that the results would be identical up to cooldown. The results after the heating stage were found to be very close between the two models (the exact matching time point could not be selected within the combined 2nd stage model (due to step size differences)). The results from the three methods are different after the heating stage. The results are slightly different between running method 1 as a combined model or running as two separate models but not significantly different. The differences between using method 1 and method 2 are much more significant. The mortar tied contact has therefore been used during cooldown for all further models as this helps prevent fibre and matrix separation which should be more representative of how the component would behave. The stress results for method 3 are presented visually in Figure 10.23 to Figure 10.24.



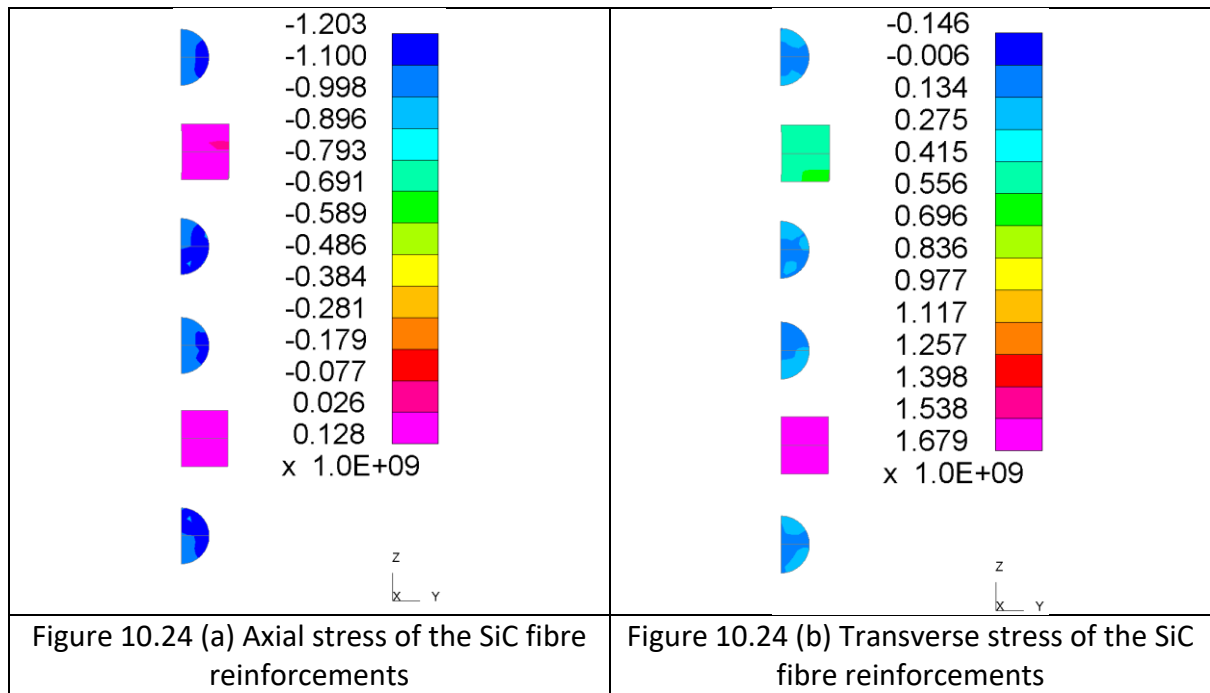
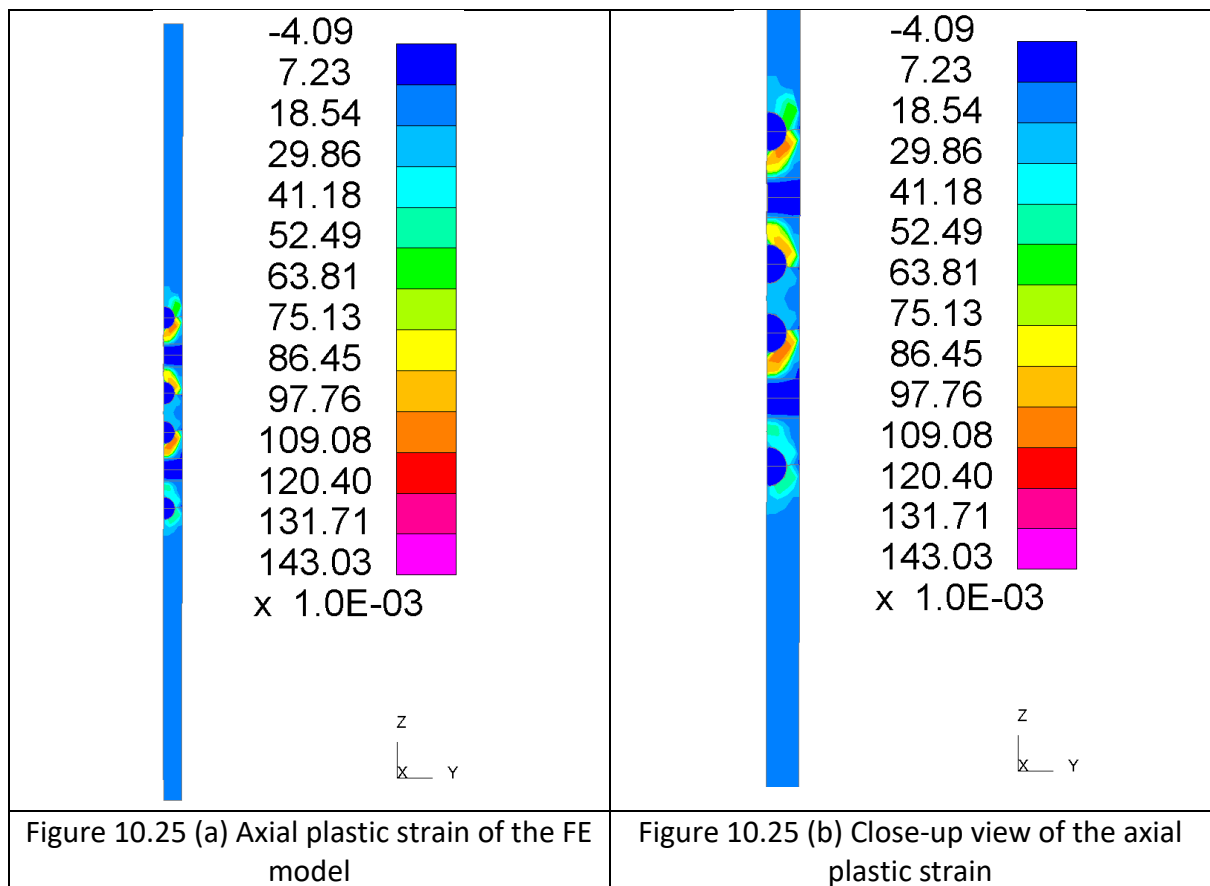


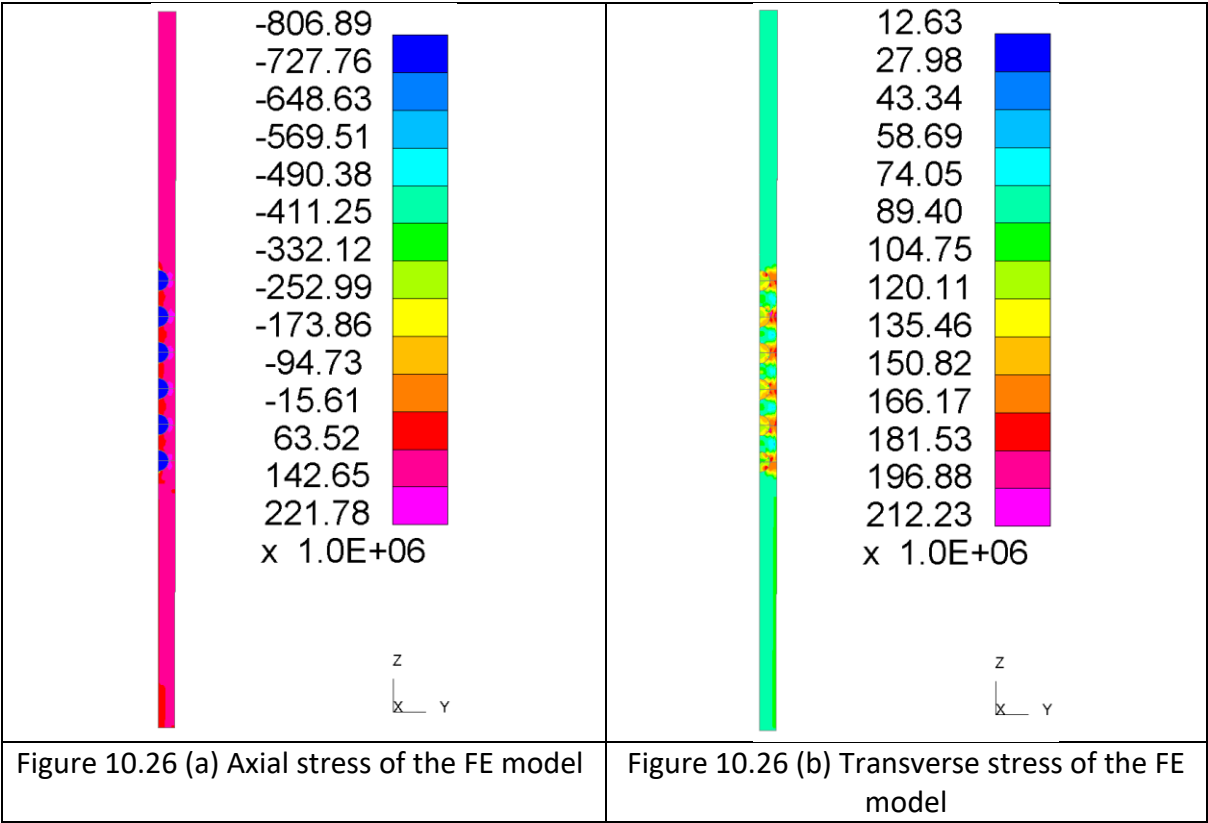
Figure 10.25 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.25(b). Plastic strain can be seen in the aluminium around the fibres.



Method 3 should give the most realistic results as it correctly considers the annealed properties that should exist after cooldown. All further models were run using method 3, including the internal pressure (burst tube) models discussed in section 10.2.5.

10.2.2.2 Component with [0]₆ fibre alignments

Figure 10.26 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stress results of the fibre reinforcements are shown in Figure 10.27. The model shows good compression of the matrix around the fibres similar to [0/90/0]_s model in section 10.2.2.1.



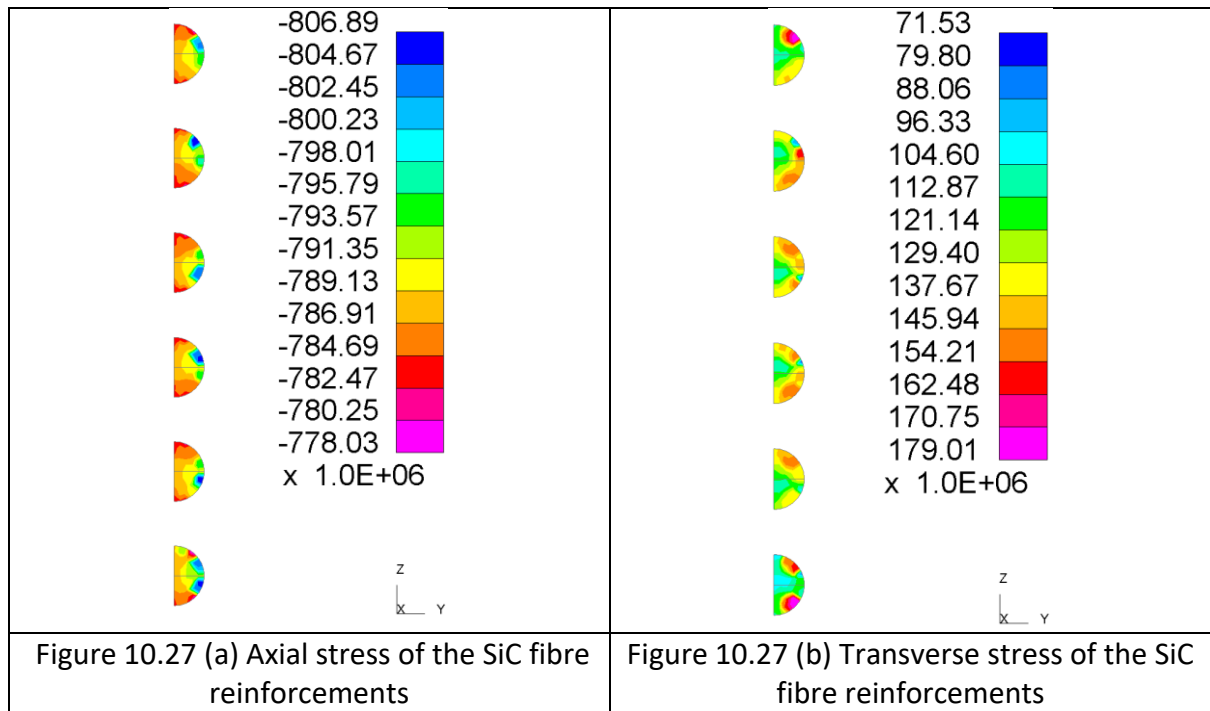
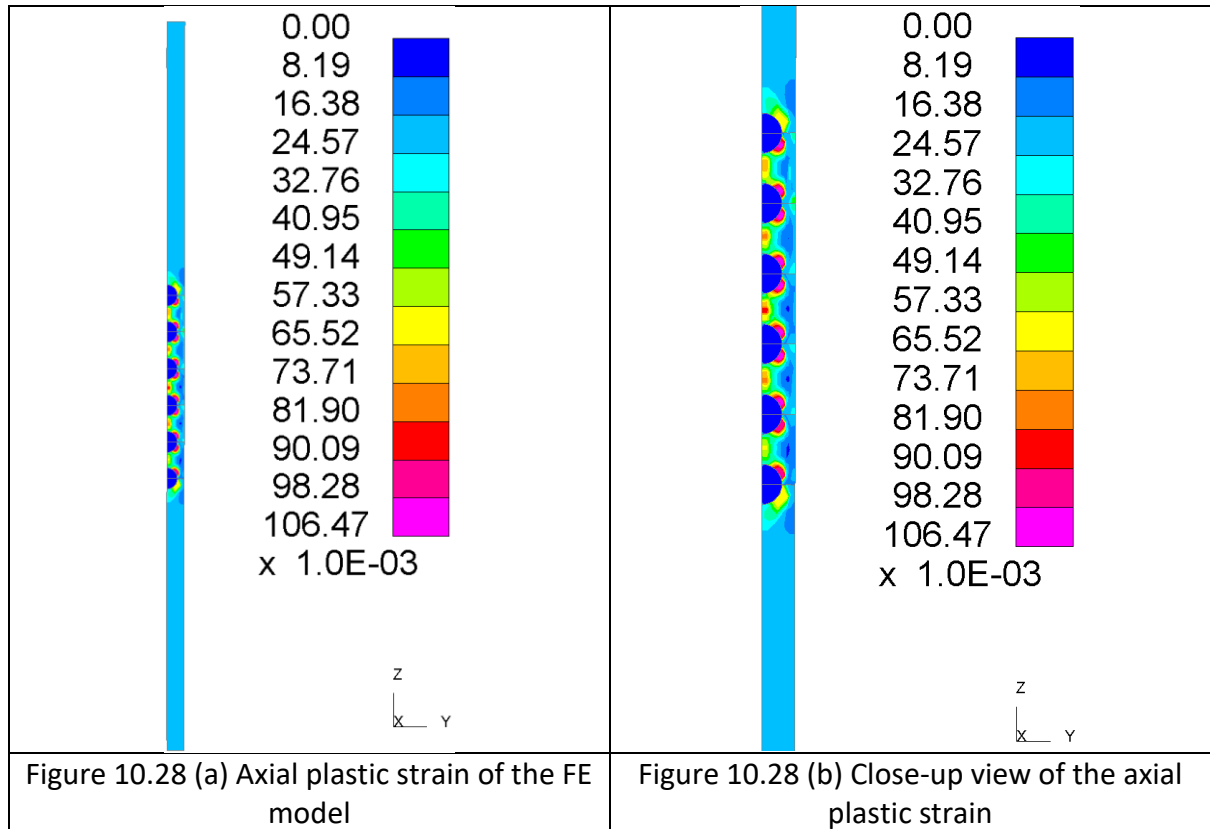
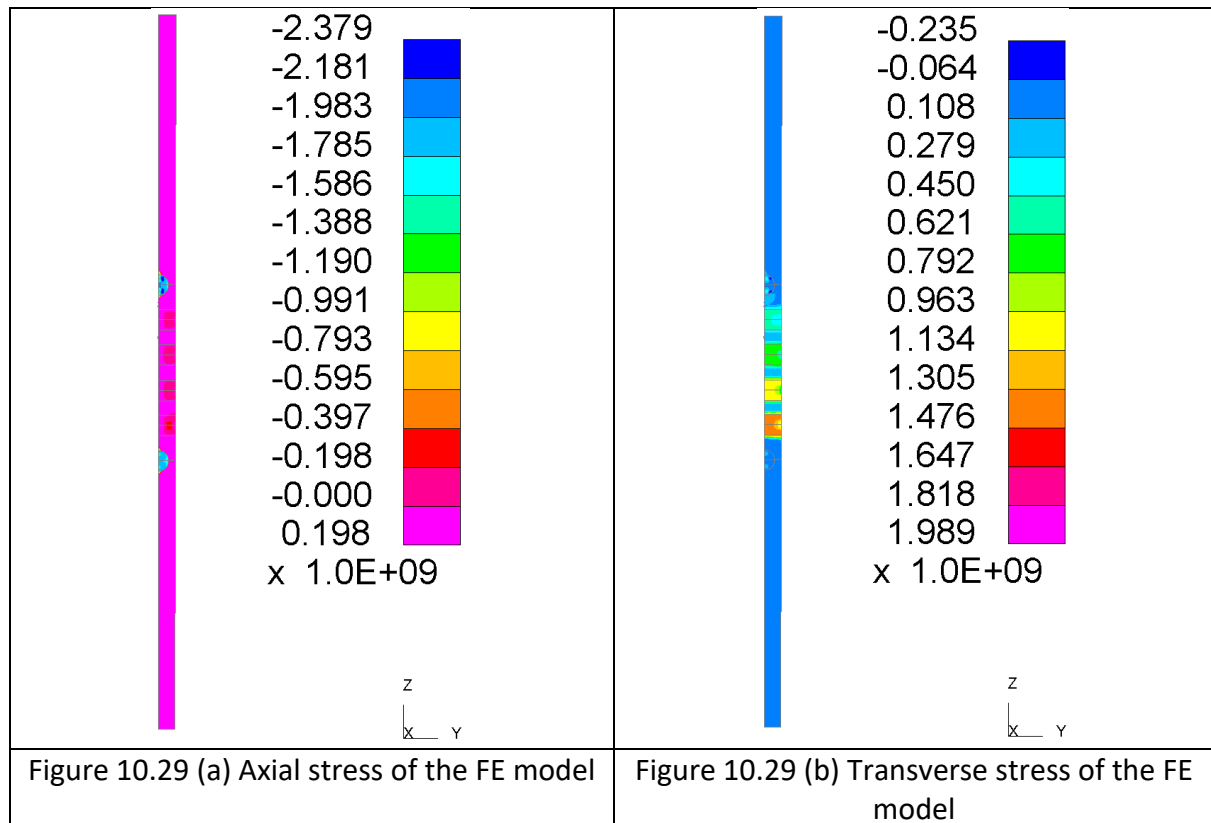


Figure 10.28 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.28 (b). Plastic strain can be seen in the aluminium around the fibres.



10.2.2.3 Component with [0/90/90]_s fibre alignments

Figure 10.29 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stresses results of the fibre reinforcements are shown in Figure 10.30. The model shows good compression of the matrix around the fibres similar to [0/90/0]_s model in section 10.2.2.1.



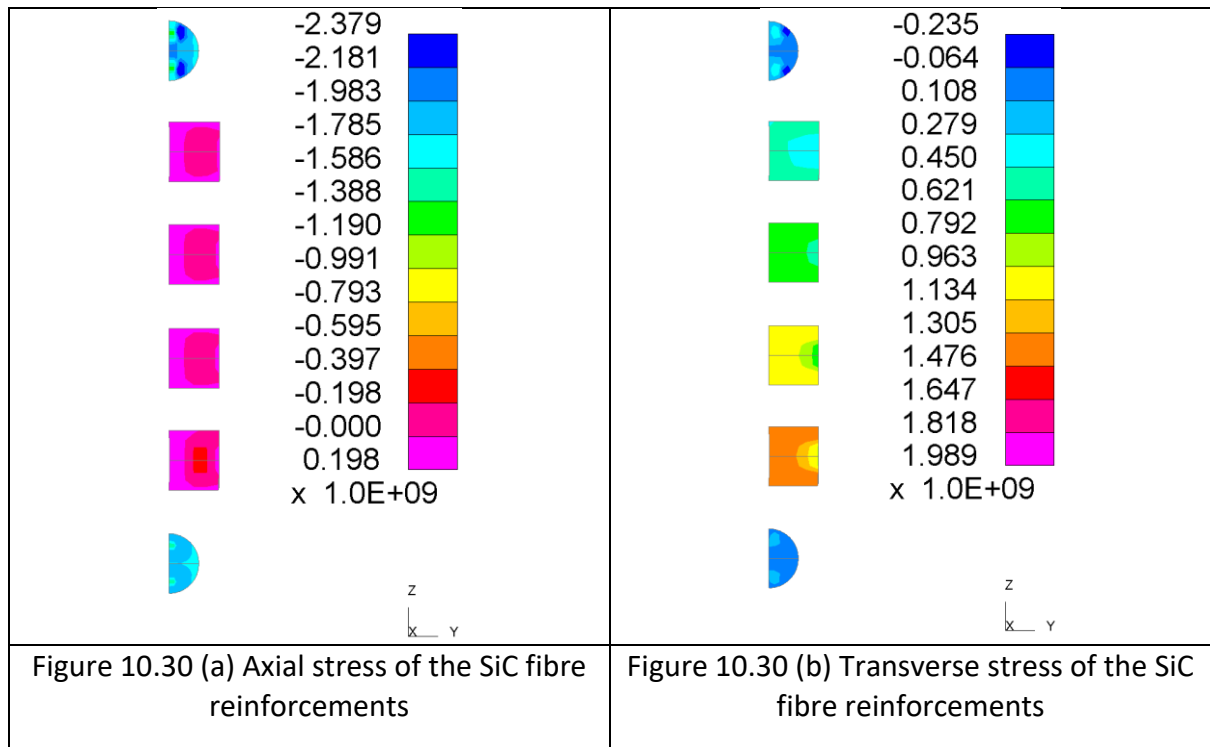
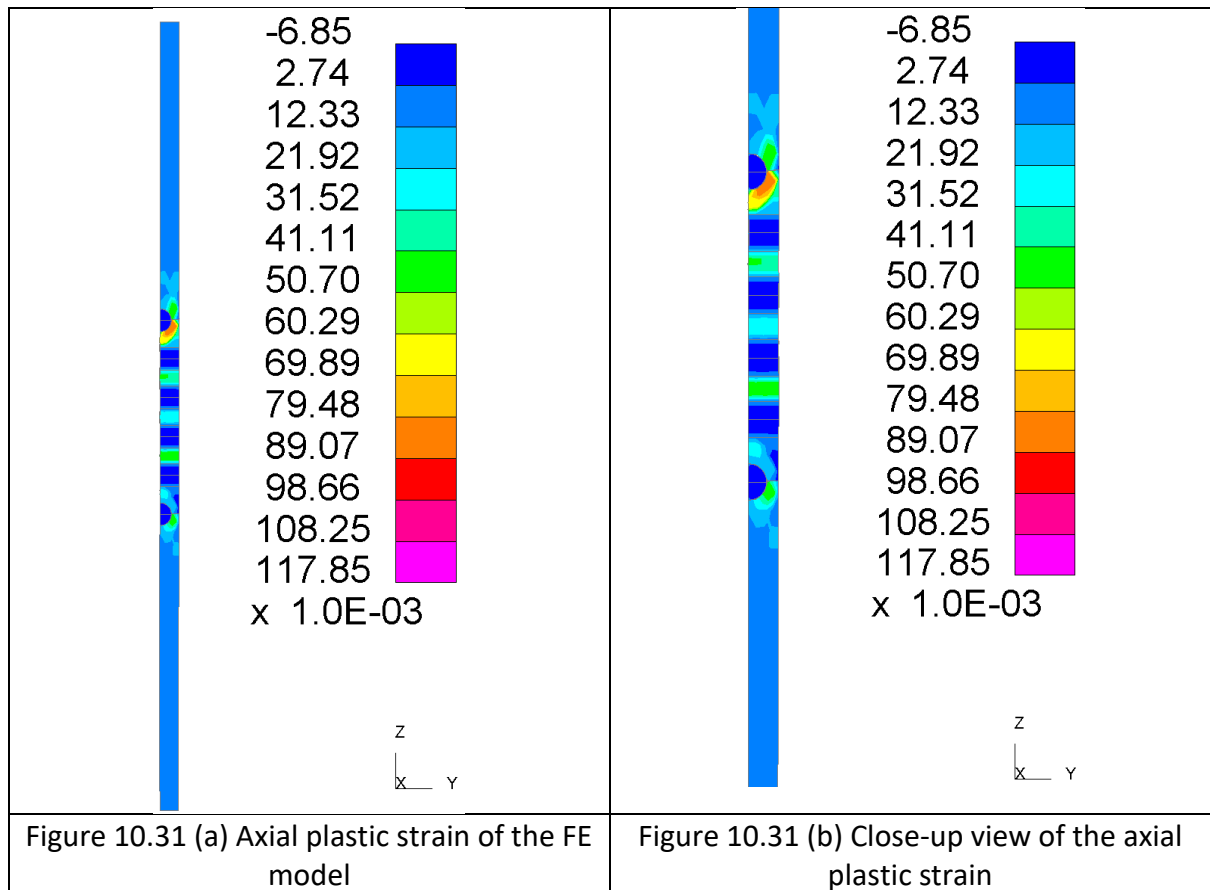


Figure 10.31 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.31 (b). Plastic strain can be seen in the aluminium around the fibres.



10.2.2.4 Component with $[90/90/0]_s$ fibre alignments

Figure 10.32 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stresses results of fibre reinforcements are shown in Figure 10.33. The model shows good compression of the matrix around the fibres similar to $[0/90/0]_s$ model in section 10.2.2.1..

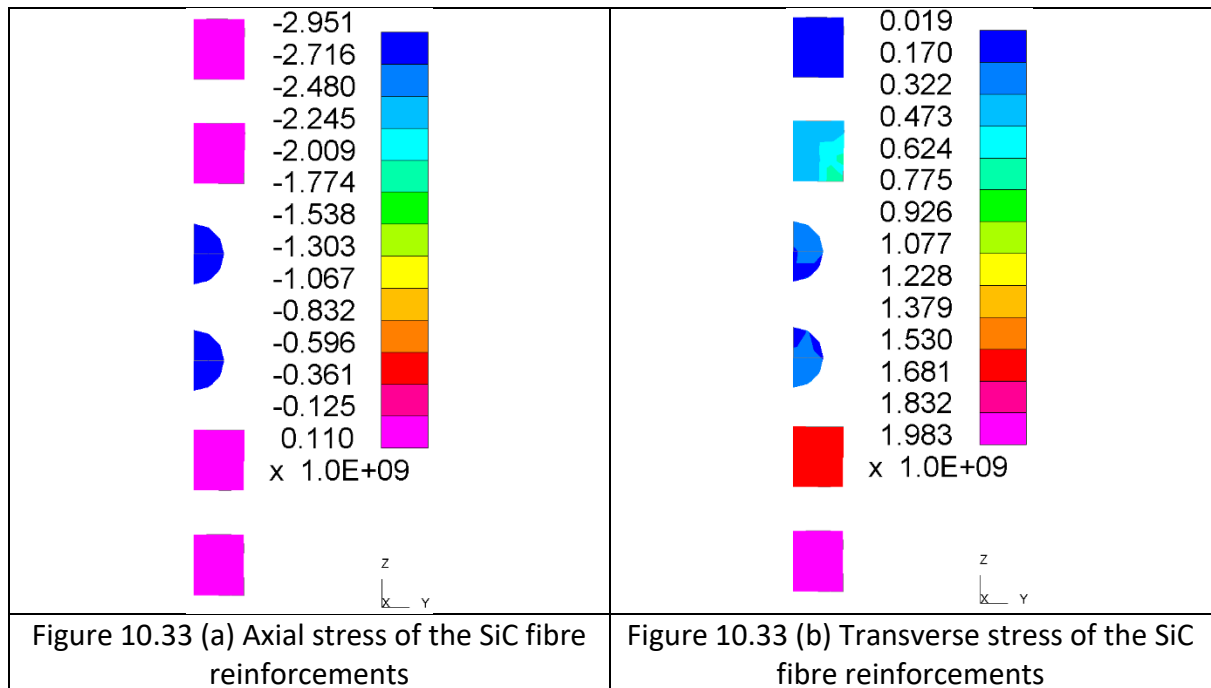
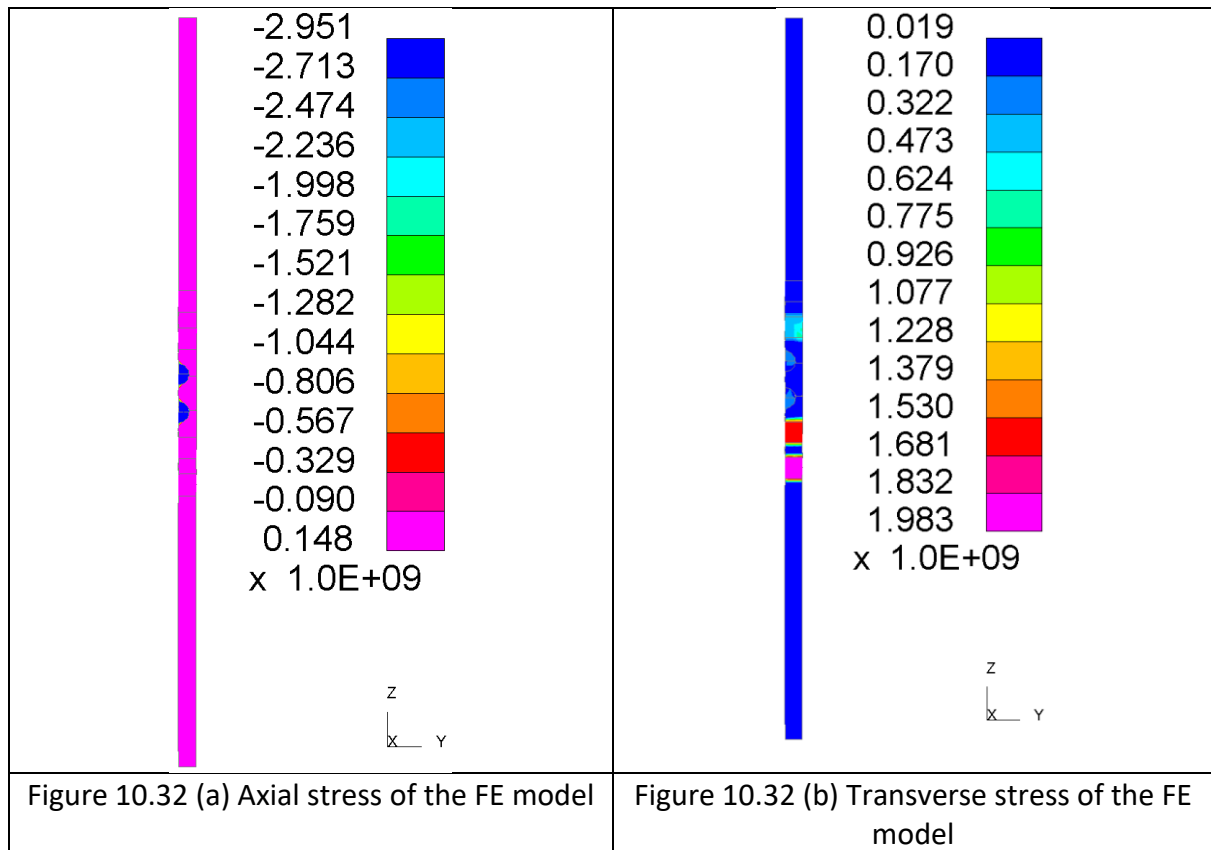
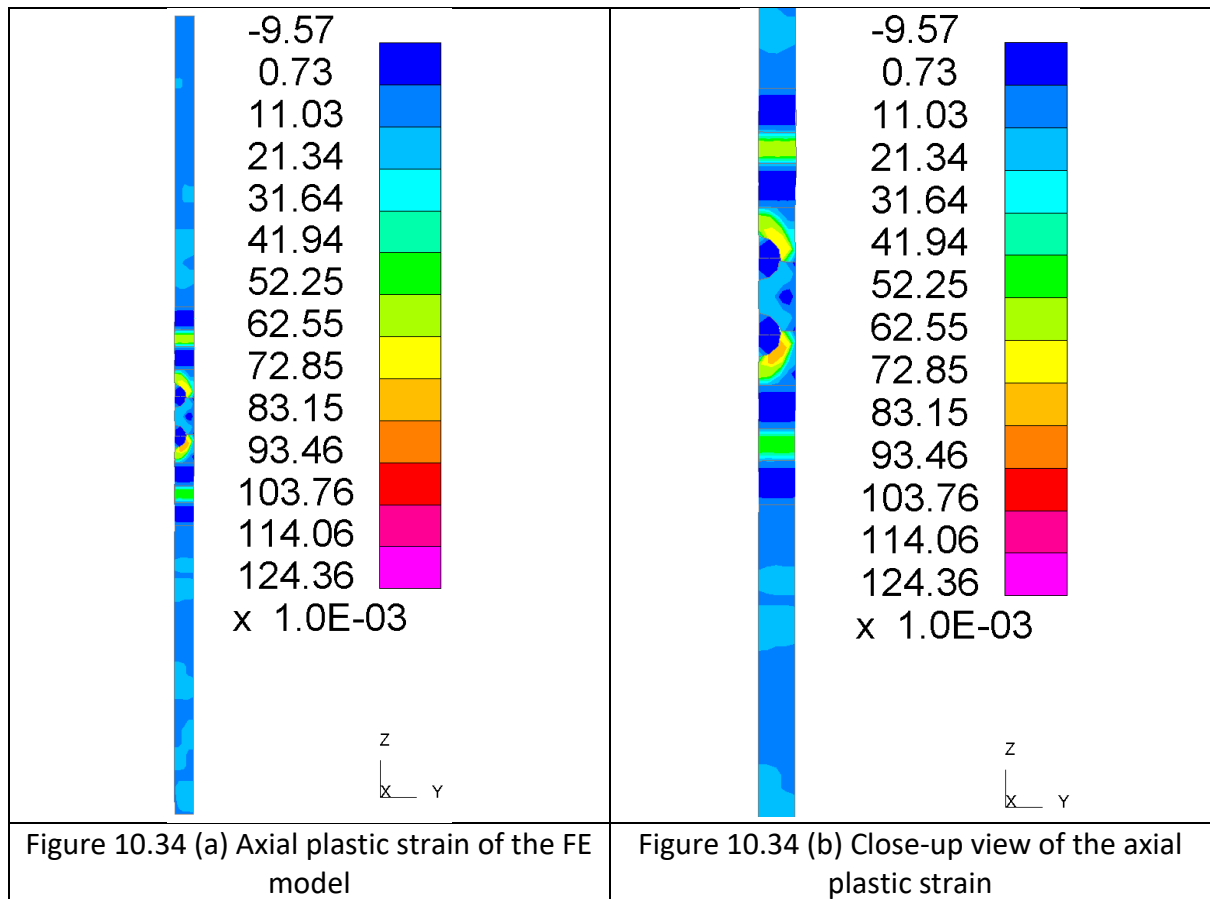


Figure 10.34 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.34 (b). Plastic strain can be seen in the aluminium around the fibres.



10.2.2.5 Component with $[90/0/90]_s$ fibre alignments

Figure 10.35 presents the axial stress and transverse stress for the composite burst tube model after heat treatment. The residual stresses results of fibre reinforcements are shown in Figure 10.36. The model shows good compression of the matrix around the fibres similar to $[0/90/0]_s$ model in section 10.2.2.1.

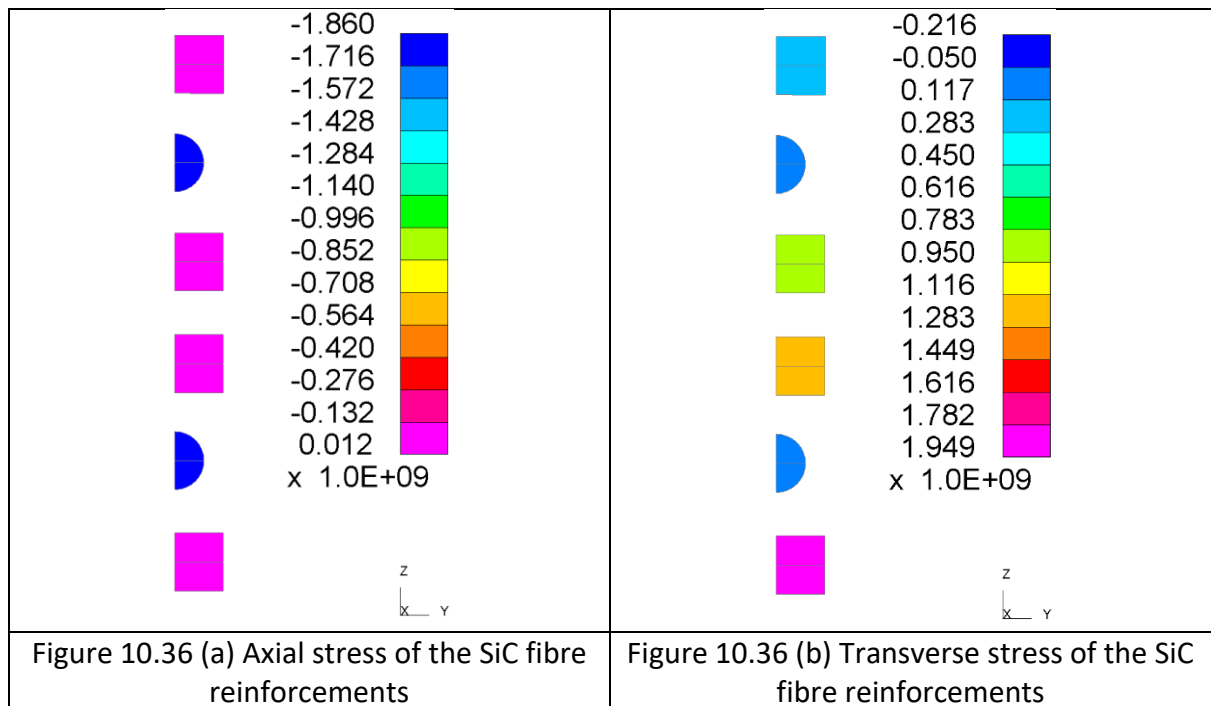
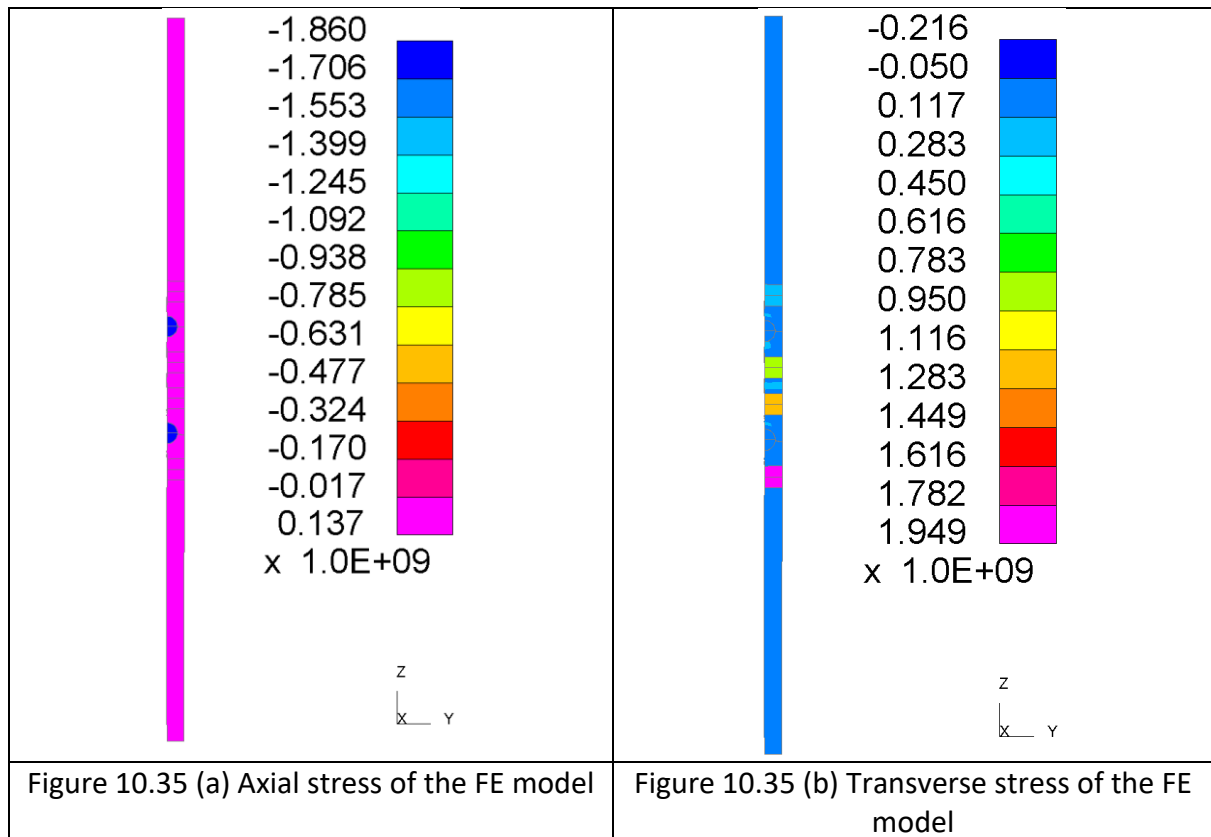
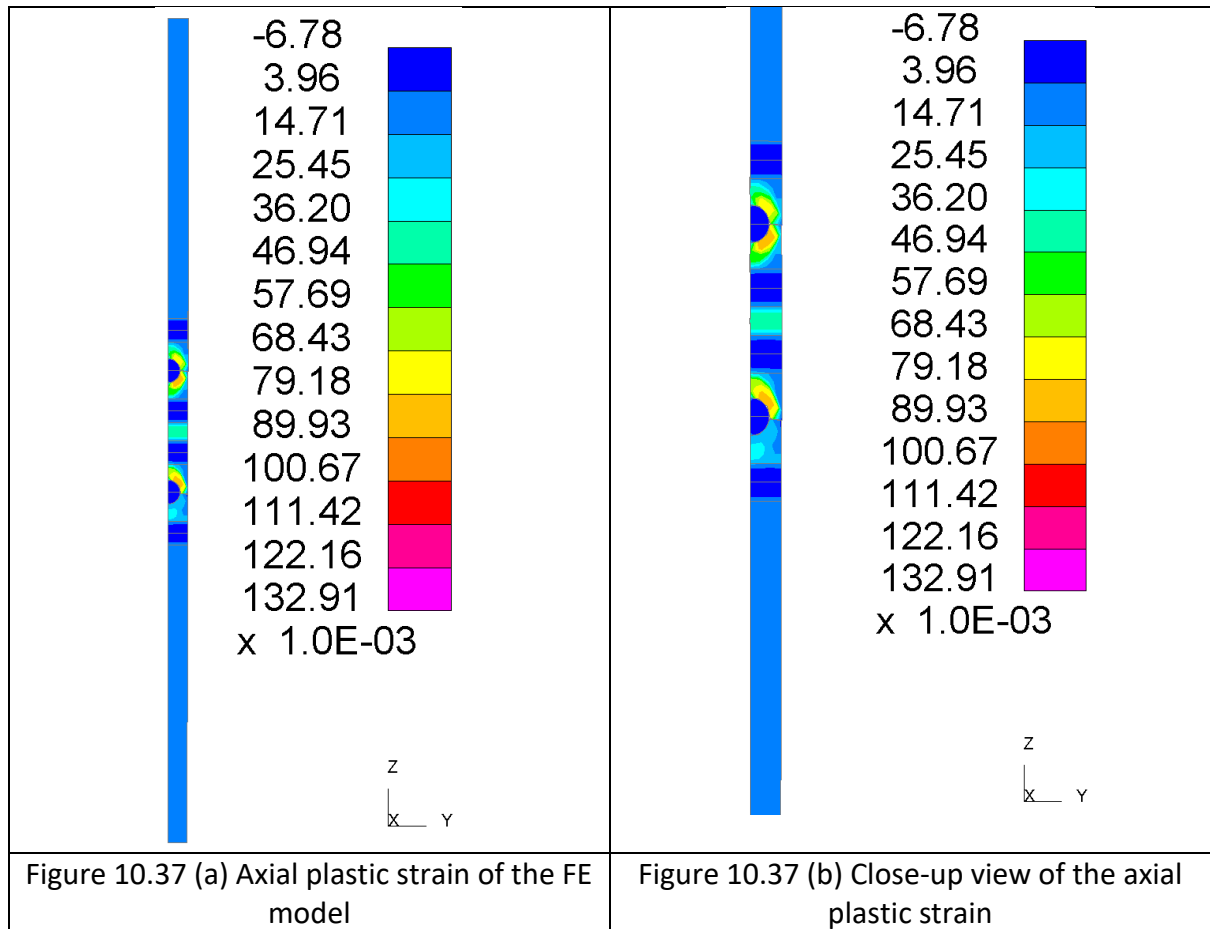


Figure 10.37 shows the axial plastic strain from the FE simulation after heat treatment. A close-up view of the results is displayed in Figure 10.37 (b). Plastic strain can be seen in the aluminium around the fibres.



10.2.2.6 Comparison of Tapered Model Results

The maximum magnitude (ignoring signs) of the residual stress and strain values at the end of the manufacturing process for the three different fibre orientation models are presented in Table 10.3.

Table 10.3 The maximum magnitude (ignoring signs) of the stress and strain results at room temperature

	Stress (MPa)				Plastic strain
	Axial	Hoop	Radial	Von Mises	
	[0/90/0]_s fibre alignments				
0° fibre	-1203	303	-196	1224	-
90° fibre	128	1679	-99	1680	-
Matrix	237	239	127	152	0.14
	[0]₆ fibre alignments				
0° fibre	-807	179	-122	863	-
Matrix	222	212	172	138	0.11
	[0/90/90]_s fibre alignments				
0° fibre	-2379	356	-235	2281	-
90° fibre	198	1989	-81	1892	-
Matrix	147	161	69	144	0.12
	[90/90/0]_s fibre alignments				
0° fibre	-2951	357	-219	3100	-
90° fibre	110	1983	-197	2019	-
Matrix	148	185	117	146	0.12
	[90/0/90]_s fibre alignments				
0° fibre	-1860	188	-172	1925	-
90° fibre	12	1949	-31	1947	-
Matrix	137	154	61	152	0.13

FE models with tapered geometry were presented in section 10.2.2 with Table 10.3 listing the stress and strain results of the models. Five different fibre alignments were analysed, these were [0]₆, [0/90/0]_s, [0/90/90]_s, [90/90/0]_s and [90/0/90]_s fibre layups. Figure 10.38 shows the comparison of the axial residual stresses within the 0° fibres, the 90° fibres and the aluminium matrix. It is clear to see from Figure 10.38 that the 0° fibres carry most of the axial stress. The models incorporating 90° fibre reinforcements have a maximum value of around -2951MPa in the 0° fibres.

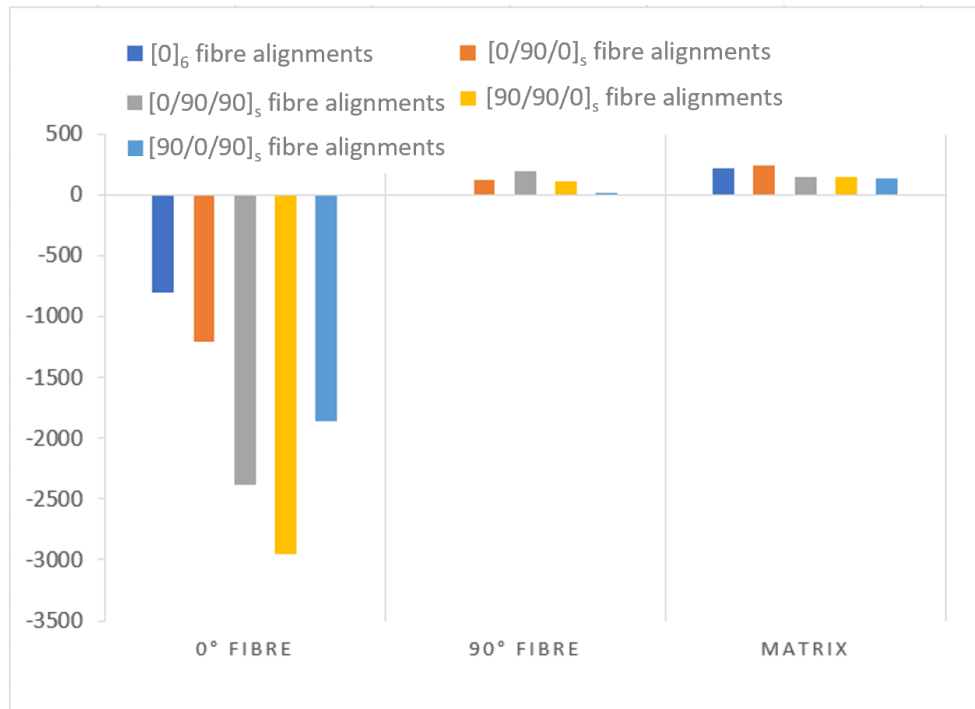


Figure 10.38 Axial stress results in comparison of the tapered models

In the hoop direction, the 90° fibres and the matrix all have positive residual stress values compared with the 0° fibres (Figure 10.39). The models with [0/90/90]_s and [90/90/0]_s fibre alignments have relatively larger stress values in the 90° fibres. The model with [0/90/90]_s fibre has the maximum hoop stress of 1989MPa. The five models have similar matrix stress values with an average of 190MPa.

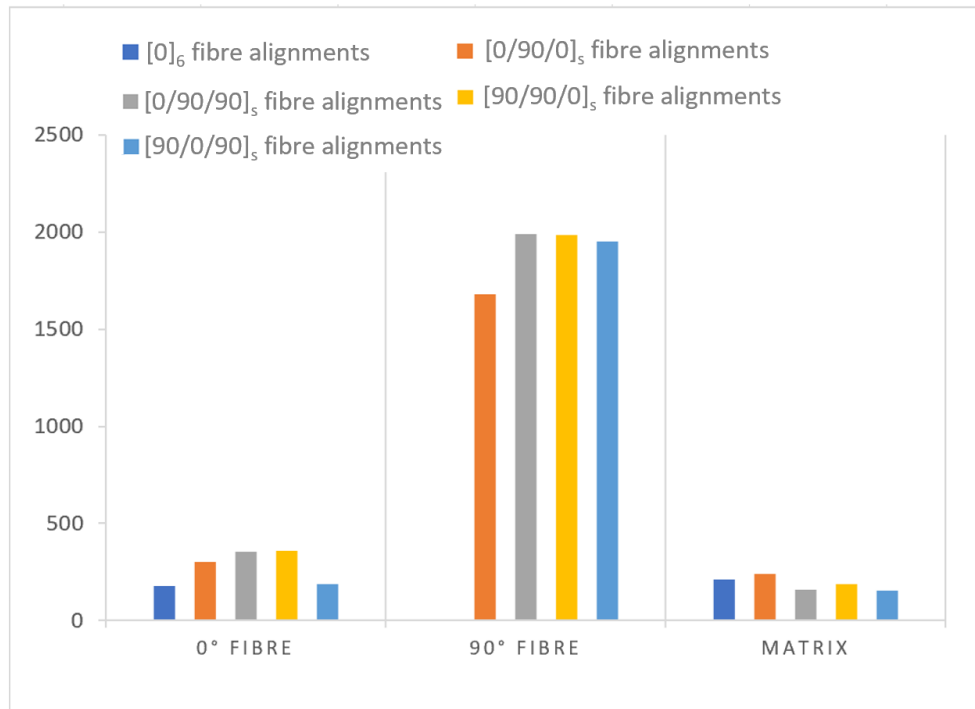


Figure 10.39 Hoop stress results for the tapered models

In the radial direction, the [90/90/0]_s and [90/90/0]_s models have similar high values in the 0° fibres. All the models have similar matrix stresses but the [90/90/0]_s model has a significantly higher stress in the 90° fibre than the other models, with a value of -197MPa (Figure 10.40).

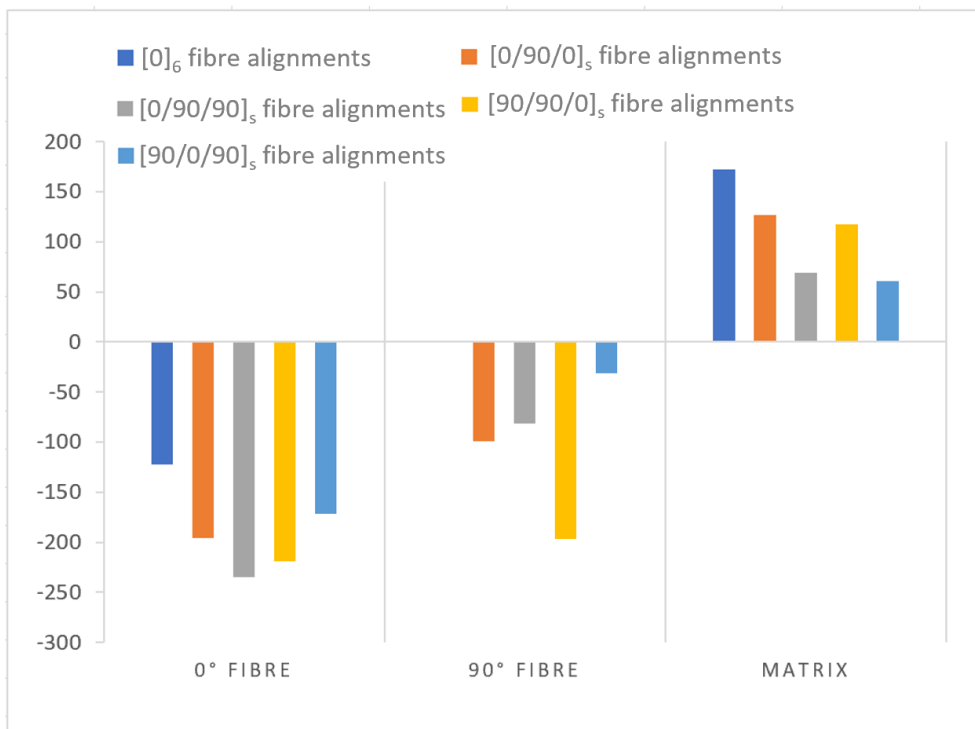


Figure 10.40 Radial stress results for the tapered models

The $[90/90/0]_s$ model has the maximum Von Mises stress with a value of 3100MPa in the 0° fibres (Figure 10.41). The 90° fibre reinforcements and the aluminium matrix within the $[90/90/0]_s$ model have maximum Von Mises stresses of 2019MPa and 152MPa respectively.

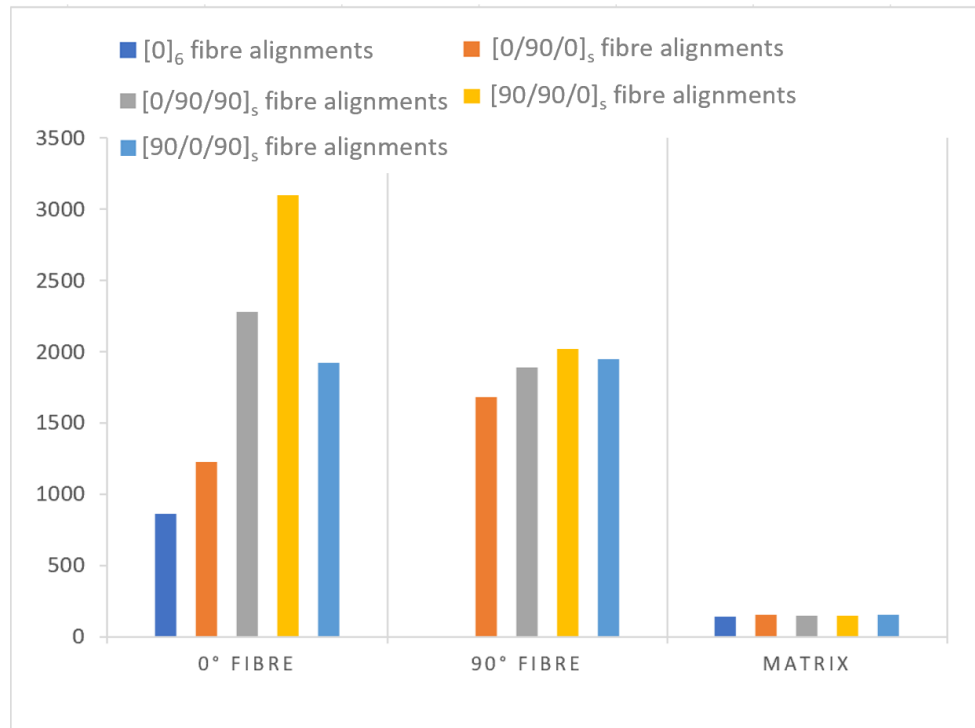


Figure 10.41 Von Mises stress results for the tapered models

In summary the models predict high compressive stresses for the fibres aligned in the axial direction and lower magnitude but high tensile stresses for the fibres aligned in the hoop direction. The matrix stresses are above the yield stress around the fibres but below that which would cause ultimate failure.

10.2.2.7 Model Run Times for Tapered Models

The run times of some of the $[0/90/0]_s$ models are shown in Table 10.4. These run times are comparable with the other layup orientations. The models are relatively quick to run but quite a lot of time is consumed setting up and checking the models. Even when proficient the setting up is probably 3 to 5 times more time consuming than the time taken to run the models.

Table 10.4 Model run times for the tapered [0/90/0]s layup configuration

LS-Dyna Version	
SSE2 Double Precision	
Processor Used	
Intel I7-6700K 16GB Ram	
4 Processors Selected	
Model Size	
Number of Elements: 4416, Number of Nodes: 6511	
Element Type: Hexahedral 8 point (Type -2, for elements with poor aspect ratios)	
Model Type	Approximate run time (mins)
Manufacturing – Combined Stage 1 and 2 (T6 Condition)	34
Stage 1 – Heating Up (T6 condition)	23
Stage 2 - Cooldown (T6 to Annealed)	14
Stage 3 – Internal pressure (Annealed)	4

10.2.3 Comparison of Non-Tapered and Tapered results

To see the difference in stresses due to adding a taper, a comparison of the residual stresses after manufacture is compared in Figure 10.42 for the [90/0/90]s FE models. Both non-tapered and tapered FE models are utilising aluminium material properties in the T6 condition. The residual stress and strain results, after the manufacturing stage, for the tapered [90/0/90]s model are shown in Table 10.5 and for the non-tapered model were presented previously in Table 10.1.

Table 10.5 Stress and strain results of tapered [90/0/90]_s model at room temperature

	Stress (MPa)				Plastic strain
	Axial	Hoop	Radial	Von Mises	
	[90/0/90]s fibre alignments				
0° fibre	-4123	-143	166	4225	-
90° fibre	-425	1997	329	2029	-
Matrix	206	456	169	363	0.12

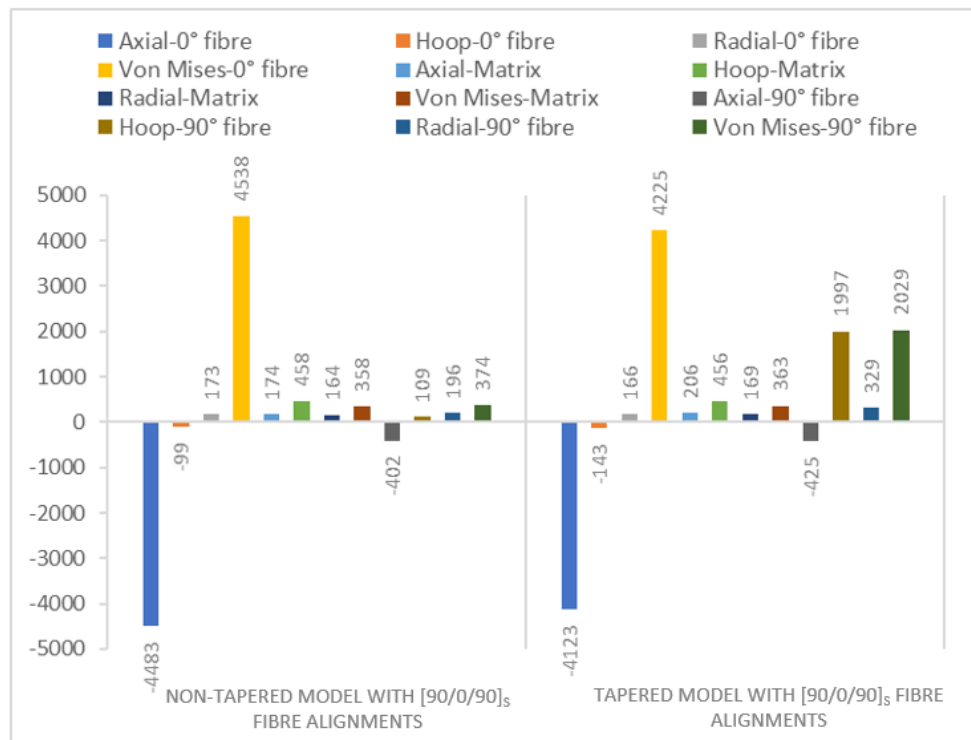


Figure 10.42 Comparison of non-tapered and tapered models with [90/0/90]_s fibre alignments

For the models with [90/0/90]_s fibre layups, there are significant difference in the stress results (Figure 10.42). The maximum axial stress within the 0° fibre reinforcements is 8% smaller in the tapered geometry, and the maximum Von-Mises stress in the axial direction is 5% smaller in the tapered geometry. However, the fibres of the tapered model have a much higher hoop and Von-Mises stress results in the 90° direction (1732% and 443%, respectively). This seems to indicate the importance of including the taper in the models even though, due to the large diameter of the burst tube, the taper angle is small.

10.2.4 Detailed analysis of stresses and strains developed during the manufacturing stage of the [90°/0°/90°]s Tapered model

The axial stress levels, through the manufacturing process, for the 90°/0°/90° fibre reinforcement are presented in Figure 10.43 and Figure 10.44 and the fibre radial and hoop stress profiles are presented in Figure 10.45 to Figure 10.48. In Figure 10.43, the 0° fibre stress starts increasing from the beginning of the manufacturing process and reaches a peak of 2311MPa at around 3.5 kilo-seconds. The axial tensile stress in the fibre is generated due to the larger CTE of the aluminium matrix and constrained movement of the fibre and matrix together in the axial direction. The component was then held at constant pressure and elevated temperature for about 3 ks. During this time, the stress decreased within the fibres from the peak value to around 742MPa. This would be due to creep happening within the aluminium matrix which releases the displacements, and therefore stresses within the fibres.

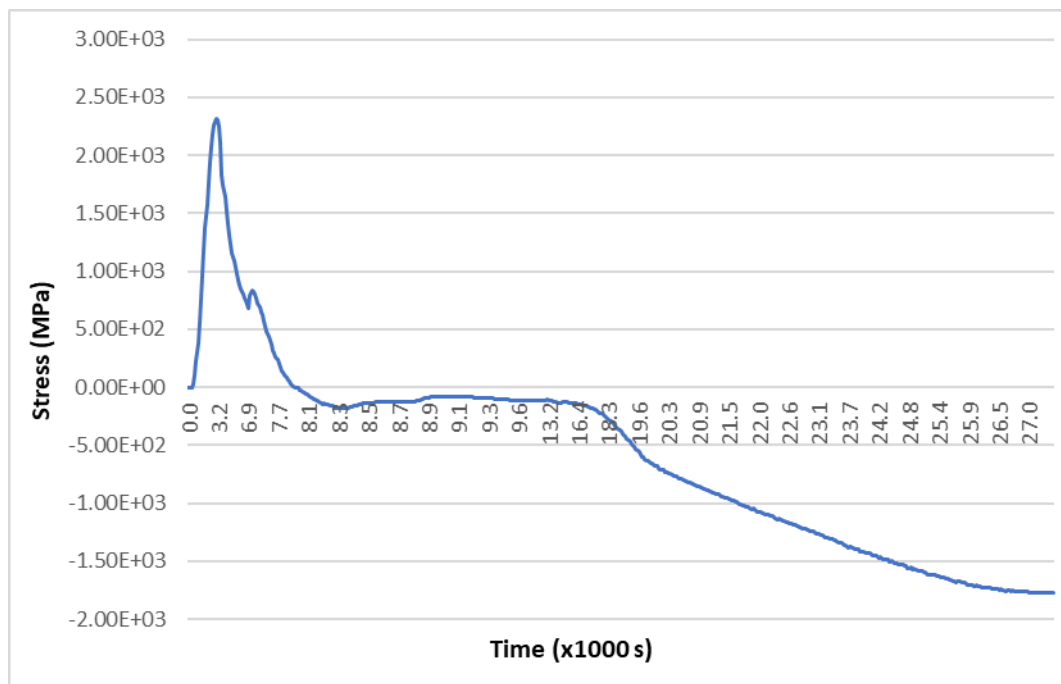


Figure 10.43 Axial stress of 0° fibre reinforcement during the manufacturing process

The hipping process starts at around 7 ks with increasing temperature and compressive loading up to around 9.5 ks. There is initial drop and then a gradual rise before plateauing in the axial stress in the fibres during this stage. The Initial drop is believed to be due to thermal lag within the system. The temperature and pressure were then held for another 4 ks. During this stage there is no noticeable drop in the axial fibre stresses in the 0° fibres. It was assumed

that there would be a decrease in the fibre stresses due to aluminium matrix creep, so it is surprising this is not seen in the 0° fibres. Following the constant temperature and pressure stage, the pressure and temperature are gradually reduced back to room pressure and temperature. As the temperature drops the creep process slows down and essentially stops. This causes the mismatch in the thermal coefficients of expansion to create negative stresses in the fibres whilst creating tension in the matrix, see Figure 10.49. The 0° fibre reinforcement had maximum residual stress of -1768MPa at room temperature.

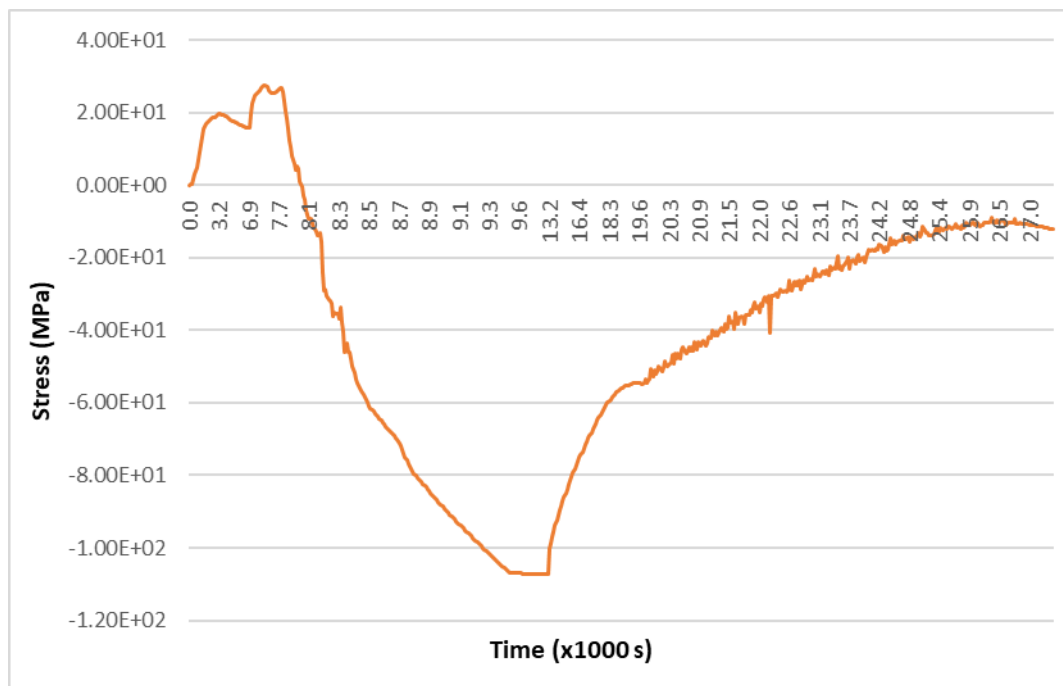


Figure 10.44 Axial stress of 90° fibre reinforcement during the manufacturing process

In Figure 10.44 a similar start is seen in terms of the axial stress in the 90° fibres. It would be expected that the axial stresses generated due to the initial temperature and pressure loading would be less in these fibres due to the matrix material, that is interspersed between the fibres in the axial direction, stretching and relieving the fibre stresses. Complete relief does not happen due to the overall containment but as can be seen the initial stress is much lower, than for the 0° fibres, reaching only 19.4MPa at around 3.5 ks. It then rises rather than drops during the hold stage reaching a maximum pressure at about 7 ks of 27.5MPa. During the hipping process and the 2nd hold stage, from roughly 7 ks to 13.5 ks the 90° fibre axial stress goes down significantly reaching about -107MPa. The slope of the decrease is steeper to start

and then shallows slightly during the hold stage (that starts at just over 9.5 ks). The drop during the hold stage is very different from the 0° fibres where the stress remained constant.

During the cooldown and pressure reduction stage, from approximately 13 ks, the 90° fibres axial stress increases and reaches a value of -12.1MPa at room temperature. The rise in stress is faster to start and then shallows out from about 14 ks onwards. Again, the cooldown results are very different to the 0° fibres where the axial stresses dropped rather than rose.

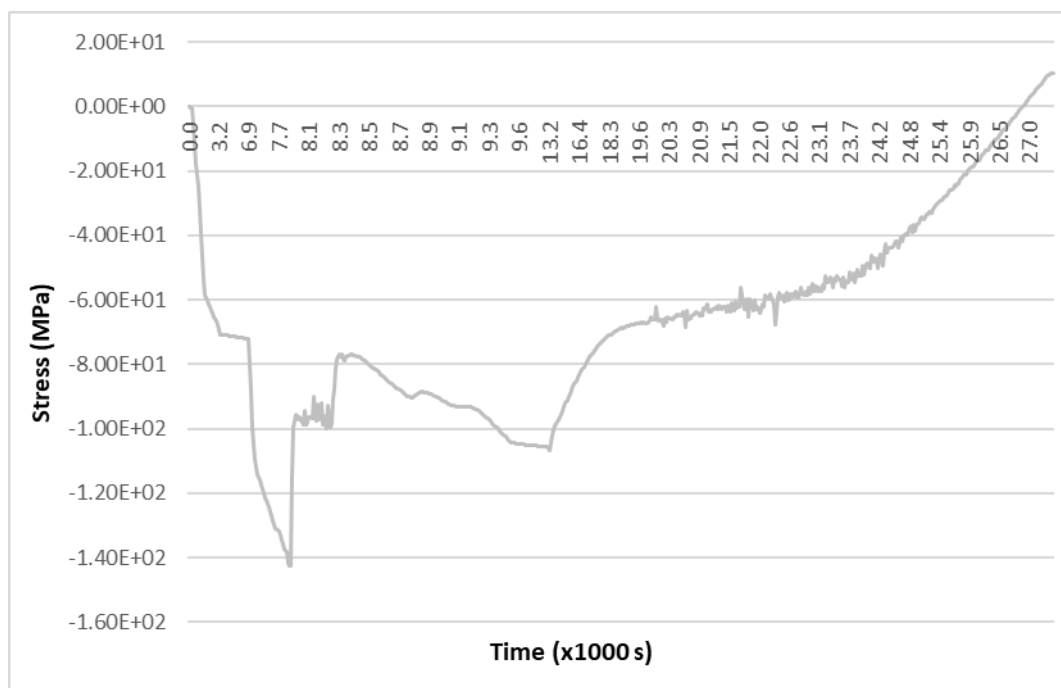


Figure 10.45 Radial stress of 0° fibre reinforcement during the manufacturing process

The radial stress results are shown in Figure 10.45 to Figure 10.46. These follow a reasonable logical path to start; with a decrease in stress up to approximately 3.5 ks, a plateau during the initial hold stage and initial decreasing stress during the start of the hipping stage. During the hipping stage though the stresses drop dramatically and then start to rise slowly again and then plateau out during the 2nd holding stage. It is not entirely obvious why the stresses should suddenly drop and then rise again but there are competing processes in terms of stress generation of creep, pressure, and temperature. As the creep is non-linear and only becomes dominant at higher temperatures it is likely that this is causing the sudden changes. During the cooldown stage the stresses decrease sharply and then linearly to 10.4MPa in the 0° fibres and 27.4MPa in the 90° fibres upon reaching room temperature. The final absolute values of

the 0° fibres (10.4MPa) and 90° fibres (27.4MPa) radial stresses were relatively small compared with the final absolute value of the axial 0° fibres stress (-1768MPa).

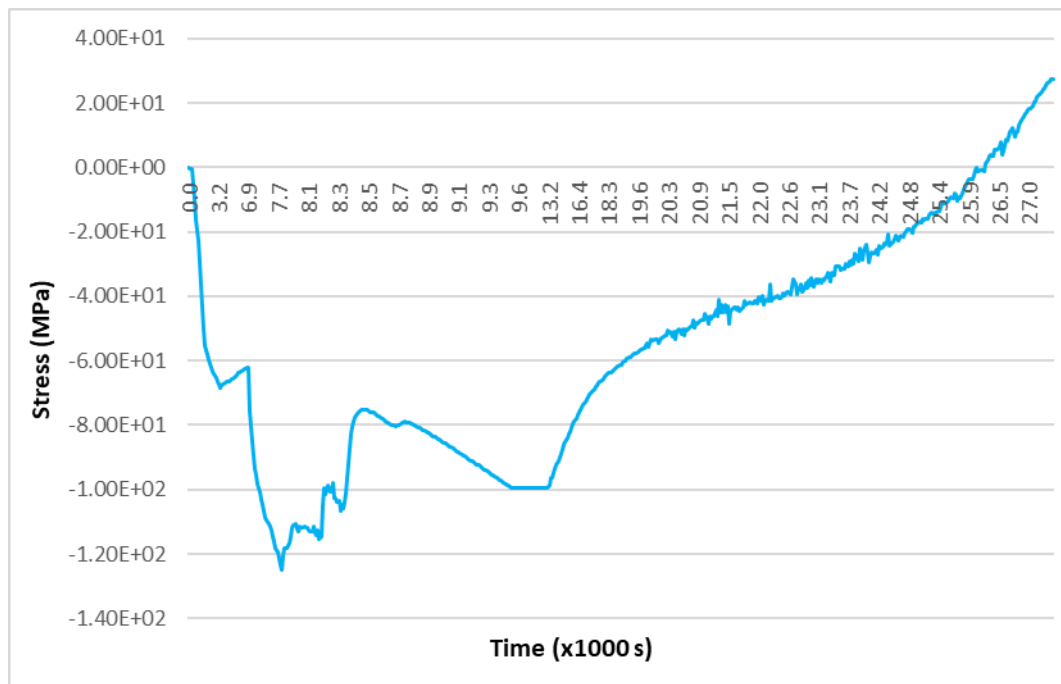


Figure 10.46 Radial stress of 90° fibre reinforcement during the manufacturing process

The hoop stress results are shown in Figure 10.47 to Figure 10.48. The hoop stresses generally react to the pressure and temperature loading in that when the temperature and pressure change so does the hoop stresses. There are some big differences though between 0° and 90° fibres. The stress goes up during the initial load stage in the 0° fibres but goes down in the 90° fibres. After the initial hold stage, they follow similar profiles although the hoop stress in the 90° fibres reaches a significantly higher value (approximately 10x higher). During the cooldown stage the stresses decrease sharply and then linearly decrease to -0.27MPa in the 0° fibres and 142MPa in the 90° fibres, upon reaching room temperature. The final absolute values of the 0° fibres (-0.27MPa) and 90° fibres (142MPa) hoop stresses are relatively small compared with the absolute value of the axial stress (-1768MPa).

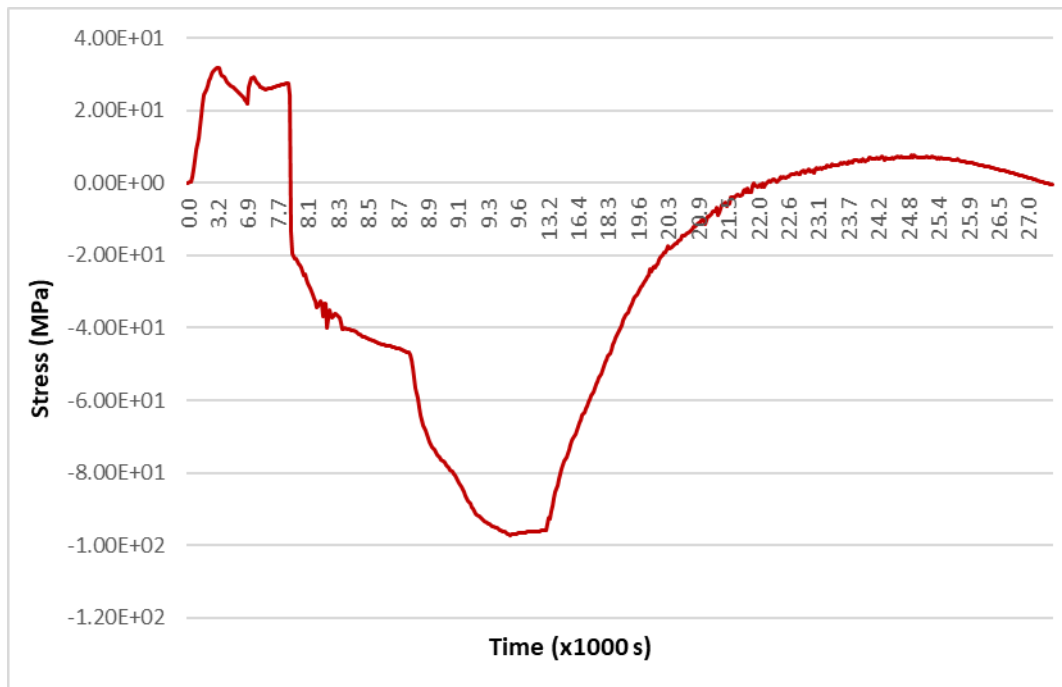


Figure 10.47 Hoop stress of 0° fibre reinforcement during the manufacturing process

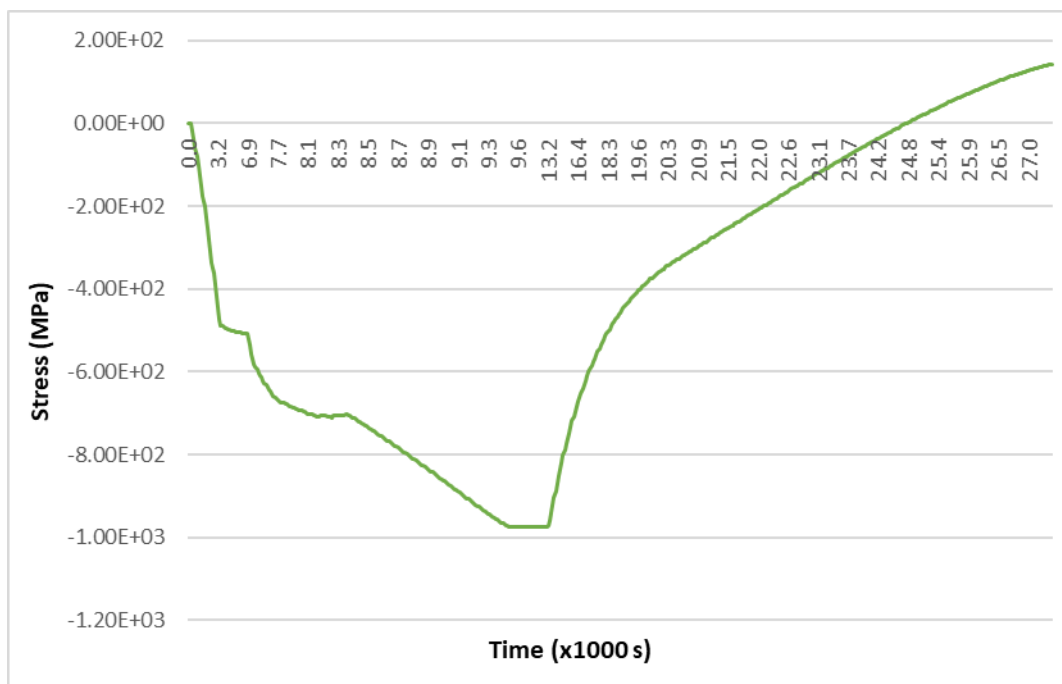


Figure 10.48 Hoop stress of 90° fibre reinforcement during the manufacturing process

The axial, hoop and radial stresses of the aluminium matrix are compared and plotted in Figure 10.49. The stresses follow each other apart from an initial sharp rise in the hoop stress and a slightly lower initial radial stress. The hoop stress initial peak reaches a maximum of -250MPa. From 7 ks onwards the stresses show a gradual linear decrease up to just over 9 ks where there is a short plateau for about 2ks before the stresses start to increasing in value

and start separating about 500s later. The stresses continue to increase until room temperature where they have final values of 21.6MPa, 41.1MPa and 76.3MPa for the radial, axial and hoop stresses respectively.

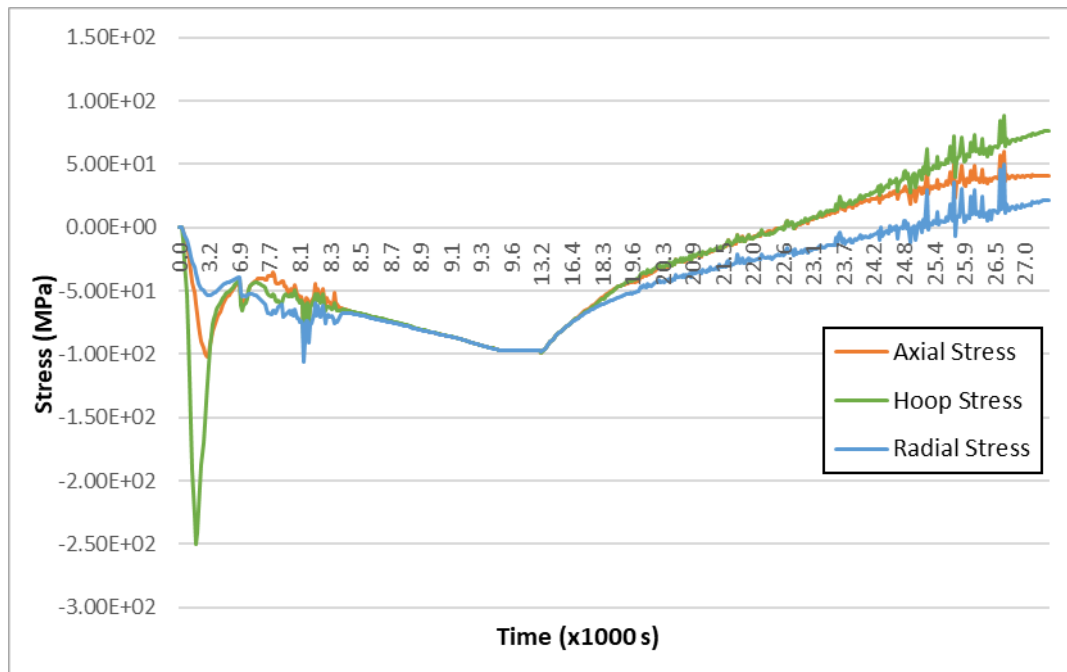


Figure 10.49 Axial, hoop, and radial stresses of the matrix during the manufacturing process

10.2.5 Internal pressure models

This section presents the results of the application of internal pressure after manufacture to the tapered burst tube models developed in sections 7.6.2.4 and section 7.6.2.5.

10.2.5.1 Component with $[0/90/0]_s$ fibre alignments

In Figure 10.50 the results for the internal pressure model with $[0/90/0]_s$ fibre alignments are shown for method 6, where the annealed properties for the aluminium are applied during cooldown and during the application of the internal pressure load, see section 7.6.2.5 for model details. The fibre reinforcements have 3.852GPa tensile stress in the y-axis at the load of 12.64MPa, thus the fibres have failed from the burst loading. The Von Mises stress in the aluminium matrix reaches 158MPa which is much lower than the UTS point. The failure of the component is caused by fibre breakage.

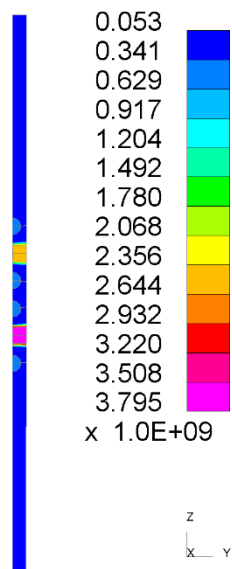
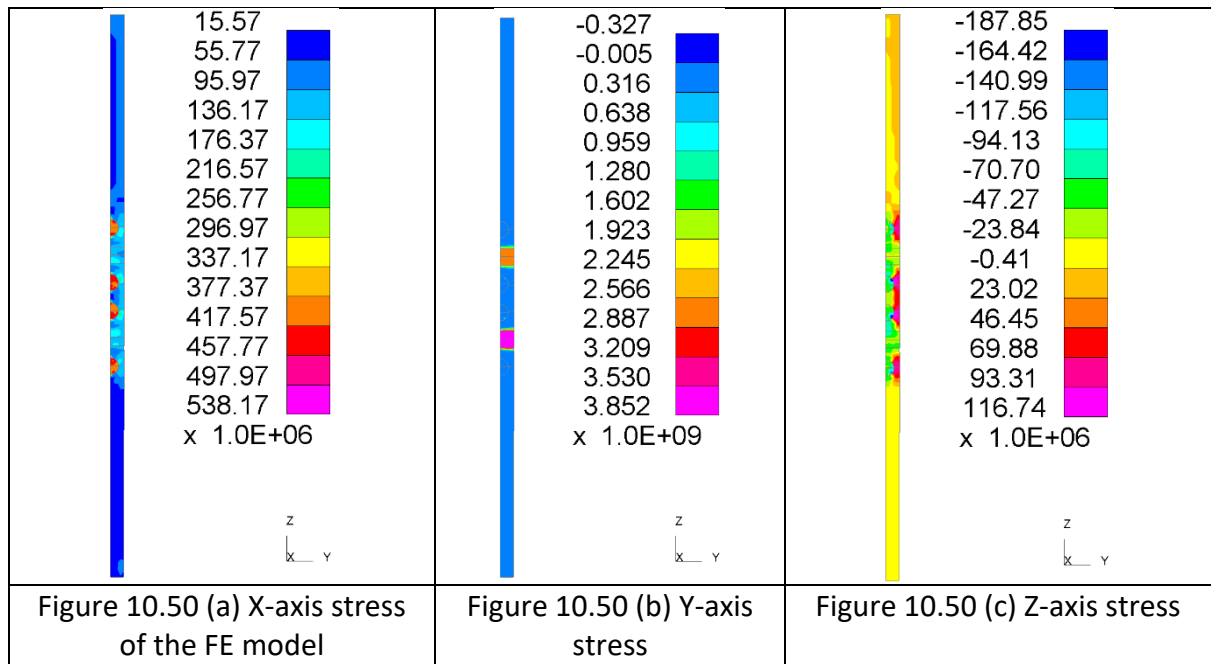


Figure 10.51 Von Mises stress of the FE model

Table 10.6 Residual stresses comparison of [0/90/0]_s fibre alignments model (MPa)

	Axial	Hoop	Radial	Von Mises	Failure load
	Method 4				
0° fibre	-2326	170	-330	2231	>20
90° fibre	68	1096	-138	1138	
Matrix	425	423	148	337	
	Method 5				
0° fibre	548	517	-212	869	12.48
90° fibre	217	3817	-130	3765	
Matrix	187	233	117	158	
	Method 6				
0° fibre	538	517	-188	870	12.64
90° fibre	210	3852	-103	3795	
Matrix	180	241	116	158	

Table 10.6 compares the residual stress results after the contact is released (prior to applying and internal pressure load). As can be seen the results are similar between method 5 and method 6. Method 4 gives very different results to Method 5 and 6 showing the importance of including the effect of annealing within the models. Although method 5 gives similar results to method 6, method 6 was applied to all the other models as it is closer to reality to what happens during the manufacturing process.

10.2.5.2 Component with [0]₆ fibre alignments

The results for the internal pressure model with [0]₆ fibre alignments are presented in Figure 10.52 to Figure 10.56. As can be seen the fibre reinforcements have a tensile stress of 334MPa (y-axis), thus the fibres have not failed from the burst loading. The Von Mises stress in the aluminium matrix reaches 207MPa which is much higher than the yield point and it is beyond the UTS. Thus, the model has failed at the load of 17.6MPa. In reality, the model should definitely be broken before this point due the large plasticity in the matrix material. The exact failure load is hard to predict due to the tapered shape of the FE model. As we can see from the Y-displacement (Figure 10.54), the +Y face does not have an even colour distribution on its edge. This is due to the majority of the edge being set to being constrained to move parrallel to the taper angle but one edge (-X edge) being constrained to move parrallel to the global Y axis (so that it could have pressure applied to it). This prevents points on this edge expanding in the Y direction when there is movement in the Z direction, producing unrealistic

results on this edge. To help counter this the stress data was collected using the results on +X edge of the face.

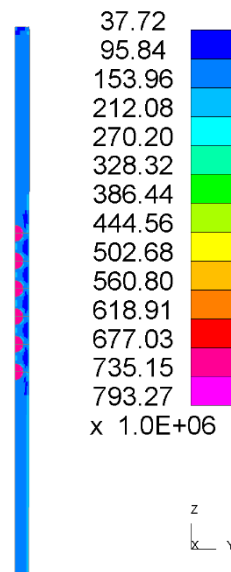
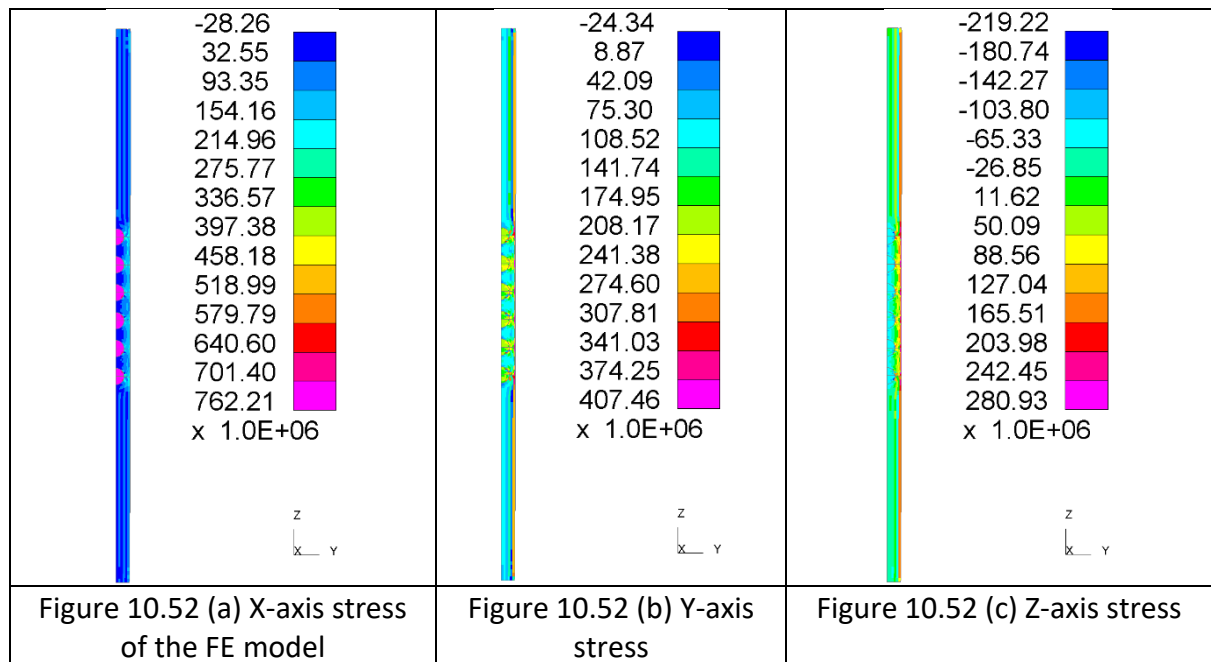
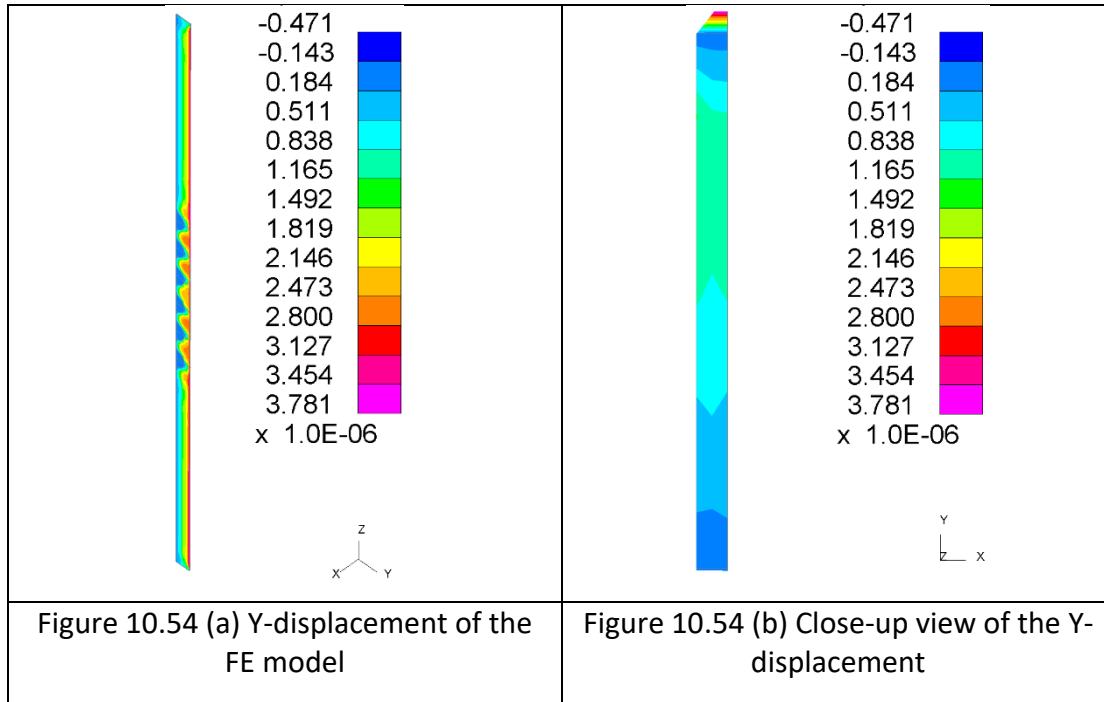
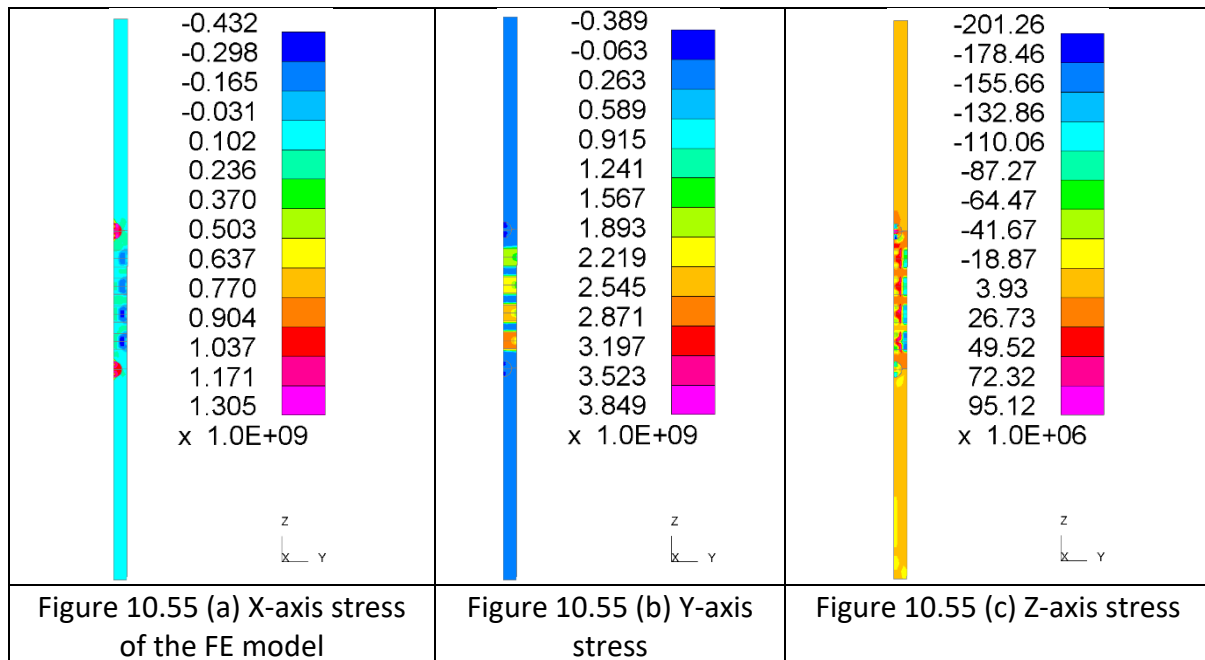


Figure 10.53 Von Mises stress of the FE model



10.2.5.3 Component with [0/90/90]_s fibre alignments

The results for the internal pressure model with [0/90/90]_s fibre alignments are presented in Figure 10.55. As we can see the fibre reinforcements has a tensile stress of 3.849GPa (y-axis) at the load of 13.92MPa, thus the fibres have failed from the burst loading. The Von Mises stress in the aluminium matrix reaches 146MPa which is lower than the UTS point.



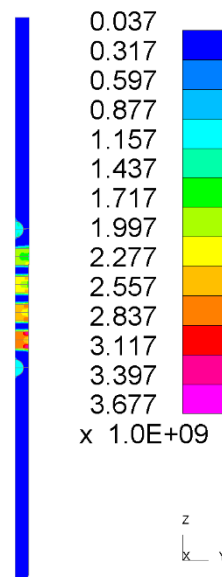
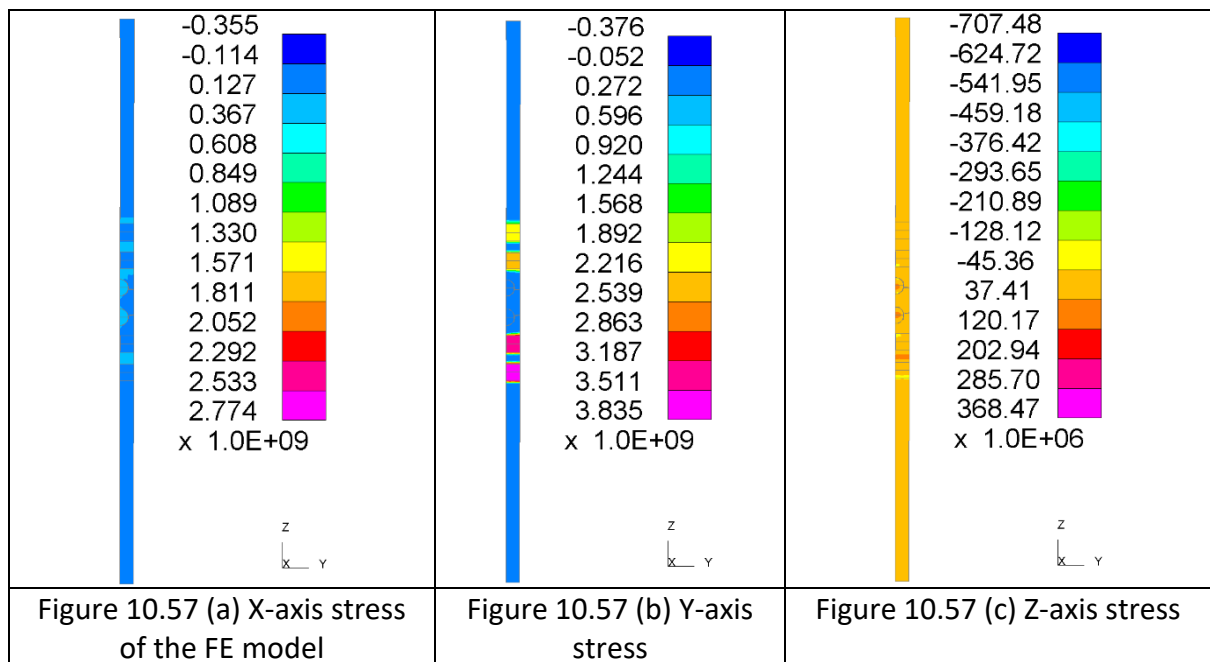


Figure 10.56 Von Mises stress of the FE model

10.2.5.4 Component with [90/90/0]_s fibre alignments

The results for the internal pressure model with [90/90/0]_s fibre alignments are presented in Figure 10.57. As we can see the fibre reinforcements has a tensile stress of 3.835GPa (y-axis) at the load of 14.72MPa, thus the fibres have failed from the burst loading. The Von Mises stress in the aluminium matrix reaches 138MPa which is much lower than the UTS point.



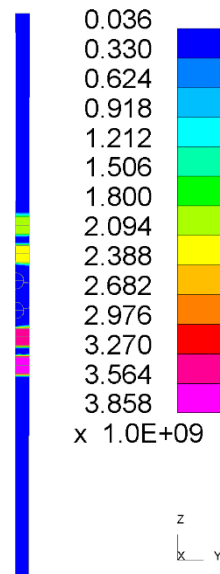
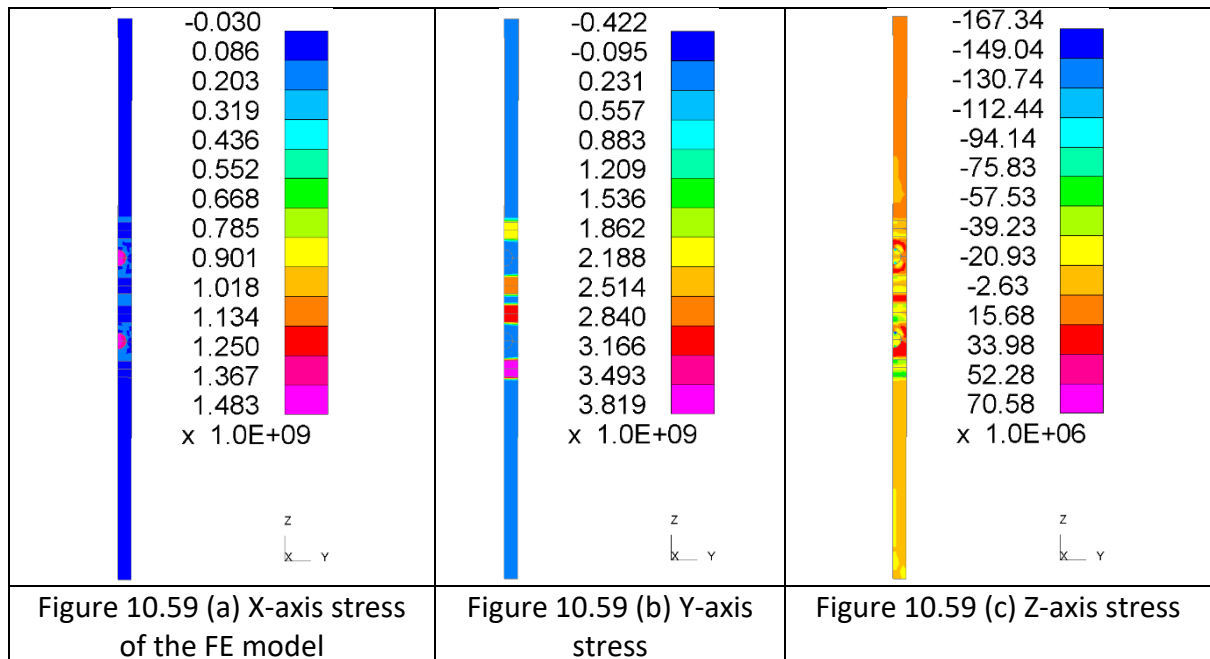


Figure 10.58 Von Mises stress of the FE model

10.2.5.5 Component with [90/0/90]_s fibre alignments

The results for the internal pressure model with [0/90/90]_s fibre alignments are presented in Figure 10.59. As we can see the fibre reinforcements has a tensile stress of 3.819GPa (y-axis) at the load of 16.16MPa, thus the fibres have failed from the burst loading. The Von Mises stress in the aluminium matrix reaches 155MPa which is higher than the yield point but much lower than the UTS point.



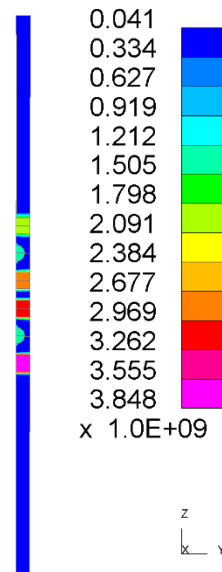


Figure 10.60 Von Mises stress of the FE model

10.2.5.6 Comparison of failure load of internal pressure models

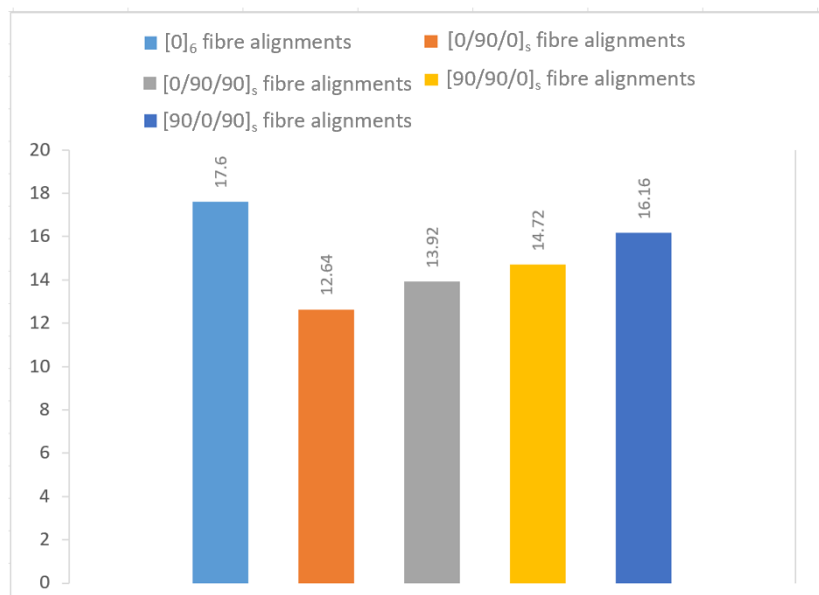


Figure 10.61 Comparison of failure load of internal pressure models

Figure 10.61 shows the failure loads of the internal pressure models with [0]₆, [0/90/0]_s, [0/90/90]_s, [90/90/0]_s and [90/0/90]_s fibre layups. Models with four layers of 90° fibre reinforcements ([0/90/90]_s, [90/90/0]_s and [90/0/90]_s) have higher failure loads. The model with the highest predicted failure load (17.16MPa) is the [0]₆ model. The failure of the [0]₆ model is in the matrix which due to high levels of plasticity. The large deformation may make the failure result unreliable and the exact failure load is hard to predict. The second highest failure load (16.16MPa) is the [90/0/90]_s model. The burst tube model with the [0/90/0]_s

layups only has predicted failure load of 12.64MPa, which is the lowest among the three models.

10.2.6 Discussion of results

Finite element models of SiC/Alu composite burst tube components with various fibre layups have been presented. The predicted residual stress and strain results throughout the manufacturing procedure are shown for tapered and non-tapered models. In comparing results between non-tapered and tapered models, it was seen that including the taper had a significant impact on the residual stresses. The effect of applying aluminium annealing during the manufacturing was shown to be even more important. Although good stress results could be obtained in the burst tube models by applying the annealing effect after the manufacturing process the results were different to when the annealing was applied prior to cooldown. It is therefore recommended that the taper angle is included, and the annealing is accounted for during the manufacturing stage. In the burst tube analysis, based on these recommendations, the maximum burst tube pressures for the different layout configurations varied from 12.64MP to 17.17MPa. Given that the models assume perfect layup the results compare well with the experimental pressure results, where 10 to 14 MPa were obtained for two burst tube tests (for which the exact layup configurations have not been supplied by the manufacturer). An aspect that is known to have significant effect on the residual stresses is the cooldown rate used at the end of the manufacturing process. In these models the same rate was used as for the plates modelled in section 8.2, for which good correlation between experiments and models was obtained. Due to the size and bulk of the burst tubes their internal temperature may not be as close to the furnace air temperature as the plates, although the burst tube wall thickness is comparable to the plate thickness. Conclusions with respect to these results are presented in Chapter 11.

CHAPTER 11

CONCLUSIONS AND FUTURE WORK

CHAPTER 11 CONCLUSIONS AND FUTURE WORK

11.1 Conclusions

This thesis presents work carried out to determine the residual stresses within metal matrix composites. The composite of interest is silicon carbide continuous fibre reinforced aluminium alloy matrices. This composite has specific application within the satellite market due to its ability to be broken down when falling back to earth at the end of the satellites' life. An issue with the manufacture of SiC/Al composites is the residual stresses generated due to the mismatch of the thermal coefficients of expansion of the constituent materials. It is difficult to measure these residual stresses and therefore this work looks at developing techniques to predict the stresses. Another area of interest is the effect of any post heat treatment in improving load performance due to stress relief within the component.

The key outputs of the work can be summarised, in order of importance, as

- Principle of using curvature to determine residual stresses in composite materials
- Validation of numerical models using curvature results
- Prediction of residual stresses in composite plates and burst tubes
- Determination of temper conditions due to manufacture
- Determination of strength change due to post manufacture heat treatment
- Introduction of creep and thermal effects into numerical material models
- Analytical model developed to determine the tensile test stress-displacement curves after manufacture and after post manufacture heat treatment.
- Method to measure experimental and numerical curvature generated by heat treatment

The following paragraphs details these outcomes in order of model development:

Manufactured Al/SiC MMC plates were made with a $[90/0]_s$ symmetric layup, a $[90/0]$ asymmetric layup and a $[+55/-55]$ asymmetric layup, as discussed in Chapter 6. The

asymmetric plates showed significant curvature due to manufacture (See 5.2). The symmetric plates did show some deformation but remained relatively flat (See Table 3.7). An EDM machine was used to cut the symmetric plates up into specimens with face dimensions of 15 x 15 mm, 15 x 30 mm, 15 x 45 mm and 15 x 75 mm (Figure 5.7), as well as dog bone shapes. Some of the specimens were then given a post-manufacture heat treatment. All the specimens were then partially sliced down through the midline, as demonstrated in Figure 5.7, using an EDM. This caused the originally manufactured specimens to open up and form palm-like shapes, but the heat-treated specimens remained closed. The curvatures of the various palms were measured, with no overall difference in curvature found due to the aspect ratio. Figure 5.11 and Figure 5.12 present the post-manufacturing heat treatment specimens. The results indicate that the post-manufacturing heat treatment seems to significantly (or maybe completely) remove the residual thermal stress. Tensile test experiments were conducted on the SiC/Al composite dog bone pieces. Tests with heat treated specimens demonstrated a much better performance under tension than the reference group. Before heat treatment, the samples had maximum tensile stress of 334MPa and 359MPa. With heat treatment, the samples achieved a maximum tensile stress of 540MPa and 595MPa. Overall, the maximum stress performance was improved by 63.5% on average. Vickers hardness tests were applied to specimens with and without heat treatment. The Vickers hardness value indicated that the aluminium alloy matrix had been heated from an annealed (O) condition to a T6 condition. The data from this work was used to inform decisions about material properties for about the FE numerical models.

An analytical tensile test model was created based on the slab analytical model, as discussed in section 6.2. The analytical model generates force versus displacement and stress versus displacement graphs for the specimens prior to and after post-manufacturing heat treatment, as presented in Figure 6.1 and Figure 6.2. To check the models' validity tensile tests were carried out on specimens prior to and after the post-manufacture heat treatment (section 5.2.4). The maximum stress and displacement results show a good correlation with 3.6% and 2.4% differences, respectively. The shapes of the experimental and analytical post-manufacture tensile curves do not match closely. The shape of the experimental curve would seem to indicate that the aluminium yields earlier than predicted in the numerical model, but

that would go against the increase in hardness measured after heat treatment and the rise in tensile strength. The improvement of the tensile strength performance was partly because of the matrix temper condition change. Another reason was the relieving of the residual thermal stress. By comparing the tensile results from both groups, thermal residual stress appears to be fully removed by the heat treatment. The results of the residual stresses from the slab model were used to check the importance of including creep in the FE models.

The method proposed to validate the residual stresses in FE models is to use the curvature of manufactured curved plates to check the numerical models are correctly working. Using classical laminate theory (APPENDIX F) a saddle-like deformation of the non-symmetrically laid up laminate plates was predicted. However, according to Hyer's study, the saddle shape is unstable, and can snap to one of two bistable cylinder shapes. Further investigation of the curvature deformation of composite laminates was carried out in Chapter 6 using 2D FE shell models. Various fibre layups ($[90_4/0_4]$, $[-30_4/30_4]$, $[60_4/30_4]$) were modelled and mesh and time-step size dependency were investigated. The final models were then run with sufficiently small meshes and time steps to ensure convergence. Saddle-like and bistable curvature deformations were obtained from all three models, as seen in Table 8.1, Table 8.2 and Table 8.3. Curvature results from the cooling process were compared with the results of the published paper in Figure 8.1. A large deflection was observed from the $[-30_4/30_4]$ laminate model in comparison with $[90_4/0_4]$ and $[60_4/30_4]$ FE models. Overall, a close match was found between FE modelling and the published work. These models though do not account for the effect of creep.

To account for creep, a hyperbolic-sin law, which has been used in literature for modelling the secondary creep rate in aluminium, was utilised. An analytical model was created based on the secondary creep rate equations and a creep strain rate curve was generated by feeding a range of external loadings at different thermal conditions into the model. An FE model, with an inbuilt material model incorporating the same theoretical creep law, was developed to mimic the analytical creep model. The same material properties and boundary conditions were applied to both models. The analytical and FE model creep strain rate results were compared and presented in Figure 7.8. A close match was found between the models, proving the effectiveness of the creep models. In section 8.2.1, the effect of creep was

introduced into FE MMC 2D shell and 3D solid laminate plate models. The residual stresses calculated in the analytical tensile test model (using the slab model) were significantly higher than the 3D FE models that include creep and thermal material property changes. This shows the importance of including creep and thermal effects in the models if accurate predictions of residual stresses are required.

In Chapter 9 the curvature results of the FE analysis results (chapter 8) are compared with the experimental results (chapter 5), see Figure 9.1 and Figure 9.2. The results show a good correlation with slight differences being expected due to manufacturing variations. The curvature results of the FE simulations give good confidence in the ability of the finite element models to predict the correct residual stresses. Section 9.3 presents the aspect ratio analysis of the SiC/Al composite. The results demonstrate a surprisingly good match that gives good confidence that the FEA models behave realistically. The 3D solid element FE models curvature and the 2D shell element results presented in Table 9.1, for the [+55/-55] cross-ply layup and [90/0] layup, show a good match. The 2D models are quicker to set up and run but do not allow the stress patterns within and around the fibres to be predicted.

In Chapter 10, burst tube models with different fibre layups are presented. The models are based on manufactured specimens made by TISICS to test the performance of different layups and materials for pressure vessel performance. Simplified cell-like FE models are presented in section 10.2 for both tapered and non-tapered models. The taper accounts for the curvature of the tubes but increases the complexity of the model. Both the taper angle and application of the annealing of the aluminium during the cooldown stage were shown to have a significant effect on the results. It was therefore recommended that both taper angle and annealed properties, during cooldown, are incorporated into these types of models. In the burst tube analysis, the maximum burst tube pressures ranged from 12.64 to 17.16MPa for five different layup configurations. This compares reasonably well with the provided experimental pressure results of 10 and 14 MPa. A big advantage of the FE models is that detailed analysis of the stress changes during manufacture can be shown as demonstrated for the [90/0/90]_s layout model in section 10.2.4.

This thesis presents and validates a finite element method for determining the residual stress profiles for metal matrix composites subject to different manufacturing processes, layups and component shapes.

11.2 Future work

Although this project has successfully validated and applied numerical models for predicting the residual stresses of composite components, due to manufacturing and due to post-manufacture heat treatment, there is still a lot more work that could be done to check the robustness of the methods and to develop the work further.

Suggestions for further work:

- Check the models' ability to consistently predict the correct curvature (and therefore residual stresses) through testing with samples made utilising different heat treatment settings and different plate layups.
- Carry out a comprehensive comparison of experimental results between the two different curvature measurement methods (full plate or palm tree) to check which gives the best results.
- Do more post-manufacture heat treatment on samples with tensile testing to see whether the results so far, that indicate increased strength and reduced residual stresses, are consistent.
- Carry out repeat destructive tests of burst tubes subjected to different manufacturing routes and layups to increase the confidence in the burst tube pressure strengths.
- Apply the burst tube model for different manufacturing routes and layups to get a prediction of the burst tube pressure strengths and validate with experimental data.
- Apply the FE method to different MMC components and destructively test to check the models' ability to correctly predict failure strength.
- Apply the models to different MMC compositions, such as a titanium alloy matrix.
- Develop models that are more realistic in terms of final fibre locations by allowing for variation in fibre layup prior to the manufacture heat treatment and by allowing sideways movement of fibres during the manufacture heat treatment.

- Create a user-defined material so that different creep models can be introduced into the models.

APPENDIX A The detailed calculation for the rule of mixtures

Rule of mixtures is mathematical expressions to calculate elastic properties of composites. They are based on several simplifying assumptions. For fibre reinforced matrix, the total volume (V) is the sum of the fibre volume (V_{fibre}) and matrix volume (V_{matrix}). Thus, we have the volume fraction,

$$V_f + V_m = 1 \quad \text{Equation A. 1}$$

For unidirectional (direction along the fibre direction) fibre reinforced matrix, we can make assumptions as follows:

- Reinforced fibres are uniform, parallel distributed and continuous
- Bonding between fibres and matrix are perfect
- Fibre and matrix experience equal strain under longitudinal load
- Fibre and matrix experience equal stress under transverse load

When a load(F_1) is applied in the fibre directions, the load is shared between fibre and matrix,

$$F_1 = F_f + F_m \quad \text{Equation A. 2}$$

The total cross-sectional area is

$$A = A_f + A_m \quad \text{Equation A. 3}$$

Thus, the load distribution can be written as

$$\sigma_1 A = \sigma_f A_f + \sigma_m A_m \quad \text{Equation A. 4}$$

Applying Hooke's law,

$$E_1 \varepsilon_1 A = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m \quad \text{Equation A. 5}$$

Because fibre and matrix experience equal strain,

$$\varepsilon_1 = \varepsilon_f = \varepsilon_m \quad \text{Equation A. 6}$$

Thus,

$$E_1 A = E_f A_f + E_m A_m \quad \text{Equation A. 7}$$

$$E_1 = E_f \frac{A_f}{A} + E_m \frac{A_m}{A} \quad \text{Equation A. 8}$$

Hence,

$$E_1 = E_f V_f + E_m V_m \quad \text{Equation A. 9}$$

$$E_1 = E_f V_f + E_m (1 - V_f) \quad \text{Equation A. 10}$$

When a load(F_2) is applied in the transverse directions, the stress is equal between fibre and matrix,

$$\sigma_2 = \sigma_f = \sigma_m \quad \text{Equation A. 11}$$

The total transverse deformation is

$$\varepsilon_2 L_2 = \varepsilon_f L_f + \varepsilon_m L_m \quad \text{Equation A. 12}$$

Dividing by L_2 on both sides,

$$\varepsilon_2 = \varepsilon_f V_f + \varepsilon_m V_m \quad \text{Equation A. 13}$$

Applying Hooke's law,

$$\frac{\sigma_2}{E_2} = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} V_m \quad \text{Equation A. 14}$$

Because $\sigma_2 = \sigma_f = \sigma_m$, the transverse tensile modulus is

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \quad \text{Equation A. 15}$$

$$E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \quad \text{Equation A. 16}$$

For shear modulus G_{12} , we have

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{F/A}{\Delta x/l} \quad \text{Equation A. 17}$$

$$G_f = \frac{F/A_f}{\Delta x/l} = \frac{Fl}{AV_f \Delta x} \quad \text{Equation A. 18}$$

$$G_m = \frac{F/A_m}{\Delta x/l} = \frac{Fl}{AV_m \Delta x} \quad \text{Equation A. 19}$$

And

$$\frac{1}{G_{12}} = \frac{\gamma_{12}}{\tau_{12}} = \frac{A \Delta x}{Fl} = \frac{A_f \Delta x}{Fl} + \frac{A_m \Delta x}{Fl} = \frac{AV_f \Delta x}{Fl} + \frac{AV_m \Delta x}{Fl} \quad \text{Equation A. 20}$$

Thus,

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \quad \text{Equation A. 21}$$

For Poisson's ratio ν_{12} ,

$$\nu_{12} = -\frac{d\varepsilon_{trans}}{d\varepsilon_{axial}} \quad \text{Equation A. 22}$$

Based on

$$\varepsilon_1 = \varepsilon_f = \varepsilon_m \quad \text{Equation A. 23}$$

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \quad \text{Equation A. 24}$$

We have

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad \text{Equation A. 25}$$

Thermal expansion in the longitudinal direction,

$$\alpha_1 = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \quad \text{Equation A. 26}$$

Based on

$$E_1 = E_f V_f + E_m V_m \quad \text{Equation A. 27}$$

We have

$$\alpha_1 \Delta T E_1 = E_1 \times \frac{\Delta V}{V} = E_1 \times \frac{\Delta V (V_f + V_m)}{V} \quad \text{Equation A. 28}$$

$$\alpha_f \Delta T E_f = E_f \times \frac{\Delta V_f}{V} \quad \text{Equation A. 29}$$

$$\alpha_m \Delta T E_m = E_m \times \frac{\Delta V_m}{V} \quad \text{Equation A. 30}$$

$$\varepsilon_1 = \varepsilon_f = \varepsilon_m \quad \text{Equation A. 31}$$

So we have

$$\alpha_f \Delta T E_f = E_f \times \frac{\Delta V}{V} \quad \text{Equation A. 32}$$

$$\alpha_m \Delta T E_m = E_m \times \frac{\Delta V}{V} \quad \text{Equation A. 33}$$

$$\begin{aligned} \alpha_1 \Delta T E_1 &= E_1 \times \frac{\Delta V}{V} = (E_f V_f + E_m V_m) \times \frac{\Delta V}{V} \\ &= E_f V_f \times \frac{\Delta V}{V} + E_m V_m \times \frac{\Delta V}{V} \\ &= \alpha_f \Delta T E_f V_f + \alpha_m \Delta T E_m V_m \end{aligned} \quad \text{Equation A. 34}$$

Thus,

$$\alpha_1 = \frac{1}{E_1} (\alpha_f E_f V_f + \alpha_m E_m V_m) \quad \text{Equation A. 35}$$

Thermal expansion in the transverse direction is the total change of volume with temperature and the change of volume with Poisson's ratio ν_{12} ,

$$\nu_{12} = - \frac{d\varepsilon_{trans}}{d\varepsilon_{axial}} \quad \text{Equation A. 36}$$

A total change of volume

$$\frac{\Delta V}{V} = \alpha_2 \Delta T - (-\alpha_1 \Delta T \nu_{12}) = \alpha_2 \Delta T + \alpha_1 \Delta T \nu_{12} \quad \text{Equation A. 37}$$

Using the same principle, the total change of fibre and matrix volume,

$$\frac{\Delta V_f}{V} = \alpha_f \Delta T V_f + \alpha_f \Delta T \nu_f V_f \quad \text{Equation A. 38}$$

$$\frac{\Delta V_m}{V} = \alpha_m \Delta T V_m + \alpha_m \Delta T \nu_m V_m \quad \text{Equation A. 39}$$

Because we have

$$\frac{\Delta V}{V} = \frac{\Delta V_f}{V} + \frac{\Delta V_m}{V} \quad \text{Equation A. 40}$$

Thus

$$\alpha_2 \Delta T + \alpha_1 \Delta T \nu_{12} \quad \text{Equation A. 41}$$

$$= \alpha_f \Delta T V_f + \alpha_f \Delta T \nu_f V_f + \alpha_m \Delta T V_m + \alpha_m \Delta T \nu_m V_m$$

$$\alpha_2 = \alpha_f V_f + \alpha_f \nu_f V_f + \alpha_m V_m + \alpha_m \nu_m V_m - \alpha_1 \nu_{12} \quad \text{Equation A. 42}$$

$$= \alpha_f V_f (1 + \nu_f) + \alpha_m V_m (1 + \nu_m) - \alpha_1 \nu_{12}$$

$$= \alpha_f V_f (1 + \nu_f) + \alpha_m (1 - V_f) (1 + \nu_m)$$

$$- \alpha_1 \nu_{12}$$

APPENDIX B Classical lamination theory (CLT)

Stress-strain relationship of ply aligned with component axis,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} & E_{16} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} & E_{26} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} & E_{36} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} & E_{46} \\ E_{51} & E_{52} & E_{53} & E_{54} & E_{55} & E_{56} \\ E_{61} & E_{62} & E_{63} & E_{64} & E_{65} & E_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} - \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \alpha_{xy} \\ \alpha_{yz} \\ \alpha_{zx} \end{bmatrix} \Delta T \quad \text{Equation B. 1}$$

$$= \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{21} & E_{22} & E_{23} & 0 & 0 & 0 \\ E_{31} & E_{32} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} - \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \alpha_{xy} \\ \alpha_{yz} \\ \alpha_{zx} \end{bmatrix} \Delta T$$

For thin laminate, we have

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} - \begin{bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \end{bmatrix} \Delta T \quad \text{Equation B. 1}$$

where $Q_{ss} = Q_{xyxy}$. In equilibrium, we have complementary shear stresses: $Q_{xy} = Q_{yx}$. Therefore:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} - \begin{bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \end{bmatrix} \Delta T \quad \text{Equation B. 1}$$

Elastic constant Q matrix is given by

$$[Q] = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} = \begin{bmatrix} \frac{E_{long}}{1 - \nu_{long}\nu_{trans}} & \frac{\nu_{long}E_{trans}}{1 - \nu_{long}\nu_{trans}} & 0 \\ \frac{\nu_{long}E_{trans}}{1 - \nu_{long}\nu_{trans}} & \frac{E_{trans}}{1 - \nu_{long}\nu_{trans}} & 0 \\ 0 & 0 & G_{Long} \end{bmatrix} \quad \text{Equation B. 1}$$

Because

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & (\cos^2 \theta - \sin^2 \theta) \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad \text{Equation B. 1}$$

Elastic constants can be written as

$$Q_{11} = \cos^4 \theta \times Q_{xx} + \sin^4 \theta \times Q_{yy} + 2 \cos^2 \theta \sin^2 \theta \times Q_{xy} + 4 \cos^2 \theta \sin^2 \theta \times Q_{ss} \quad \text{Equation B. 6}$$

$$Q_{22} = \sin^4 \theta \times Q_{xx} + \cos^4 \theta \times Q_{yy} + 2 \cos^2 \theta \sin^2 \theta \times Q_{xy} + 4 \cos^2 \theta \sin^2 \theta \times Q_{ss} \quad \text{Equation B. 7}$$

$$Q_{12} = \cos^2 \theta \sin^2 \theta \times Q_{xx} + \cos^2 \theta \sin^2 \theta \times Q_{yy} + (\cos^4 \theta + \sin^4 \theta) \times Q_{xy} - 4 \cos^2 \theta \sin^2 \theta \times Q_{ss} \quad \text{Equation B. 8}$$

$$Q_{44} = \cos^2 \theta \sin^2 \theta \times Q_{xx} + \cos^2 \theta \sin^2 \theta \times Q_{yy} - 2 \cos^2 \theta \sin^2 \theta \times Q_{xy} + (\cos^2 \theta - \sin^2 \theta)^2 \times Q_{ss} \quad \text{Equation B. 9}$$

$$Q_{14} = \cos^3 \theta \sin \theta \times Q_{xx} - \cos \theta \sin^3 \theta \times Q_{yy} + (\cos \theta \sin^3 \theta - \cos^3 \theta \sin \theta) \times Q_{xy} + 2(\cos \theta \sin^3 \theta - \cos^3 \theta \sin \theta) \times Q_{ss} \quad \text{Equation B. 10}$$

$$Q_{24} = \cos \theta \sin^3 \theta \times Q_{xx} - \cos^3 \theta \sin \theta \times Q_{yy} + (\cos^3 \theta \sin \theta - \cos \theta \sin^3 \theta) \times Q_{xy} + 2(\cos^3 \theta \sin \theta - \cos \theta \sin^3 \theta) \times Q_{ss} \quad \text{Equation B. 11}$$

Stresses in the direction of ply orientation are

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{14} \\ Q_{12} & Q_{22} & Q_{24} \\ Q_{14} & Q_{24} & Q_{44} \end{bmatrix} \left\{ \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} - \begin{bmatrix} \alpha_{11} \\ \alpha_{22} \\ \alpha_{12} \end{bmatrix} \Delta T \right\} \quad \text{Equation B. 12}$$

A, B, and D matrices are

$$A_{ij} = \sum_{k=1}^n (Q_{ij})_k (z_k - z_{k-1}) \quad \text{Equation B. 13}$$

$$B_{ij} = \sum_{k=1}^n (Q_{ij})_k \frac{(z_k^2 - z_{k-1}^2)}{2} \quad \text{Equation B. 14}$$

$$D_{ij} = \sum_{k=1}^n (Q_{ij})_k \frac{(z_k^3 - z_{k-1}^3)}{3} \quad \text{Equation B. 15}$$

where k represents the number of the current layer, z_k the top position of that layer (relative to the bottom layer z_0) and n is the total number of layers. Assembly to form [ABD] matrix, we have:

$$[ABD] = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} \\ B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} \end{bmatrix} \quad \text{Equation B. 16}$$

$$[\alpha\beta\delta] = [ABD]^{-1} \quad \text{Equation B. 17}$$

Results loads [N] and resultant moments [M] are expressed as:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} F_{11} & F_{12} & F_{14} \\ F_{12} & F_{22} & F_{24} \\ F_{14} & F_{24} & F_{44} \end{bmatrix} \Delta T \quad \text{Equation B. 18}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{12} & E_{14} \\ E_{12} & E_{22} & E_{24} \\ E_{14} & E_{24} & E_{44} \end{bmatrix} \Delta T \quad \text{Equation B. 19}$$

Combing the moment and force distribution expression gives:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} \\ B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{12} & E_{14} \\ E_{12} & E_{22} & E_{24} \\ E_{14} & E_{24} & E_{44} \\ F_{11} & F_{12} & F_{14} \\ F_{12} & F_{22} & F_{24} \\ F_{14} & F_{24} & F_{44} \end{bmatrix} \Delta T \quad \text{Equation B. 20}$$

Midplane strains and curvatures are calculated as:

$$\left\{ \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{12} & E_{14} \\ E_{12} & E_{22} & E_{24} \\ E_{14} & E_{24} & E_{44} \\ F_{11} & F_{12} & F_{14} \\ F_{12} & F_{22} & F_{24} \\ F_{14} & F_{24} & F_{44} \end{bmatrix} \Delta T \right\} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{14} & \beta_{11} & \beta_{12} & \beta_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{24} & \beta_{12} & \beta_{22} & \beta_{24} \\ \alpha_{14} & \alpha_{24} & \alpha_{44} & \beta_{14} & \beta_{24} & \beta_{44} \\ \beta_{11} & \beta_{12} & \beta_{14} & \delta_{11} & \delta_{12} & \delta_{14} \\ \beta_{12} & \beta_{22} & \beta_{24} & \delta_{12} & \delta_{22} & \delta_{24} \\ \beta_{14} & \beta_{24} & \beta_{44} & \delta_{14} & \delta_{24} & \delta_{44} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \quad \text{Equation B. 21}$$

ε_{xo} , ε_{yo} and γ_{xyo} represent the strains at the midplane and given that the normal of the midplane remains perpendicular and that the laminate thickness does not change. κ_x , κ_y , and κ_{xy} represents the laminate curvatures.

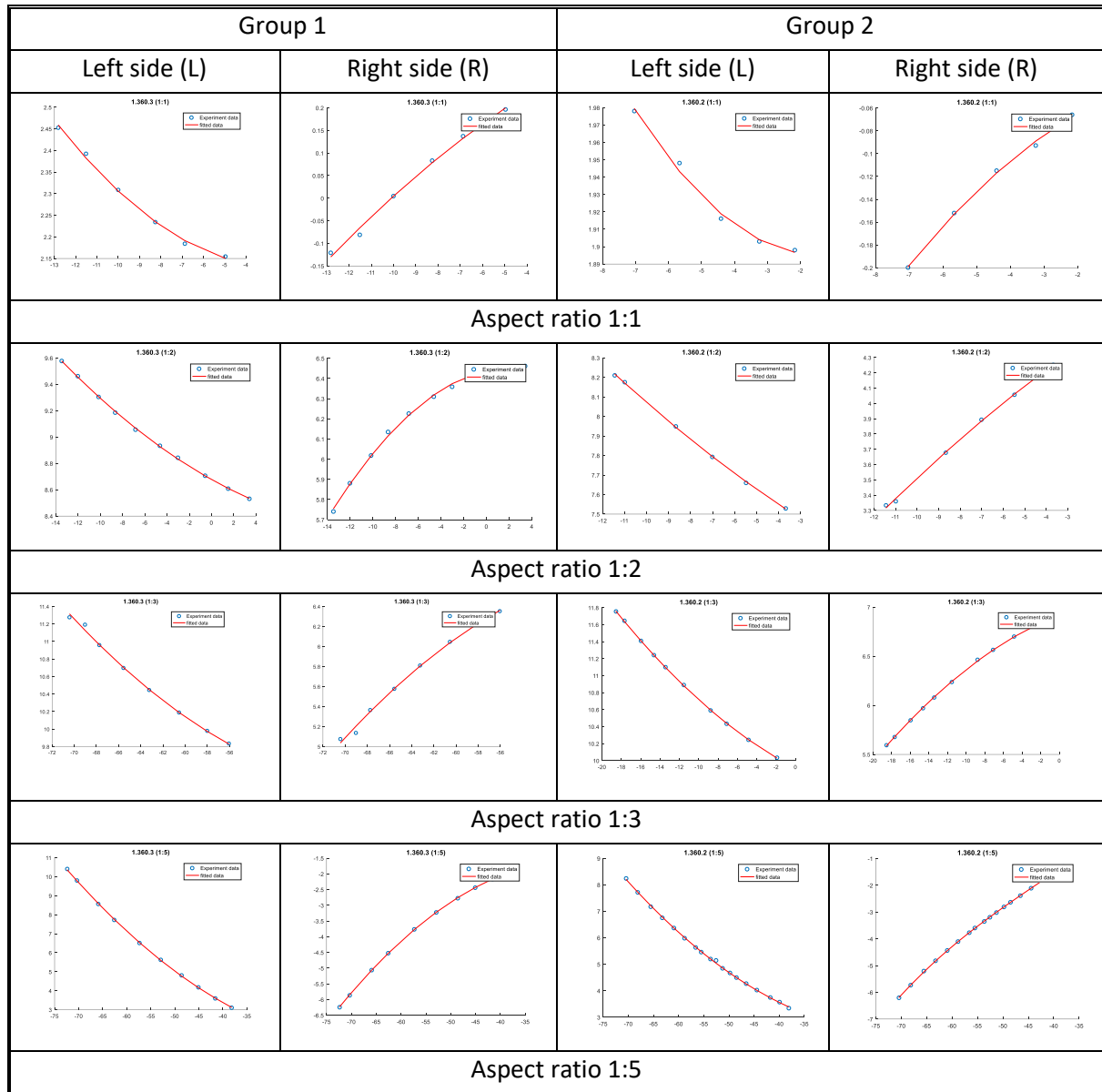
For symmetric laminates, the B matrix will be zero, midplane strains and curvatures are written as:

$$\left\{ \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{12} & E_{14} \\ E_{12} & E_{22} & E_{24} \\ E_{14} & E_{24} & E_{44} \\ F_{11} & F_{12} & F_{14} \\ F_{12} & F_{22} & F_{24} \\ F_{14} & F_{24} & F_{44} \end{bmatrix} \Delta T \right\} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{14} & \beta_{11} & \beta_{12} & \beta_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{24} & \beta_{12} & \beta_{22} & \beta_{24} \\ \alpha_{14} & \alpha_{24} & \alpha_{44} & \beta_{14} & \beta_{24} & \beta_{44} \\ \beta_{11} & \beta_{12} & \beta_{14} & \delta_{11} & \delta_{12} & \delta_{14} \\ \beta_{12} & \beta_{22} & \beta_{24} & \delta_{12} & \delta_{22} & \delta_{24} \\ \beta_{14} & \beta_{24} & \beta_{44} & \delta_{14} & \delta_{24} & \delta_{44} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \quad \text{Equation B. 22}$$

Detailed calculation process using classical lamination theory is presented in the table.

Classical Lamination Theory (CLT) Calculations										
A CLT Analysis when loads are known										
Material properties for each material										
Mat Name	Mat No.	E11 (pa)	E22 (pa)	V12	G12	α11	α22	Ply thick		
SiC/Al_6061	1	1.22E+11	7.95E+10	0.297	3.52E+10	1.43E-05	2.77E-05	1.00E-04		
SiC/Al_6061	1	1.22E+11	7.95E+10	0.297	3.52E+10	1.43E-05	2.77E-05	1.00E-04		
Laminate description					Total laminate thickness			0.0004		
Lamina no.	fibre angle (deg)	material no.	thickness(t)		z0	-0.0002				
1	0	1	1.00E-04		z1	-0.0001				
2	90	1	1.00E-04		z2	0				
3	90	1	1.00E-04		z3	0.0001				
4	0	1	1.00E-04		z4	0.0002				
Mechanical and thermal loads										
Nxx	0		Tcure	293						
Nyy	0		Tservice	859						
Nxy	0		ΔT	566						
Mxx	0									
Myy	0									
Mxy	0									
Q matrix for each material										
Q(mat1)	1.29E+11	2.51E+10	0	Q(mat2)	1.29E+11	2.51E+10	0			
	2.51E+10	8.43E+10	0		2.51E+10	8.43E+10	0			
	0	0	3.52E+10		0	0	3.52E+10			
Q matrix for each ply				thermal expansion coefficients for each ply						
Q(Lamina1)	1.29E+11	2.5051E+10	0	Ply no.	αxx	αyy	αxy			
	25051469885	8.43E+10	0	ply 1	1.43E-05	2.77E-05	0.00E+00			
	0	0	35200000000	ply 2	2.77E-05	1.43E-05	-1.64E-21			
Q(Lamina2)	8.43E+10	2.5051E+10	0	ply 3	2.77E-05	1.43E-05	-1.64E-21			
	2.51E+10	1.29E+11	0	ply 4	1.43E-05	2.77E-05	0.00E+00			
	0	0	35200000000							
Q(Lamina3)	8.43E+10	2.5051E+10	0	Q(Lamina4)	1.2944E+11	2.5051E+10	0			
	25051469885	1.29E+11	0		25051469885	8.4348E+10	0			
	0	0	35200000000		0	0	35200000000			
A, B, D matrices										
42757734	10020587.95	0	0	0	Nxx+Nxx(T)	591729.6722				
10020587.95	42757734	0	0	0	Nyy+Nyy(T)	591729.6722				
0	0	14080000	0	0	Nxy+Nxy(T)	-6.5428E-12				
0	0	0	0.66	0.13	Mxx+Mxx(T)	0				
0	0	0	0.13	0.48	Myy+Myy(T)	0				
0	0	0	0	0	Mxy+Mxy(T)	0				

abd marix						
2.47468E-08	-5.79958E-09	0	0	0	0	0
-5.79958E-09	2.47468E-08	0	0	0	0	0
0	0	7.1023E-08	0	0	0	0
0	0	0	1.600533511	-0.43347783	0	0
0	0	0	-0.43347783	2.200733578	0	0
0	0	0	0	0	0	5.263157895
Thermal stress and moment resultants			Midplane strains and curvatures			
Nxx(T)	591729.6722		εxx	0.011211605		
Nyy(T)	591729.6722		εyy	0.011211605		
Nxy(T)	-6.5428E-12		γxy	-4.6469E-19		
Mxx(T)	0		κxx	0		
Myy(T)	0		κyy	0		
Mxy(T)	0		κxy	0		
	Ply1			Ply2		
εxx-ΔT*αxx	3.14E-03	3.14E-03	-4.44E-03	-4.44E-03		
εyy-ΔT*αyy	-4.44E-03	-4.44E-03	3.14E-03	3.14E-03		
εxy-ΔT*αxy	-4.65E-19	-4.65E-19	4.65E-19	4.65E-19		
SIGxx	295810277.8	295810278	-295810278	-295810278		
SIGyy	-295810277.5	-295810278	295810277.8	295810277.8		
TAUxy	-1.6357E-08	-1.6357E-08	1.6357E-08	1.6357E-08		
	Ply3			Ply4		
	-4.44E-03	-4.44E-03	3.14E-03	3.14E-03		
	3.14E-03	3.14E-03	-4.44E-03	-4.44E-03		
	4.65E-19	4.65E-19	-4.65E-19	-4.65E-19		
	-295810277.5	-295810278	295810277.8	295810277.8		
	295810277.8	295810278	-295810278	-295810278		
	1.6357E-08	1.6357E-08	-1.6357E-08	-1.6357E-08		
Ply strains in x-y coordination system						
Ply no.	Z-COORD	EPSxx	EPSyy	GAMxy		
1	-0.0002	0.0112116	0.011211605	-4.6469E-19		
	-0.0001	0.0112116	0.011211605	-4.6469E-19		
	0	0.0112116	0.011211605	-4.6469E-19		
	0.0001	0.0112116	0.011211605	-4.6469E-19		
	0.0002	0.0112116	0.011211605	-4.6469E-19		
Ply stresses in x-y coordination system						
Ply no.	Z-COORD	SIGxx	SIGyy	TAUxy		
1	-0.01	295810278	-295810278	-1.6357E-08		
	-0.005	295810278	-295810278	-1.6357E-08		
2	-0.005	-295810278	295810277.8	1.6357E-08		
	0	-295810278	295810277.8	1.6357E-08		
3	0	-295810278	295810277.8	1.6357E-08		
	0.005	-295810278	295810277.8	1.6357E-08		
4	0.005	295810278	-295810278	-1.6357E-08		
	0.1	295810278	-295810278	-1.6357E-08		
Composite Ply stresses in x-y coordination system						
Layer No.	Fibre angel (deg)	Stress x	Stress y			
Layer 1	0	295810278	-295810278			
Layer 2	90	-295810278	295810277.8			
Layer 3	90	-295810278	295810277.8			
Layer 4	0	295810278	-295810278			



APPENDIX C Detailed composite layups calculation for burst tube

The aluminium SiC composites used for the burst tube component are made of 6 layers of SiC fibres and 9 layers of aluminium foils. The overall thickness of the composite is 3.542mm (Original data received from TISICS wall thickness spreadsheets). All the data are scientifically measured by TISICS, detailed parameters of the composite are listed in the table.

Diameter of the SiC fibre	100 um
The thickness of the Aluminium alloy foil	127 um
The thickness of the Aluminium cladding	1.02mm
Fibre density	158 fibres per inch

The gaps between each fibre are filled by the aluminium foils, the foils located between the fibre layers (Middle foil layer) filled both top and bottom gaps.

Fibre volume Equation C. 1

$$= 6220.472441 \times \pi r^2 \times 1 \times 1 = 6220.472441 \times \pi \left(\frac{100 \times 10^{-6}}{2} \right)^2$$

$$= 4.89E^{-5} m^3$$

Fibre layer volume Equation C. 2

$$= 100um \times 1 \times 1 = 1E^{-4} m^3$$

Aluminium foil Layer volume Equation C. 3

$$= 127um \times 1 \times 1 = 1.27E^{-4} m^3$$

Outer aluminium layer thickness Equation C. 4

$$= \frac{\text{Aluminium Foil Layer Volume} - 0.5 \times \text{Layer Volume Difference}}{1m^2}$$

$$= 1.01E^{-4} m$$

Middle foil layer thickness Equation C. 5

$$= \frac{\text{Aluminium Foil Layer Volume} - 2 \times 0.5 \times \text{Layer Volume Difference}}{1 m^2}$$

$$= 7.59E^{-5} m$$

Total volume

Equation C. 6

$$= 2 \times (\text{Cladding} + \text{Aluminium Foil Layer Volume} + \text{Outer aluminium layer thickness} + 3 \times \text{Fibre layer volume} + 2 \times \text{Middle foil layer thickness}) + \text{Middle foil layer thickness} = 3.47 \times 10^{-3} \text{ m}^3$$

$$\text{Volume Fraction} = \frac{6 \times \text{Fibre Layer Volume}}{\text{Total volume}} = 8.45\%$$

Equation C. 7

APPENDIX D Composite Manufacturing Methods

- **Liquid state processes**

The manufacturing methods of liquid state processes include stir casting, gas pressure infiltration, squeeze casting, or pressure die infiltration.

a) Stir casting

During the stir casting method, reinforcements like ceramic particles or short fibres are mixed with melted matrix metal by stirring. The composite mixture in the liquid state is cast by conventional casting methods, followed by metal forming procedures. A schematic diagram of the stir casting technique is shown in Figure D.1.

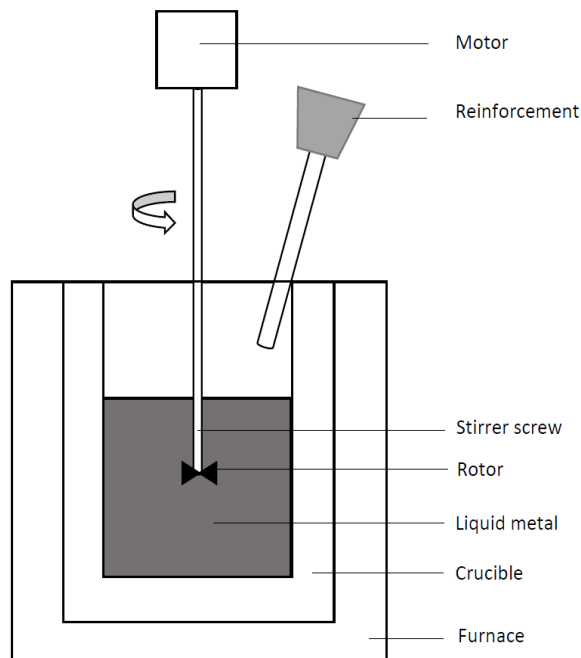


Figure D.1 Schematic diagram of the stir casting set-up

b) Gas pressure infiltration

The gas pressure infiltration method applies a pressurised gas to force the metal liquid into a dispersed reinforcement (preform) (Figure D.2). This method is used for the fabrication of large composite parts (Bhaskar *et al.*, 2014).

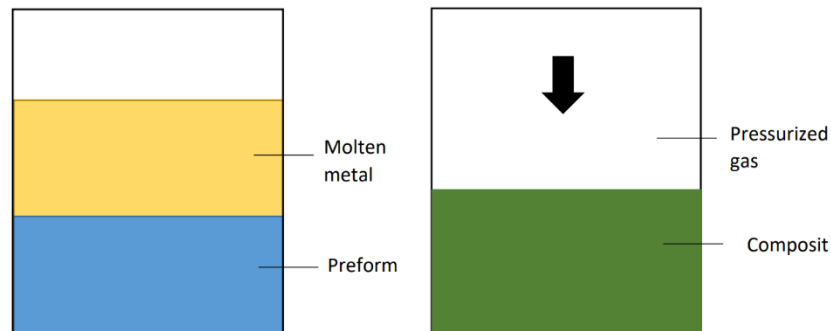


Figure D.2 Schematic of the gas pressure infiltration technique

c) Squeeze casting infiltration (pressure die infiltration)

The squeeze casting infiltration method is similar to the gas pressure infiltration technique, which applies pressure to force the liquid matrix into the preform. However, this method uses a movable die to apply pressure instead of using pressurised gas. The squeeze casting infiltration method fabricates small components, such as engine pistons (Bhaskar *et al.*, 2014). Figure D.3 presented a brief process diagram.

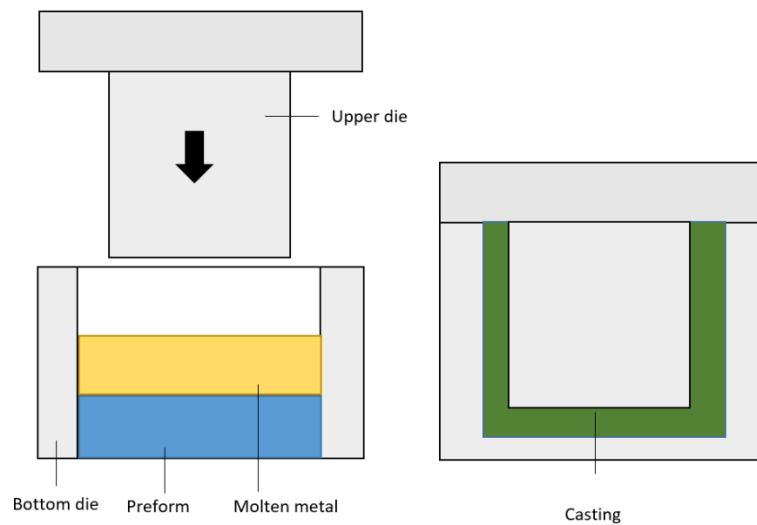


Figure D.3 Schematic of the squeeze casting infiltration technique

- **Solid-liquid processes**

Solid-liquid techniques diffuse the matrix metal and the reinforcements under high pressure and high temperature. The standard techniques are diffusion bonding and power processing (powder metallurgy).

a) Diffusion bonding

Diffusion bonding is a solid-state welding technique used to join metal components. It has been widely used within aerospace and nuclear industries to join metal parts that cannot be combined with conventional welding, such as metal components with precise structures or unique properties. During the bonding process, fibre layers and metal foils are aligned into layups. Fibres are placed with designed orientation, and the volume fraction can be tailored during the manufacturing process. The foil and fibre layups are then placed into a mould under pressure and elevated temperature, resulting in bonded composite materials (Figure D.4).

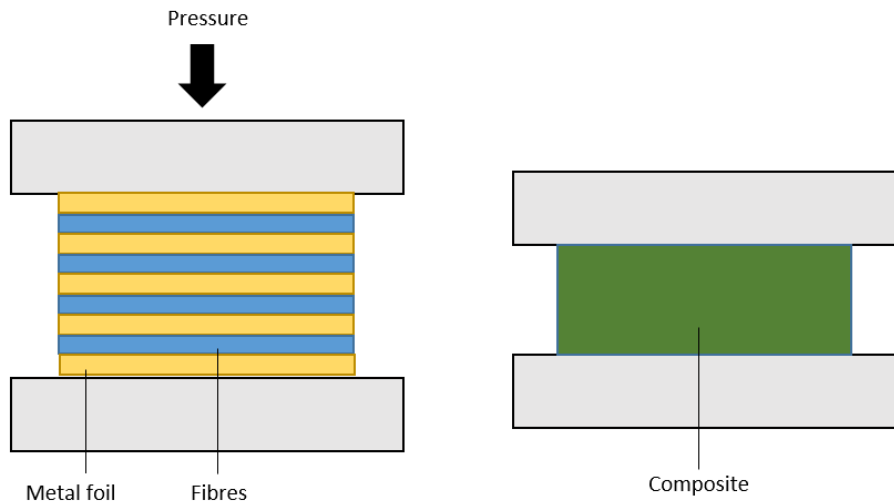


Figure D.4 Schematic of the diffusion bonding technique

Three parameters determine the success of the diffusion bonding: bonding temperature, pressure and holding time. The bonding temperature is generally set at 50%-70% of the melting temperature of the most fusible components. The bonding pressure is applied to break the oxide surface between the bonding components. It should be sufficient to extinguish the voids between the welding parts without changing their macro structures.

b) Power processing (powder metallurgy)

Powder metallurgy (PM) is a manufacturing process using powder metal and is used to manufacture MMC with particulate or short fibre reinforcements (Bhaskar *et al.*, 2014). The benefits of powder metallurgy consist of the ability to produce net-shaped products with excellent microstructures, low manufacturing cost, and minimal secondary machining process requirements (Arifin *et al.*, 2014)

The conventional powder metallurgy process usually includes material blending, powder compaction in the desired mould, sintering, secondary and finishing operations, as shown in Figure D.5 (Arifin *et al.*, 2014).

Non-conventional processes include hot isostatic pressing (hipping) and powder injection moulding (PIM). Hot isostatic pressing manufactures components with pressure and heat. With heat alone, the process is called sintering; with pressure alone, the process is referred to as cold pressing.

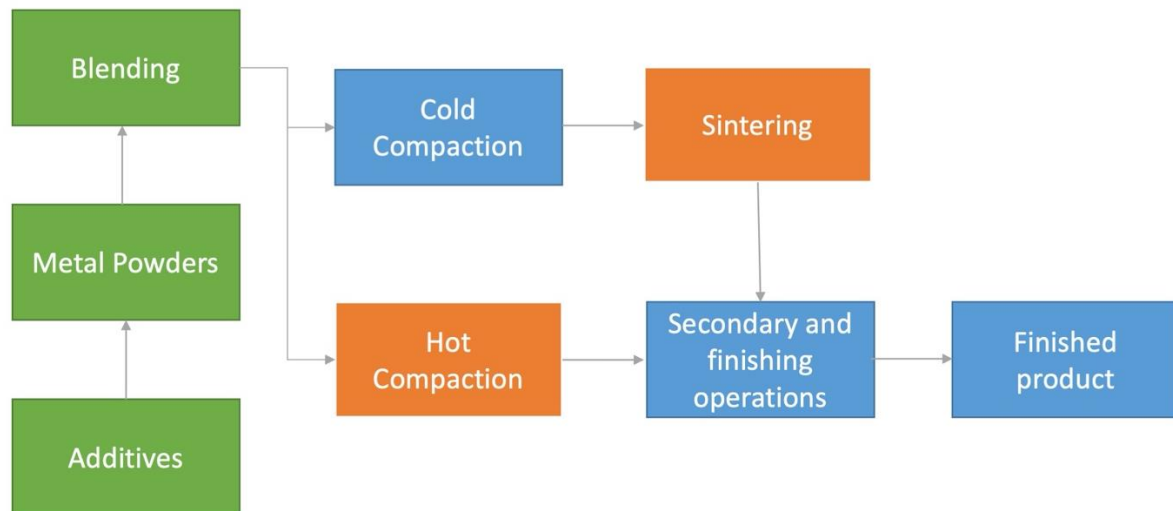


Figure D.5 Powder metallurgy processes

During the conventional PM fabrication process, the metal powder is compressed by uniaxial pressure in a die. The compacts are removed through the same in-line movement of the punch before they are transported into a furnace. The pressing lubricant will then be removed by evaporation and burning of the vapours. The surface oxides will need to be reduced before sintering.

- **Deposition techniques**

Thin-film deposition applies a thin layer of material or coating to the substrate. The techniques involve coating reinforced fibres (in tow) with the matrix metal to form the composite for metal matrix composites. The composite material will go through diffusion bonding for consolidation. The commonly used techniques are immersion plating, electroplating, spray deposition and chemical vapour deposition (CVD) (Bhaskar et al., 2014).

- **In-situ processes**

The in-situ fabrication technique is a process to form the reinforcement phase in-situ during cooling and solidification. It only takes one step to fabricate the composite from the initial alloy in the solid state.

- **Two-phase (solid-liquid) processes**

Two-phase processes blend the reinforcements and the matrix where the matrix metal contains liquid and solid states, such as compo-casting (Bhaskar *et al.*, 2014).

- **Comparison of fabrication methods**

Table 1 compares the fabrication methods discussed in this section regarding reinforcing morphology and MMC systems. SiC_w represents silicon carbide fibre whisker reinforcement.

Table 1 Comparison of fabrication methods (Taya and Arsenault, 2016)

Fabrication method	Reinforcing morphology	Examples
Diffusion bonding	Laminated composites	B/Al
Squeeze casting	All types of reinforcement	C/Mg SiC _w /Al
Compo-casting	Particulates, short fibres composites	Al ₂ O ₃ /Al SiC _w /Al
Powder metallurgy	Particulates, short fibres composites	SiC _w /Al

APPENDIX E Micromechanical FEA

Microstructure models were investigated to study the distribution of residual thermal stresses. The magnitude and distribution of the residual thermal stresses are evaluated with the finite element model.

Thermal residual stresses in SiC/Al composite are introduced during the cooling of the manufacturing process. When the temperature decreases from HIP temperature to room temperature, tensile stress is generated in the matrix, and compressive stress is generated in the fibres. It is because the coefficient of thermal expansion (CTE) of the aluminium 6061 matrix is larger than that of SiC fibre.

In the micromechanical models, we assume that both fibre and matrix properties are elastic and temperature independent. The selection of thermal modelling conditions is based on the study of Durodola (Durodola and Derby, 1994). The authors investigated the structure and mechanism of the diffusion bonding process. A temperature needs to be set just below the metal's melting point during the diffusion bonding process. They suggested that a temperature above 859K will strongly bond with the interfacial phase formed. For microstructure composites models, the temperature of 859K is used to study the residual thermal stresses after the cooling down process.

- **Unidirectional fibre reinforced matrix slab model**

a) Analytical model

The one-dimensional model can estimate the thermal stresses generated in the composite (see Figure 2.26). The strain generated in the fibre is given by Equation E.1,

$$\varepsilon_f = \frac{\sigma_f}{E_f} + \alpha_f \Delta T \quad \text{Equation E.1}$$

where σ_f is the stress in the continuous fibre.

The strain generated in the matrix is given by Equation E.2,

$$\varepsilon_m = \frac{\sigma_m}{E_m} + \alpha_m \Delta T \quad \text{Equation E.2}$$

where σ_m is the stress in the metal matrix.

Because of the strain of the fibre equal to that of the matrix:

$$\varepsilon_f = \varepsilon_m \quad \text{Equation E.3}$$

Using Equation E.1, Equation E.2, we have

$$\frac{\sigma_f}{E_f} + \alpha_f \Delta T = \frac{\sigma_m}{E_m} + \alpha_m \Delta T \quad \text{Equation E.4}$$

Due to equilibrium along the axial direction, the internal forces (due to stresses) must balance with the external force. The external force equals zero; hence we have:

$$F_f + F_m = 0 \quad \text{Equation E.5}$$

$$\sigma_f V_f + \sigma_m V_m = 0 \quad \text{Equation E.6}$$

where V_f is the volume fraction of the reinforced fibre, and V_m is the volume fraction of the matrix ($V_m = 1 - V_f$).

Using Equation E.4, Equation E.6, we have

$$\sigma_m = \frac{-\Delta\alpha\Delta TV_f E_f E_m}{E_c} \quad \text{Equation E.7}$$

$$\sigma_f = \frac{\Delta\alpha\Delta TV_m E_f E_m}{E_c} \quad \text{Equation E.8}$$

where $\Delta\alpha = \alpha_m - \alpha_f$, and E_c is the elastic modulus of the composite, given by the law of mixtures.

$$E_c = E_f V_f + E_m V_m \quad \text{Equation E.9}$$

The analytical model was written in an excel spreadsheet, with the results as shown in Table 5.1. The change of temperature causes a change in the volume of matrix and fibre within the composite. Because of the difference in the coefficients of thermal expansion, the cooling down temperature generates positive residual stress within the matrix and compressive residual stress within the fibre. As seen in Table E.1, when the temperate change is -561K (859K to 298K), the residual stresses of the matrix and fibre are 418MPa and -1670MPa, respectively.

Table E.1 Analytical model of unidirectional fibre reinforced matrix slab model

Slab model	
CTE of fibre ($^{\circ}\text{C}^{-1}$)	4.50E-06
CTE of the matrix ($^{\circ}\text{C}^{-1}$)	2.30E-05
Uniform Temperature Drop (ΔT)	-561
Fibre volume fraction (V_f)	0.2
Matrix volume fraction (V_m)	0.8
Young's modulus of the fibre (GPa)	380
Young's modulus of the matrix (GPa)	70
Young's modulus of composite (GPa)	132
Residual Stress of Matrix (Axial) (MPa)	418
Residual Stress of Fibre (Axial) (MPa)	-1670

b) FEA model

A unidirectional fibre reinforced matrix slab CAD model utilising 3D geometry was created using Solidworks. The volume fraction of the SiC fibre in the model is 20%. Meshed structure for finite element analysis is presented in Figure E.1. The FEA model is created using the same parameters as the analytical model. In Figure E.2, we have the result of residual stresses left after the cooling down process.

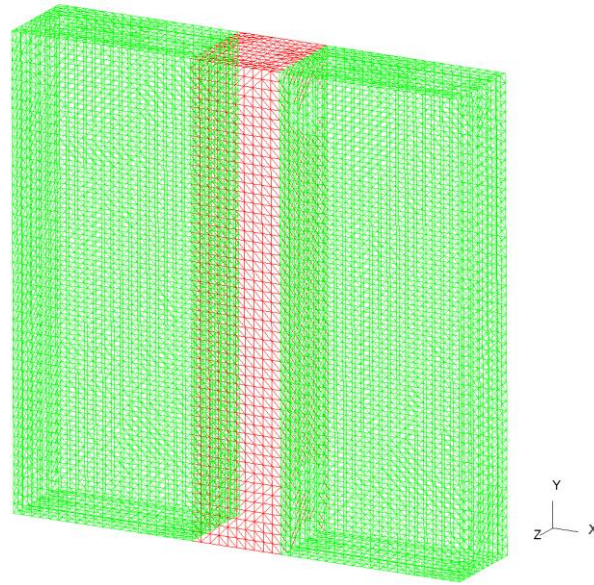


Figure E.1 Mesh structure of unidirectional fibre reinforced matrix slab model

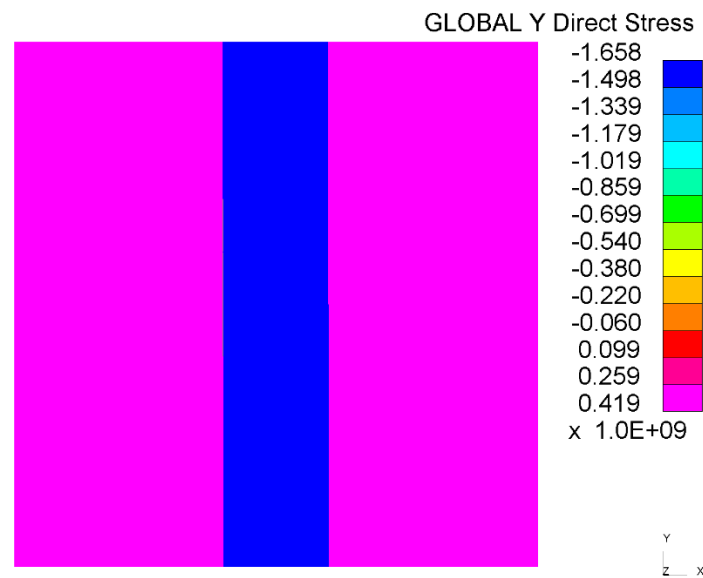


Figure E.2 FEA results of unidirectional fibre reinforced matrix slab model

As shown in Figure E.2, the fibre is under compression, whilst the matrix material is under tension. The residual stresses of matrix and fibre are 419MPa and -1658MPa, respectively.

c) Comparison of FEA and analytical results of slab model

Table E.2 compared the results of analytical and FEA models. Both models were assumed to have a fibre volume fraction of 0.2 and experience a cooling down of 561K. The results have

a 99% similarity, showing a good correlation between the analytical and the finite element analysis.

Table E.2 Comparison of FEA and analytical results of the slab model

	σ_f (MPa)	σ_m (MPa)
Analytical results	-1670	418
FEA results	-1658	419

- **Unidirectional fibre reinforced matrix cylinder model**

The cylinder model is based on the same assumptions used in the slab model. The fundamental equations for the analytical model were discussed in detail in chapter 2.6.5. The analytical model results and the FEA model of the composite cylinder are discussed in this section.

a) Analytical model

The analytical cylinder model contains a long cylinder fibre with perfectly bonded surrounding matrix material. The principal stresses in the cylinder model can be expressed as (Choy *et al.*, 1995):

Radial stress:

$$\sigma_r = A_i + \frac{B_i}{r^2} \quad \text{Equation E.10}$$

Hoop stress:

$$\sigma_\theta = A_i - \frac{B_i}{r^2} \quad \text{Equation E.11}$$

Axial stress:

$$\sigma_z = C_i \quad \text{Equation E.12}$$

where i represents fibre material properties when $i = f$, and matrix material properties when $i = c$. The constants A_i , B_i , and C_i , as given by Equation E.13 - E.18, are using elastic stress-strain relations, continuity, compatibility, and boundary conditions (Choy *et al.*, 1995).

$$A_f = A_c \left(1 - \frac{r_c^2}{r_f^2} \right) + \frac{r_c^2}{r_f^2} P_0 \quad \text{Equation E.13}$$

$$B_f = 0 \quad \text{Equation E.14}$$

$$C_f = \frac{F}{\pi r_f^2} + C_c \left(1 - \frac{r_c^2}{r_f^2} \right) \quad \text{Equation E.15}$$

$$A_c = (a_{22}d_1 - a_{12}d_2)/\Delta \quad \text{Equation E.16}$$

$$B_c = r_c^2 (P_0 - A_c) \quad \text{Equation E.17}$$

$$C_c = (a_{11}d_2 - a_{21}d_1)/\Delta \quad \text{Equation E.18}$$

F and P_0 are the externally applied load and radial pressure, respectively.

a_{ii} , d_i and Δ are material properties constants:

$$a_{11} = \frac{-2\nu_c}{E_c} + \frac{2\nu_f}{E_f} \left(1 - \frac{r_c^2}{r_f^2} \right) \quad \text{Equation E.19}$$

$$a_{12} = \frac{1}{E_c} - \frac{1}{E_f} \left(1 - \frac{r_c^2}{r_f^2} \right) \quad \text{Equation E.20}$$

$$a_{21} = \frac{1 - \nu_c}{E_c} + \frac{r_c^2}{r_f^2} \left(\frac{1 + \nu_c}{E_c} \right) - \left(1 - \frac{r_c^2}{r_f^2} \right) \left(\frac{1 - \nu_f}{E_f} \right) \quad \text{Equation E.21}$$

$$a_{22} = \frac{-\nu_c}{E_c} + \frac{\nu_f}{E_f} \left(1 - \frac{r_c^2}{r_f^2} \right) \quad \text{Equation E.22}$$

$$d_1 = \frac{F}{\pi r_f^2 E_f} - \frac{2 \nu_f r_c^2 P_0}{E_f r_f^2} - \Delta \alpha \Delta T \quad \text{Equation E.23}$$

$$d_2 = \frac{r_c^2 P_0}{r_f^2} \left[\frac{1 - \nu_f}{E_f} + \frac{1 + \nu_c}{E_c} \right] - \frac{\nu_f F}{\pi r_f^2 E_f} - \Delta \alpha \Delta T \quad \text{Equation E.24}$$

$$\Delta = a_{11}a_{22} - a_{12}a_{21} \quad \text{Equation E.25}$$

where

$$\Delta \alpha = \alpha_c - \alpha_f \quad \text{Equation E.26}$$

ΔT is the temperature difference from the manufacturing process to room temperature. E , ν and α are young's modulus, Poisson's ratio and coefficient of thermal expansion, respectively (Choy *et al.*, 1995).

A unidirectional (direction along the fibre direction) fibre reinforced matrix model was generated based on the assumptions of:

- Reinforced fibres are uniform, parallel distributed and continuous

- Bonding between fibres and matrix are perfect

The calculations for the analytical cylinder model are shown in Table E.3. The cooling down temperature generates tensile residual stress within the matrix and compressive residual stress within the fibre. When the thermal loading is -561K, the residual stresses of matrix and fibre are 613MPa and -1430MPa.

Table E.3 Analytical model of unidirectional fibre reinforced matrix cylinder model

Cylinder model			
CTE of fibre ($^{\circ}\text{C}^{-1}$)	4.50E-06	a22	-5.94E-12
CTE of the matrix ($^{\circ}\text{C}^{-1}$)	2.30E-05	d1	1.04E-02
Radius of fibre (mm)	0.00612	d2	1.04E-02
Radius of matrix (mm)	0.01118	Delta	-1.52E-21
Radius of composite (mm)	0.01118	Ac	1.80E+08
Uniform Temperature Drop (ΔT)	-561	Cc	6.13E+08
Young's modulus of fibre (GPa)	380	Bc	-2.25E+04
Young's modulus of composite (GPa)	70	Af	-4.21E+08
Poisson's ratio of fibre (V_f)	2.00E-01	Bf	0.00E+00
Poisson's ratio of matrix (V_m)	3.30E-01	Cf	-1.43E+09
External loading (F)	0	StressF (Radial)	-4.21E+08
Radial pressure (Pa)	0	StressF (Hoop)	-4.21E+08
CTE difference ($^{\circ}\text{C}^{-1}$)	-1.85E-05	StressF (Axial) (MPa)	-1430
a11	-1.19E-11	StressC (Radial)	0.00E+00
a12	2.04E-11	StressC (Hoop)	3.60E+08
a21	7.79E-11	StressC (Axial) (MPa)	613

b) FEA model

A cylinder CAD model with 3D structure geometry was created using Solidworks, as presented in Figure E.3. The fibre volume fraction used is 30%, and the temperature difference was -561K.

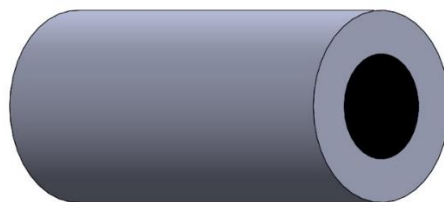


Figure E.3 Geometry of cylinder model in Solidworks

The FEA models were created using the same parameters as the analytical cylinder model. The residual stress results from thermal loading are shown in Figure E.4, which shows that the matrix and fibre generated residual stresses are 623Mpa and -1414MPa, respectively.

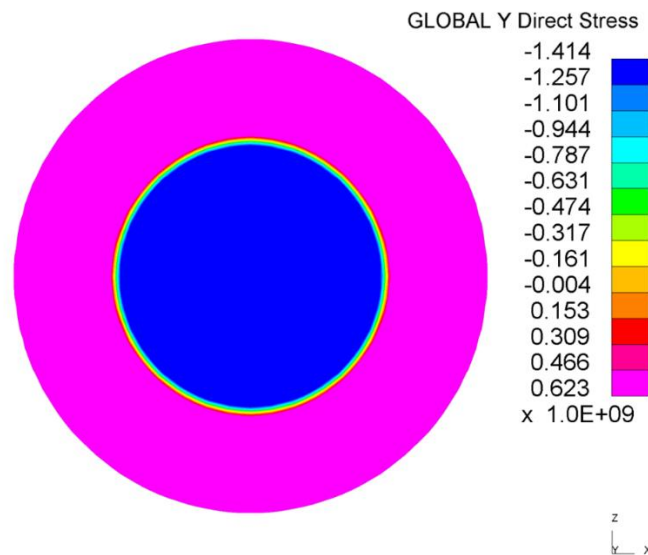


Figure E.4 FEA model results of cylinder model

c) Comparison of FEA and analytical results of the cylinder model

Residual stresses from the analytical model and FEA models are compared in Table E.4. Analytical prediction and finite element analysis results have 98% similarity, which presents a good correlation between the FE results and the analytical model.

Table E.4 Comparison of FEA and analytical results of the cylinder model

	σ_f (MPa)	σ_m (MPa)
Cylinder model analytical results	-1430	613
FEA model results	-1414	623

- **Comparison of single fibre and multi-fibre models**

This section compares the finite element analysis of a single fibre unit cell, a double fibre model and a three-layer fibre model. All three models have circular SiC fibre reinforcements with aluminium matrix, with a fibre volume fraction of 19.63%. The thermal and mechanical properties employed are presented in Table E.5. A linear elastic model is applied for the reinforcements, and the matrix is treated as elastic-perfect plastic. Figure E.7 shows the thermal loading of the finite element models — two thermal cycles of 500°C to 550°C and then back to 500°C.

a) Boundary conditions

All three FE models applied the same boundary conditions. Surfaces in -X, -Y, -Z directions were fixed in the corresponding directions. It means the -X surface (left) can only move in Y and Z directions, but not the X direction. The -Y surface (bottom) cannot move in the Y direction but can move in the X and Z directions. Furthermore -the Z surface (back) can only move in X and Y directions, but not the Z direction. Because of the cyclic thermal loadings, the models expand and contract correspondingly. The +X, +Y, +Z surface nodes were constrained to move in their axial directions together. For example, nodes on the +Z surface (front) move together in the Z direction. Thus, the three models simulated infinite length (due to boundary constraints) even though the models had finite length geometries.

b) Single fibre unit cell model

The single fibre FEA model was created with a continuous reinforcement fibre within the aluminium matrix. Figure E.5 and Figure E.6 illustrate the cross-sectional and isometric view. The single fibre unit cell model developed in this section is compared with the same model presented in the paper (Giugliano, 2016). The published paper investigated the cyclic plastic behaviour of unidirectional fibre-reinforced aluminium composite under thermal conditions. Abaqus finite element model was applied for the analysis. The micromechanical unit cell model has circular cross-section fibre and square matrix. Ceramic fibre reinforcements were assumed to be perfectly bonded to the aluminium matrix. The fibre was treated as a linear elastic material, and an elastic-perfect plastic model was employed for the matrix.

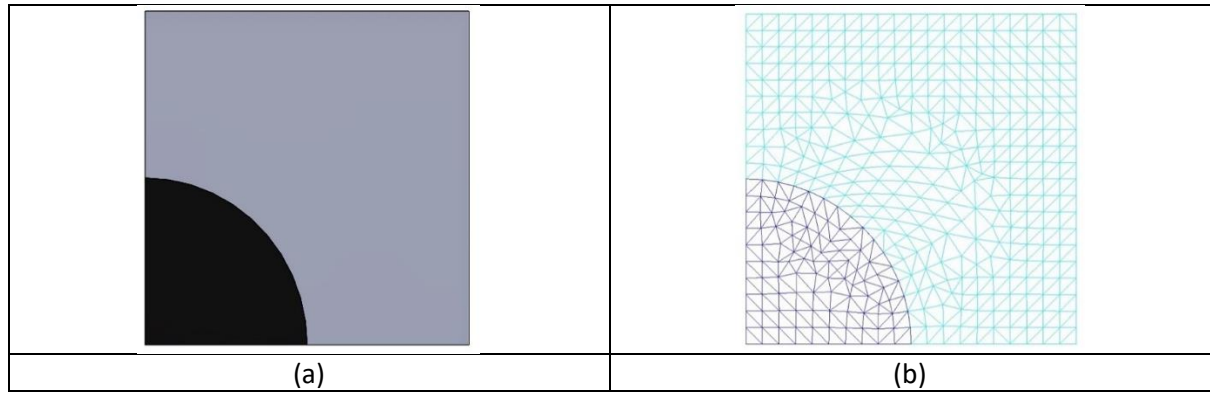


Figure E.5 (a) Geometry of single fibre unit cell model (b) mesh structure for FEA model

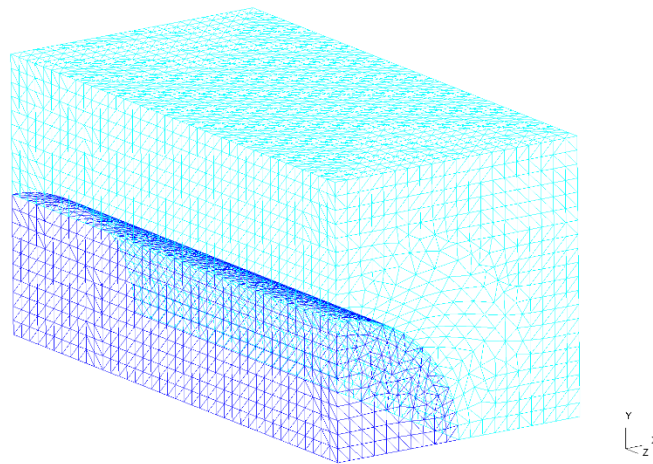


Figure E.6 Isometric view of single fibre unit cell model

The single fibre unit cell model has a structure as shown in Figure E.5 (a) with a volume fraction of 19.63%. The meshed elements of the model are displayed in Figure E.5 (b). The material properties used for the FEA model are presented in Table E.5.

Table E.5 Material properties of single fibre unit cell model (Giugliano, 2016)

	Al Matrix	SiC Fibre
CTE ($^{\circ}\text{C}^{-1}$)	2.2E-05	4.0E-06
Uniform Temperature Drop (ΔT)	-50	
Volume Fraction (V_f)	0.8037	0.1963
Young's Modulus E (GPa)	70	450
Possion's Ratio ν	0.34	0.15
Yield Stress (MPa)	80	10,000

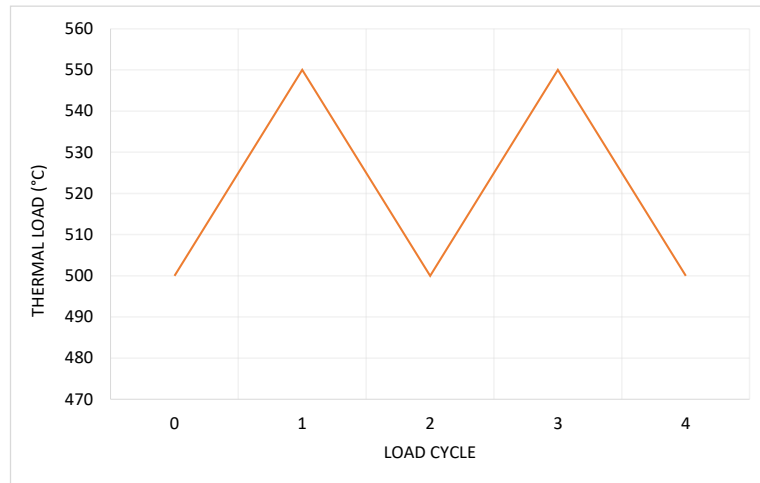


Figure E.7 Thermal loadings applied for FEA analysis

Two thermal cycles are applied to the model, as shown in Figure E.7.

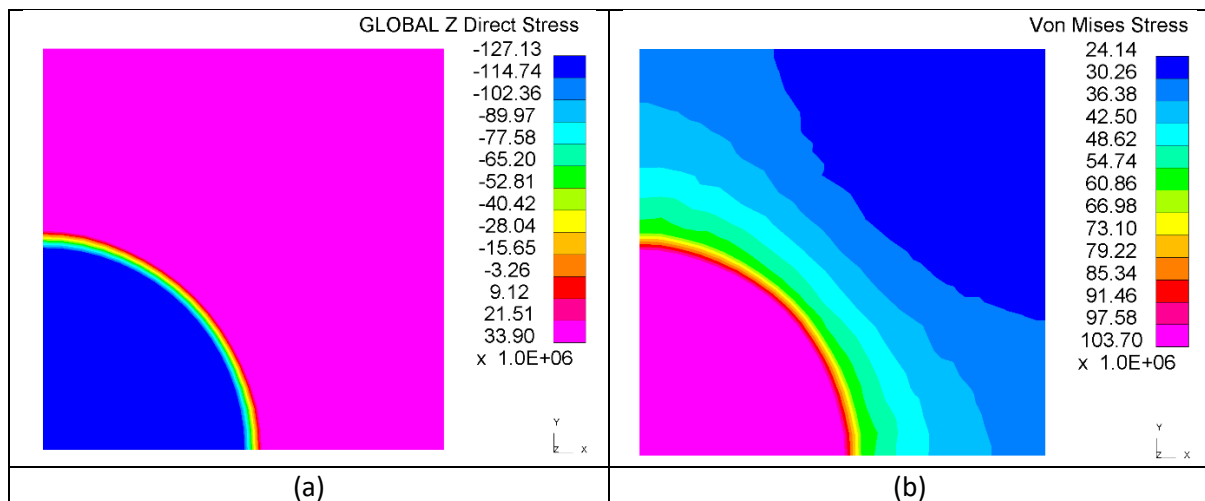


Figure E.8 (a) Axial thermal stress results of single fibre model (b) Von Mises stress distribution

Figure E.8(a) illustrates the thermal stresses generated in the fibre direction after the loadings. Because of the difference in the mechanical properties and coefficient of thermal expansions between matrix and fibre, the fibre part is experiencing compressive stress while the matrix part is experiencing tensile stress. The residual stresses of the matrix and fibre in the fibre direction (z-direction) are 33.9MPa and -127.13MPa, respectively. Figure E.8(b) presents the Von Mises stress conditions after the thermal loading. The minimum von Mises' stress is in

the matrix at 24.14MPa, and the maximum von Mises' stress is in the fibre at 103.7MPa. The FEA results compared with the results from the paper are presented in Table E.6.

Table E.6 Comparison of FEA and paper results of single fibre model

Thermal residual stresses (MPa)				
	Axial Min Value	Axial Max Value	Von Mises Stress Min Value (MPa)	Von Mises Stress Max Value (MPa)
Paper results (Giugliano, 2016)	-	-	22.68	103.5
FEA results	-127.13	33.9	24.14	103.7

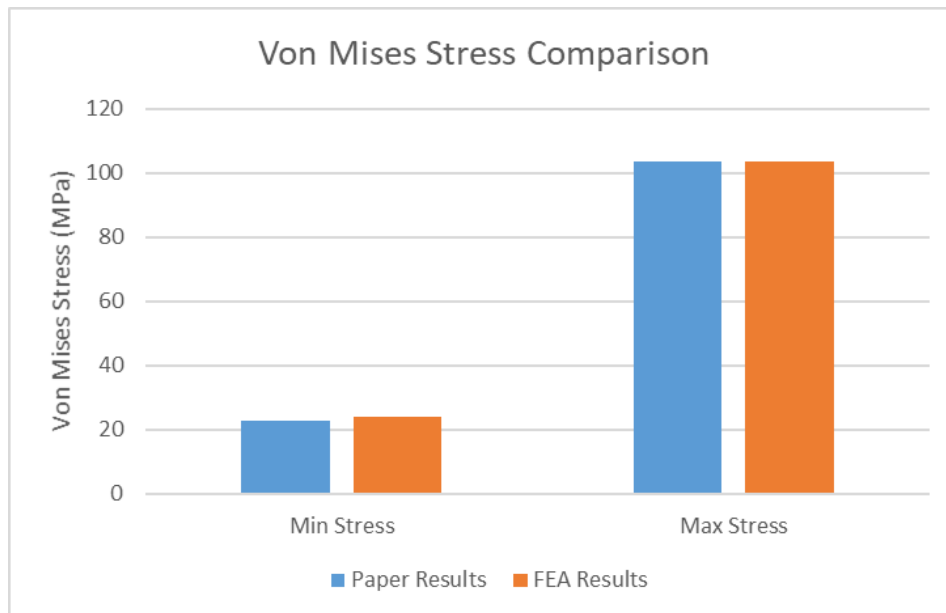


Figure E.9 Von Mises stress comparison of FEA analysis results and paper results

The single fibre unit cell model was investigated under uniform thermal loading conditions. Figure E.9 compares the von Mises results of the FEA model and the paper, and the paper stress results along the fibre direction are presented in Table E.6. The maximum result for the von Mises stress in the fibre is very similar between the FEA model and the paper. The minimum results show more divergence but are still similar. The von Mises stresses in the matrix vary across the matrix, and therefore there will be some sensitivity to mesh size. The results are though considered to be significantly close that the correctness of the FEA model is validated.

c) Double fibre model

The double fibre model presented in this section has two fibre reinforcements within the aluminium alloy matrix. The composite has a volume fraction of 19.63%. The movement was constraint at the fibre cross-sectional surface. Thus, the lengths of reinforcement fibres can be treated as infinite. The model employed the same material properties as shown in section 5.3.3.2. Cyclic thermal loadings, same as the single-cell model, were applied to the FE analysis. The meshed structure for the finite element analysis is presented in Figure E.10.

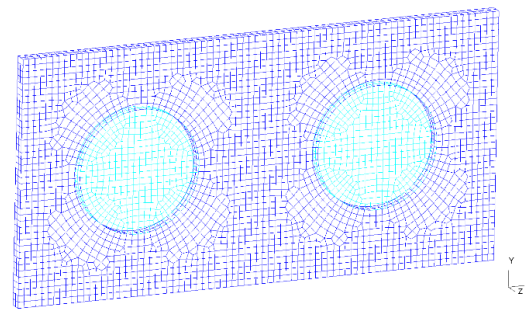


Figure E.10 Mesh structures of double fibre model

As shown in Figure E.11, when the composite experiences cyclic thermal loading, the fibre part is compressed, and the matrix part experiences tension. The residual stresses of the matrix and fibre were 34.17MPa and -134.28MPa, respectively.

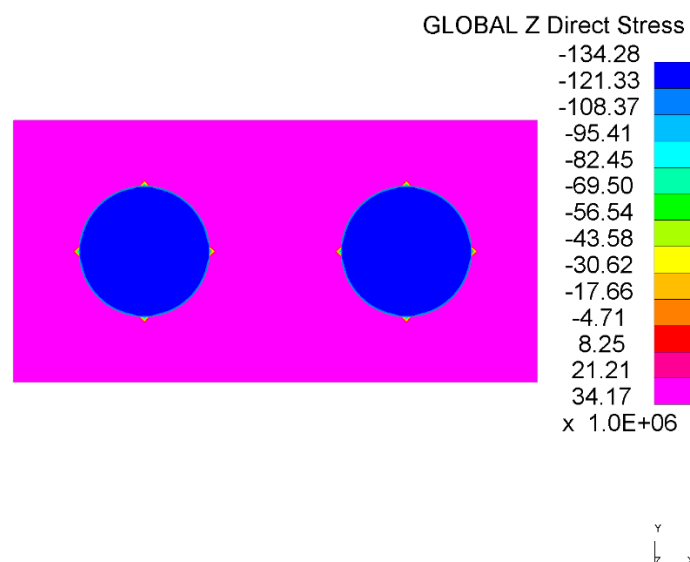


Figure E.11 Thermal stress results of double fibre model on fibre direction

d) Three-layer fibre model

Three-layer fibre models investigated the thermal stress with the same material properties as the single fibre unit cell model (5.3.3.2) and the double fibre model (section 5.3.3.3). Same cyclic thermal loading was employed for this analysis. The mesh structure of the model is shown in Figure E.12. The composite is made of three layers with three continuous fibres evenly distributed in each layer. Nine fibres are reinforcing the composite, with a volume fraction of 19.63%. The boundary conditions applied for this model are presented in detail in section 5.3.3.1.

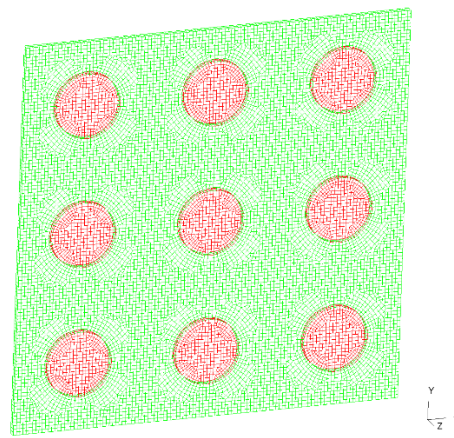


Figure E.12 Mesh structures of three layers fibre model

When the composite experiences a cyclic temperature loading from 550° to 500° , all the fibres experienced a compression. The reason is the difference in the coefficient of thermal expansion with the matrix. The aluminium alloy matrix is under tension, as shown in Figure E.13. The residual stresses of matrix and fibres are 34.17Mpa and -134.28MPa.

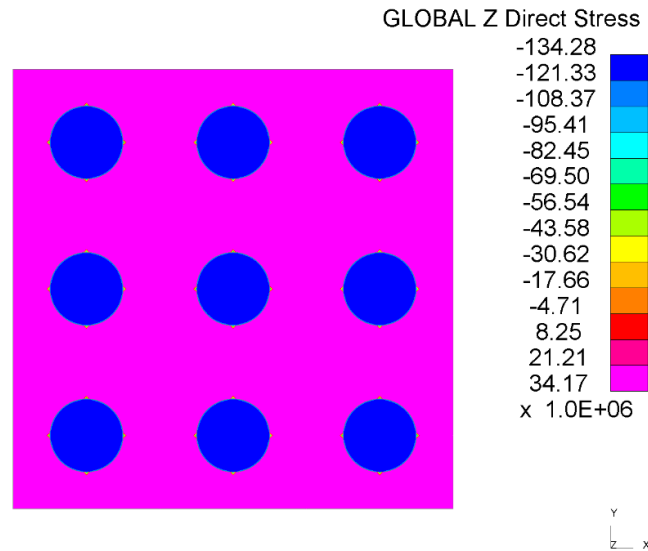


Figure E.13 Thermal stress results of three layers fibre model on fibre direction

e) Results comparison

Table E.7 presents the thermal stresses in the Z direction of the single fibre unit cell, the double fibre and the three-layer fibre models. The double fibre and the single fibre model results have an average of 97% similarity. Compared with the double fibre model, the three-layer fibre model has 100% thermal stress results similarity. The results prove the effectiveness of double fibre and three-layer models in thermal stress prediction under cyclic thermal loading.

Table E.7 Thermal stresses of single fibre and multi-fibre models

	Volume Fraction (V_f)	Thermal residual stresses (MPa)	
		Axial Min Value	Axial Max Value
Single fibre unit cell model	19.63%	-127.13	33.9
Double fibre model		-134.28	34.17
Three-layer fibre model		-134.28	34.17

APPENDIX F Classical lamination theory (CLT) model

This section demonstrates the residual stresses calculation using classical laminate theory when a composite with different layups is cooled down to room temperature. Simplified material properties are used for the CLT model, as shown in Table F.1.

Table F.1 Simplified material properties of CLT models (Choy et al., 1995)

Material Property	SiC fibre	6061 Aluminium alloy	Composite material properties estimation using Rule of mixtures (vf=16%)
Young's modulus, E (GPa)	400	69	E1=122 E2=79.5 G12=35.2
Possion's ratio, ν	0.20	0.33	0.3092
Coefficient of thermal expansion, α ($10^{-6} K^{-1}$)	4.7	24.8	$\alpha_1 = 14.3$ $\alpha_2 = 27.7$

The SiC fibre reinforcements used in the finite element models are assumed isotropic linear elastic materials without considering creep and failure properties. A uniform temperature drop from 859K to 293K with zero initial external stress was applied to the composite. The composite material has four lamina layers, with a [0/90/90/0] fibre angle layup. The detailed calculation process is presented in Appendix B.

Table F.2 Residual thermal stress using classic lamination theory

Composite Ply stresses in the x-y coordination system				
Layer No.	Fibre angel (degree)	Layer thickness (mm)	Stress X	Stress Y
Layer 1	0	0.1	296 MPa	-296MPa
Layer 2	90	0.1	-296 MPa	296 MPa
Layer 3	90	0.1	-296 MPa	296MPa
Layer 4	0	0.1	296 MPa	-296 MPa
Axial stresses in fibre and matrix components				
Layer No.	Fibre	Matrix		
Layer 1	-2960 MPa	296 MPa		
Layer 2	-2960 MPa	296 MPa		
Layer 3	-2960 MPa	296 MPa		
Layer 4	-2960 MPa	296 MPa		

- **2D shell CTE model**

A 2D FEA shell model with identical fibre layups and material properties was created in LS-Dyna for comparison. The residual stresses in the fibre directions after the cooling down process are listed in Table F.2. Laminate layers one and four, with zero degree fibre angles, have 296MPa residual stress in the x-direction and -296MPa stress in the y-direction. It means the layer is under tension in the fibre direction and compression in the transverse direction. It can be seen that the residual fibre stresses in layers two and three, where the fibre angle is 90 degrees, is the other way round relative to the stresses in fibres one and four. The mesh structure and stress results of the FEA model are presented in Figure F.1.

Table F.3 Results comparison of Classic lamination theory with 2D FEA models

Composite Ply stresses in the x-y coordination system			
Layer No.	Fibre angel (degree)	Stress X	Stress Y
Layer 1	0	296 MPa	-296 MPa
Layer 2	90	-296 MPa	296 MPa
Layer 3	90	-296 MPa	296 MPa
Layer 4	0	296 MPa	-296 MPa
Composite Ply stresses in FEA models			
Layer No.	Stress X	Stress Y	
Layer 1	296.34 MPa	-296.34 MPa	
Layer 2	-296.34 MPa	296.34 MPa	
Layer 3	-296.34 MPa	296.34 MPa	
Layer 4	296.34 MPa	-296.34 MPa	

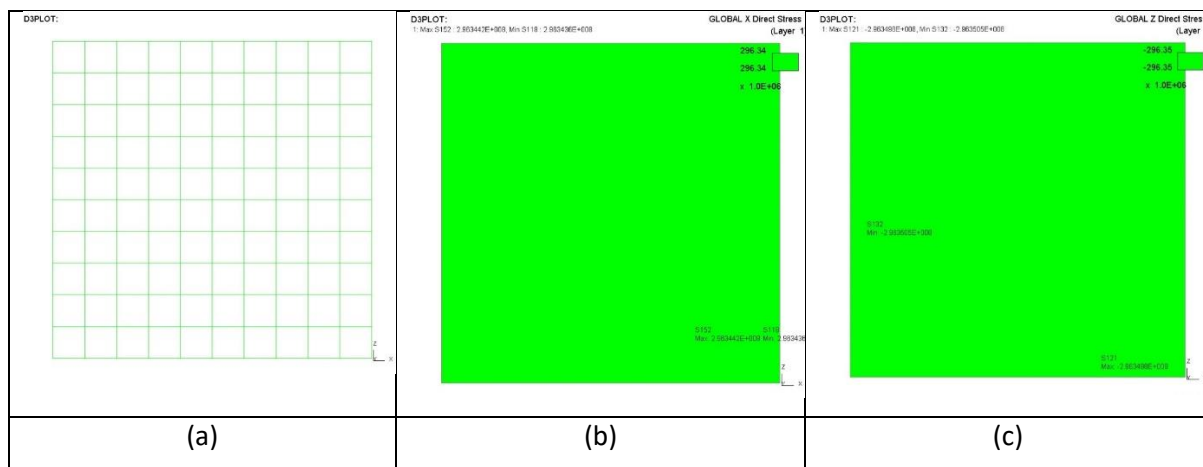


Figure F.1 (a) FEA model mesh structure (b) stress results of layer one in the X direction (c) stress results of layer one in the Z direction

The results from the FEA models matched well with the classical lamination theory results. The 2D shell model provides an efficient way to predict the residual thermal stresses in the composite laminates. However, the model lacks the ability to give detailed thermal stresses within the fibre and the matrix components due to structural limitations. A 3D finite element model was created to validate with the CTE analytical and 2D models.

- **3D laminate composite CTE model**

The [0/90/90/0] laminate model comprises four unidirectional continuous fibres with an aluminium alloy matrix. The composite had a fibre volume fraction set of 16%. The geometry is shown in Figure F.2. The meshed component for the finite element analysis is presented in Figure F.3.

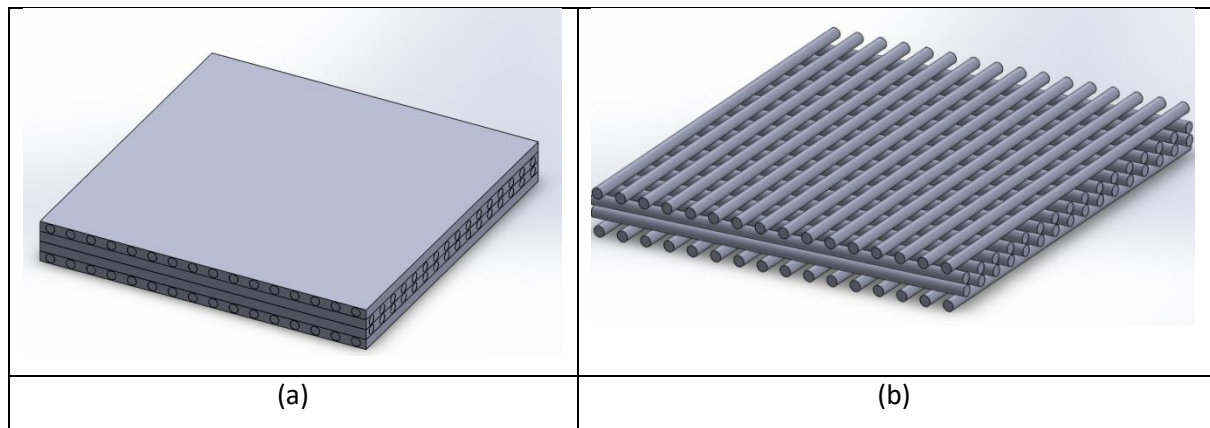


Figure F.2 Geometry of [0/90/90/0] laminate composite (a) fibre with matrix structure (b) fibre only structure

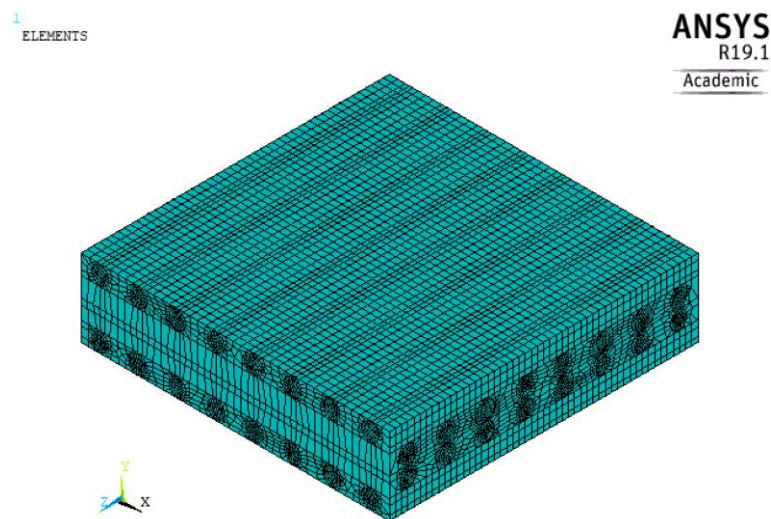


Figure F.3 Meshed structure of [0/90/90/0] laminate

The model has square dimensions of 2.25mm x 2.25mm with a total thickness of 0.56mm. The middle point of the plates is restricted from all degrees of freedom in the FEA model. The cooling down temperature was set from 859K to 293K.

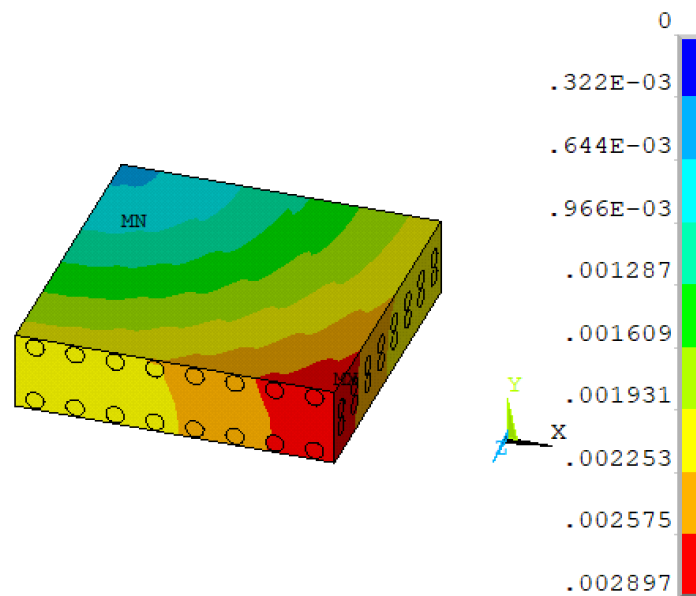
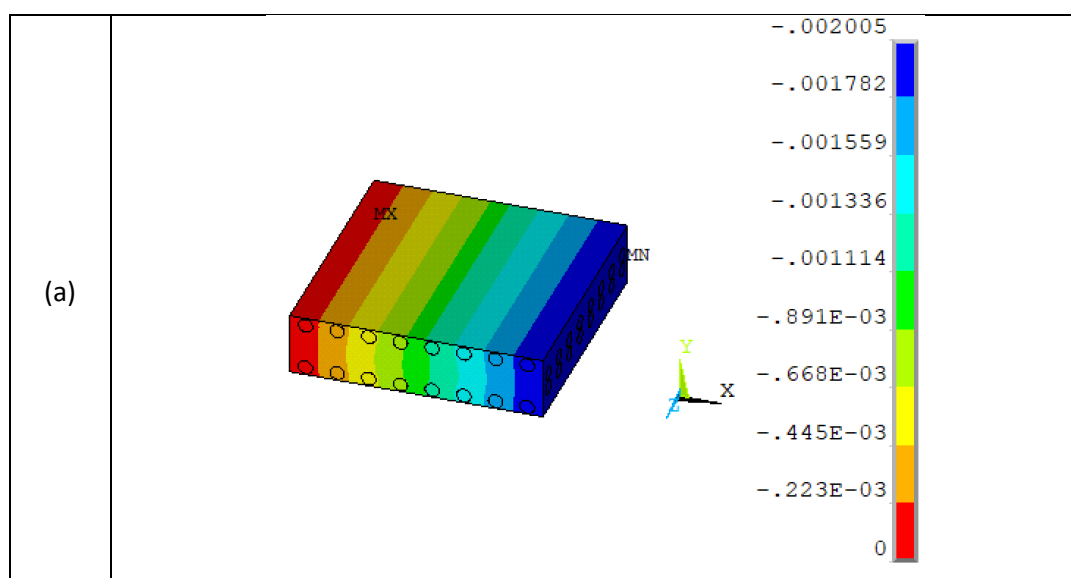


Figure F.4 Total displacements of [0/90/90/0] laminate

Figure F.4 shows that the composite has uniform deformation when cooled down to room temperature. The maximum displacement is 0.029mm. The magnitude of the X, Y and Z displacements are 0.02mm, 0.006mm and 0.02mm, respectively.



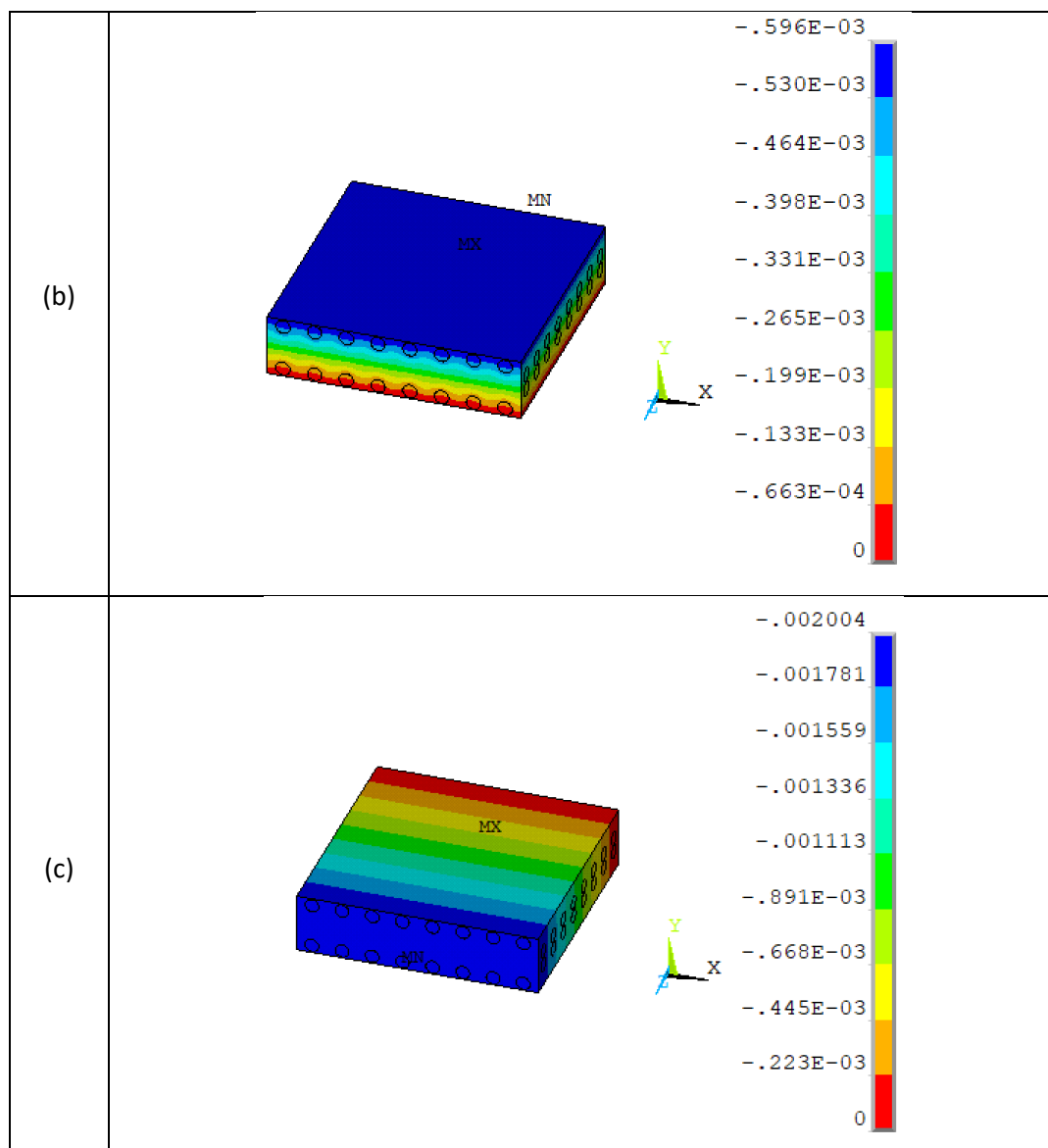
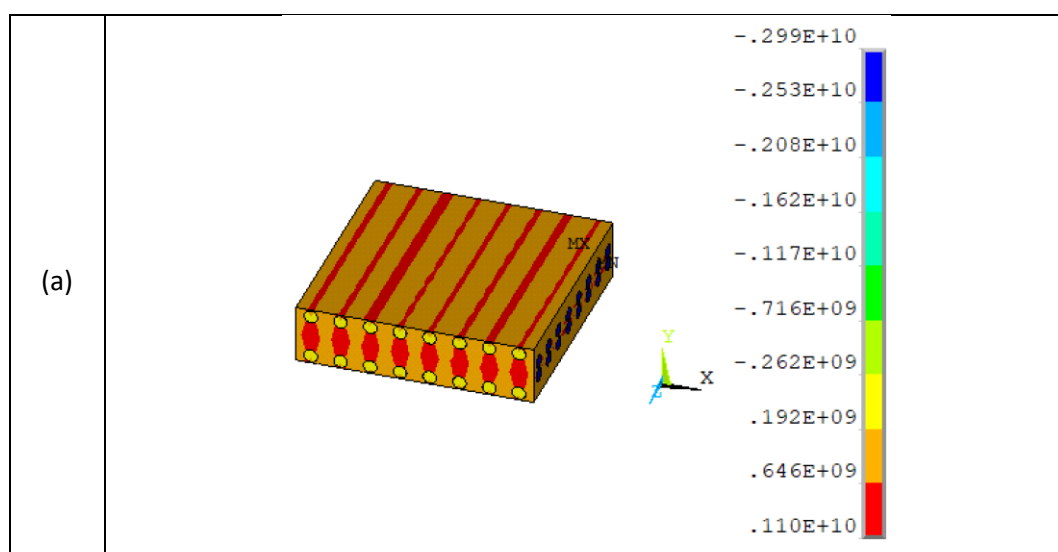


Figure F.5 Displacement of [0/90/90/0] laminate at (a) X direction (b) Y direction (c) Z direction



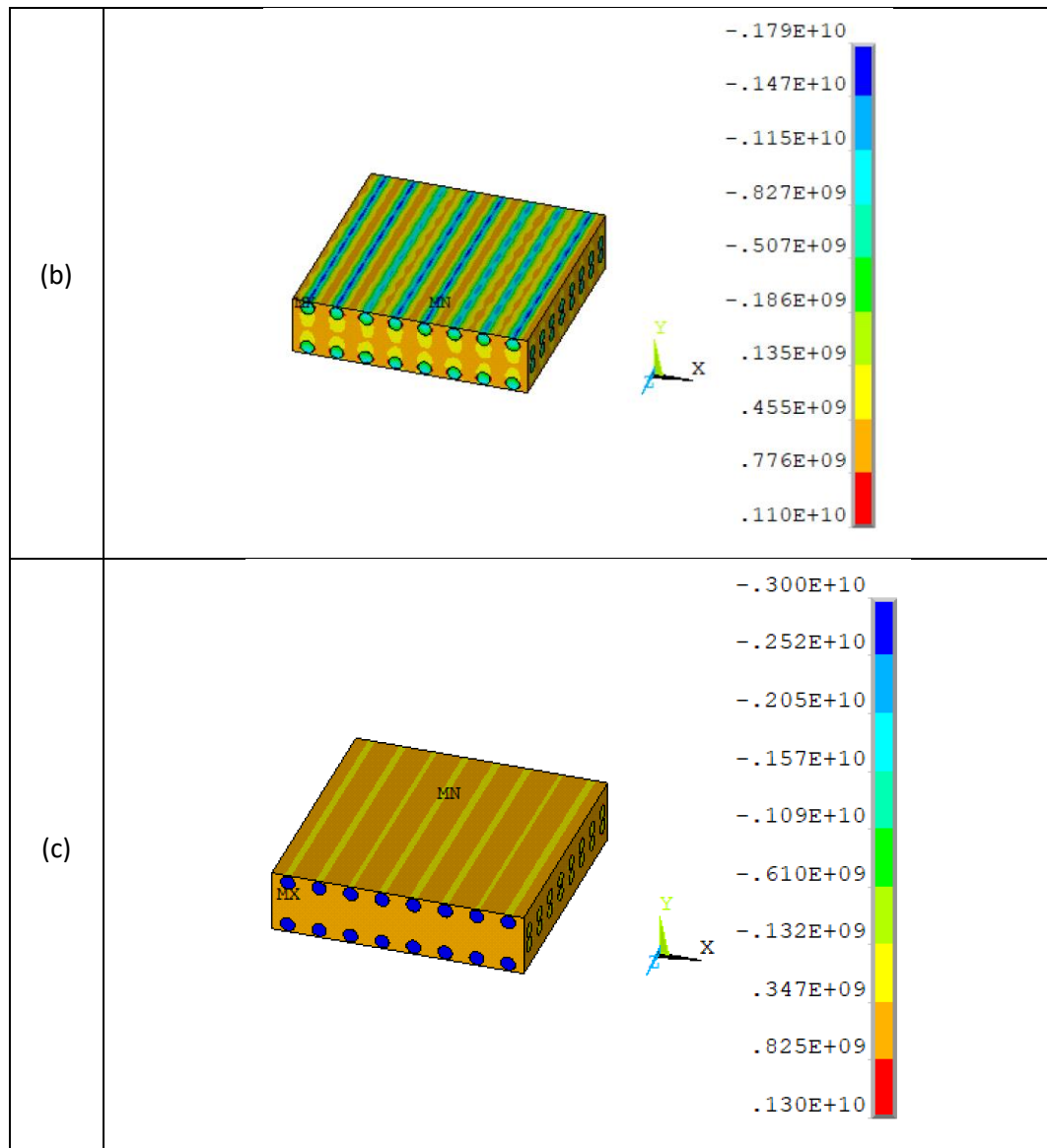


Figure F.6 Thermal stress of [0/90/90/0] laminate at (a) X direction (b) Y direction (c) Z direction

The thermal stress has a maximum value of 1100MPa and a minimum value of -2990Mpa in direction X. In the Y direction, the maximum and minimum thermal stress values are 1100MPa and -1790MPa, respectively. In the Z direction, the thermal stress has similar values for the maximum and minimum values of 1300MPa and -3000MPa.

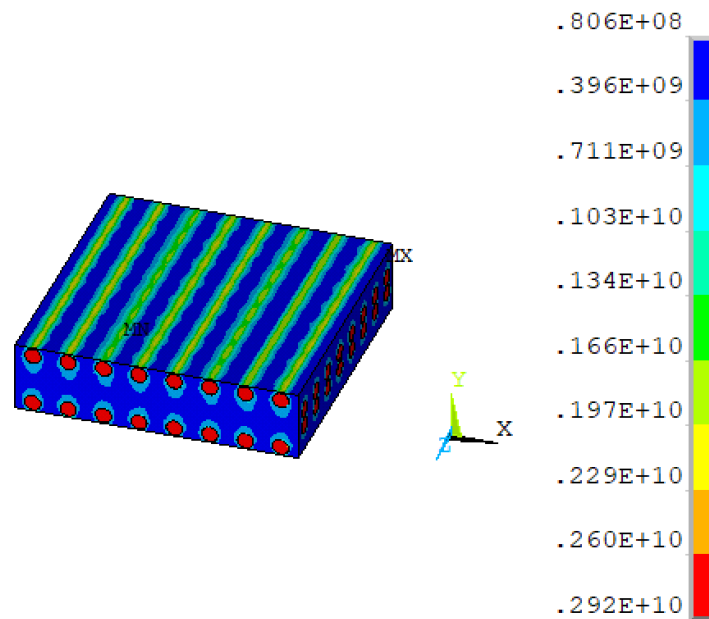


Figure F.7 Von Mises thermal stress of [0/90/90/0] laminate (a) top view (b) isometric view

Figure F.7 shows the von Mises stresses within the [0/90/90/0] laminate. The maximum von Mises stress is 2920MPa, and the minimum von Mises stress is 80.6MPa.

Table F.4 shows the residual thermal stresses comparison from the classic lamination theory analytical prediction and 3D FEA model results. Residual stresses of fibre reinforcement have a great match with the CTE model. The 3D finite element models show stress scale as illustrated in Figure F.6. Thus, an average stress value from Figure F.6 (b) was applied for the comparison.

Three models have been created; they are the classic lamination theory analytical mode, the 2D FE shell model and the 3D FE composite lamination model. Residual thermal residual stress within the laminates and the localised components were compared between the models, and good match was obtained. Both the 2D and 3D finite element models were validated with the results of the CTE analytical model.

Table F.4 Results comparison of Classic lamination theory with 3D FEA models

CTE analytical axial stresses in fibre and matrix components		
Layer No.	Fibre	Matrix
Layer 1	-2960 MPa	296 MPa
Layer 2	-2960 MPa	296 MPa
Layer 3	-2960 MPa	296 MPa
Layer 4	-2960 MPa	296 MPa
FE axial stresses in fibre and matrix components		
Layer No.	Fibre	Matrix
Layer 1	-3000 MPa	295MPa
Layer 2	-2990 MPa	295MPa
Layer 3	-2990 MPa	295MPa
Layer 4	-3000 MPa	295MPa

APPENDIX G Analytical tensile test model

(a) Analytical tensile test model before heat treatment

Material	Modulus	Yield strength	Volume fraction	Original Length (gauge)	CTE	Specimen Dimensions		Cooldown Temperature
Aluminium 6061 Alloy	7.0000E+10	5.5000E+07	9.1950E-01	6.0000E-02	2.3000E-05	Thick	Width	°C
Silicon Carbide Fibre	3.8000E+11	3.6000E+09	8.0500E-02	6.0000E-02	4.5000E-06	mm	mm	-500
Delta alpha					1.85000000E-05	1.6	12.4	
Composite	9.50E+10					m	m	
						0.0016	0.0124	
Residual	Residual							
Al	Fibre							
5.50E+07	-6.28E+08							

Total Area	Al Area	Fibre Area					
0.00001984	1.82E-05	1.60E-06					
Loading disp	Stress Al Pa	Stress F Pa	Force N	Elongation (Strain)	Al Theoretical stress	SiC Theoretical stress	
0.0E+00	5.5E+07	-6.3E+08	0.0E+00	0.0E+00	5.5E+07	-6.3E+08	
1.0E-05	5.5E+07	-5.6E+08	1.0E+02	1.7E-02	5.5E+07	-5.6E+08	
2.0E-05	5.5E+07	-5.0E+08	2.1E+02	3.3E-02	6.7E+07	-5.0E+08	
3.0E-05	5.5E+07	-4.4E+08	3.1E+02	5.0E-02	7.8E+07	-4.4E+08	
4.0E-05	5.6E+07	-3.7E+08	4.1E+02	6.7E-02	9.0E+07	-3.7E+08	
5.0E-05	5.6E+07	-3.1E+08	5.2E+02	8.3E-02	1.0E+08	-3.1E+08	
6.0E-05	5.6E+07	-2.5E+08	6.2E+02	1.0E-01	1.1E+08	-2.5E+08	
7.0E-05	5.6E+07	-1.8E+08	7.2E+02	1.2E-01	1.3E+08	-1.8E+08	
8.0E-05	5.6E+07	-1.2E+08	8.3E+02	1.3E-01	1.4E+08	-1.2E+08	
9.0E-05	5.6E+07	-5.8E+07	9.3E+02	1.5E-01	1.5E+08	-5.8E+07	
1.0E-04	5.6E+07	5.1E+06	1.0E+03	1.7E-01	1.6E+08	5.1E+06	
1.1E-04	5.6E+07	6.8E+07	1.1E+03	1.8E-01	1.7E+08	6.8E+07	
1.2E-04	5.7E+07	1.3E+08	1.2E+03	2.0E-01	1.8E+08	1.3E+08	
1.3E-04	5.7E+07	2.0E+08	1.3E+03	2.2E-01	2.0E+08	2.0E+08	
1.4E-04	5.7E+07	2.6E+08	1.4E+03	2.3E-01	2.1E+08	2.6E+08	
1.5E-04	5.7E+07	3.2E+08	1.6E+03	2.5E-01	2.2E+08	3.2E+08	
1.6E-04	5.7E+07	3.9E+08	1.7E+03	2.7E-01	2.3E+08	3.9E+08	
1.7E-04	5.7E+07	4.5E+08	1.8E+03	2.8E-01	2.4E+08	4.5E+08	
1.8E-04	5.7E+07	5.1E+08	1.9E+03	3.0E-01	2.5E+08	5.1E+08	
1.9E-04	5.7E+07	5.8E+08	2.0E+03	3.2E-01	2.7E+08	5.8E+08	
2.0E-04	5.8E+07	6.4E+08	2.1E+03	3.3E-01	2.8E+08	6.4E+08	
2.1E-04	5.8E+07	7.0E+08	2.2E+03	3.5E-01	2.9E+08	7.0E+08	
2.2E-04	5.8E+07	7.7E+08	2.3E+03	3.7E-01	3.0E+08	7.7E+08	
2.3E-04	5.8E+07	8.3E+08	2.4E+03	3.8E-01	3.1E+08	8.3E+08	
2.4E-04	5.8E+07	8.9E+08	2.5E+03	4.0E-01	3.2E+08	8.9E+08	
2.5E-04	5.8E+07	9.6E+08	2.6E+03	4.2E-01	3.4E+08	9.6E+08	
2.6E-04	5.8E+07	1.0E+09	2.7E+03	4.3E-01	3.5E+08	1.0E+09	
2.7E-04	5.8E+07	1.1E+09	2.8E+03	4.5E-01	3.6E+08	1.1E+09	
2.8E-04	5.9E+07	1.1E+09	2.9E+03	4.7E-01	3.7E+08	1.1E+09	
2.9E-04	5.9E+07	1.2E+09	3.0E+03	4.8E-01	3.8E+08	1.2E+09	
3.0E-04	5.9E+07	1.3E+09	3.1E+03	5.0E-01	3.9E+08	1.3E+09	
3.1E-04	5.9E+07	1.3E+09	3.2E+03	5.2E-01	4.1E+08	1.3E+09	

3.2E-04	5.9E+07	1.4E+09	3.3E+03	5.3E-01	4.2E+08	1.4E+09
3.3E-04	5.9E+07	1.5E+09	3.4E+03	5.5E-01	4.3E+08	1.5E+09
3.4E-04	5.9E+07	1.5E+09	3.5E+03	5.7E-01	4.4E+08	1.5E+09
3.5E-04	5.9E+07	1.6E+09	3.6E+03	5.8E-01	4.5E+08	1.6E+09
3.6E-04	6.0E+07	1.7E+09	3.7E+03	6.0E-01	4.6E+08	1.7E+09
3.7E-04	6.0E+07	1.7E+09	3.8E+03	6.2E-01	4.8E+08	1.7E+09
3.8E-04	6.0E+07	1.8E+09	3.9E+03	6.3E-01	4.9E+08	1.8E+09
3.9E-04	6.0E+07	1.8E+09	4.0E+03	6.5E-01	5.0E+08	1.8E+09
4.0E-04	6.0E+07	1.9E+09	4.1E+03	6.7E-01	5.1E+08	1.9E+09
4.1E-04	6.0E+07	2.0E+09	4.2E+03	6.8E-01	5.2E+08	2.0E+09
4.2E-04	6.0E+07	2.0E+09	4.3E+03	7.0E-01	5.3E+08	2.0E+09
4.3E-04	6.0E+07	2.1E+09	4.4E+03	7.2E-01	5.5E+08	2.1E+09
4.4E-04	6.1E+07	2.2E+09	4.6E+03	7.3E-01	5.6E+08	2.2E+09
4.5E-04	6.1E+07	2.2E+09	4.7E+03	7.5E-01	5.7E+08	2.2E+09
4.6E-04	6.1E+07	2.3E+09	4.8E+03	7.7E-01	5.8E+08	2.3E+09
4.7E-04	6.1E+07	2.3E+09	4.9E+03	7.8E-01	5.9E+08	2.3E+09
4.8E-04	6.1E+07	2.4E+09	5.0E+03	8.0E-01	6.0E+08	2.4E+09
4.9E-04	6.1E+07	2.5E+09	5.1E+03	8.2E-01	6.2E+08	2.5E+09
5.0E-04	6.1E+07	2.5E+09	5.2E+03	8.3E-01	6.3E+08	2.5E+09
5.1E-04	6.1E+07	2.6E+09	5.3E+03	8.5E-01	6.4E+08	2.6E+09
5.2E-04	6.2E+07	2.7E+09	5.4E+03	8.7E-01	6.5E+08	2.7E+09
5.3E-04	6.2E+07	2.7E+09	5.5E+03	8.8E-01	6.6E+08	2.7E+09
5.4E-04	6.2E+07	2.8E+09	5.6E+03	9.0E-01	6.7E+08	2.8E+09
5.5E-04	6.2E+07	2.9E+09	5.7E+03	9.2E-01	6.9E+08	2.9E+09
5.6E-04	6.2E+07	2.9E+09	5.8E+03	9.3E-01	7.0E+08	2.9E+09
5.7E-04	6.2E+07	3.0E+09	5.9E+03	9.5E-01	7.1E+08	3.0E+09
5.8E-04	6.2E+07	3.0E+09	6.0E+03	9.7E-01	7.2E+08	3.0E+09
5.9E-04	6.2E+07	3.1E+09	6.1E+03	9.8E-01	7.3E+08	3.1E+09
6.0E-04	6.3E+07	3.2E+09	6.2E+03	1.0E+00	7.4E+08	3.2E+09
6.1E-04	6.3E+07	3.2E+09	6.3E+03	1.0E+00	7.6E+08	3.2E+09
6.2E-04	6.3E+07	3.3E+09	6.4E+03	1.0E+00	7.7E+08	3.3E+09
6.3E-04	6.3E+07	3.4E+09	6.5E+03	1.1E+00	7.8E+08	3.4E+09
6.4E-04	6.3E+07	3.4E+09	6.6E+03	1.1E+00	7.9E+08	3.4E+09
6.5E-04	6.3E+07	3.5E+09	6.7E+03	1.1E+00	8.0E+08	3.5E+09
6.6E-04	6.3E+07	3.6E+09	6.8E+03	1.1E+00	8.1E+08	3.6E+09
6.7E-04	6.3E+07	3.6E+09	6.9E+03	1.1E+00	8.3E+08	3.6E+09
6.8E-04	6.4E+07	3.6E+09	6.9E+03	1.1E+00	8.4E+08	3.7E+09
6.9E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	8.5E+08	3.7E+09
7.0E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	8.6E+08	3.8E+09
7.1E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	8.7E+08	3.9E+09
7.2E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	8.8E+08	3.9E+09
7.3E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	9.0E+08	4.0E+09
7.4E-04	6.4E+07	3.6E+09	6.9E+03	1.2E+00	9.1E+08	4.1E+09
7.5E-04	6.4E+07	3.6E+09	6.9E+03	1.3E+00	9.2E+08	4.1E+09
7.6E-04	6.5E+07	3.6E+09	6.9E+03	1.3E+00	9.3E+08	4.2E+09
7.7E-04	6.5E+07	3.6E+09	6.9E+03	1.3E+00	9.4E+08	4.2E+09
7.8E-04	6.5E+07	3.6E+09	6.9E+03	1.3E+00	9.5E+08	4.3E+09
7.9E-04	6.5E+07	3.6E+09	6.9E+03	1.3E+00	9.7E+08	4.4E+09

8.0E-04	6.5E+07	3.6E+09	6.9E+03	1.3E+00	9.8E+08	4.4E+09
8.1E-04	6.5E+07	3.6E+09	6.9E+03	1.4E+00	9.9E+08	4.5E+09
8.2E-04	6.5E+07	3.6E+09	6.9E+03	1.4E+00	1.0E+09	4.6E+09
8.3E-04	6.5E+07	3.6E+09	6.9E+03	1.4E+00	1.0E+09	4.6E+09
8.4E-04	6.6E+07	3.6E+09	6.9E+03	1.4E+00	1.0E+09	4.7E+09

Loading									
Disp	Stress Al	Stress F	Load	Load Al	Load Fibre	Total Stress	Loading	Loading	
mm	Mpa	Mpa	kN	kN	kN	Mpa	Disp	Disp	
0.0E+00	5.5E+01	-6.3E+02	0.0E+00	1.0E+00	-1.0E+00	0.0E+00	mm	mm	
1.0E-02	5.5E+01	-5.6E+02	1.0E-01	1.0E+00	-9.0E-01	5.2E+00	0.0E+00	0.0E+00	
2.0E-02	5.5E+01	-5.0E+02	2.1E-01	1.0E+00	-8.0E-01	1.0E+01	1.0E-02	4.4E-02	
3.0E-02	5.5E+01	-4.4E+02	3.1E-01	1.0E+00	-7.0E-01	1.6E+01	2.0E-02	8.7E-02	
4.0E-02	5.6E+01	-3.7E+02	4.1E-01	1.0E+00	-6.0E-01	2.1E+01	3.0E-02	1.3E-01	
5.0E-02	5.6E+01	-3.1E+02	5.2E-01	1.0E+00	-5.0E-01	2.6E+01	4.0E-02	1.7E-01	
6.0E-02	5.6E+01	-2.5E+02	6.2E-01	1.0E+00	-4.0E-01	3.1E+01	5.0E-02	2.2E-01	
7.0E-02	5.6E+01	-1.8E+02	7.2E-01	1.0E+00	-3.0E-01	3.6E+01	6.0E-02	2.6E-01	
8.0E-02	5.6E+01	-1.2E+02	8.3E-01	1.0E+00	-1.9E-01	4.2E+01	7.0E-02	3.0E-01	
9.0E-02	5.6E+01	-5.8E+01	9.3E-01	1.0E+00	-9.3E-02	4.7E+01	8.0E-02	3.5E-01	
1.0E-01	5.6E+01	5.1E+00	1.0E+00	1.0E+00	8.2E-03	5.2E+01	9.0E-02	3.9E-01	
1.1E-01	5.6E+01	6.8E+01	1.1E+00	1.0E+00	1.1E-01	5.7E+01	1.0E-01	4.4E-01	
1.2E-01	5.7E+01	1.3E+02	1.2E+00	1.0E+00	2.1E-01	6.3E+01	1.1E-01	4.8E-01	
1.3E-01	5.7E+01	2.0E+02	1.3E+00	1.0E+00	3.1E-01	6.8E+01	1.2E-01	5.2E-01	
1.4E-01	5.7E+01	2.6E+02	1.4E+00	1.0E+00	4.1E-01	7.3E+01	1.3E-01	5.7E-01	
1.5E-01	5.7E+01	3.2E+02	1.6E+00	1.0E+00	5.1E-01	7.8E+01	1.4E-01	6.1E-01	
1.6E-01	5.7E+01	3.9E+02	1.7E+00	1.0E+00	6.2E-01	8.3E+01	1.5E-01	6.5E-01	
1.7E-01	5.7E+01	4.5E+02	1.8E+00	1.0E+00	7.2E-01	8.9E+01	1.6E-01	7.0E-01	
1.8E-01	5.7E+01	5.1E+02	1.9E+00	1.0E+00	8.2E-01	9.4E+01	1.7E-01	7.4E-01	
1.9E-01	5.7E+01	5.8E+02	2.0E+00	1.0E+00	9.2E-01	9.9E+01	1.8E-01	7.8E-01	
2.0E-01	5.8E+01	6.4E+02	2.1E+00	1.0E+00	1.0E+00	1.0E+02	1.9E-01	8.3E-01	
2.1E-01	5.8E+01	7.0E+02	2.2E+00	1.1E+00	1.1E+00	1.1E+02	2.0E-01	8.7E-01	
2.2E-01	5.8E+01	7.7E+02	2.3E+00	1.1E+00	1.2E+00	1.1E+02	2.1E-01	9.1E-01	
2.3E-01	5.8E+01	8.3E+02	2.4E+00	1.1E+00	1.3E+00	1.2E+02	2.2E-01	9.6E-01	
2.4E-01	5.8E+01	8.9E+02	2.5E+00	1.1E+00	1.4E+00	1.3E+02	2.3E-01	1.0E+00	
2.5E-01	5.8E+01	9.6E+02	2.6E+00	1.1E+00	1.5E+00	1.3E+02	2.4E-01	1.0E+00	
2.6E-01	5.8E+01	1.0E+03	2.7E+00	1.1E+00	1.6E+00	1.4E+02	2.5E-01	1.1E+00	
2.7E-01	5.8E+01	1.1E+03	2.8E+00	1.1E+00	1.7E+00	1.4E+02	2.6E-01	1.1E+00	
2.8E-01	5.9E+01	1.1E+03	2.9E+00	1.1E+00	1.8E+00	1.5E+02	2.7E-01	1.2E+00	
2.9E-01	5.9E+01	1.2E+03	3.0E+00	1.1E+00	1.9E+00	1.5E+02	2.8E-01	1.2E+00	
3.0E-01	5.9E+01	1.3E+03	3.1E+00	1.1E+00	2.0E+00	1.6E+02	2.9E-01	1.3E+00	
3.1E-01	5.9E+01	1.3E+03	3.2E+00	1.1E+00	2.1E+00	1.6E+02	3.0E-01	1.3E+00	
3.2E-01	5.9E+01	1.4E+03	3.3E+00	1.1E+00	2.2E+00	1.7E+02	3.1E-01	1.3E+00	
3.3E-01	5.9E+01	1.5E+03	3.4E+00	1.1E+00	2.3E+00	1.7E+02	3.2E-01	1.4E+00	
3.4E-01	5.9E+01	1.5E+03	3.5E+00	1.1E+00	2.4E+00	1.8E+02	3.3E-01	1.4E+00	
3.5E-01	5.9E+01	1.6E+03	3.6E+00	1.1E+00	2.5E+00	1.8E+02	3.4E-01	1.5E+00	
3.6E-01	6.0E+01	1.7E+03	3.7E+00	1.1E+00	2.6E+00	1.9E+02	3.5E-01	1.5E+00	
3.7E-01	6.0E+01	1.7E+03	3.8E+00	1.1E+00	2.7E+00	1.9E+02	3.6E-01	1.6E+00	

3.8E-01	6.0E+01	1.8E+03	3.9E+00	1.1E+00	2.8E+00	2.0E+02	3.7E-01	1.6E+00
3.9E-01	6.0E+01	1.8E+03	4.0E+00	1.1E+00	2.9E+00	2.0E+02	3.8E-01	1.7E+00
4.0E-01	6.0E+01	1.9E+03	4.1E+00	1.1E+00	3.0E+00	2.1E+02	3.9E-01	1.7E+00
4.1E-01	6.0E+01	2.0E+03	4.2E+00	1.1E+00	3.1E+00	2.1E+02	4.0E-01	1.7E+00
4.2E-01	6.0E+01	2.0E+03	4.3E+00	1.1E+00	3.2E+00	2.2E+02	4.1E-01	1.8E+00
4.3E-01	6.0E+01	2.1E+03	4.4E+00	1.1E+00	3.3E+00	2.2E+02	4.2E-01	1.8E+00
4.4E-01	6.1E+01	2.2E+03	4.6E+00	1.1E+00	3.4E+00	2.3E+02	4.3E-01	1.9E+00
4.5E-01	6.1E+01	2.2E+03	4.7E+00	1.1E+00	3.5E+00	2.3E+02	4.4E-01	1.9E+00
4.6E-01	6.1E+01	2.3E+03	4.8E+00	1.1E+00	3.6E+00	2.4E+02	4.5E-01	2.0E+00
4.7E-01	6.1E+01	2.3E+03	4.9E+00	1.1E+00	3.8E+00	2.5E+02	4.6E-01	2.0E+00
4.8E-01	6.1E+01	2.4E+03	5.0E+00	1.1E+00	3.9E+00	2.5E+02	4.7E-01	2.0E+00
4.9E-01	6.1E+01	2.5E+03	5.1E+00	1.1E+00	4.0E+00	2.6E+02	4.8E-01	2.1E+00
5.0E-01	6.1E+01	2.5E+03	5.2E+00	1.1E+00	4.1E+00	2.6E+02	4.9E-01	2.1E+00
5.1E-01	6.1E+01	2.6E+03	5.3E+00	1.1E+00	4.2E+00	2.7E+02	5.0E-01	2.2E+00
5.2E-01	6.2E+01	2.7E+03	5.4E+00	1.1E+00	4.3E+00	2.7E+02	5.1E-01	2.2E+00
5.3E-01	6.2E+01	2.7E+03	5.5E+00	1.1E+00	4.4E+00	2.8E+02	5.2E-01	2.3E+00
5.4E-01	6.2E+01	2.8E+03	5.6E+00	1.1E+00	4.5E+00	2.8E+02	5.3E-01	2.3E+00
5.5E-01	6.2E+01	2.9E+03	5.7E+00	1.1E+00	4.6E+00	2.9E+02	5.4E-01	2.4E+00
5.6E-01	6.2E+01	2.9E+03	5.8E+00	1.1E+00	4.7E+00	2.9E+02	5.5E-01	2.4E+00
5.7E-01	6.2E+01	3.0E+03	5.9E+00	1.1E+00	4.8E+00	3.0E+02	5.6E-01	2.4E+00
5.8E-01	6.2E+01	3.0E+03	6.0E+00	1.1E+00	4.9E+00	3.0E+02	5.7E-01	2.5E+00
5.9E-01	6.2E+01	3.1E+03	6.1E+00	1.1E+00	5.0E+00	3.1E+02	5.8E-01	2.5E+00
6.0E-01	6.3E+01	3.2E+03	6.2E+00	1.1E+00	5.1E+00	3.1E+02	5.9E-01	2.6E+00
6.1E-01	6.3E+01	3.2E+03	6.3E+00	1.1E+00	5.2E+00	3.2E+02	6.0E-01	2.6E+00
6.2E-01	6.3E+01	3.3E+03	6.4E+00	1.1E+00	5.3E+00	3.2E+02	6.1E-01	2.7E+00
6.3E-01	6.3E+01	3.4E+03	6.5E+00	1.1E+00	5.4E+00	3.3E+02	6.2E-01	2.7E+00
6.4E-01	6.3E+01	3.4E+03	6.6E+00	1.1E+00	5.5E+00	3.3E+02	6.3E-01	2.7E+00
6.5E-01	6.3E+01	3.5E+03	6.7E+00	1.2E+00	5.6E+00	3.4E+02	6.4E-01	2.8E+00
6.6E-01	6.3E+01	3.6E+03	6.8E+00	1.2E+00	5.7E+00	3.4E+02	6.5E-01	2.8E+00
6.7E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.6E-01	2.9E+00
6.8E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.7E-01	2.9E+00
6.9E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.8E-01	3.0E+00
7.0E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.9E-01	3.0E+00
7.1E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.0E-01	3.0E+00
7.2E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.1E-01	3.1E+00
7.3E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.2E-01	3.1E+00
7.4E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.3E-01	3.2E+00
7.5E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.4E-01	3.2E+00
7.6E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.5E-01	3.3E+00
7.7E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.6E-01	3.3E+00
7.8E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.7E-01	3.4E+00
7.9E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.8E-01	3.4E+00
8.0E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	7.9E-01	3.4E+00
8.1E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	8.0E-01	3.5E+00
8.2E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	8.1E-01	3.5E+00

(b) Analytical tensile test model after heat treatment

Material	Modulus	Yield strength	Volume fraction	Original Length (gauge)	CTE	Specimen Dimensions	
Aluminium 6061 Alloy	7.0000E+10	2.7500E+08	9.1950E-01	6.0000E-02	2.3000E-05	Thick	Width
Silicon Carbide Fibre	3.8000E+11	3.6000E+09	8.0500E-02	6.0000E-02	4.5000E-06	mm	mm
Delta alpha					1.85000000E-05	1.6	12.4
Composite	9.50E+10					m	m
						0.0016	0.0124
Cooldown Temperature		Residual	Residual				
°C		Al	Fibre				
0		0.00E+00	0.00E+00				

Total Area	Al Area	Fibre Area				
0.00001984	1.82E-05	1.60E-06				
Loading	Stress Al	Stress F	Force	Elongation (Strain)	Al Theoretical stress	SiC Theoretical stress
disp	Pa	Pa	N			
0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00
1.0E-05	0.0E+00	6.3E+07	1.0E+02	1.7E-02	0.0E+00	6.3E+07
2.0E-05	1.2E+07	1.3E+08	4.2E+02	3.3E-02	1.2E+07	1.3E+08
3.0E-05	2.3E+07	1.9E+08	7.3E+02	5.0E-02	2.3E+07	1.9E+08
4.0E-05	3.5E+07	2.5E+08	1.0E+03	6.7E-02	3.5E+07	2.5E+08
5.0E-05	4.7E+07	3.2E+08	1.4E+03	8.3E-02	4.7E+07	3.2E+08
6.0E-05	5.8E+07	3.8E+08	1.7E+03	1.0E-01	5.8E+07	3.8E+08
7.0E-05	7.0E+07	4.4E+08	2.0E+03	1.2E-01	7.0E+07	4.4E+08
8.0E-05	8.2E+07	5.1E+08	2.3E+03	1.3E-01	8.2E+07	5.1E+08
9.0E-05	9.3E+07	5.7E+08	2.6E+03	1.5E-01	9.3E+07	5.7E+08
1.0E-04	1.1E+08	6.3E+08	2.9E+03	1.7E-01	1.1E+08	6.3E+08
1.1E-04	1.2E+08	7.0E+08	3.2E+03	1.8E-01	1.2E+08	7.0E+08
1.2E-04	1.3E+08	7.6E+08	3.6E+03	2.0E-01	1.3E+08	7.6E+08
1.3E-04	1.4E+08	8.2E+08	3.9E+03	2.2E-01	1.4E+08	8.2E+08
1.4E-04	1.5E+08	8.9E+08	4.2E+03	2.3E-01	1.5E+08	8.9E+08
1.5E-04	1.6E+08	9.5E+08	4.5E+03	2.5E-01	1.6E+08	9.5E+08
1.6E-04	1.8E+08	1.0E+09	4.8E+03	2.7E-01	1.8E+08	1.0E+09
1.7E-04	1.9E+08	1.1E+09	5.1E+03	2.8E-01	1.9E+08	1.1E+09
1.8E-04	2.0E+08	1.1E+09	5.4E+03	3.0E-01	2.0E+08	1.1E+09
1.9E-04	2.1E+08	1.2E+09	5.8E+03	3.2E-01	2.1E+08	1.2E+09
2.0E-04	2.2E+08	1.3E+09	6.1E+03	3.3E-01	2.2E+08	1.3E+09
2.1E-04	2.3E+08	1.3E+09	6.4E+03	3.5E-01	2.3E+08	1.3E+09
2.2E-04	2.5E+08	1.4E+09	6.7E+03	3.7E-01	2.5E+08	1.4E+09
2.3E-04	2.6E+08	1.5E+09	7.0E+03	3.8E-01	2.6E+08	1.5E+09
2.4E-04	2.7E+08	1.5E+09	7.3E+03	4.0E-01	2.7E+08	1.5E+09
2.5E-04	2.8E+08	1.6E+09	7.5E+03	4.2E-01	2.8E+08	1.6E+09
2.6E-04	2.8E+08	1.6E+09	7.8E+03	4.3E-01	2.9E+08	1.6E+09
2.7E-04	3.0E+08	1.7E+09	8.2E+03	4.5E-01	3.0E+08	1.7E+09
2.8E-04	3.1E+08	1.8E+09	8.5E+03	4.7E-01	3.2E+08	1.8E+09
2.9E-04	3.1E+08	1.8E+09	8.6E+03	4.8E-01	3.3E+08	1.8E+09
3.0E-04	3.1E+08	1.9E+09	8.7E+03	5.0E-01	3.4E+08	1.9E+09
3.1E-04	3.1E+08	2.0E+09	8.8E+03	5.2E-01	3.5E+08	2.0E+09
3.2E-04	3.1E+08	2.0E+09	8.9E+03	5.3E-01	3.6E+08	2.0E+09
3.3E-04	3.1E+08	2.1E+09	9.0E+03	5.5E-01	3.7E+08	2.1E+09

3.4E-04	3.1E+08	2.2E+09	9.1E+03	5.7E-01	3.9E+08	2.2E+09
3.5E-04	3.1E+08	2.2E+09	9.3E+03	5.8E-01	4.0E+08	2.2E+09
3.6E-04	3.1E+08	2.3E+09	9.4E+03	6.0E-01	4.1E+08	2.3E+09
3.7E-04	3.1E+08	2.3E+09	9.5E+03	6.2E-01	4.2E+08	2.3E+09
3.8E-04	3.2E+08	2.4E+09	9.6E+03	6.3E-01	4.3E+08	2.4E+09
3.9E-04	3.2E+08	2.5E+09	9.7E+03	6.5E-01	4.4E+08	2.5E+09
4.0E-04	3.2E+08	2.5E+09	9.8E+03	6.7E-01	4.6E+08	2.5E+09
4.1E-04	3.2E+08	2.6E+09	9.9E+03	6.8E-01	4.7E+08	2.6E+09
4.2E-04	3.2E+08	2.7E+09	1.0E+04	7.0E-01	4.8E+08	2.7E+09
4.3E-04	3.2E+08	2.7E+09	1.0E+04	7.2E-01	4.9E+08	2.7E+09
4.4E-04	3.2E+08	2.8E+09	1.0E+04	7.3E-01	5.0E+08	2.8E+09
4.5E-04	3.2E+08	2.9E+09	1.0E+04	7.5E-01	5.1E+08	2.9E+09
4.6E-04	3.2E+08	2.9E+09	1.0E+04	7.7E-01	5.3E+08	2.9E+09
4.7E-04	3.2E+08	3.0E+09	1.1E+04	7.8E-01	5.4E+08	3.0E+09
4.8E-04	3.2E+08	3.0E+09	1.1E+04	8.0E-01	5.5E+08	3.0E+09
4.9E-04	3.2E+08	3.1E+09	1.1E+04	8.2E-01	5.6E+08	3.1E+09
5.0E-04	3.2E+08	3.2E+09	1.1E+04	8.3E-01	5.7E+08	3.2E+09
5.1E-04	3.2E+08	3.2E+09	1.1E+04	8.5E-01	5.8E+08	3.2E+09
5.2E-04	3.2E+08	3.3E+09	1.1E+04	8.7E-01	6.0E+08	3.3E+09
5.3E-04	3.2E+08	3.4E+09	1.1E+04	8.8E-01	6.1E+08	3.4E+09
5.4E-04	3.2E+08	3.4E+09	1.1E+04	9.0E-01	6.2E+08	3.4E+09
5.5E-04	3.2E+08	3.5E+09	1.1E+04	9.2E-01	6.3E+08	3.5E+09
5.6E-04	3.2E+08	3.5E+09	1.1E+04	9.3E-01	6.4E+08	3.5E+09
5.7E-04	3.2E+08	3.6E+09	1.2E+04	9.5E-01	6.5E+08	3.6E+09
5.8E-04	3.3E+08	3.6E+09	1.2E+04	9.7E-01	6.7E+08	3.7E+09
5.9E-04	3.4E+08	3.6E+09	1.2E+04	9.8E-01	6.8E+08	3.7E+09
6.0E-04	3.5E+08	3.6E+09	1.2E+04	1.0E+00	6.9E+08	3.8E+09
6.1E-04	3.6E+08	3.6E+09	1.2E+04	1.0E+00	7.0E+08	3.9E+09
6.2E-04	3.6E+08	3.6E+09	1.2E+04	1.0E+00	7.1E+08	3.9E+09
6.3E-04	3.6E+08	3.6E+09	1.2E+04	1.1E+00	7.2E+08	4.0E+09
6.4E-04	3.6E+08	3.6E+09	1.2E+04	1.1E+00	7.4E+08	4.1E+09
6.5E-04	3.6E+08	3.6E+09	1.2E+04	1.1E+00	7.5E+08	4.1E+09

Loading							Loading		
Disp	Stress Al	Stress F	Load	Load Al	Load Fibre	Total Stress	Disp		
mm	Mpa	Mpa	kN	kN	kN	Mpa	mm		
0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	
1.0E-02	0.0E+00	6.3E+01	0.0E+00	0.0E+00	1.0E-01	0.0E+00	1.0E-02	6.0E-02	
2.0E-02	1.2E+01	1.3E+02	4.2E-01	2.1E-01	2.0E-01	2.1E+01	2.0E-02	1.2E-01	
3.0E-02	2.3E+01	1.9E+02	7.3E-01	4.3E-01	3.0E-01	3.7E+01	3.0E-02	1.8E-01	
4.0E-02	3.5E+01	2.5E+02	1.0E+00	6.4E-01	4.0E-01	5.3E+01	4.0E-02	2.4E-01	
5.0E-02	4.7E+01	3.2E+02	1.4E+00	8.5E-01	5.1E-01	6.8E+01	5.0E-02	3.0E-01	
6.0E-02	5.8E+01	3.8E+02	1.7E+00	1.1E+00	6.1E-01	8.4E+01	6.0E-02	3.6E-01	
7.0E-02	7.0E+01	4.4E+02	2.0E+00	1.3E+00	7.1E-01	1.0E+02	7.0E-02	4.2E-01	
8.0E-02	8.2E+01	5.1E+02	2.3E+00	1.5E+00	8.1E-01	1.2E+02	8.0E-02	4.8E-01	
9.0E-02	9.3E+01	5.7E+02	2.6E+00	1.7E+00	9.1E-01	1.3E+02	9.0E-02	5.4E-01	
1.0E-01	1.1E+02	6.3E+02	2.9E+00	1.9E+00	1.0E+00	1.5E+02	1.0E-01	6.0E-01	
1.1E-01	1.2E+02	7.0E+02	3.2E+00	2.1E+00	1.1E+00	1.6E+02	1.1E-01	6.6E-01	

1.2E-01	1.3E+02	7.6E+02	3.6E+00	2.3E+00	1.2E+00	1.8E+02	1.2E-01	7.2E-01
1.3E-01	1.4E+02	8.2E+02	3.9E+00	2.6E+00	1.3E+00	2.0E+02	1.3E-01	7.8E-01
1.4E-01	1.5E+02	8.9E+02	4.2E+00	2.8E+00	1.4E+00	2.1E+02	1.4E-01	8.4E-01
1.5E-01	1.6E+02	9.5E+02	4.5E+00	3.0E+00	1.5E+00	2.3E+02	1.5E-01	9.0E-01
1.6E-01	1.8E+02	1.0E+03	4.8E+00	3.2E+00	1.6E+00	2.4E+02	1.6E-01	9.6E-01
1.7E-01	1.9E+02	1.1E+03	5.1E+00	3.4E+00	1.7E+00	2.6E+02	1.7E-01	1.0E+00
1.8E-01	2.0E+02	1.1E+03	5.4E+00	3.6E+00	1.8E+00	2.7E+02	1.8E-01	1.1E+00
1.9E-01	2.1E+02	1.2E+03	5.8E+00	3.8E+00	1.9E+00	2.9E+02	1.9E-01	1.1E+00
2.0E-01	2.2E+02	1.3E+03	6.1E+00	4.0E+00	2.0E+00	3.1E+02	2.0E-01	1.2E+00
2.1E-01	2.3E+02	1.3E+03	6.4E+00	4.3E+00	2.1E+00	3.2E+02	2.1E-01	1.3E+00
2.2E-01	2.5E+02	1.4E+03	6.7E+00	4.5E+00	2.2E+00	3.4E+02	2.2E-01	1.3E+00
2.3E-01	2.6E+02	1.5E+03	7.0E+00	4.7E+00	2.3E+00	3.5E+02	2.3E-01	1.4E+00
2.4E-01	2.7E+02	1.5E+03	7.3E+00	4.9E+00	2.4E+00	3.7E+02	2.4E-01	1.4E+00
2.5E-01	2.8E+02	1.6E+03	7.5E+00	5.0E+00	2.5E+00	3.8E+02	2.5E-01	1.5E+00
2.6E-01	2.8E+02	1.6E+03	7.8E+00	5.2E+00	2.6E+00	3.9E+02	2.6E-01	1.6E+00
2.7E-01	3.0E+02	1.7E+03	8.2E+00	5.4E+00	2.7E+00	4.1E+02	2.7E-01	1.6E+00
2.8E-01	3.1E+02	1.8E+03	8.5E+00	5.6E+00	2.8E+00	4.3E+02	2.8E-01	1.7E+00
2.9E-01	3.1E+02	1.8E+03	8.6E+00	5.6E+00	2.9E+00	4.3E+02	2.9E-01	1.7E+00
3.0E-01	3.1E+02	1.9E+03	8.7E+00	5.7E+00	3.0E+00	4.4E+02	3.0E-01	1.8E+00
3.1E-01	3.1E+02	2.0E+03	8.8E+00	5.7E+00	3.1E+00	4.4E+02	3.1E-01	1.9E+00
3.2E-01	3.1E+02	2.0E+03	8.9E+00	5.7E+00	3.2E+00	4.5E+02	3.2E-01	1.9E+00
3.3E-01	3.1E+02	2.1E+03	9.0E+00	5.7E+00	3.3E+00	4.6E+02	3.3E-01	2.0E+00
3.4E-01	3.1E+02	2.2E+03	9.1E+00	5.7E+00	3.4E+00	4.6E+02	3.4E-01	2.0E+00
3.5E-01	3.1E+02	2.2E+03	9.3E+00	5.7E+00	3.5E+00	4.7E+02	3.5E-01	2.1E+00
3.6E-01	3.1E+02	2.3E+03	9.4E+00	5.7E+00	3.6E+00	4.7E+02	3.6E-01	2.2E+00
3.7E-01	3.1E+02	2.3E+03	9.5E+00	5.7E+00	3.7E+00	4.8E+02	3.7E-01	2.2E+00
3.8E-01	3.2E+02	2.4E+03	9.6E+00	5.7E+00	3.8E+00	4.8E+02	3.8E-01	2.3E+00
3.9E-01	3.2E+02	2.5E+03	9.7E+00	5.7E+00	3.9E+00	4.9E+02	3.9E-01	2.3E+00
4.0E-01	3.2E+02	2.5E+03	9.8E+00	5.7E+00	4.0E+00	4.9E+02	4.0E-01	2.4E+00
4.1E-01	3.2E+02	2.6E+03	9.9E+00	5.7E+00	4.1E+00	5.0E+02	4.1E-01	2.5E+00
4.2E-01	3.2E+02	2.7E+03	1.0E+01	5.7E+00	4.2E+00	5.0E+02	4.2E-01	2.5E+00
4.3E-01	3.2E+02	2.7E+03	1.0E+01	5.7E+00	4.3E+00	5.1E+02	4.3E-01	2.6E+00
4.4E-01	3.2E+02	2.8E+03	1.0E+01	5.7E+00	4.5E+00	5.1E+02	4.4E-01	2.6E+00
4.5E-01	3.2E+02	2.9E+03	1.0E+01	5.7E+00	4.6E+00	5.2E+02	4.5E-01	2.7E+00
4.6E-01	3.2E+02	2.9E+03	1.0E+01	5.7E+00	4.7E+00	5.2E+02	4.6E-01	2.8E+00
4.7E-01	3.2E+02	3.0E+03	1.1E+01	5.7E+00	4.8E+00	5.3E+02	4.7E-01	2.8E+00
4.8E-01	3.2E+02	3.0E+03	1.1E+01	5.7E+00	4.9E+00	5.3E+02	4.8E-01	2.9E+00
4.9E-01	3.2E+02	3.1E+03	1.1E+01	5.7E+00	5.0E+00	5.4E+02	4.9E-01	2.9E+00
5.0E-01	3.2E+02	3.2E+03	1.1E+01	5.7E+00	5.1E+00	5.4E+02	5.0E-01	3.0E+00
5.1E-01	3.2E+02	3.2E+03	1.1E+01	5.7E+00	5.2E+00	5.5E+02	5.1E-01	3.1E+00
5.2E-01	3.2E+02	3.3E+03	1.1E+01	5.9E+00	5.3E+00	5.5E+02	5.2E-01	3.1E+00
5.3E-01	3.3E+02	3.4E+03	1.1E+01	6.0E+00	5.4E+00	5.6E+02	5.3E-01	3.2E+00
5.4E-01	3.4E+02	3.4E+03	1.1E+01	6.2E+00	5.5E+00	5.6E+02	5.4E-01	3.2E+00
5.5E-01	3.5E+02	3.5E+03	1.1E+01	6.4E+00	5.6E+00	5.7E+02	5.5E-01	3.3E+00
5.6E-01	3.6E+02	3.5E+03	1.1E+01	6.5E+00	5.7E+00	5.8E+02	5.6E-01	3.4E+00
5.7E-01	3.6E+02	0.0E+00	1.2E+01	6.6E+00	0.0E+00	5.8E+02	5.7E-01	3.4E+00
5.8E-01	3.6E+02	0.0E+00	1.2E+01	6.6E+00	0.0E+00	5.9E+02	5.8E-01	3.5E+00
5.9E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	5.9E-01	3.5E+00

6.0E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.0E-01	3.6E+00
6.1E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.1E-01	3.7E+00
6.2E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.2E-01	3.7E+00
6.3E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.3E-01	3.8E+00
6.4E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.4E-01	3.8E+00
6.5E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.5E-01	3.9E+00
6.6E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.6E-01	4.0E+00
6.7E-01	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	6.7E-01	4.0E+00

APPENDIX H Detailed volume fraction calculation for tensile specimens

Detailed volume fraction calculation for tensile specimens is presented in this section.

Total volume Equation E. 2

$$= 2 \times (\text{Aluminium Foil Layer Volume} + 2 \times \text{Outer aluminium layer thickness} + 2 \times \text{Fibre layer volume} + \text{Middle foil layer thickness}) = 1.21 \text{E}^{-3} \text{m}^3$$

Volume Fraction Equation E. 3

$$= \frac{4 \times \text{Fibre Layer Volume}}{\text{Total volume}} = 16.1\%$$

Volume Fraction at the direction of pulling Equation E. 4

$$= \frac{\text{Volume fraction}}{2} = 8.05\%$$

APPENDIX I Matlab code for curvature measurement processing

```
clear;

clc;

[C]=xlsread('Coordinate measurement Excel file location');

%Need to change address of read and write statements to reflect spreadsheet
being used.

sf = fit([C(:,2), C(:,1)],C(:,3),'poly33') %curve fit data to a polynomial
surface

plot(sf,[C(:,2), C(:,1)],C(:,3))%plot data

coeffs =
coeffvalues(NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NAS
A) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) (NASA) ('Hubble in orbit
') (Hubble in orbit ); %output coefficient data

%xlswrite('E:\PhD research-backup weekly\Research models\FE laminate\[0-15]
Laminate\[0-15]2-11\Data results\Displacement Matrix Bottom
face.xlsx',coeffs,'sheet1','T1')

%write coefficient into Excel speicified sheet

syms x y

%Copy coefficients (p numbers) from command window into here

[p00]=coeffs(1,1);

[p10]=coeffs(1,2);

[p01]=coeffs(Summers et al., 2015);

[p20]=coeffs(1,4);

[p11]=coeffs(1,5);

[p02]=coeffs(1,6);

[p30]=coeffs(1,7);

[p21]=coeffs(1,8);

[p12]=coeffs(1,9);

[p03]=coeffs(1,10);
```

```
K = -((p11 + 2*p21*x + 2*p12*y)^2 - (2*p02 + 2*p12*x + 6*p03*y)*(2*p20 +
6*p30*x + 2*p21*y))/((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)^2 + (3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 +
1)^2;
```

```
% Gaussian curvatures
```

```
H=((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 + 1)*(2*p20
+ 6*p30*x + 2*p21*y) + ((3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y
+ p10)^2 + 1)*(2*p02 + 2*p12*x + 6*p03*y) - (p11 + 2*p21*x +
2*p12*y)*(p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)*(6*p30*x^2 + 4*p21*x*y + 4*p20*x + 2*p12*y^2 + 2*p11*y +
2*p10))/(2*((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^(3/2));
```

```
% Mean curvatures
```

```
k1((((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
1)*(2*p20 + 6*p30*x + 2*p21*y) + ((3*p30*x^2 + 2*p21*x*y + 2*p20*x +
p12*y^2 + p11*y + p10)^2 + 1)*(2*p02 + 2*p12*x + 6*p03*y) - (p11 + 2*p21*x
+ 2*p12*y)*(p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)*(6*p30*x^2 + 4*p21*x*y + 4*p20*x + 2*p12*y^2 + 2*p11*y +
2*p10))/(4*((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^3) + ((p11
+ 2*p21*x + 2*p12*y)^2 - (2*p02 + 2*p12*x + 6*p03*y)*(2*p20 + 6*p30*x +
2*p21*y))/((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^2)^(1/2) +
(((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 + 1)*(2*p20 +
6*p30*x + 2*p21*y) + ((3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y +
p10)^2 + 1)*(2*p02 + 2*p12*x + 6*p03*y) - (p11 + 2*p21*x +
2*p12*y)*(p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)*(6*p30*x^2 + 4*p21*x*y + 4*p20*x + 2*p12*y^2 + 2*p11*y +
2*p10))/(2*((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^(3/2));
```

```
% k1 principle curvature
```

```
k2=((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
1)*(2*p20 + 6*p30*x + 2*p21*y) + ((3*p30*x^2 + 2*p21*x*y + 2*p20*x +
p12*y^2 + p11*y + p10)^2 + 1)*(2*p02 + 2*p12*x + 6*p03*y) - (p11 + 2*p21*x
+ 2*p12*y)*(p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)*(6*p30*x^2 + 4*p21*x*y + 4*p20*x + 2*p12*y^2 + 2*p11*y +
2*p10))/(2*((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^(3/2)) -
```

```

(((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 + 1)*(2*p20
+ 6*p30*x + 2*p21*y) + ((3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y
+ p10)^2 + 1)*(2*p02 + 2*p12*x + 6*p03*y) - (p11 + 2*p21*x +
2*p12*y)*(p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y +
p01)*(6*p30*x^2 + 4*p21*x*y + 4*p20*x + 2*p12*y^2 + 2*p11*y +
2*p10))/(4*((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^3) + ((p11
+ 2*p21*x + 2*p12*y)^2 - (2*p02 + 2*p12*x + 6*p03*y)*(2*p20 + 6*p30*x +
2*p21*y))/((p21*x^2 + 2*p12*x*y + p11*x + 3*p03*y^2 + 2*p02*y + p01)^2 +
(3*p30*x^2 + 2*p21*x*y + 2*p20*x + p12*y^2 + p11*y + p10)^2 + 1)^2)^(1/2);

```

```

% k2 principle curvature

```

```

A=[C(:,1)];

```

```

B=[C(:,3)];

```

```

S=subs(K,{x, y}, {A, B});%give value for x and y

```

```

D=subs(H,{x, y}, {A, B});

```

```

K1=subs(k1,{x, y}, {A, B});

```

```

K2=subs(k2,{x, y}, {A, B});

```

```

[Kmax,KmaxL]=max(Rohatgi); %return max value and location

```

```

[KMax]=double(Kmax);

```

```

[Hmax,HmaxL]=max(Rohatgi);

```

```

[HMax]=double(Hmax);

```

```

[K1max,K1maxL]=max(K1);

```

```

[K1Max]=double(K1max);

```

```

[K1min,K1minL] = min(K1);%return min value and location

```

```

[K1Min]=double(K1min);

```

```

[K2max,K2maxL]=max(K2);

```

```

[K2Max]=double(K2max);

```

```

[K2min,K2minL] = min(K2);%return min value and location

```

```

[K2Min]=double(K2min);

```

```

[E]=[KMax HMax K1Max K1Min K2Max K2Min KmaxL HmaxL K1maxL K2minL];

```

```

xlswrite('Data file location')

```


APPENDIX J Thermomechanical behaviour analysis

Elasticity

Under some load limits, elasticity describes the ability of a material to revert to its original shape and size when it is unloaded. For anisotropic materials, the relation between stress and strain is given by Equation G. 1.

$$\sigma = c \cdot \varepsilon \quad \text{Equation G. 1}$$

Where σ is the stress tensor, ε is the strain tensor, and c is the elastic stiffness tensor. For components in an orthonormal coordinate system, elastic relation can be expressed in Equation G. 2.

$$\sigma_{ij} = c_{ijkl} \cdot \varepsilon_{kl} \quad \text{Equation G. 2}$$

Because the stress and strain tensors are symmetric, and the elastic stiffness tensor c follows the second law of thermodynamics (Taya and Arsenault, 2016), we have,

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij} \quad \text{Equation G. 3}$$

The stress tensors are:

$$[\sigma] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \quad \text{Equation G. 4}$$

The strain tensors are:

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \quad \text{Equation G. 5}$$

The stiffness tensors are given by Equation G. 6.

$$[c] = \begin{bmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1131} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2231} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3331} & c_{3312} \\ c_{2311} & c_{2322} & c_{2333} & c_{2323} & c_{2331} & c_{2312} \\ c_{3111} & c_{3122} & c_{3133} & c_{3123} & c_{3131} & c_{3112} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1231} & c_{1212} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \quad \text{Equation G. 6}$$

The stress-strain relation for linear elastic materials is given by Equation G. 7

$$[\sigma] = [c] \cdot [\varepsilon] \quad \text{Equation G. 7}$$

And the inverse of this relation is

$$[\varepsilon] = [S] \cdot [\sigma] \quad \text{Equation G. 8}$$

where S is the compliance tensor.

For orthotropic materials (three orthogonal planes of symmetry), the relationship between stress and strain is given by Equation G. 9.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \quad \text{Equation G. 9}$$

The inverse of this relationship is given by Equation G. 10.

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} \quad \text{Equation G. 10}$$

For transversely isotropic materials, whose properties are isotropic in one plane, the relationship is given by Equation G. 11.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \quad \text{Equation G. 11}$$

The inverse relationship is given by Equation G. 12.

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{yz}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2+2\nu_{yz}}{E_y} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} \quad \text{Equation G. 12}$$

For isotropic materials whose properties are identical in all directions, the stress-strain relationship is given by Equation G. 13.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \quad \text{Equation G. 13}$$

Hooke's law for isotropic materials is given by Equation G. 14.

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} \quad \text{Equation G. 14}$$

where $\gamma_{ij} = 2\varepsilon_{ij}$, γ_{ij} is the engineering shear strain.

Plasticity

Plasticity describes the irreversible deformation of a material in response to applied stress, usually, beyond a limit known as the yield stress of the material (Lubliner, 2008). Uniaxial tensile test is applied to determine the stress strain behaviour of metal. In uniaxial testing,

the test specimen is usually placed in grips in the tensile testing machine, and loads are then slowly applied to extend the sample until it fractures. True strain is given by Equation G. 15.

$$\varepsilon = \ln\left(\frac{L}{L_0}\right) \quad \text{Equation G. 15}$$

where L_0 is the original length and L is the final length.

In the actual tensile test, the elongation of the specimen is recorded using a strain gauge or an extensometer. Engineering strain is calculated using the elongation data, which is given by Equation G. 16.

$$\varepsilon = \frac{\Delta L}{L} = \frac{L - L_0}{L_0} \quad \text{Equation G. 16}$$

where ΔL is the elongation of the specimen.

Engineering stress is expressed as

$$\sigma = \frac{F}{A} \quad \text{Equation G. 17}$$

where F is the applied forced, and A is the original cross-sectional area of the specimen.

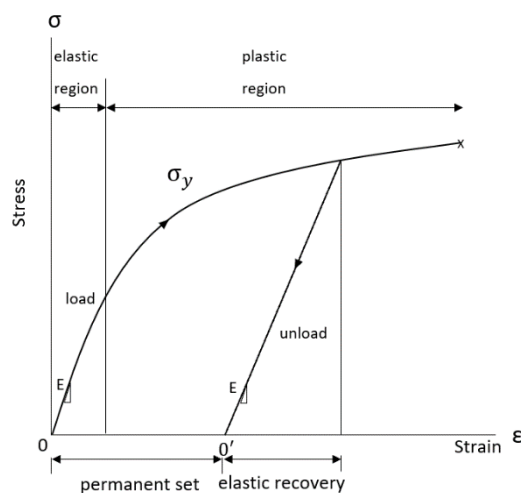


Figure G. 1 Diagram of a stress-strain curve of a ductile material

Figure G. 1 demonstrated a stress-strain curve of a ductile material. In the elastic region, the shape of the material is reversible when the load is removed. When the elastic area is passed, as the load increases, the material will be deformed such that it will have a permanent set when the load or stress is removed as illustrated in the figure (plastic deformation).

The total strain is the elastic and the plastic strain summation, as Equation G. 18.

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad \text{Equation G. 18}$$

where ε_e is the elastic strain and ε_p is the plastic strain.

Elastic strain is given by Equation G. 19.

$$\varepsilon_e = \frac{\sigma}{E} \quad (\sigma < \sigma_y) \quad \text{Equation G. 19}$$

and the plastic strain can be expressed using the nonlinear Equation G. 20

$$\varepsilon_p = \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \quad (\sigma \geq \sigma_y) \quad \text{Equation G. 20}$$

where σ_y is the yield stress, H is a material constant, and n is a hardening exponent.

The total strain is using the Ramberg Osgood equation given in Equation G. 21

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \quad \text{Equation G. 21}$$

Poisson's ratio

Poisson's ratio is the ratio of the lateral strain to longitudinal strain in a material (see Figure G. 2 and Figure G. 3). When a material expands (or compresses) in one direction, it tends to contract (or expand) in the perpendicular direction.

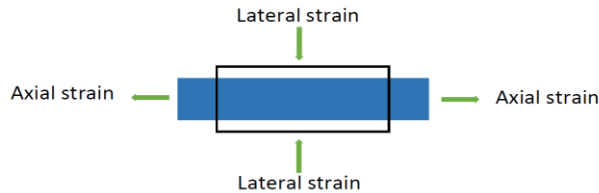


Figure G. 2 2D sketch of Poisson's effect

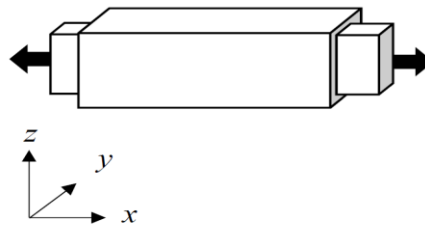


Figure G. 3 3D sketch of material stretches along the axial (x) direction

Poisson's ratio is given by Equation G. 22,

$$\nu = -\frac{\text{Lateral strain}}{\text{Axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \quad \text{Equation G. 22}$$

ε_y and ε_z are the transverse strain. The transverse strain is negative for axial stretching and positive for axial compression. ε_x is the axial strain; it is positive for axial tension and negative for axial compression.

Thus, we have

$$\varepsilon_x = \frac{\sigma_x}{E} \quad \text{Equation G. 23}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\nu\sigma_x}{E} \quad \text{Equation G. 24}$$

where E is the young's modulus.

Yield strength

Yield strength (stress) is the stress level at which the material deforms plastically. Before the applied force reaches the yield strength, the material deforms elastically, where the component can reverse to its original shape after the load is removed. When the applied stress increases above the yield point, the material will deform plastically, and part of the deformation becomes permanent. Figure G. 4 demonstrated a typical stress-strain curve.

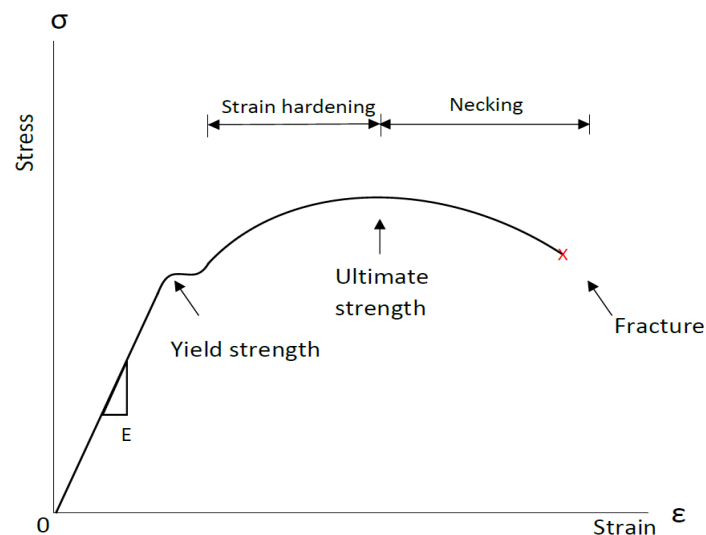


Figure G. 4 Diagram of the typical stress-strain curve

In actual tests, the yield point is often difficult to define. Thus, an offset yield point is applied, commonly set as 0.2% of the plastic strain (Mazzolani, 2001). The stress-strain curve allows us to define many properties of the material. Also, we can distinguish between brittle and ductile behaviour due to the shape of the curve.

At room temperature, ductile materials can yield. However, plastic deformation is significant before fracture. Common ductile materials are structural steel, aluminium alloy and many other metal alloys. Brittle materials such as carbon fibre or silicon carbide fibre do not have a

yield point. The ultimate strength is the same as the fracture strength for this type of material. Figure G. 5 demonstrates the failure mode of brittle and ductile materials after uniaxial loading. When the applied stress reaches the ultimate strength, the material will fracture with little or no plastic deformation. The typical stress-strain curve for brittle material is displayed in Figure G. 7. Figure G. 6 demonstrates the failure of the aluminium bar after uniaxial loading, and the necking stage is presented in the picture.

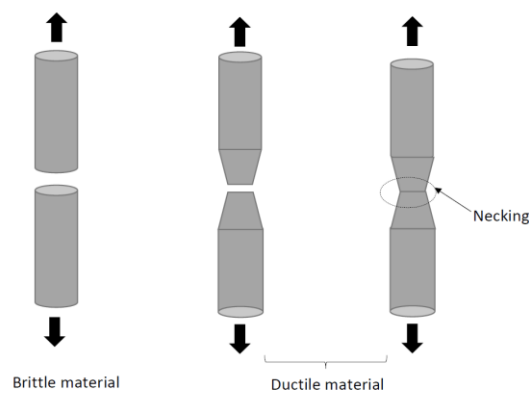


Figure G. 5 Schematic failure mode of brittle and ductile materials after uniaxial testing



Figure G. 6 Failure of ductile materials after uniaxial testing

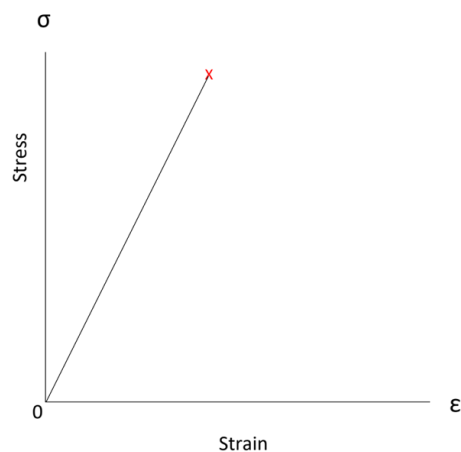


Figure G. 7 Typical stress-strain curve for brittle material

APPENDIX K Curvature measurement results

(A) Sample 1.360.3 with an aspect ratio of 1:5

1.360.3 (1:5)			
X	Y	X(Rohatgi)	Y(Rohatgi)
10.986	-74.387	-6.675	-74.387
10.848	-73.813	-6.538	-73.813
10.4	-72.405	-6.24	-72.405
9.788	-70.362	-5.859	-70.362
8.57	-65.962	-5.065	-65.962
7.73	-62.603	-4.522	-62.603
6.52	-57.318	-3.766	-57.318
5.622	-52.951	-3.224	-52.951
4.802	-48.631	-2.77	-48.631
4.173	-45.087	-2.445	-45.087
3.604	-41.581	-2.166	-41.581
3.098	-38.164	-1.935	-38.164
2.678	-35.028	-1.754	-35.028
2.321	-32.025	-1.59	-32.025
1.893	-27.948	-1.38	-27.948
1.637	-24.919	-1.231	-24.919
1.426	-21.871	-1.103	-21.871
1.259	-18.895	-0.997	-18.895
1.127	-15.921	-0.923	-15.921
1.051	-12.952	-0.884	-12.952
1.021	-9.914	-0.863	-9.914
1.005	-6.922	-0.872	-6.922
0.955	-3.995	-0.948	-3.995
0.841	-0.393	-0.952	-0.393

(B) Sample 1.360.2 with an aspect ratio of 1:5

1.360.2 (1:5)			
X	Y	X(Rohatgi)	Y(Rohatgi)
9.265	-74.85	-7.095	-74.85
8.756	-72.784	-6.679	-72.784
8.242	-70.476	-6.202	-70.476
7.722	-68.098	-5.739	-68.098
7.174	-65.52	-5.208	-65.52
6.756	-63.244	-4.825	-63.244
6.362	-60.97	-4.44	-60.97
5.98	-58.785	-4.107	-58.785
5.63	-56.61	-3.778	-56.61
5.449	-55.466	-3.604	-55.466

5.19	-53.704	-3.346	-53.704
5.15	-52.594	-3.199	-52.594
4.837	-51.292	-3.012	-51.292
4.662	-49.79	-2.81	-49.79
4.491	-48.52	-2.64	-48.52
4.268	-46.601	-2.384	-46.601
4.032	-44.46	-2.113	-44.46
3.75	-41.79	-1.762	-41.79
3.561	-39.968	-1.524	-39.968
3.348	-38.094	-1.312	-38.094
3.147	-36.39	-1.15	-36.39
2.925	-32.332	-0.975	-32.332
2.782	-32.524	-0.828	-32.524
2.8	-29.563	-0.594	-29.563
2.414	-26.609	-0.374	-26.609
2.302	-24.175	-0.21	-24.175
2.219	-21.252	-0.02	-21.252
2.161	-19.429	0.074	-19.429
2.108	-17.195	0.168	-17.195
2.107	-15.373	0.236	-15.373
2.107	-13.049	0.326	-13.049
2.155	-10.666	0.39	-10.666
2.218	-5.944	0.431	-5.944
2.212	-2.465	0.396	-2.465
2.215	-0.41	0.389	-0.41

(C) Sample 1.360.3 with an aspect ratio of 1:3

1.360.3 (1:3)			
X	Y	X(Rohatgi)	Y(Rohatgi)
11.278	-70.429	5.076	-70.429
11.194	-69.028	5.135	-69.028
10.961	-67.724	5.366	-67.724
10.698	-65.564	5.577	-65.564
10.447	-63.231	5.812	-63.231
10.188	-60.546	6.046	-60.546
9.98	-57.992	6.229	-57.992
9.836	-56.04	6.355	-56.04
9.657	-53.17	6.538	-53.17
9.508	-50.191	6.706	-50.191
9.365	-46.779	6.863	-46.779
9.306	-44.169	6.973	-44.169
9.248	-41.157	7.077	-41.157
9.221	-39.026	7.139	-39.026

9.202	-36.503	7.183	-36.503
9.199	-34.427	7.228	-34.427
9.203	-32.108	7.272	-32.108
9.223	-30.278	7.321	-30.278
9.239	-27.643	7.377	-27.643
9.258	-25.523	7.414	-25.523

(D) Sample 1.360.2 with an aspect ratio of 1:3

1.360.2 (1:3)			
X	Y	X(Rohatgi)	Y(Rohatgi)
11.753	-18.569	5.595	-18.569
11.646	-17.685	5.675	-17.685
11.411	-15.985	5.847	-15.985
11.24	-14.617	5.971	-14.617
11.099	-13.456	6.079	-13.456
10.887	-11.559	6.237	-11.559
10.59	-8.761	6.465	-8.761
10.435	-7.129	6.567	-7.129
10.245	-4.857	6.699	-4.857
10.032	-1.925	6.84	-1.925
9.867	0.609	6.967	0.609
9.726	3.016	7.066	3.016
9.611	5.353	7.133	5.353
9.509	7.878	7.206	7.878
9.441	10.071	7.225	10.071
9.377	12.209	7.256	12.209
9.31	15.153	7.285	15.153
9.285	16.861	7.295	16.861
9.255	18.747	7.296	18.747
9.235	20.38	7.287	20.38

(E) Sample 1.360.3 with an aspect ratio of 1:2

1.360.3 (1:2)			
X	Y	X(Rohatgi)	Y(Rohatgi)
9.576	-13.484	5.741	-13.484
9.463	-12.048	5.88	-12.048
9.306	-10.149	6.018	-10.149
9.186	-8.668	6.136	-8.668
9.055	-6.851	6.227	-6.851
8.933	-4.667	6.309	-4.667
8.843	-3.024	6.358	-3.024
8.707	-0.597	6.427	-0.597

8.609	1.493	6.453	1.493
8.53	3.405	6.462	3.405
8.444	6.878	6.452	6.878
8.378	9.351	6.436	9.351
8.321	12.769	6.43	12.769
8.279	15.231	6.438	15.231

(F) Sample 1.360.2 with an aspect ratio of 1:2

1.360.2 (1:2)			
X	Y	X(Rohatgi)	Y(Rohatgi)
8.21	-11.452	3.333	-11.452
8.176	-10.991	3.36	-10.991
7.95	-8.679	3.679	-8.679
7.793	-7.015	3.893	-7.015
7.66	-5.487	4.058	-5.487
7.53	-3.68	4.253	-3.68
7.427	-2.09	4.398	-2.09
7.27	0.79	4.593	0.79
7.157	3.062	4.717	3.062
7.087	4.573	4.789	4.573
7.024	6.708	4.868	6.708
6.96	9.585	4.953	9.585
6.939	13.048	4.994	13.048
6.921	15	5.009	15
6.896	18.536	5.021	18.536

(G) Sample 1.360.3 with aspect ratio of 1:1

1.360.3 (1:1)			
X	Y	X(Rohatgi)	Y(Rohatgi)
2.453	-12.82	-0.121	-12.82
2.392	-11.526	-0.081	-11.526
2.309	-10	0.005	-10
2.234	-8.259	0.083	-8.259
2.184	-6.877	0.137	-6.877
2.155	-4.966	0.196	-4.966
2.14	-3.621	0.225	-3.621
2.156	-1.995	0.265	-1.995
2.169	-0.803	0.269	-0.803
2.177	0.372	0.281	0.372
2.197	1.653	0.293	1.653
2.195	2.807	0.323	2.807

(H) Sample 1.360.2 with an aspect ratio of 1:1

1.360.2 (1:1)			
X	Y	X(Rohatgi)	Y(Rohatgi)
2.117	-10.778	-0.456	-10.778
2.078	-9.866	-0.372	-9.866
2.025	-8.335	-0.264	-8.335
1.978	-7.043	-0.2	-7.043
1.948	-5.67	-0.152	-5.67
1.916	-4.413	-0.115	-4.413
1.903	-3.25	-0.093	-3.25
1.898	-2.178	-0.066	-2.178
1.895	-1.215	-0.064	-1.215
1.892	0.869	-0.057	0.869
1.893	1.866	-0.043	1.866
1.894	2.801	-0.042	2.801
1.894	3.629	-0.03	3.629
1.895	4.282	-0.025	4.282

APPENDIX L Vickers hardness test results

(A) Before heat treatment

13/12/2018	1.360.3 (1:5)	Tensile specimen (1.360.3)	Tensile specimen (1.360.2)
Points	Vickers hardness	Vickers hardness	Vickers hardness
1	43.8	48.6	46.6
2	44	46.1	45.3
3	43.5	37.8	47.2
4	44.7	44.8	43
5	43	48.4	45.4
6	47	43.2	45.6
7	45.8	44.3	44.3
8	44.4	43.2	47.3
9	44.3	43.4	50.7
10	43.4	43	46.9
Average	44.39	44.28	46.23

(B) After heat treatment

14/12/2018	1.360.3 (1:5)	Tensile specimen (1.360.3)	Tensile specimen (1.360.2)
Points	Vickers hardness	Vickers hardness	Vickers hardness
1	120	130	119
2	114	107	116
3	111	105	124
4	114	106	133
5	117	117	155
6	108	102	105
7	102	113	116
8	98	125	115.5
9	115	127	115
10	121	116	105
Average	112	114.8	120.35

APPENDIX M Tensile test results

Test Name: MMC Dogbone Flat Plates
Test Type : Tensile
Test Date : 20/12/2018 13:40
Test Speed : 2.000 mm/min
Pretension : Off
Width : 12.400 mm
Thickness : 1.600 mm
Sample Length : 60.000 mm

Test Number		Stress at Peak (N/mm ²)	Elongation at Peak
1	1.360.2 Before heat treatment	334.042	2.688
2	1.360.2 After heat treatment	540.489	3.444
3	1.360.3 Before heat treatment	359.622	2.83
4	1.360.3 After heat treatment	595.066	3.483

Tensile test results of samples before heat treatment

Before heat treatment			
1.360.2		1.360.3	
Elongation	Stress (N/mm ²)	Elongation	Stress (N/mm ²)
0	2.3589	0	10.5444
0	2.6109	0	10.5494
0.001	2.994	0	10.5494
0.003	3.2712	0.002	10.7812
0.007	3.3972	0.004	11.996
0.009	4.0927	0.007	12.7671
0.01	4.2389	0.008	13.3165
0.013	4.5968	0.01	13.7903
0.014	5.0151	0.011	14.2288
0.016	5.8871	0.013	14.6623
0.017	6.6431	0.014	15.0504
0.019	6.9607	0.016	15.3679
0.02	7.2782	0.017	15.6603
0.021	7.5958	0.018	15.9325
0.023	8.8911	0.02	16.1492
0.024	9.2641	0.023	16.5625
0.026	9.3901	0.024	16.749
0.029	7.8125	0.026	16.9153
0.03	6.5222	0.027	17.0817
0.032	8.7349	0.028	17.2429
0.033	8.4073	0.03	17.4143
0.034	8.9567	0.031	17.5554
0.036	9.4556	0.033	17.7067
0.038	9.9194	0.034	17.8831
0.039	10.4234	0.036	18.0292
0.04	10.8317	0.039	18.3367
0.042	11.6179	0.04	18.5081
0.043	11.8952	0.042	18.6694
0.044	11.8498	0.043	18.8306
0.046	12.0111	0.044	18.9919
0.048	12.2933	0.046	19.1683
0.049	12.6815	0.047	19.3397
0.05	13.0192	0.049	19.5161
0.052	13.3871	0.05	19.6724
0.055	13.5685	0.052	19.8488
0.056	13.8357	0.053	20.0101
0.058	14.0776	0.055	20.1663
0.061	13.9718	0.056	20.3327
0.062	14.3044	0.057	20.4889
0.063	14.7581	0.059	20.6452
0.065	14.9597	0.06	20.7913
0.066	15.4032	0.062	20.9879
0.068	15.4335	0.063	21.129
0.069	15.2823	0.065	21.2651
0.072	15.247	0.066	21.4415
0.073	14.5111	0.068	21.6079
0.076	14.6522	0.069	21.7742
0.078	14.8589	0.07	21.9254
0.079	13.7702	0.072	22.0817
0.081	12.3337	0.073	22.253
0.082	11.2903	0.075	22.4194

0.084	11.8548	0.078	22.7369
0.086	12.1371	0.079	22.9032
0.088	12.8629	0.081	23.0595
0.089	13.372	0.082	23.2308
0.092	13.8962	0.084	23.3972
0.094	14.3901	0.085	23.5585
0.095	14.8488	0.086	23.6895
0.096	15.2671	0.089	23.9919
0.098	15.5796	0.091	24.128
0.1	15.8317	0.092	24.2944
0.101	16.0484	0.094	24.4254
0.102	16.2298	0.095	24.5917
0.104	16.5575	0.096	24.7429
0.106	16.7087	0.098	24.879
0.107	16.8448	0.1	25.0353
0.108	16.996	0.101	25.1764
0.11	17.1573	0.104	25.4435
0.111	17.2782	0.105	25.6099
0.113	17.4194	0.107	25.756
0.114	17.5605	0.109	26.0232
0.115	17.6815	0.111	26.1694
0.117	17.8175	0.113	26.3105
0.118	18.0746	0.114	26.4415
0.12	18.2258	0.115	26.5575
0.121	18.3821	0.117	26.7187
0.123	18.5383	0.118	26.8599
0.125	18.6744	0.119	27.0111
0.127	18.8155	0.121	27.1421
0.128	18.9768	0.123	27.2782
0.13	19.123	0.125	27.5403
0.131	19.2792	0.127	27.6714
0.133	19.4254	0.129	27.7974
0.134	19.5413	0.13	27.9284
0.136	19.6673	0.131	28.0544
0.137	19.7883	0.133	28.1905
0.139	19.9194	0.134	28.3165
0.141	20.0605	0.136	28.4325
0.143	20.1865	0.137	28.5484
0.144	20.2923	0.138	28.6694
0.146	20.4183	0.14	28.8054
0.147	20.5494	0.143	29.0625
0.149	20.6905	0.144	29.1885
0.152	20.8317	0.146	29.3196
0.153	20.9577	0.147	29.4506
0.154	21.0837	0.148	29.5363
0.157	21.2198	0.15	29.6724
0.159	21.3609	0.152	29.7984
0.16	21.4869	0.153	29.9143
0.162	21.6079	0.154	30.0504
0.163	21.7238	0.157	30.2722
0.164	21.8347	0.159	30.4032
0.166	21.9758	0.16	30.494
0.167	22.1169	0.162	30.6502
0.169	22.248	0.163	30.756
0.17	22.369	0.164	30.8669
0.173	22.505	0.166	31.003
0.175	22.621	0.168	31.1139
0.176	22.7571	0.169	31.2248
0.177	22.8679	0.17	31.3357
0.179	23.13	0.172	31.4516
0.181	23.256	0.174	31.5625
0.182	23.3821	0.175	31.6532
0.183	23.5181	0.176	31.7742
0.185	23.6391	0.178	31.9002
0.186	23.7198	0.179	32.001
0.188	23.8407	0.182	32.2077
0.189	23.9768	0.183	32.3337

0.191	24.0927	0.185	32.4345
0.192	24.2087	0.186	32.5504
0.193	24.3498	0.188	32.6563
0.195	24.4607	0.189	32.7571
0.196	24.5917	0.191	32.8831
0.198	24.7127	0.193	33.0847
0.199	24.8286	0.195	33.2006
0.201	24.9294	0.198	33.4173
0.202	25.0655	0.199	33.5181
0.205	25.1663	0.201	33.624
0.206	25.2823	0.202	33.745
0.208	25.4133	0.204	33.8206
0.209	25.5444	0.205	33.9415
0.211	25.6502	0.207	34.0575
0.214	25.7661	0.208	34.1683
0.215	25.8871	0.209	34.2792
0.216	26.0131	0.211	34.375
0.218	26.1341	0.212	34.4808
0.221	26.2651	0.215	34.6673
0.222	26.5171	0.217	34.7782
0.224	26.6331	0.218	34.8538
0.225	26.8044	0.221	35.0907
0.227	26.9103	0.223	35.1865
0.228	27.1472	0.224	35.2671
0.229	27.2984	0.226	35.3831
0.231	27.4042	0.227	35.4889
0.232	27.5554	0.228	35.5796
0.234	27.6613	0.23	35.6754
0.237	27.9083	0.231	35.7813
0.238	28.0242	0.232	35.8871
0.24	28.1603	0.234	35.9929
0.241	28.2762	0.237	36.1895
0.243	28.3821	0.238	36.2853
0.244	28.503	0.24	36.381
0.245	28.6492	0.241	36.4869
0.247	28.755	0.243	36.5978
0.248	28.9063	0.244	36.6986
0.25	29.0272	0.246	36.8044
0.253	29.1583	0.247	36.9052
0.254	29.2742	0.249	37.001
0.256	29.4052	0.25	37.1069
0.257	29.5313	0.253	37.2933
0.258	29.6573	0.254	37.4042
0.26	29.9143	0.256	37.5101
0.261	30.0252	0.257	37.6109
0.263	30.1663	0.259	37.7268
0.264	30.2974	0.26	37.8175
0.266	30.3982	0.262	37.9234
0.267	30.5192	0.263	38.0192
0.27	30.7661	0.265	38.12
0.272	30.8972	0.266	38.246
0.273	31.0232	0.269	38.4325
0.276	31.1492	0.271	38.5383
0.277	31.3962	0.272	38.624
0.279	31.5222	0.273	38.745
0.28	31.6683	0.275	38.8659
0.282	31.7843	0.276	38.9415
0.283	31.9153	0.277	39.0675
0.284	32.0413	0.279	39.1633
0.286	32.1673	0.281	39.2742
0.287	32.2933	0.282	39.38
0.289	32.4395	0.283	39.4456
0.292	32.6915	0.286	39.6925
0.293	32.9385	0.288	39.7833
0.295	33.0746	0.289	39.8942
0.296	33.2056	0.291	40.0202
0.297	33.3216	0.292	40.126

0.299	33.4425	0.294	40.2016
0.3	33.5837	0.295	40.3125
0.303	33.7198	0.298	40.5141
0.305	33.8558	0.299	40.6351
0.306	33.9819	0.301	40.756
0.307	34.1129	0.302	40.8619
0.309	34.244	0.304	40.9577
0.311	34.37	0.305	41.0786
0.312	34.5161	0.306	41.1794
0.313	34.6573	0.31	41.3861
0.315	34.9244	0.311	41.497
0.316	35.0806	0.312	41.6079
0.318	35.1915	0.314	41.7137
0.319	35.3327	0.315	41.8196
0.321	35.5897	0.317	41.9304
0.324	35.7308	0.318	42.0413
0.325	35.877	0.319	42.1623
0.326	36.0081	0.321	42.2681
0.328	36.1442	0.322	42.3942
0.329	36.3962	0.325	42.5756
0.331	36.5272	0.327	42.6815
0.332	36.6532	0.328	42.7772
0.334	36.7994	0.33	42.8982
0.335	36.9204	0.331	43.0292
0.336	37.0514	0.333	43.1401
0.34	37.1673	0.334	43.256
0.341	37.3135	0.335	43.377
0.342	37.4446	0.337	43.4728
0.344	37.5756	0.339	43.5938
0.345	37.8427	0.341	43.8105
0.347	37.9486	0.343	43.9466
0.348	38.1149	0.344	44.0524
0.35	38.2359	0.346	44.1683
0.351	38.3417	0.347	44.2944
0.352	38.498	0.349	44.3851
0.354	38.6442	0.35	44.5111
0.357	38.7702	0.351	44.6169
0.358	38.8911	0.353	44.748
0.36	39.0474	0.354	44.8538
0.361	39.1835	0.356	44.9849
0.363	39.3095	0.359	45.2117
0.364	39.4304	0.36	45.3327
0.366	39.5615	0.362	45.4234
0.369	39.6976	0.363	45.5595
0.37	39.9647	0.364	45.6653
0.371	40.1008	0.366	45.8014
0.373	40.2167	0.367	45.8871
0.374	40.3478	0.369	46.0282
0.376	40.4839	0.37	46.1593
0.377	40.6502	0.373	46.3508
0.38	40.756	0.374	46.4768
0.381	40.8972	0.376	46.5978
0.383	41.0181	0.378	46.7238
0.384	41.1391	0.379	46.8448
0.386	41.4113	0.38	46.9708
0.387	41.5272	0.382	47.0615
0.389	41.6734	0.383	47.1774
0.39	41.8196	0.385	47.3236
0.392	41.9607	0.386	47.4395
0.393	42.0766	0.388	47.5302
0.396	42.2329	0.389	47.6411
0.397	42.5	0.391	47.7823
0.399	42.621	0.392	47.8881
0.4	42.9032	0.394	47.9889
0.402	43.0343	0.396	48.2056
0.403	43.1653	0.398	48.3216
0.404	43.3014	0.399	48.4375

0.406	43.4224	0.401	48.5282
0.408	43.5685	0.402	48.6391
0.409	43.7147	0.404	48.745
0.412	43.8306	0.405	48.8407
0.414	43.9869	0.407	48.9315
0.415	44.1028	0.408	49.0524
0.416	44.254	0.409	49.2036
0.418	44.5161	0.411	49.3044
0.419	44.6673	0.412	49.4103
0.422	44.8286	0.414	49.5212
0.423	44.9446	0.415	49.6421
0.425	45.0756	0.417	49.748
0.426	45.2117	0.418	49.869
0.428	45.3579	0.419	49.9899
0.429	45.4839	0.421	50.1109
0.431	45.625	0.423	50.2319
0.432	45.8972	0.424	50.3478
0.433	46.0433	0.425	50.4587
0.435	46.2147	0.427	50.5847
0.437	46.3206	0.428	50.6804
0.438	46.4768	0.43	50.8266
0.439	46.7389	0.431	50.9526
0.441	46.8649	0.433	51.0635
0.444	47.0111	0.434	51.1643
0.445	47.1522	0.437	51.4214
0.447	47.2933	0.438	51.5423
0.448	47.4395	0.44	51.6633
0.449	47.5857	0.441	51.7843
0.451	47.7067	0.443	51.8851
0.452	47.8528	0.444	52.001
0.454	47.999	0.446	52.1119
0.455	48.2762	0.447	52.2329
0.457	48.4224	0.449	52.369
0.458	48.5685	0.45	52.4698
0.46	48.6996	0.452	52.5806
0.461	48.8407	0.453	52.7218
0.463	48.9718	0.454	52.8226
0.464	49.1179	0.456	52.9435
0.467	49.244	0.457	53.0746
0.468	49.4204	0.459	53.1905
0.47	49.5514	0.46	53.3115
0.472	49.6724	0.463	53.5282
0.474	49.8135	0.464	53.6542
0.476	49.9345	0.466	53.7752
0.477	50.0806	0.469	54.0222
0.478	50.1966	0.47	54.1331
0.48	50.373	0.472	54.2591
0.481	50.5141	0.473	54.3599
0.483	50.6552	0.475	54.506
0.484	50.7863	0.476	54.622
0.486	50.9274	0.477	54.7581
0.487	51.0635	0.479	54.869
0.489	51.2198	0.48	55.0151
0.49	51.3558	0.482	55.121
0.491	51.497	0.485	55.3579
0.493	51.6431	0.486	55.4839
0.494	51.7692	0.488	55.6149
0.496	52.0413	0.489	55.756
0.497	52.1774	0.491	55.8518
0.5	52.3387	0.492	55.9829
0.502	52.4849	0.493	56.119
0.503	52.6109	0.495	56.2298
0.505	52.747	0.496	56.3558
0.506	52.8982	0.498	56.5171
0.509	53.0343	0.499	56.6331
0.51	53.1603	0.502	56.8851
0.512	53.3115	0.504	57.0262

0.513	53.4476	0.505	57.1522
0.516	53.5988	0.508	57.3942
0.517	53.7147	0.509	57.5202
0.519	53.8609	0.511	57.6512
0.52	54.002	0.512	57.8024
0.522	54.1331	0.514	57.9234
0.523	54.2792	0.515	58.0544
0.526	54.5716	0.516	58.1704
0.528	54.7228	0.518	58.3317
0.529	54.8639	0.52	58.4627
0.531	54.9748	0.521	58.5938
0.532	55.1512	0.524	58.8659
0.534	55.2873	0.525	59.002
0.535	55.4234	0.527	59.123
0.536	55.5645	0.528	59.2591
0.538	55.7006	0.53	59.4052
0.539	55.9728	0.531	59.5615
0.541	56.1391	0.532	59.6673
0.542	56.2853	0.534	59.8034
0.544	56.4214	0.536	59.9345
0.545	56.5222	0.537	60.0706
0.546	56.6986	0.54	60.3377
0.55	56.8448	0.541	60.4687
0.551	56.9808	0.543	60.625
0.552	57.127	0.544	60.7863
0.554	57.2782	0.545	60.9073
0.555	57.4093	0.547	61.0181
0.557	57.5756	0.549	61.1744
0.558	57.7319	0.55	61.2954
0.559	57.8679	0.551	61.4365
0.561	57.9839	0.553	61.5827
0.562	58.1452	0.555	61.8548
0.564	58.4325	0.557	61.9909
0.565	58.5685	0.559	62.1321
0.567	58.7097	0.56	62.2782
0.568	58.8558	0.561	62.4093
0.57	59.0071	0.564	62.6915
0.571	59.1633	0.566	62.8125
0.573	59.2994	0.567	62.9587
0.574	59.4556	0.569	63.0998
0.575	59.6119	0.572	63.372
0.577	59.748	0.573	63.498
0.579	60.0252	0.574	63.6442
0.58	60.1865	0.576	63.7752
0.581	60.3226	0.577	63.9415
0.583	60.4738	0.579	64.0625
0.584	60.625	0.58	64.1986
0.587	60.7762	0.582	64.3448
0.589	61.0837	0.583	64.4808
0.59	61.2298	0.584	64.6069
0.591	61.3811	0.586	64.7581
0.593	61.5222	0.589	65.0605
0.595	61.6835	0.59	65.1966
0.596	61.8397	0.592	65.3579
0.597	61.9808	0.593	65.4889
0.599	62.1169	0.595	65.6048
0.6	62.2933	0.596	65.746
0.602	62.5706	0.598	65.9173
0.604	62.7167	0.599	66.0585
0.606	62.8831	0.6	66.1794
0.607	63.0292	0.602	66.3357
0.609	63.3065	0.603	66.4718
0.61	63.4728	0.606	66.7439
0.612	63.6341	0.608	66.9052
0.613	63.8004	0.609	67.0413
0.614	63.9415	0.611	67.1623
0.616	64.0877	0.612	67.3034

0.618	64.2742	0.613	67.4395
0.619	64.3851	0.615	67.6109
0.62	64.5111	0.617	67.7621
0.622	64.8286	0.618	67.8881
0.623	64.9647	0.619	68.0393
0.625	65.126	0.621	68.1754
0.626	65.2823	0.622	68.3266
0.628	65.4435	0.624	68.4526
0.629	65.5746	0.625	68.5988
0.63	65.7107	0.628	68.8659
0.632	65.872	0.629	69.0121
0.634	66.0282	0.631	69.1583
0.635	66.1845	0.633	69.3044
0.638	66.3357	0.634	69.4556
0.639	66.4819	0.635	69.6018
0.641	66.6431	0.637	69.7379
0.643	66.7843	0.638	69.8942
0.645	66.9405	0.64	70.0302
0.647	67.1018	0.641	70.1563
0.648	67.3942	0.644	70.4637
0.649	67.5403	0.645	70.5998
0.651	67.6764	0.647	70.7611
0.652	67.8226	0.648	70.9425
0.654	67.9738	0.65	71.0585
0.655	68.13	0.651	71.1694
0.657	68.3115	0.653	71.3206
0.659	68.4677	0.654	71.4819
0.661	68.619	0.656	71.628
0.662	68.7802	0.657	71.7944
0.664	68.9163	0.658	71.9405
0.665	69.0423	0.662	72.2379
0.667	69.1683	0.663	72.3841
0.668	69.37	0.664	72.5454
0.67	69.5464	0.666	72.6764
0.671	69.7026	0.667	72.8276
0.673	69.874	0.668	72.9687
0.674	70.005	0.67	73.125
0.677	70.1663	0.672	73.2661
0.678	70.2974	0.673	73.4123
0.68	70.4687	0.674	73.5484
0.683	70.7964	0.676	73.6895
0.684	70.9526	0.677	73.8558
0.686	71.1139	0.679	74.0171
0.687	71.2702	0.68	74.1633
0.688	71.4314	0.682	74.3145
0.692	71.5776	0.683	74.4708
0.693	71.754	0.685	74.6119
0.694	72.0464	0.686	74.753
0.696	72.1825	0.687	74.9143
0.699	72.4798	0.689	75.0706
0.7	72.6714	0.69	75.2319
0.702	72.7873	0.692	75.3679
0.703	72.9687	0.693	75.5141
0.704	73.1452	0.695	75.6704
0.706	73.2964	0.696	75.8115
0.709	73.4325	0.697	75.9778
0.71	73.5988	0.699	76.129
0.712	73.7601	0.701	76.2752
0.713	73.8911	0.702	76.4264
0.715	74.0423	0.703	76.5927
0.716	74.375	0.705	76.754
0.717	74.5212	0.707	76.8851
0.719	74.6724	0.708	77.0413
0.72	74.995	0.709	77.2077
0.722	75.126	0.711	77.3488
0.723	75.2974	0.712	77.4849
0.725	75.4587	0.713	77.626

0.726	75.6099	0.717	77.9587
0.728	75.9173	0.718	78.0847
0.731	76.0937	0.719	78.2409
0.732	76.2399	0.721	78.3972
0.733	76.3659	0.722	78.5786
0.735	76.6784	0.724	78.7097
0.736	76.87	0.725	78.881
0.738	77.0212	0.727	79.0474
0.739	77.1673	0.728	79.1835
0.741	77.3185	0.73	79.3095
0.742	77.4496	0.732	79.6421
0.743	77.7671	0.734	79.7833
0.745	77.9335	0.735	79.9597
0.746	78.1099	0.737	80.1311
0.748	78.251	0.738	80.2823
0.749	78.4123	0.74	80.4083
0.751	78.5736	0.741	80.5696
0.752	78.6946	0.742	80.751
0.755	78.8458	0.744	80.877
0.757	79.0171	0.746	81.0534
0.758	79.1683	0.747	81.2097
0.76	79.3296	0.75	81.5373
0.761	79.4859	0.752	81.6835
0.762	79.6573	0.753	81.8246
0.764	79.8185	0.754	81.9708
0.765	79.9496	0.756	82.1472
0.767	80.126	0.757	82.3135
0.768	80.2873	0.758	82.4647
0.771	80.4133	0.76	82.6109
0.773	80.5746	0.761	82.7621
0.774	80.7157	0.764	83.0292
0.776	80.8871	0.766	83.2359
0.777	81.0534	0.767	83.4123
0.778	81.3861	0.769	83.5585
0.781	81.5171	0.771	83.8558
0.783	81.6784	0.773	84.002
0.784	81.9708	0.775	84.1683
0.786	82.1472	0.776	84.3498
0.787	82.2833	0.777	84.5161
0.788	82.4143	0.779	84.6673
0.79	82.5857	0.78	84.8387
0.792	82.7671	0.782	85
0.794	82.9133	0.783	85.1563
0.796	83.0292	0.785	85.2974
0.797	83.2006	0.786	85.4536
0.799	83.3468	0.789	85.7964
0.8	83.4929	0.791	85.9425
0.802	83.6593	0.792	86.1139
0.803	83.8155	0.793	86.2702
0.805	83.9768	0.796	86.5776
0.806	84.128	0.798	86.754
0.807	84.2944	0.799	86.9103
0.809	84.4708	0.801	87.0514
0.81	84.6018	0.802	87.2127
0.812	84.7429	0.803	87.3438
0.813	84.9093	0.805	87.5252
0.815	85.0554	0.807	87.6815
0.816	85.2167	0.808	87.8327
0.818	85.378	0.809	87.9738
0.821	85.5494	0.811	88.1603
0.822	85.6905	0.812	88.3014
0.823	85.8518	0.814	88.4677
0.826	86.0081	0.815	88.6442
0.828	86.1593	0.817	88.7954
0.829	86.3256	0.818	88.9365
0.831	86.502	0.82	89.0877
0.832	86.6431	0.821	89.254

0.833	86.7893	0.822	89.4304
0.835	86.9355	0.824	89.6018
0.836	87.0968	0.825	89.748
0.838	87.2329	0.828	90.0756
0.839	87.3891	0.83	90.2117
0.842	87.5554	0.831	90.378
0.844	87.7117	0.832	90.5494
0.845	88.0141	0.834	90.7056
0.846	88.1905	0.836	90.877
0.848	88.3115	0.837	91.0635
0.85	88.4829	0.838	91.1895
0.851	88.6593	0.84	91.3357
0.852	88.9768	0.841	91.502
0.854	89.128	0.843	91.6885
0.855	89.2893	0.844	91.8347
0.857	89.4355	0.846	92.0111
0.86	89.753	0.847	92.1724
0.861	89.9042	0.848	92.3286
0.862	90.0605	0.85	92.4698
0.864	90.3982	0.851	92.6159
0.865	90.5544	0.853	92.7772
0.867	90.7006	0.854	92.9587
0.868	90.8619	0.856	93.12
0.87	91.0333	0.857	93.2863
0.871	91.1794	0.859	93.4375
0.873	91.3306	0.861	93.7702
0.874	91.628	0.863	93.9163
0.876	91.7792	0.864	94.0524
0.877	91.9304	0.867	94.4052
0.878	92.253	0.869	94.5917
0.88	92.4294	0.87	94.7581
0.883	92.5706	0.872	94.9093
0.884	92.7016	0.873	95.0806
0.885	92.878	0.875	95.2218
0.887	93.0091	0.876	95.4183
0.889	93.1603	0.877	95.5948
0.89	93.3115	0.879	95.7409
0.891	93.4677	0.88	95.9022
0.893	93.6341	0.882	96.0484
0.894	93.9919	0.883	96.2198
0.896	94.1431	0.885	96.381
0.899	94.2742	0.886	96.5575
0.9	94.4204	0.888	96.7389
0.901	94.5716	0.889	96.8851
0.903	94.7379	0.891	97.0464
0.905	94.8942	0.892	97.1976
0.906	95.0756	0.893	97.3639
0.907	95.2067	0.895	97.5454
0.91	95.3881	0.896	97.6966
0.912	95.5393	0.898	97.8327
0.914	95.7056	0.901	98.2006
0.916	95.8367	0.902	98.3669
0.918	95.9879	0.904	98.5181
0.919	96.1542	0.906	98.8558
0.92	96.2903	0.908	99.0071
0.922	96.4567	0.909	99.1482
0.923	96.7641	0.911	99.3196
0.925	96.9103	0.912	99.4859
0.926	97.0817	0.914	99.6673
0.928	97.2631	0.915	99.8387
0.929	97.4143	0.917	100.005
0.932	97.5756	0.918	100.1562
0.934	97.7319	0.92	100.2974
0.935	97.8931	0.922	100.6552
0.936	98.0242	0.924	100.8317
0.938	98.1905	0.925	101.0081
0.939	98.3821	0.927	101.1744

0.941	98.5433	0.928	101.3458
0.942	98.7046	0.93	101.4869
0.944	98.8659	0.931	101.6532
0.945	99.1532	0.933	101.8044
0.946	99.3196	0.934	101.9859
0.948	99.496	0.936	102.1673
0.949	99.6472	0.937	102.3135
0.951	99.8135	0.94	102.6411
0.952	99.9647	0.941	102.8075
0.954	100.1411	0.943	102.9587
0.955	100.3024	0.944	103.1048
0.957	100.4486	0.945	103.2661
0.958	100.6351	0.947	103.4476
0.959	100.7762	0.949	103.619
0.961	100.9375	0.95	103.7903
0.963	101.0937	0.951	103.9617
0.964	101.2399	0.954	104.2792
0.965	101.4264	0.956	104.4708
0.967	101.7238	0.957	104.6321
0.968	101.8851	0.959	104.8085
0.971	102.0716	0.96	104.9647
0.973	102.2429	0.961	105.1159
0.974	102.3992	0.963	105.2621
0.975	102.5706	0.964	105.4637
0.977	102.7621	0.966	105.62
0.979	103.004	0.967	105.7812
0.98	103.2107	0.969	105.9526
0.981	103.3871	0.97	106.1189
0.983	103.5282	0.972	106.2853
0.986	103.7097	0.973	106.4718
0.987	103.8861	0.975	106.6532
0.989	104.0272	0.976	106.7994
0.99	104.1835	0.979	107.1321
0.991	104.3347	0.981	107.3135
0.993	104.5161	0.982	107.4849
0.994	104.6724	0.983	107.631
0.996	104.8286	0.985	107.7873
0.997	104.9899	0.986	107.9637
0.999	105.1764	0.988	108.1351
1	105.3377	0.989	108.3165
1.003	105.499	0.99	108.4879
1.004	105.6502	0.992	108.6492
1.006	105.8216	0.994	108.8206
1.007	105.9879	0.995	109.0171
1.009	106.3004	0.996	109.1734
1.01	106.4768	0.998	109.3347
1.012	106.7641	0.999	109.4909
1.015	107.1472	1.002	109.8286
1.016	107.3135	1.004	110.005
1.017	107.4597	1.005	110.1915
1.019	107.621	1.006	110.378
1.02	107.7823	1.008	110.5091
1.021	108.0796	1.009	110.6603
1.023	108.2712	1.011	110.8367
1.025	108.4375	1.012	110.9778
1.026	108.7903	1.014	111.1794
1.027	108.9415	1.015	111.3659
1.029	109.0978	1.018	111.6885
1.03	109.254	1.02	111.875
1.032	109.3901	1.021	112.0464
1.033	109.5917	1.023	112.2026
1.035	109.9194	1.024	112.3538
1.036	110.1058	1.025	112.5403
1.038	110.2772	1.027	112.7167
1.039	110.4335	1.028	112.8679
1.041	110.6099	1.03	113.0343
1.042	110.7611	1.031	113.2006

1.043	110.9425	1.034	113.5433
1.045	111.0887	1.035	113.7248
1.046	111.2298	1.037	113.9062
1.048	111.3911	1.038	114.0827
1.049	111.5776	1.04	114.2339
1.05	111.7288	1.041	114.3851
1.052	111.8851	1.043	114.5464
1.054	112.0766	1.044	114.7278
1.055	112.2429	1.046	114.9194
1.056	112.3992	1.047	115.0857
1.058	112.5857	1.048	115.2419
1.059	112.7369	1.051	115.5696
1.061	112.9083	1.053	115.751
1.062	113.0595	1.054	115.9073
1.064	113.2006	1.056	116.0988
1.065	113.3921	1.057	116.25
1.068	113.5585	1.058	116.4365
1.069	113.7147	1.06	116.5978
1.071	113.8861	1.062	116.8044
1.072	114.0575	1.063	116.9657
1.074	114.37	1.064	117.127
1.075	114.5514	1.066	117.3085
1.077	114.7177	1.069	117.6714
1.078	114.8992	1.07	117.8327
1.079	115.0907	1.072	117.994
1.081	115.247	1.073	118.1502
1.083	115.4234	1.075	118.3317
1.084	115.5696	1.076	118.4728
1.085	115.7409	1.077	118.6341
1.087	115.9173	1.079	118.8508
1.088	116.0786	1.08	119.0222
1.09	116.4214	1.083	119.37
1.091	116.5776	1.085	119.5565
1.092	116.7439	1.086	119.7026
1.094	116.9103	1.088	119.8387
1.095	117.0716	1.091	120.2066
1.097	117.253	1.092	120.378
1.098	117.4597	1.093	120.5796
1.1	117.6109	1.095	120.7561
1.101	117.7823	1.096	120.9123
1.102	117.9536	1.098	121.0685
1.104	118.1048	1.099	121.2349
1.106	118.2409	1.101	121.4163
1.107	118.3972	1.102	121.5927
1.108	118.5887	1.103	121.7439
1.11	118.9617	1.107	122.1119
1.111	119.1179	1.108	122.2681
1.113	119.2994	1.109	122.4496
1.116	119.4556	1.111	122.6462
1.117	119.622	1.112	122.8075
1.118	119.8337	1.114	122.999
1.12	119.9849	1.115	123.1603
1.121	120.1462	1.117	123.3014
1.123	120.3125	1.118	123.4778
1.124	120.4738	1.119	123.6593
1.126	120.6452	1.121	123.7903
1.127	121.0181	1.122	123.9819
1.13	121.1744	1.124	124.1835
1.131	121.3256	1.125	124.3599
1.133	121.4768	1.127	124.5212
1.134	121.6331	1.13	124.8488
1.136	121.8196	1.131	125.0151
1.137	122.0111	1.132	125.2016
1.139	122.1472	1.134	125.3881
1.141	122.3337	1.136	125.5595
1.143	122.5202	1.137	125.751
1.145	122.7167	1.138	125.9274

1.146	122.878	1.14	126.0937
1.147	123.0091	1.141	126.25
1.149	123.2107	1.143	126.4062
1.15	123.3468	1.144	126.6079
1.152	123.7046	1.146	126.7893
1.153	123.871	1.147	126.9607
1.155	124.0524	1.148	127.1371
1.156	124.2238	1.15	127.3135
1.158	124.4153	1.152	127.5101
1.159	124.5917	1.153	127.6613
1.16	124.8841	1.154	127.8478
1.162	125.0907	1.156	128.0292
1.163	125.2268	1.157	128.1855
1.165	125.6149	1.159	128.377
1.166	125.7712	1.16	128.5333
1.169	125.9375	1.162	128.6744
1.17	126.1038	1.163	128.8609
1.172	126.25	1.164	129.0272
1.174	126.4113	1.166	129.1986
1.175	126.6129	1.167	129.3851
1.176	126.7843	1.169	129.5615
1.178	126.9355	1.17	129.748
1.179	127.1018	1.172	129.9143
1.181	127.2933	1.173	130.1109
1.182	127.4446	1.175	130.2722
1.183	127.5958	1.176	130.4183
1.185	127.7823	1.177	130.6149
1.186	127.9738	1.179	130.7913
1.188	128.3266	1.181	130.9728
1.189	128.4778	1.182	131.1391
1.191	128.6391	1.183	131.3206
1.192	128.7954	1.186	131.6381
1.195	128.9768	1.188	131.8145
1.197	129.1431	1.189	132.001
1.198	129.3347	1.191	132.1573
1.201	129.5111	1.192	132.3437
1.202	129.6875	1.193	132.5403
1.204	129.8488	1.195	132.7016
1.205	130.1915	1.197	132.8881
1.207	130.373	1.198	133.0645
1.208	130.5141	1.199	133.2611
1.209	130.6855	1.202	133.5837
1.211	130.8669	1.204	133.7752
1.212	131.0534	1.205	133.9365
1.214	131.2097	1.207	134.0827
1.217	131.3911	1.208	134.249
1.218	131.5776	1.209	134.4304
1.22	131.744	1.211	134.627
1.221	132.0766	1.212	134.8135
1.222	132.2429	1.214	135.005
1.224	132.4093	1.215	135.1764
1.226	132.5806	1.218	135.5192
1.227	132.7318	1.22	135.6754
1.228	132.9385	1.221	135.8367
1.23	133.1351	1.222	136.0131
1.233	133.2964	1.226	136.4012
1.234	133.4627	1.227	136.5877
1.236	133.624	1.228	136.754
1.237	133.8004	1.23	136.9254
1.238	134.1028	1.231	137.0867
1.24	134.2994	1.234	137.4546
1.241	134.4809	1.236	137.6361
1.243	134.6522	1.237	137.7974
1.244	135.0504	1.238	137.9688
1.246	135.1815	1.24	138.1552
1.249	135.3125	1.241	138.3417
1.25	135.504	1.243	138.5232

1.251	135.6855	1.244	138.7248
1.253	135.877	1.246	138.876
1.254	136.0534	1.247	139.0625
1.256	136.2298	1.249	139.2036
1.257	136.4415	1.252	139.5817
1.26	136.6129	1.253	139.7379
1.261	136.7641	1.254	139.8942
1.263	137.1119	1.256	140.0907
1.266	137.2732	1.259	140.504
1.267	137.4446	1.26	140.6603
1.269	137.621	1.262	140.8417
1.27	137.7974	1.263	140.998
1.272	137.9587	1.265	141.1794
1.275	138.1452	1.266	141.3508
1.276	138.3266	1.267	141.5474
1.278	138.5333	1.269	141.7187
1.279	138.8911	1.271	141.9002
1.282	139.0323	1.272	142.0867
1.283	139.2137	1.273	142.2682
1.285	139.3599	1.275	142.4546
1.286	139.748	1.276	142.6159
1.288	139.9446	1.278	142.8024
1.289	140.0756	1.279	142.994
1.29	140.2369	1.281	143.1603
1.292	140.3982	1.282	143.3468
1.293	140.5847	1.283	143.5484
1.295	140.7913	1.285	143.6996
1.296	140.9778	1.286	143.8861
1.299	141.1492	1.288	144.0675
1.301	141.3206	1.291	144.4355
1.302	141.502	1.292	144.627
1.304	141.8649	1.294	144.8034
1.305	142.0363	1.295	144.9546
1.306	142.2177	1.298	145.2974
1.308	142.3992	1.299	145.4536
1.309	142.5806	1.301	145.6452
1.311	142.7369	1.302	145.8165
1.312	142.9133	1.304	146.0181
1.314	143.0746	1.305	146.2097
1.315	143.256	1.307	146.371
1.317	143.4224	1.308	146.5171
1.318	143.5786	1.31	146.6935
1.319	143.7802	1.311	146.88
1.321	143.9919	1.312	147.0817
1.322	144.1331	1.314	147.2631
1.324	144.3246	1.315	147.4597
1.325	144.5262	1.317	147.6462
1.327	144.6875	1.318	147.8024
1.328	144.8337	1.32	147.9637
1.329	145.005	1.321	148.1452
1.331	145.2016	1.322	148.3568
1.332	145.3831	1.324	148.5434
1.334	145.5696	1.326	148.6996
1.335	145.8871	1.327	148.8861
1.337	146.0484	1.33	149.2591
1.338	146.245	1.331	149.4355
1.34	146.5978	1.333	149.6522
1.341	146.7691	1.334	149.8236
1.343	146.9506	1.337	150.1563
1.344	147.1472	1.339	150.3427
1.346	147.3236	1.34	150.5091
1.347	147.505	1.342	150.6754
1.348	147.6815	1.343	150.8619
1.35	147.8579	1.344	151.0232
1.353	148.0494	1.346	151.2198
1.354	148.2409	1.347	151.4163
1.356	148.4073	1.349	151.5776

1.357	148.5635	1.35	151.7641
1.358	148.755	1.352	151.9456
1.361	148.9163	1.355	152.2782
1.364	149.0675	1.356	152.4546
1.366	149.2994	1.357	152.6361
1.369	149.4859	1.359	152.8276
1.37	149.6421	1.36	153.0191
1.372	149.8286	1.362	153.2208
1.373	150	1.363	153.4274
1.374	150.1512	1.365	153.5786
1.376	150.3377	1.366	153.7349
1.377	150.7308	1.367	153.9315
1.379	150.9073	1.37	154.3095
1.38	151.0937	1.372	154.496
1.382	151.4466	1.373	154.6875
1.383	151.5978	1.375	154.869
1.386	151.7692	1.376	155.0504
1.387	151.9708	1.378	155.252
1.389	152.1623	1.379	155.4234
1.391	152.3438	1.381	155.5796
1.392	152.5403	1.382	155.7611
1.393	152.8982	1.384	155.9375
1.395	153.0847	1.385	156.1038
1.396	153.4425	1.386	156.3004
1.397	153.6089	1.388	156.4869
1.401	153.7752	1.389	156.6532
1.402	153.9617	1.391	156.8448
1.403	154.1633	1.394	157.248
1.405	154.3397	1.395	157.4345
1.406	154.4607	1.396	157.5958
1.408	154.6825	1.398	157.7722
1.409	154.8841	1.4	157.9436
1.411	155.0655	1.401	158.1452
1.412	155.2268	1.402	158.3518
1.414	155.4385	1.404	158.5282
1.415	155.5897	1.405	158.7046
1.418	155.7308	1.407	158.8861
1.419	155.9073	1.41	159.2389
1.421	156.1189	1.411	159.4204
1.422	156.3155	1.412	159.6169
1.424	156.5272	1.414	159.7933
1.425	156.6784	1.415	159.9748
1.426	156.87	1.417	160.1663
1.428	157.0464	1.42	160.5444
1.429	157.2177	1.421	160.7107
1.431	157.3942	1.423	160.8871
1.432	157.7671	1.426	161.2752
1.434	157.9536	1.427	161.4415
1.435	158.125	1.429	161.6179
1.437	158.2863	1.43	161.8095
1.438	158.4778	1.433	162.1623
1.44	158.6492	1.434	162.3639
1.441	158.8306	1.436	162.5605
1.442	159.0222	1.437	162.747
1.444	159.2238	1.439	162.9133
1.445	159.3851	1.44	163.1149
1.447	159.5565	1.442	163.3367
1.448	159.7379	1.443	163.4778
1.449	159.8992	1.445	163.6139
1.451	160.0806	1.446	163.8155
1.453	160.2974	1.449	164.1885
1.454	160.6804	1.45	164.3952
1.457	160.8568	1.452	164.5917
1.458	161.0232	1.453	164.7631
1.46	161.1946	1.455	164.9345
1.461	161.3357	1.456	165.126
1.463	161.5222	1.459	165.4889

1.464	161.7188	1.46	165.6552
1.466	161.9304	1.462	165.8468
1.467	162.122	1.463	166.0333
1.469	162.3085	1.465	166.2298
1.47	162.6563	1.466	166.4113
1.473	162.873	1.468	166.6028
1.474	163.0393	1.469	166.8044
1.476	163.3972	1.47	166.9859
1.477	163.5837	1.472	167.1673
1.479	163.7702	1.474	167.3387
1.48	163.9516	1.475	167.5101
1.482	164.1482	1.476	167.6865
1.483	164.375	1.478	167.8528
1.484	164.5363	1.479	168.0544
1.486	164.6925	1.481	168.2611
1.487	164.8841	1.482	168.4476
1.49	165.0554	1.484	168.6139
1.492	165.4083	1.485	168.7853
1.493	165.5746	1.486	168.9567
1.494	165.7661	1.488	169.1331
1.496	165.9778	1.489	169.3246
1.497	166.1996	1.491	169.5514
1.499	166.3659	1.492	169.7581
1.5	166.5323	1.494	169.9446
1.502	166.7087	1.495	170.121
1.505	166.87	1.498	170.4788
1.506	167.0514	1.499	170.6149
1.508	167.2228	1.501	170.8165
1.509	167.626	1.502	171.0333
1.512	167.8024	1.504	171.2046
1.513	168.0091	1.505	171.4062
1.515	168.1804	1.507	171.6028
1.516	168.3619	1.508	171.8095
1.518	168.5333	1.509	171.9758
1.519	168.7198	1.511	172.1724
1.521	168.9113	1.514	172.5605
1.522	169.0978	1.515	172.7218
1.523	169.4909	1.517	172.9032
1.525	169.6774	1.518	173.0696
1.527	169.8437	1.52	173.2712
1.528	170.0302	1.521	173.4577
1.529	170.2268	1.523	173.6492
1.531	170.4133	1.524	173.8609
1.532	170.6048	1.526	174.0474
1.534	170.9375	1.527	174.2389
1.535	171.1391	1.528	174.4204
1.537	171.3306	1.531	174.7883
1.538	171.5121	1.533	174.9496
1.539	171.6784	1.534	175.1462
1.541	171.8851	1.537	175.5494
1.544	172.0917	1.538	175.7359
1.545	172.2732	1.54	175.9022
1.547	172.4647	1.542	176.0837
1.548	172.626	1.543	176.2601
1.55	172.7974	1.544	176.4314
1.551	173.004	1.546	176.6381
1.552	173.2258	1.547	176.8347
1.555	173.4173	1.549	177.0161
1.557	173.7954	1.55	177.2077
1.558	173.9718	1.553	177.5857
1.561	174.1381	1.554	177.7823
1.562	174.3498	1.556	177.9889
1.564	174.6976	1.557	178.1855
1.566	174.8841	1.559	178.3669
1.567	175.0807	1.56	178.5585
1.568	175.2722	1.562	178.7248
1.57	175.4637	1.563	178.8911

1.571	175.6603	1.565	179.0524
1.573	175.872	1.566	179.2339
1.574	176.0433	1.569	179.6673
1.577	176.2399	1.57	179.8589
1.579	176.3911	1.572	180.0353
1.58	176.5524	1.573	180.2319
1.581	176.7188	1.575	180.4234
1.583	176.9355	1.576	180.5847
1.584	177.1018	1.577	180.7611
1.586	177.3085	1.579	180.9425
1.587	177.5353	1.581	181.1593
1.589	177.7319	1.582	181.3508
1.59	177.9032	1.585	181.7339
1.592	178.0948	1.586	181.9405
1.595	178.2813	1.588	182.0968
1.596	178.4425	1.589	182.2933
1.597	178.7954	1.592	182.7066
1.6	179.0071	1.594	182.878
1.602	179.2036	1.595	183.0796
1.603	179.4052	1.597	183.251
1.605	179.6068	1.598	183.4274
1.606	179.7833	1.599	183.6139
1.607	179.9446	1.601	183.8458
1.609	180.121	1.602	184.0575
1.61	180.3075	1.604	184.2238
1.612	180.5343	1.605	184.4052
1.613	180.7308	1.608	184.7581
1.616	181.1038	1.61	184.9546
1.618	181.3054	1.611	185.121
1.619	181.5121	1.612	185.3125
1.621	181.6986	1.614	185.5292
1.622	181.8548	1.615	185.7409
1.624	182.0565	1.617	185.9325
1.625	182.248	1.618	186.0887
1.626	182.4143	1.62	186.2651
1.628	182.5907	1.621	186.4516
1.629	182.7772	1.624	186.8246
1.632	183.1502	1.626	187.0212
1.634	183.3619	1.627	187.2228
1.635	183.5887	1.628	187.4193
1.636	183.9617	1.63	187.6311
1.638	184.1381	1.631	187.8276
1.639	184.2843	1.633	187.999
1.641	184.4859	1.636	188.3871
1.642	184.6875	1.637	188.5685
1.644	184.8841	1.639	188.755
1.645	185.0857	1.641	189.1432
1.647	185.4687	1.643	189.3347
1.648	185.6351	1.644	189.5262
1.65	185.8266	1.646	189.7329
1.651	186.0333	1.647	189.9244
1.652	186.2248	1.649	190.1411
1.654	186.4264	1.65	190.3276
1.655	186.6179	1.651	190.5242
1.657	186.8044	1.653	190.7107
1.658	186.9808	1.655	190.8821
1.66	187.3639	1.656	191.0686
1.661	187.5907	1.657	191.2702
1.663	187.752	1.659	191.4617
1.664	188.1401	1.66	191.6482
1.666	188.2913	1.662	191.8649
1.667	188.503	1.663	192.0716
1.669	188.6794	1.665	192.2782
1.67	189.0675	1.666	192.4597
1.671	189.2843	1.667	192.6311
1.673	189.4808	1.669	192.8377
1.674	189.6472	1.67	193.0141

1.675	189.879	1.672	193.2359
1.677	190.0454	1.675	193.5988
1.679	190.1966	1.676	193.8004
1.68	190.5948	1.678	193.9869
1.681	190.7964	1.68	194.3649
1.683	190.9879	1.682	194.5313
1.684	191.2097	1.684	194.7177
1.687	191.4012	1.685	194.9395
1.689	191.5675	1.686	195.1361
1.69	191.749	1.688	195.3679
1.692	191.9607	1.689	195.5494
1.693	192.1623	1.692	195.9526
1.695	192.3438	1.694	196.1442
1.696	192.5353	1.696	196.502
1.697	192.7117	1.698	196.6986
1.699	192.9284	1.699	196.9052
1.7	193.125	1.701	197.1018
1.702	193.5282	1.702	197.2883
1.703	193.7248	1.704	197.4899
1.705	193.9315	1.705	197.7117
1.706	194.1179	1.707	197.8881
1.708	194.4556	1.708	198.0746
1.709	194.627	1.71	198.2661
1.71	194.8286	1.712	198.624
1.712	195.0504	1.714	198.8105
1.714	195.252	1.715	199.0121
1.715	195.63	1.717	199.2036
1.716	195.8065	1.72	199.622
1.718	195.998	1.721	199.8286
1.719	196.1794	1.723	200.0101
1.72	196.3861	1.724	200.1865
1.722	196.6179	1.725	200.3679
1.724	196.8196	1.727	200.5645
1.725	197.0111	1.728	200.7762
1.728	197.2228	1.73	200.9677
1.729	197.4294	1.731	201.1643
1.731	197.5806	1.733	201.3911
1.732	197.7671	1.736	201.7994
1.735	197.9788	1.737	201.9556
1.737	198.1804	1.739	202.122
1.738	198.3921	1.74	202.3286
1.739	198.5786	1.741	202.5151
1.741	198.7802	1.743	202.7016
1.744	198.997	1.744	202.8931
1.745	199.1935	1.746	203.0796
1.747	199.3952	1.747	203.2762
1.748	199.6018	1.749	203.4879
1.75	199.7984	1.75	203.6845
1.751	199.9597	1.753	204.0675
1.753	200.1361	1.754	204.2641
1.754	200.3377	1.756	204.4405
1.755	200.5393	1.757	204.627
1.757	200.746	1.759	204.8286
1.758	200.9476	1.76	205.0504
1.76	201.1694	1.762	205.247
1.761	201.3659	1.763	205.4234
1.763	201.5524	1.764	205.63
1.764	201.754	1.766	205.8165
1.767	201.9405	1.769	206.2097
1.769	202.0968	1.77	206.4063
1.77	202.2883	1.772	206.6028
1.771	202.5101	1.773	206.7943
1.773	202.7268	1.775	206.996
1.774	203.0998	1.776	207.1976
1.777	203.3115	1.779	207.5706
1.779	203.5081	1.78	207.8024
1.78	203.6694	1.782	207.9738

1.782	203.881	1.784	208.1804
1.784	204.0978	1.785	208.3417
1.786	204.2692	1.786	208.5383
1.787	204.4909	1.788	208.7399
1.789	204.6825	1.789	208.9315
1.79	204.8942	1.792	209.3296
1.792	205.0857	1.794	209.5464
1.793	205.2923	1.795	209.748
1.795	205.494	1.797	209.9244
1.796	205.6653	1.798	210.126
1.799	205.872	1.799	210.3276
1.802	206.0887	1.801	210.4839
1.803	206.255	1.802	210.6754
1.805	206.4315	1.804	210.8669
1.806	206.6079	1.805	211.0736
1.808	207.0262	1.808	211.5121
1.809	207.2379	1.81	211.7238
1.81	207.4546	1.811	211.87
1.812	207.6562	1.813	212.0464
1.813	207.8478	1.814	212.2581
1.815	208.0242	1.815	212.4496
1.816	208.2157	1.818	212.8226
1.818	208.3871	1.82	213.0544
1.819	208.5938	1.821	213.251
1.821	208.8055	1.823	213.4375
1.823	209.0222	1.825	213.8558
1.825	209.2137	1.827	214.0474
1.827	209.4002	1.829	214.2692
1.828	209.5766	1.83	214.4708
1.831	210.0151	1.831	214.6371
1.832	210.2117	1.833	214.8085
1.834	210.3982	1.834	214.9849
1.835	210.7661	1.836	215.1764
1.837	210.9728	1.837	215.4234
1.839	211.1694	1.838	215.6502
1.841	211.4113	1.84	215.8518
1.843	211.6381	1.842	216.0333
1.844	211.8145	1.843	216.2248
1.845	212.0061	1.844	216.4063
1.847	212.1976	1.846	216.5928
1.848	212.374	1.847	216.8044
1.85	212.5655	1.848	217.0312
1.851	212.9587	1.85	217.2278
1.853	213.1804	1.852	217.4395
1.855	213.4073	1.853	217.6411
1.857	213.624	1.854	217.8427
1.858	213.8105	1.857	218.1855
1.86	213.9718	1.859	218.4022
1.861	214.1684	1.86	218.6341
1.863	214.37	1.862	218.8407
1.864	214.5816	1.863	219.0323
1.866	214.7581	1.865	219.2389
1.867	214.9698	1.866	219.4405
1.868	215.3881	1.868	219.6321
1.87	215.5948	1.869	219.8488
1.872	215.7762	1.87	220.0554
1.873	215.9627	1.872	220.2621
1.874	216.1895	1.873	220.4385
1.876	216.3861	1.875	220.5847
1.879	216.5877	1.876	220.7913
1.88	216.8044	1.878	220.9929
1.882	217.0161	1.879	221.1895
1.883	217.1875	1.881	221.3962
1.884	217.3841	1.882	221.6129
1.886	217.5958	1.883	221.8296
1.888	217.7923	1.885	222.0061
1.889	218.004	1.887	222.2077

1.89	218.2157	1.888	222.4445
1.892	218.4123	1.889	222.5907
1.895	218.5937	1.891	222.7571
1.896	218.7752	1.892	222.9486
1.897	218.9718	1.893	223.1804
1.899	219.1986	1.897	223.5837
1.9	219.3901	1.898	223.8054
1.902	219.5917	1.899	223.9818
1.903	219.8085	1.901	224.1683
1.906	220.0101	1.902	224.37
1.907	220.2167	1.904	224.5867
1.911	220.4032	1.905	224.7681
1.912	220.7712	1.907	224.9597
1.913	221.0081	1.908	225.1865
1.915	221.2298	1.91	225.3931
1.916	221.4214	1.911	225.5897
1.918	221.6381	1.913	225.7913
1.919	221.8296	1.914	226.003
1.921	222.0061	1.915	226.2097
1.922	222.2278	1.917	226.381
1.923	222.4294	1.918	226.6129
1.925	222.8226	1.92	226.7843
1.928	223.0242	1.921	226.9708
1.929	223.2157	1.922	227.1421
1.931	223.4224	1.924	227.3488
1.932	223.6492	1.926	227.5756
1.934	224.0726	1.927	227.7823
1.935	224.249	1.928	227.9889
1.937	224.4456	1.93	228.1855
1.938	224.6371	1.932	228.3921
1.94	224.7984	1.933	228.5887
1.942	224.9899	1.936	228.9264
1.944	225.1966	1.937	229.1482
1.945	225.4133	1.938	229.3548
1.947	225.6502	1.94	229.5766
1.948	225.8569	1.941	229.7782
1.951	226.2601	1.943	229.9698
1.952	226.4516	1.944	230.1865
1.954	226.623	1.946	230.3679
1.956	226.8347	1.947	230.5645
1.957	227.0312	1.949	230.7762
1.958	227.2429	1.952	231.1946
1.96	227.4395	1.953	231.371
1.961	227.6714	1.955	231.5726
1.963	227.8831	1.956	231.7742
1.964	228.0796	1.957	231.9909
1.967	228.2611	1.959	232.2379
1.968	228.4425	1.96	232.4395
1.97	228.6441	1.962	232.6159
1.972	228.8357	1.965	232.9889
1.973	229.0575	1.966	233.1754
1.974	229.2792	1.967	233.3669
1.976	229.4909	1.969	233.5736
1.977	229.6825	1.971	233.7551
1.979	229.9042	1.972	233.9869
1.98	230.121	1.973	234.1935
1.982	230.3175	1.975	234.38
1.983	230.4889	1.976	234.5766
1.985	230.8669	1.978	234.7732
1.986	231.0585	1.979	234.9748
1.987	231.2349	1.981	235.1865
1.989	231.6885	1.983	235.6048
1.99	231.8851	1.985	235.8165
1.992	232.1018	1.987	236.0232
1.993	232.3135	1.988	236.2198
1.995	232.505	1.989	236.4113
1.996	232.6966	1.992	236.8044

1.997	232.878	1.994	237.0111
1.999	233.2913	1.995	237.2077
2.001	233.503	1.997	237.4193
2.002	233.8861	1.998	237.6512
2.003	234.1079	2.001	237.999
2.006	234.2843	2.002	238.2208
2.008	234.4758	2.004	238.4426
2.009	234.6875	2.005	238.6492
2.011	234.8992	2.008	239.0827
2.012	235.1159	2.01	239.2288
2.013	235.3125	2.011	239.4506
2.015	235.5242	2.012	239.6371
2.016	235.7409	2.014	239.8438
2.018	235.9123	2.015	239.9798
2.019	236.119	2.017	240.1966
2.021	236.381	2.018	240.4183
2.022	236.5625	2.02	240.6452
2.024	236.7692	2.021	240.8569
2.025	236.9657	2.023	241.0585
2.026	237.1522	2.025	241.4466
2.028	237.3538	2.027	241.6784
2.029	237.5202	2.028	241.8901
2.032	237.7218	2.03	242.0766
2.034	237.9587	2.031	242.2934
2.035	238.3921	2.033	242.4899
2.036	238.5837	2.034	242.6764
2.038	238.7903	2.036	242.8579
2.04	238.997	2.037	243.0847
2.041	239.1633	2.039	243.3014
2.042	239.5766	2.04	243.4929
2.044	239.7933	2.041	243.6895
2.047	240.0051	2.043	243.9163
2.048	240.2369	2.045	244.1331
2.05	240.4435	2.047	244.5565
2.051	240.62	2.049	244.7682
2.053	240.8266	2.05	244.9698
2.054	241.0585	2.052	245.1613
2.055	241.255	2.053	245.3579
2.057	241.4415	2.055	245.5444
2.058	241.8296	2.056	245.7409
2.06	242.0564	2.057	245.9274
2.061	242.2833	2.059	246.1341
2.063	242.495	2.06	246.3811
2.064	242.7167	2.062	246.5927
2.065	242.9536	2.063	246.7994
2.067	243.1502	2.065	247.0061
2.069	243.3317	2.066	247.1875
2.071	243.5333	2.068	247.3992
2.073	243.7298	2.069	247.5706
2.074	243.9113	2.07	247.7722
2.076	244.3246	2.072	247.9889
2.077	244.5565	2.073	248.1905
2.078	244.7732	2.075	248.4073
2.08	244.9698	2.076	248.6442
2.081	245.3579	2.078	248.8558
2.083	245.5645	2.079	249.0222
2.084	245.7812	2.08	249.1986
2.087	246.1946	2.082	249.4103
2.088	246.4012	2.084	249.6119
2.09	246.623	2.086	250.0101
2.092	246.8044	2.088	250.2268
2.093	246.9859	2.089	250.4486
2.094	247.3941	2.091	250.6754
2.097	247.6159	2.092	250.8921
2.099	247.8276	2.094	251.0837
2.1	248.0444	2.095	251.2702
2.102	248.246	2.098	251.6784

2.103	248.4778	2.1	251.87
2.105	248.6845	2.101	252.0665
2.106	248.9062	2.102	252.248
2.107	249.123	2.104	252.4647
2.109	249.2994	2.105	252.6865
2.112	249.501	2.107	252.8831
2.113	249.8589	2.108	253.1048
2.115	250.0958	2.11	253.3065
2.116	250.2722	2.111	253.5282
2.119	250.4939	2.113	253.7046
2.121	250.9829	2.114	253.871
2.122	251.1542	2.115	254.0877
2.123	251.371	2.118	254.5262
2.125	251.5675	2.12	254.7379
2.126	251.754	2.121	254.9496
2.128	251.9506	2.123	255.1562
2.129	252.1774	2.124	255.3377
2.131	252.4093	2.125	255.5393
2.133	252.6109	2.127	255.7561
2.135	252.8226	2.13	256.1643
2.136	253.0393	2.131	256.376
2.138	253.4224	2.134	256.7994
2.139	253.629	2.136	256.9708
2.141	253.8508	2.137	257.2026
2.142	254.2692	2.139	257.4345
2.144	254.4808	2.14	257.6462
2.145	254.6976	2.141	257.7873
2.146	254.8891	2.143	257.9688
2.148	255.0857	2.144	258.1804
2.15	255.5192	2.146	258.3921
2.151	255.7157	2.147	258.5786
2.152	255.9224	2.149	258.7954
2.154	256.1139	2.152	259.2691
2.155	256.4567	2.153	259.4657
2.158	256.6683	2.154	259.6925
2.16	256.9002	2.156	259.8992
2.161	257.1371	2.158	260.0857
2.162	257.5605	2.159	260.2268
2.164	257.7671	2.16	260.4335
2.165	257.9536	2.162	260.6552
2.167	258.12	2.163	260.872
2.168	258.3316	2.165	261.0988
2.17	258.5484	2.166	261.3256
2.171	258.9819	2.168	261.5272
2.173	259.2087	2.169	261.7188
2.174	259.4304	2.17	261.9506
2.175	259.7782	2.172	262.1371
2.177	259.9597	2.175	262.5101
2.178	260.2067	2.176	262.7319
2.18	260.4234	2.178	262.9687
2.181	260.6149	2.179	263.1754
2.183	260.8266	2.181	263.3821
2.184	261.0434	2.182	263.5988
2.185	261.2651	2.183	263.8155
2.187	261.4869	2.185	264.0423
2.19	261.749	2.186	264.2641
2.191	262.1321	2.188	264.4506
2.193	262.3286	2.191	264.8438
2.194	262.5151	2.192	265.0303
2.196	262.7067	2.193	265.247
2.197	262.9385	2.195	265.4788
2.199	263.1199	2.197	265.7057
2.201	263.3417	2.198	265.9123
2.203	263.5938	2.199	266.129
2.206	263.8206	2.201	266.3407
2.207	264.0121	2.202	266.5323
2.209	264.2087	2.204	266.7238

2.21	264.5867	2.205	266.9657
2.212	264.8085	2.208	267.4042
2.213	265.0202	2.209	267.631
2.214	265.247	2.211	267.8528
2.216	265.6552	2.214	268.2006
2.218	265.8972	2.215	268.4073
2.219	266.2802	2.217	268.629
2.22	266.497	2.218	268.8508
2.223	266.6885	2.22	269.0474
2.224	266.9052	2.221	269.2691
2.226	267.1321	2.223	269.501
2.229	267.379	2.224	269.7278
2.23	267.5655	2.226	269.9345
2.232	267.757	2.227	270.1512
2.233	268.0091	2.23	270.5393
2.235	268.2107	2.231	270.7107
2.238	268.4173	2.233	270.9274
2.239	268.6492	2.234	271.1542
2.241	268.8357	2.236	271.3659
2.242	269.0423	2.237	271.5474
2.245	269.2339	2.238	271.7943
2.246	269.4254	2.24	272.0161
2.248	269.6673	2.242	272.2127
2.249	269.8639	2.243	272.4244
2.251	270.3075	2.246	272.868
2.252	270.5292	2.247	273.0393
2.255	270.7359	2.249	273.246
2.256	270.9173	2.25	273.4627
2.258	271.3407	2.252	273.6694
2.259	271.5575	2.253	273.8861
2.261	271.7641	2.254	274.1129
2.262	271.996	2.256	274.3347
2.264	272.1976	2.257	274.5363
2.265	272.4093	2.259	274.7177
2.267	272.5857	2.26	274.9294
2.268	272.8125	2.263	275.3175
2.269	273.0343	2.265	275.5595
2.271	273.2258	2.266	275.8065
2.272	273.4425	2.268	276.0182
2.274	273.871	2.269	276.2248
2.275	274.0877	2.271	276.4264
2.277	274.2994	2.272	276.6331
2.278	274.5464	2.273	276.8448
2.28	274.7732	2.275	277.0363
2.281	274.9698	2.276	277.2329
2.283	275.1764	2.278	277.4244
2.284	275.3478	2.281	277.8226
2.286	275.5091	2.282	277.999
2.288	275.7107	2.283	278.2208
2.29	275.9375	2.286	278.6895
2.291	276.1593	2.288	278.8962
2.294	276.4163	2.289	279.1079
2.296	276.628	2.291	279.3145
2.297	277.0413	2.292	279.506
2.298	277.2429	2.294	279.6774
2.3	277.4244	2.295	279.9042
2.302	277.6311	2.298	280.3427
2.303	277.8427	2.299	280.5494
2.304	278.0494	2.302	280.9778
2.306	278.2762	2.304	281.1895
2.307	278.4677	2.305	281.4113
2.309	278.7097	2.307	281.623
2.31	278.9163	2.308	281.8246
2.311	279.0827	2.31	282.0363
2.313	279.5111	2.312	282.4647
2.314	279.7026	2.314	282.6915
2.316	279.9294	2.316	282.9184

2.317	280.1512	2.317	283.13
2.319	280.3579	2.32	283.5433
2.32	280.5847	2.321	283.745
2.321	280.8115	2.323	283.9415
2.323	281.2147	2.326	284.3347
2.325	281.4315	2.327	284.5565
2.326	281.6381	2.328	284.8085
2.327	281.8145	2.33	285.0554
2.329	282.0313	2.332	285.252
2.331	282.2178	2.334	285.6452
2.332	282.4194	2.336	285.8468
2.333	282.6411	2.337	286.0534
2.335	282.8629	2.339	286.2802
2.336	283.1048	2.341	286.6935
2.338	283.3316	2.343	286.9254
2.339	283.5181	2.344	287.1573
2.341	283.7298	2.346	287.3538
2.342	283.8911	2.347	287.5252
2.343	284.3599	2.349	287.752
2.345	284.5968	2.35	287.9839
2.346	284.7984	2.351	288.1552
2.348	285.2117	2.353	288.3569
2.351	285.4184	2.355	288.5988
2.352	285.63	2.356	288.7954
2.354	285.8417	2.359	289.2339
2.355	286.2903	2.361	289.4456
2.356	286.497	2.362	289.6522
2.359	286.9153	2.363	289.8841
2.361	287.127	2.365	290.1109
2.362	287.3236	2.366	290.2923
2.364	287.5807	2.368	290.4788
2.366	287.7772	2.369	290.7107
2.368	287.9788	2.371	290.9073
2.37	288.1804	2.372	291.1341
2.372	288.377	2.373	291.3306
2.374	288.5837	2.376	291.8044
2.375	288.7954	2.378	292.006
2.377	289.0071	2.379	292.2278
2.378	289.2238	2.381	292.4294
2.38	289.4758	2.382	292.621
2.381	289.6774	2.384	292.8427
2.384	289.8841	2.385	293.0847
2.386	290.1059	2.386	293.3065
2.387	290.3175	2.388	293.5131
2.39	290.4889	2.389	293.7298
2.391	290.9173	2.391	293.9466
2.393	291.1643	2.392	294.1431
2.394	291.3861	2.394	294.3548
2.396	291.628	2.395	294.5867
2.397	291.8548	2.398	294.9748
2.399	292.0464	2.4	295.2016
2.4	292.2329	2.401	295.4284
2.401	292.4395	2.402	295.6552
2.403	292.6613	2.404	295.8871
2.406	292.8629	2.406	296.1139
2.407	293.0796	2.407	296.3256
2.409	293.2813	2.408	296.5222
2.41	293.503	2.41	296.7339
2.411	293.7349	2.411	296.9304
2.413	293.9567	2.412	297.0968
2.414	294.1835	2.416	297.5454
2.416	294.6018	2.417	297.7823
2.417	294.8236	2.418	298.004
2.419	295.0202	2.42	298.2359
2.42	295.2268	2.421	298.4425
2.422	295.4184	2.423	298.6694
2.423	295.6048	2.424	298.8861

2.425	295.8669	2.426	299.0877
2.426	296.1089	2.427	299.2742
2.427	296.3256	2.429	299.4909
2.429	296.5272	2.43	299.7026
2.432	296.9304	2.431	299.9294
2.433	297.1069	2.433	300.1663
2.434	297.3538	2.434	300.4183
2.436	297.5907	2.437	300.756
2.438	297.8226	2.439	300.9577
2.439	298.0444	2.44	301.1542
2.44	298.2611	2.441	301.3861
2.442	298.4577	2.443	301.6129
2.443	298.624	2.445	301.8347
2.445	298.8407	2.446	302.0716
2.446	299.0726	2.449	302.5252
2.448	299.5111	2.451	302.7117
2.449	299.7278	2.453	303.0948
2.451	299.9647	2.455	303.3115
2.454	300.3982	2.456	303.503
2.455	300.6603	2.457	303.7097
2.456	300.877	2.459	303.9315
2.458	301.0887	2.46	304.1381
2.459	301.2702	2.463	304.5514
2.461	301.4617	2.465	304.8085
2.462	301.6684	2.466	305.0151
2.464	302.0816	2.468	305.2016
2.465	302.2933	2.47	305.6149
2.467	302.5202	2.472	305.8165
2.468	302.747	2.474	306.0534
2.47	302.9536	2.475	306.2651
2.471	303.1754	2.476	306.502
2.472	303.4022	2.478	306.6986
2.474	303.5736	2.479	306.9204
2.477	303.7802	2.481	307.122
2.478	304.0071	2.482	307.3387
2.48	304.2339	2.484	307.5605
2.481	304.4506	2.485	307.7671
2.483	304.6522	2.486	307.9839
2.484	304.869	2.488	308.2006
2.485	305.2772	2.49	308.4173
2.488	305.4738	2.491	308.6391
2.49	305.6905	2.492	308.8559
2.491	305.8972	2.494	309.0726
2.493	306.1089	2.495	309.3145
2.494	306.3559	2.497	309.501
2.496	306.5776	2.498	309.7177
2.497	306.7893	2.5	309.9345
2.499	307.0161	2.501	310.1361
2.5	307.4597	2.502	310.3579
2.501	307.873	2.504	310.5645
2.503	308.0696	2.505	310.746
2.504	308.2661	2.507	310.9829
2.506	308.4677	2.508	311.2399
2.507	308.6794	2.51	311.4415
2.509	308.9063	2.511	311.6734
2.51	309.1331	2.513	311.8901
2.512	309.38	2.514	312.1018
2.513	309.5766	2.515	312.2883
2.514	309.8085	2.517	312.4698
2.517	310.0252	2.518	312.6764
2.519	310.1815	2.52	312.9133
2.522	310.3982	2.521	313.1351
2.524	310.625	2.523	313.3972
2.526	311.0635	2.524	313.6139
2.527	311.2752	2.526	313.8155
2.529	311.507	2.527	314.0222
2.53	311.7036	2.529	314.249

2.533	311.9153	2.53	314.4556
2.535	312.1522	2.531	314.6522
2.536	312.3589	2.533	314.8891
2.539	312.5807	2.534	315.0958
2.541	312.8024	2.536	315.3226
2.542	312.999	2.537	315.5242
2.543	313.246	2.539	315.7309
2.545	313.4577	2.542	316.2046
2.546	313.6845	2.543	316.4214
2.549	313.9466	2.545	316.623
2.551	314.1432	2.546	316.8397
2.552	314.3548	2.547	317.0413
2.553	314.5564	2.549	317.2329
2.555	314.748	2.55	317.4395
2.557	314.9647	2.553	317.9234
2.558	315.1815	2.554	318.1351
2.559	315.3831	2.556	318.3417
2.561	315.8518	2.559	318.755
2.562	316.0988	2.56	318.9567
2.564	316.3004	2.562	319.1885
2.565	316.4869	2.563	319.4304
2.567	316.8952	2.565	319.6421
2.568	317.1169	2.568	320.121
2.569	317.3438	2.569	320.3276
2.571	317.5756	2.571	320.5141
2.572	317.8226	2.572	320.7208
2.574	318.0444	2.575	321.1694
2.575	318.2863	2.576	321.4063
2.577	318.4677	2.578	321.6129
2.578	318.6643	2.579	321.8498
2.58	318.871	2.581	322.0917
2.581	319.2994	2.582	322.3135
2.582	319.5464	2.584	322.5353
2.584	319.7732	2.585	322.752
2.586	319.9849	2.587	322.9587
2.588	320.1915	2.588	323.1452
2.59	320.3982	2.589	323.3417
2.591	320.6401	2.592	323.7802
2.593	320.8619	2.594	324.002
2.594	321.0837	2.595	324.2036
2.596	321.2903	2.598	324.6724
2.597	321.6986	2.599	324.8639
2.598	321.9254	2.601	325.0857
2.6	322.1472	2.603	325.3075
2.602	322.3538	2.604	325.5191
2.603	322.5857	2.605	325.7157
2.606	323.0494	2.607	325.9426
2.607	323.4526	2.61	326.381
2.609	323.6442	2.611	326.5978
2.61	324.1028	2.613	326.8145
2.612	324.3246	2.614	327.0312
2.613	324.1835	2.616	327.248
2.614	321.9355	2.617	327.4546
2.616	322.3689	2.618	327.6764
2.617	322.8226	2.62	327.8831
2.619	323.3064	2.621	328.1099
2.62	323.8004	2.623	328.3367
2.622	324.2137	2.624	328.5534
2.623	324.622	2.626	328.7702
2.625	324.9899	2.627	328.9919
2.627	325.7157	2.629	329.2288
2.629	326.0434	2.63	329.4456
2.63	326.3659	2.632	329.6573
2.632	326.6179	2.633	329.874
2.633	326.6129	2.634	330.0706
2.635	325.2772	2.636	330.2621
2.636	325.0101	2.637	330.4688

2.637	325.4234	2.639	330.6905
2.639	325.8417	2.64	330.9123
2.641	326.3054	2.642	331.1593
2.642	326.6936	2.643	331.4063
2.645	327.0968	2.644	331.6129
2.647	327.5353	2.646	331.8296
2.648	327.878	2.647	332.0262
2.649	328.1754	2.649	332.2026
2.651	328.7349	2.65	332.4345
2.652	328.9516	2.653	332.8579
2.654	329.1684	2.655	333.0696
2.655	329.4052	2.656	333.3014
2.657	329.6371	2.657	333.5232
2.658	329.874	2.659	333.7349
2.659	330.131	2.66	333.9617
2.661	330.3579	2.662	334.1935
2.662	330.5343	2.663	334.4304
2.664	330.7409	2.665	334.6321
2.665	331.2046	2.666	334.8437
2.668	331.4365	2.668	335.0554
2.67	331.6784	2.671	335.494
2.671	331.9002	2.672	335.7409
2.672	332.127	2.674	335.998
2.674	332.369	2.675	336.1895
2.675	332.621	2.677	336.3962
2.678	332.8276	2.678	336.5978
2.68	333.0292	2.679	336.8095
2.681	333.2308	2.681	337.0313
2.684	333.6341	2.682	337.2429
2.686	333.8307	2.685	337.6815
2.687	334.0373	2.687	337.9435
2.688	334.0423	2.688	338.1704
2.69	164.375	2.689	338.3619
2.692	141.124	2.692	338.7752
2.693	141.3357	2.694	338.9718
2.694	141.4516	2.695	339.1835
2.696	141.5423	2.697	339.4204
2.697	141.7036	2.698	339.6572
2.699	144.1936	2.7	339.8841
2.7	145.0605	2.701	340.1109
2.702	145.9022	2.703	340.3478
2.703	146.7641	2.704	340.5393
2.704	148.3014	2.705	340.751
2.707	148.8659	2.707	340.9728
2.709	148.7853	2.71	341.4012
2.71	148.4778	2.711	341.6784
2.711	147.7823	2.713	341.9002
2.713	146.4214	2.714	342.1119
2.715	144.9597	2.716	342.3236
2.716	143.4375	2.717	342.5655
2.717	141.7692	2.719	342.7722
2.719	140.6351	2.722	343.2057
2.72	139.6421	2.723	343.4123
2.723	138.4929	2.724	343.6139
2.725	137.2429	2.726	343.8256
2.726	135.6351	2.727	344.0373
2.727	133.7651	2.729	344.2742
2.729	133.0595	2.73	344.4758
2.731	132.1472	2.733	344.9496
2.732	129.7934	2.734	345.1764
2.733	128.1653	2.736	345.3932
2.735	126.0081	2.737	345.5897
2.737	123.4274	2.739	345.7863
2.739	116.9909	2.74	346.0383
2.741	113.12	2.742	346.2702
2.742	106.0736	2.743	346.5071
2.744	99.254	2.745	346.7389

2.745	74.8992	2.746	346.9304
2.747	2.3286	2.749	347.3236
2.748	2.4042	2.75	347.5504
2.749	2.2732	2.752	347.7873
2.751	2.3488	2.753	347.9839
2.753	2.3034	2.755	348.1905
2.754	2.3438	2.756	348.3821
2.757	2.3488	2.758	348.6391
2.758	2.3286	2.759	348.8861
2.76	2.2984	2.761	349.128
2.761	2.2782	2.762	349.3649
2.764	2.2782	2.764	349.5867
2.765	2.253	2.765	349.7833
2.767	2.2127	2.767	349.9899
2.768	2.1774	2.768	350.2016
2.77	2.1623	2.769	350.4032
2.771	2.127	2.771	350.6048
2.773	2.1321	2.772	350.8216
2.774	2.0716	2.774	351.0534
2.776	2.0514	2.775	351.3105
2.777	2.0464	2.777	351.5675
2.779	2.006	2.778	351.744
2.782	1.996	2.779	351.9859
2.783	1.9708	2.781	352.2228
2.784	1.9304	2.783	352.4042
2.786	1.9103	2.784	352.6059
2.787	1.8952	2.785	352.8125
2.789	1.87	2.787	353.0343
2.79	1.8196	2.79	353.4778
2.792	1.7994	2.791	353.7399
2.793	1.7692	2.793	353.9718
2.796	1.7389	2.794	354.1633
2.798	1.7187	2.795	354.3548
2.799	1.6835	2.797	354.5665
2.8	1.6683	2.798	354.7934
2.803	1.6532	2.8	355.0101
2.805	1.623	2.801	355.2419
2.806	1.5927	2.803	355.4788
2.808	1.5524	2.804	355.7309
2.809	1.5323	2.806	355.9829
2.81	1.497	2.807	356.1895
2.812	1.4768	2.809	356.3911
2.813	1.4365	2.81	356.5827
2.815	1.3155	2.813	357.0212
2.816	1.3508	2.814	357.2228
2.818	1.2903	2.816	357.4143
2.821	1.3105	2.817	357.6361
2.822	1.3458	2.819	357.8881
2.823	1.3357	2.82	358.1149
2.825	1.3206	2.822	358.3468
2.827	1.3054	2.823	358.5736
2.828	1.3054	2.824	358.7853
2.829	1.2853	2.826	359.002
2.831	1.2853	2.829	359.4052
2.832	1.2651	2.83	359.622
2.834	1.2601	2.834	204.2036
2.835	1.255	2.836	142.0665
2.837	1.2349	2.836	142.2429
2.838	1.2198	2.836	142.379
2.839	1.2198	2.837	142.5504
2.841	1.2046	2.839	143.4073
2.843	1.1794	2.842	144.7984
2.844	1.1744	2.843	145.7561
2.845	1.1542	2.846	147.4093
2.847	1.1341	2.848	148.3014
2.848	1.1391	2.849	149.0927
2.85	0.8165	2.851	149.9244

2.851	0.7813	2.852	150.7863
2.854	0.7712	2.853	151.6028
2.855	0.6804	2.855	152.4244
2.857	0.6855	2.857	153.2258
2.86	0.7006	2.858	154.0071
2.861	0.6855	2.861	155.4587
2.863	0.6855	2.862	155.9073
2.864	0.6905	2.863	156.0181
2.866	0.6956	2.865	156.0282
2.867	0.6754	2.868	156.0685
2.868	0.6754	2.869	155.9274
2.87	0.6855	2.871	155.6552
2.872	0.6905	2.872	154.9546
2.873	0.6754	2.874	154.1431
2.874	0.6855	2.875	153.2057
2.876	0.6603	2.877	152.0464
2.877	0.6754	2.878	150.9325
2.878	0.6804	2.88	149.6976
2.88	0.6653	2.881	148.3518
2.882	0.6704	2.884	145.6603
2.883	0.6552	2.885	143.9819
2.884	0.6552	2.887	141.9405
2.886	0.6653	2.888	139.8639
2.887	0.6552	2.89	137.9234
2.889	0.6653	2.893	134.2288
2.89	0.6401	2.894	131.9052
2.893	0.6401	2.896	129.3145
2.895	0.5746	2.897	126.0232
2.896	0.5847	2.902	-0.2419
2.899	0.5796	2.902	-0.1815
2.9	0.5796	2.903	-0.1411
2.902	0.5696	2.904	0
2.903	0.5645	2.906	0.1411
2.905	0.5494	2.907	0.2067
2.906	0.5696	2.908	0.2319
2.907	0.5645	2.91	0.2319
2.911	0.5645	2.911	0.2268
2.912	0.5645	2.913	0.2722
2.915	0.5696	2.914	0.2319
2.917	0.5444	2.917	0.2772
2.918	0.5393	2.919	0.2621
2.919	0.5292	2.92	0.252
2.921	0.5343	2.921	0.2571
2.922	0.5091	2.923	0.2571
2.923	0.5091	2.925	0.2571
2.925	0.5091	2.926	0.2419
2.927	0.499	2.927	0.2419
2.928	0.494	2.929	0.2571
2.929	0.5091	2.93	0.2369
2.931	0.499	2.932	0.2419
2.932	0.504	2.933	0.2369
2.934	0.494	2.935	0.2369
2.935	0.499	2.936	0.2319
2.937	0.4889	2.937	0.2419
2.938	0.494	2.939	0.2167
2.94	0.5091	2.941	0.2117
2.941	0.494	2.942	0.2167
2.942	0.499	2.943	0.2218
2.944	0.494	2.945	0.2268
2.945	0.499	2.946	0.2117
2.947	0.4839	2.948	0.2167
2.95	0.494	2.951	0.2067
2.951	0.494	2.952	0.2067
2.952	0.4889	2.953	0.2067
2.954	0.494	2.956	0.1966
2.956	0.4839	2.958	0.1915
2.957	0.4839	2.959	0.2067

2.958	0.4889	2.961	0.2067
2.96	0.494	2.962	0.2117
2.962	0.494	2.964	0.2016
2.963	0.494	2.965	0.2167
2.964	0.4889	2.966	0.2117
2.966	0.4738	2.968	0.2117
2.967	0.4788	2.97	0.2067

Tensile test results of samples after heat treatment

After heat treatment			
1.360.2		1.360.3	
Elongation	Stress (N/mm ²)	Elongation	Stress (N/mm ²)
0	2.8427	0	9.732
0	2.8327	0	9.712
0	2.8528	0	9.712
0	2.9083	0.001	9.808
0.002	3.0343	0.004	10.967
0.004	3.5282	0.005	11.376
0.008	4.7883	0.006	11.592
0.009	5.1915	0.01	12.187
0.011	5.5746	0.011	12.318
0.012	5.9224	0.012	12.439
0.014	6.2248	0.014	12.464
0.015	6.5121	0.015	12.479
0.017	6.749	0.017	12.454
0.018	6.9456	0.02	12.484
0.02	7.127	0.021	12.515
0.021	7.3034	0.023	12.555
0.023	7.4446	0.024	12.605
0.024	7.6159	0.025	12.671
0.025	7.7621	0.027	12.696
0.027	7.8982	0.028	12.711
0.028	8.004	0.03	12.747
0.03	8.1452	0.031	12.787
0.031	8.2813	0.033	12.883
0.033	8.4073	0.035	13.215
0.034	8.5131	0.037	13.356
0.035	8.6593	0.039	13.487
0.037	8.7651	0.041	13.704
0.04	9.0272	0.043	13.825
0.041	9.1381	0.044	13.961
0.043	9.2641	0.046	14.052
0.044	9.38	0.047	14.158
0.046	9.4909	0.049	14.264
0.047	9.6018	0.05	14.364
0.048	9.7177	0.053	14.556
0.051	9.9294	0.054	14.647
0.053	10.0353	0.057	14.858
0.056	10.2419	0.059	14.959
0.057	10.3629	0.06	15.065
0.059	10.4587	0.062	15.166
0.06	10.5696	0.063	15.277
0.062	10.6603	0.064	15.347
0.063	10.751	0.066	15.438
0.066	10.9325	0.068	15.539
0.067	11.0333	0.069	15.64
0.069	11.119	0.07	15.746
0.072	11.2903	0.072	15.841
0.073	11.371	0.074	16.078
0.074	11.4718	0.076	16.164
0.076	11.5423	0.078	16.28
0.077	11.628	0.079	16.396
0.079	11.6986	0.08	16.497
0.08	11.7792	0.082	16.623
0.082	11.8448	0.083	16.718

0.083	11.9456	0.085	16.839
0.085	12.001	0.086	16.95
0.087	12.1421	0.089	17.187
0.089	12.2026	0.091	17.288
0.09	12.2883	0.092	17.414
0.092	12.3337	0.093	17.535
0.093	12.4042	0.095	17.636
0.095	12.4698	0.096	17.757
0.096	12.5302	0.098	17.857
0.098	12.5907	0.099	17.963
0.099	12.6764	0.101	18.079
0.101	12.7419	0.102	18.165
0.102	12.8024	0.103	18.276
0.105	12.9335	0.105	18.361
0.106	13.0091	0.107	18.462
0.108	13.0544	0.108	18.563
0.109	13.12	0.109	18.654
0.111	13.1804	0.111	18.734
0.112	13.2661	0.112	18.84
0.114	13.3115	0.114	18.931
0.115	13.3821	0.115	19.007
0.117	13.4425	0.117	19.082
0.119	13.5585	0.118	19.163
0.121	13.624	0.119	19.213
0.122	13.6845	0.121	19.279
0.124	13.7248	0.122	19.375
0.125	13.7903	0.124	19.465
0.126	13.8407	0.125	19.551
0.128	13.9163	0.127	19.682
0.13	13.9617	0.13	19.914
0.131	14.0272	0.131	20.035
0.132	14.0726	0.132	20.156
0.134	14.128	0.134	20.287
0.135	14.1784	0.136	20.393
0.137	14.2339	0.137	20.524
0.138	14.2843	0.14	20.771
0.14	14.3397	0.141	20.902
0.141	14.3851	0.143	21.013
0.143	14.4355	0.146	21.26
0.144	14.4708	0.147	21.376
0.145	14.5312	0.148	21.502
0.147	14.5766	0.15	21.628
0.148	14.622	0.151	21.749
0.15	14.6673	0.154	21.975
0.153	14.748	0.156	22.081
0.154	14.8135	0.157	22.217
0.155	14.8438	0.159	22.333
0.157	14.879	0.161	22.57
0.159	14.9093	0.163	22.701
0.16	14.9546	0.164	22.807
0.161	14.995	0.166	22.913
0.163	15.0504	0.167	23.044
0.164	15.1109	0.169	23.155
0.166	15.1462	0.17	23.266
0.167	15.1714	0.171	23.382
0.17	15.2369	0.173	23.538
0.171	15.2823	0.175	23.659
0.173	15.3478	0.176	23.77
0.174	15.3982	0.177	23.906
0.176	15.4234	0.179	24.042
0.177	15.4637	0.18	24.143
0.178	15.5141	0.182	24.254
0.18	15.5645	0.183	24.364
0.182	15.5897	0.185	24.485
0.183	15.625	0.186	24.606
0.184	15.6905	0.188	24.737
0.186	15.7208	0.189	24.833

0.187	15.7661	0.19	24.944
0.189	15.8165	0.192	25.07
0.19	15.8417	0.193	25.211
0.192	15.8871	0.195	25.327
0.193	15.9425	0.196	25.458
0.194	15.9728	0.198	25.564
0.196	16.0081	0.2	25.796
0.197	16.0635	0.202	25.942
0.199	16.1038	0.204	26.078
0.2	16.1341	0.205	26.174
0.202	16.1845	0.206	26.315
0.203	16.25	0.208	26.441
0.205	16.2853	0.209	26.562
0.206	16.3206	0.211	26.693
0.207	16.3861	0.212	26.849
0.209	16.4063	0.214	26.965
0.21	16.4365	0.215	27.081
0.212	16.4869	0.216	27.217
0.213	16.5423	0.218	27.358
0.215	16.5675	0.219	27.484
0.216	16.623	0.221	27.615
0.218	16.6784	0.222	27.777
0.219	16.7288	0.224	27.888
0.221	16.7692	0.227	28.17
0.222	16.7994	0.228	28.296
0.223	16.8397	0.229	28.422
0.225	16.8901	0.232	28.709
0.226	16.9355	0.234	28.86
0.228	16.9808	0.235	28.997
0.229	17.0212	0.237	29.133
0.23	17.0565	0.238	29.259
0.232	17.122	0.239	29.395
0.234	17.1573	0.241	29.556
0.235	17.2077	0.243	29.707
0.236	17.253	0.244	29.853
0.239	17.3488	0.245	30.005
0.241	17.3942	0.247	30.141
0.242	17.4294	0.249	30.277
0.244	17.5	0.25	30.438
0.245	17.5302	0.251	30.574
0.246	17.5655	0.253	30.73
0.249	17.6613	0.254	30.856
0.251	17.6865	0.256	31.008
0.252	17.7419	0.257	31.179
0.254	17.7873	0.259	31.325
0.255	17.8327	0.26	31.461
0.257	17.8881	0.261	31.607
0.258	17.9183	0.263	31.759
0.259	17.9435	0.266	32.066
0.261	17.994	0.267	32.197
0.262	18.0343	0.269	32.374
0.264	18.0998	0.27	32.51
0.265	18.1653	0.273	32.807
0.267	18.1905	0.274	32.963
0.268	18.2409	0.276	33.114
0.269	18.2964	0.277	33.251
0.271	18.3417	0.279	33.402
0.273	18.3871	0.28	33.558
0.274	18.4577	0.283	33.876
0.275	18.503	0.284	34.032
0.277	18.5484	0.286	34.173
0.28	18.6694	0.289	34.49
0.281	18.7349	0.29	34.642
0.283	18.7702	0.292	34.828
0.284	18.8357	0.293	34.964
0.287	18.9315	0.295	35.136
0.288	18.9667	0.296	35.312

0.29	19.0272	0.298	35.448
0.291	19.0927	0.299	35.594
0.293	19.1482	0.301	35.761
0.294	19.2087	0.302	35.917
0.296	19.2692	0.305	36.239
0.297	19.3246	0.306	36.391
0.298	19.3851	0.308	36.567
0.3	19.4456	0.309	36.708
0.303	19.5665	0.311	36.87
0.304	19.622	0.312	37.051
0.306	19.6673	0.314	37.197
0.309	19.7933	0.315	37.333
0.31	19.8639	0.317	37.505
0.312	19.9143	0.318	37.636
0.313	19.9798	0.319	37.797
0.314	20.0454	0.322	38.114
0.316	20.1008	0.324	38.291
0.319	20.2419	0.325	38.482
0.32	20.2823	0.327	38.634
0.322	20.3579	0.328	38.77
0.323	20.4284	0.329	38.936
0.325	20.494	0.331	39.117
0.326	20.5595	0.332	39.274
0.327	20.625	0.334	39.415
0.329	20.6855	0.335	39.591
0.331	20.7661	0.337	39.768
0.332	20.8266	0.338	39.934
0.335	20.9577	0.34	40.095
0.336	21.0383	0.341	40.262
0.337	21.129	0.344	40.574
0.339	21.1895	0.346	40.735
0.34	21.2399	0.347	40.902
0.342	21.3155	0.348	41.068
0.343	21.4012	0.35	41.214
0.345	21.4617	0.351	41.386
0.346	21.5323	0.353	41.562
0.348	21.6028	0.354	41.708
0.35	21.744	0.356	41.87
0.352	21.8196	0.357	42.051
0.354	21.88	0.358	42.212
0.355	21.9657	0.362	42.56
0.356	22.0313	0.363	42.716
0.358	22.0968	0.364	42.873
0.359	22.1925	0.366	43.029
0.361	22.2782	0.367	43.195
0.362	22.3538	0.369	43.351
0.364	22.4395	0.37	43.533
0.365	22.5101	0.372	43.694
0.366	22.6008	0.373	43.886
0.368	22.6663	0.374	44.062
0.37	22.7319	0.377	44.359
0.371	22.8125	0.379	44.531
0.372	22.8881	0.38	44.687
0.374	22.9738	0.382	44.848
0.375	23.0494	0.383	45.04
0.377	23.125	0.385	45.196
0.378	23.2056	0.386	45.352
0.38	23.2863	0.387	45.524
0.381	23.3518	0.389	45.665
0.382	23.4224	0.391	45.841
0.384	23.5232	0.392	46.008
0.385	23.629	0.395	46.325
0.387	23.6946	0.396	46.486
0.388	23.7853	0.397	46.653
0.391	23.9466	0.401	46.975
0.393	24.0323	0.402	47.132
0.394	24.1079	0.403	47.313

0.395	24.1935	0.405	47.454
0.397	24.2843	0.406	47.585
0.399	24.3599	0.408	47.736
0.401	24.5363	0.409	47.903
0.403	24.6119	0.411	48.029
0.404	24.7026	0.412	48.18
0.407	24.874	0.414	48.341
0.408	24.9798	0.415	48.503
0.41	25.0605	0.417	48.664
0.411	25.1411	0.418	48.825
0.413	25.247	0.419	48.956
0.414	25.3276	0.421	49.112
0.416	25.4284	0.422	49.254
0.417	25.5393	0.424	49.42
0.419	25.6099	0.425	49.576
0.42	25.6804	0.427	49.727
0.423	25.8619	0.428	49.879
0.424	25.9577	0.43	50.035
0.426	26.0635	0.431	50.206
0.427	26.1341	0.434	50.463
0.429	26.2147	0.435	50.655
0.43	26.3306	0.437	50.801
0.432	26.3962	0.438	50.937
0.433	26.5071	0.44	51.108
0.434	26.6028	0.441	51.255
0.436	26.6885	0.442	51.421
0.439	26.87	0.444	51.552
0.44	26.9657	0.446	51.713
0.442	27.0766	0.448	52.036
0.443	27.1623	0.45	52.207
0.445	27.253	0.45	42.666
0.446	27.3337	0.45	42.676
0.447	27.4294	0.453	43.266
0.449	27.5353	0.455	44.874
0.45	27.621	0.456	45.559
0.452	27.7218	0.458	46.098
0.453	27.8377	0.46	46.769
0.456	28.0091	0.461	47.384
0.458	28.1099	0.464	48.624
0.459	28.2056	0.465	49.198
0.461	28.2913	0.467	49.742
0.462	28.3921	0.468	50.216
0.463	28.503	0.469	50.71
0.465	28.6089	0.471	51.169
0.467	28.6845	0.472	51.567
0.468	28.8004	0.474	51.935
0.469	28.8911	0.475	52.248
0.472	29.0978	0.477	52.55
0.474	29.1986	0.478	52.802
0.475	29.2994	0.48	53.034
0.477	29.4103	0.481	53.266
0.478	29.5111	0.483	53.477
0.479	29.6018	0.484	53.689
0.481	29.7127	0.485	53.911
0.482	29.8135	0.487	54.117
0.484	29.8992	0.489	54.349
0.485	30.0202	0.49	54.551
0.488	30.2218	0.491	54.773
0.49	30.3276	0.493	54.995
0.491	30.4385	0.495	55.408
0.492	30.5242	0.497	55.625
0.494	30.625	0.499	55.831
0.495	30.7409	0.5	56.013
0.497	30.8468	0.501	56.239
0.498	30.9425	0.503	56.451
0.5	31.0333	0.504	56.668
0.501	31.1341	0.506	56.885

0.502	31.245	0.507	57.081
0.506	31.4315	0.508	57.278
0.507	31.5575	0.51	57.489
0.508	31.6734	0.512	57.666
0.511	31.87	0.513	57.852
0.513	31.9657	0.514	58.059
0.514	32.0716	0.516	58.271
0.516	32.1875	0.517	58.462
0.517	32.2681	0.519	58.649
0.519	32.3639	0.52	58.881
0.52	32.4798	0.522	59.092
0.521	32.5907	0.523	59.269
0.523	32.7016	0.524	59.46
0.524	32.8075	0.527	59.869
0.527	33.0192	0.529	60.08
0.529	33.13	0.53	60.292
0.53	33.246	0.532	60.483
0.531	33.3619	0.533	60.7
0.533	33.4526	0.535	60.917
0.534	33.5736	0.536	61.119
0.536	33.6794	0.537	61.315
0.537	33.7903	0.539	61.522
0.539	33.8962	0.54	61.723
0.54	34.0171	0.543	62.127
0.542	34.128	0.545	62.323
0.543	34.244	0.546	62.52
0.544	34.3649	0.547	62.736
0.546	34.4657	0.55	63.165
0.547	34.5917	0.552	63.366
0.549	34.7127	0.553	63.578
0.55	34.8286	0.555	63.78
0.552	34.9294	0.556	63.997
0.553	35.0605	0.558	64.193
0.555	35.1563	0.559	64.42
0.556	35.2772	0.56	64.642
0.558	35.373	0.562	64.833
0.559	35.499	0.563	65.05
0.56	35.62	0.565	65.257
0.562	35.7258	0.566	65.468
0.564	35.8468	0.568	65.66
0.566	36.0837	0.569	65.866
0.568	36.1794	0.57	66.093
0.569	36.3256	0.572	66.295
0.57	36.4466	0.575	66.718
0.572	36.5423	0.576	66.935
0.574	36.6683	0.578	67.142
0.575	36.7944	0.579	67.338
0.576	36.9153	0.581	67.565
0.578	37.0363	0.583	67.978
0.579	37.1573	0.585	68.155
0.581	37.2732	0.587	68.372
0.582	37.374	0.588	68.568
0.584	37.4798	0.591	68.956
0.585	37.621	0.592	69.163
0.587	37.7419	0.594	69.39
0.588	37.8629	0.595	69.586
0.589	37.999	0.597	69.778
0.591	38.12	0.598	70.005
0.592	38.246	0.601	70.413
0.594	38.3669	0.602	70.589
0.595	38.503	0.604	70.796
0.598	38.7601	0.606	71.214
0.6	38.9113	0.608	71.426
0.601	39.0474	0.61	71.653
0.603	39.1532	0.611	71.859
0.604	39.2792	0.612	72.041
0.605	39.4204	0.614	72.242

0.607	39.5312	0.615	72.454
0.608	39.6522	0.617	72.671
0.61	39.7883	0.618	72.857
0.611	39.9294	0.62	73.059
0.613	40.0605	0.622	73.513
0.614	40.2067	0.624	73.699
0.615	40.3276	0.625	73.911
0.617	40.4385	0.627	74.117
0.618	40.5645	0.628	74.329
0.621	40.8317	0.63	74.546
0.623	40.9627	0.631	74.753
0.624	41.0786	0.633	74.929
0.626	41.2147	0.634	75.115
0.627	41.3357	0.635	75.322
0.628	41.4617	0.637	75.529
0.632	41.7389	0.639	75.751
0.633	41.8599	0.64	75.972
0.634	41.996	0.641	76.169
0.637	42.2681	0.643	76.376
0.639	42.3992	0.644	76.577
0.64	42.5403	0.645	76.769
0.642	42.6966	0.647	76.96
0.643	42.7873	0.648	77.182
0.646	43.0444	0.65	77.394
0.647	43.1754	0.651	77.6
0.649	43.2964	0.653	77.822
0.65	43.4173	0.654	78.024
0.653	43.6744	0.656	78.21
0.655	43.8004	0.657	78.397
0.656	43.9567	0.658	78.598
0.658	44.0625	0.66	78.81
0.659	44.2087	0.663	79.228
0.66	44.3296	0.664	79.445
0.662	44.4607	0.666	79.667
0.663	44.5867	0.667	79.894
0.665	44.7278	0.669	80.075
0.666	44.8438	0.67	80.297
0.668	45	0.672	80.504
0.671	45.252	0.673	80.705
0.672	45.373	0.674	80.907
0.673	45.5192	0.676	81.088
0.675	45.6603	0.678	81.315
0.676	45.7812	0.679	81.522
0.678	45.9022	0.68	81.713
0.679	46.0433	0.682	81.93
0.681	46.1996	0.683	82.152
0.682	46.3105	0.685	82.348
0.684	46.4365	0.686	82.55
0.687	46.7137	0.688	82.777
0.688	46.8397	0.689	82.994
0.689	46.9859	0.69	83.185
0.691	47.1169	0.692	83.382
0.692	47.2429	0.693	83.603
0.694	47.3841	0.695	83.825
0.695	47.505	0.696	84.012
0.697	47.6361	0.698	84.244
0.698	47.7823	0.699	84.45
0.7	47.9032	0.702	84.848
0.701	48.0696	0.703	85.06
0.702	48.2006	0.705	85.292
0.704	48.3417	0.706	85.504
0.705	48.4677	0.708	85.7
0.707	48.6089	0.709	85.917
0.71	48.8911	0.711	86.134
0.711	49.0171	0.713	86.562
0.712	49.1683	0.715	86.784
0.714	49.2893	0.718	87.187

0.716	49.4456	0.719	87.394
0.717	49.5716	0.721	87.59
0.718	49.7127	0.722	87.777
0.721	49.9849	0.724	87.999
0.723	50.131	0.725	88.215
0.726	50.4335	0.726	88.432
0.727	50.5444	0.728	88.674
0.728	50.7107	0.73	88.871
0.73	50.8417	0.731	89.067
0.732	50.9879	0.734	89.49
0.733	51.1391	0.735	89.692
0.736	51.4012	0.737	89.879
0.737	51.5222	0.738	90.1
0.739	51.6784	0.739	90.332
0.741	51.9758	0.741	90.514
0.743	52.1069	0.742	90.735
0.745	52.2631	0.744	90.937
0.746	52.4143	0.745	91.144
0.747	52.5756	0.747	91.335
0.749	52.6966	0.75	91.759
0.75	52.8427	0.751	91.98
0.752	52.9637	0.753	92.187
0.753	53.0998	0.754	92.399
0.755	53.2611	0.755	92.615
0.756	53.4022	0.757	92.827
0.759	53.7046	0.758	93.059
0.761	53.8458	0.76	93.281
0.762	53.9819	0.761	93.492
0.763	54.1129	0.763	93.714
0.765	54.2893	0.765	94.097
0.766	54.4204	0.767	94.319
0.768	54.5817	0.769	94.526
0.769	54.7228	0.77	94.727
0.771	54.874	0.771	94.934
0.772	55.0202	0.773	95.156
0.775	55.3226	0.774	95.388
0.776	55.4536	0.776	95.579
0.778	55.6048	0.777	95.796
0.779	55.756	0.779	96.018
0.782	56.0433	0.782	96.421
0.784	56.1845	0.783	96.638
0.785	56.3306	0.784	96.849
0.786	56.4718	0.786	97.076
0.789	56.8145	0.787	97.298
0.791	56.9405	0.789	97.52
0.792	57.1069	0.79	97.711
0.794	57.2379	0.792	97.913
0.795	57.3992	0.793	98.13
0.797	57.5252	0.794	98.351
0.798	57.6815	0.796	98.558
0.8	57.8427	0.799	99.007
0.801	58.004	0.8	99.218
0.802	58.1452	0.802	99.42
0.804	58.3014	0.803	99.642
0.805	58.4476	0.805	99.838
0.807	58.619	0.806	100.05
0.808	58.7752	0.808	100.272
0.81	58.9365	0.809	100.483
0.811	59.0877	0.81	100.695
0.813	59.249	0.812	100.917
0.815	59.5665	0.813	101.139
0.817	59.7228	0.816	101.557
0.818	59.879	0.817	101.774
0.82	60.0302	0.819	102.001
0.821	60.1966	0.821	102.222
0.823	60.3629	0.822	102.444
0.824	60.5141	0.823	102.681

0.825	60.6754	0.825	102.873
0.827	60.8367	0.826	103.089
0.829	60.998	0.828	103.316
0.83	61.1694	0.831	103.77
0.831	61.3206	0.832	103.971
0.833	61.4869	0.833	104.208
0.835	61.6431	0.835	104.42
0.836	61.8044	0.836	104.632
0.837	61.9808	0.838	104.858
0.839	62.1472	0.839	105.07
0.84	62.2883	0.84	105.267
0.842	62.4194	0.842	105.494
0.843	62.5958	0.844	105.746
0.844	62.7571	0.845	105.972
0.846	62.9183	0.846	106.204
0.847	63.0948	0.848	106.441
0.849	63.2611	0.85	106.658
0.85	63.4274	0.851	106.89
0.853	63.7399	0.852	107.091
0.855	63.9365	0.854	107.298
0.856	64.0675	0.855	107.52
0.858	64.2238	0.858	107.948
0.859	64.4002	0.86	108.165
0.86	64.5917	0.861	108.387
0.862	64.753	0.862	108.588
0.863	64.9194	0.864	108.79
0.865	65.0907	0.865	109.057
0.866	65.2571	0.867	109.274
0.869	65.5948	0.868	109.485
0.87	65.756	0.87	109.682
0.872	65.9224	0.871	109.904
0.873	66.0786	0.873	110.105
0.875	66.2399	0.874	110.327
0.876	66.4163	0.875	110.569
0.878	66.5877	0.877	110.806
0.879	66.7742	0.88	111.275
0.881	66.9355	0.881	111.486
0.882	67.1169	0.883	111.703
0.883	67.2933	0.885	112.157
0.885	67.4395	0.887	112.389
0.886	67.6159	0.888	112.615
0.888	67.8075	0.89	112.862
0.889	67.9587	0.891	113.109
0.891	68.125	0.894	113.573
0.892	68.3165	0.896	113.785
0.894	68.4829	0.897	114.007
0.895	68.6442	0.898	114.233
0.897	68.8256	0.901	114.692
0.898	69.0121	0.903	114.919
0.899	69.1936	0.904	115.136
0.901	69.3649	0.906	115.367
0.904	69.7278	0.907	115.589
0.905	69.9042	0.908	115.841
0.907	70.0756	0.911	116.305
0.91	70.4536	0.913	116.512
0.911	70.62	0.914	116.733
0.913	70.8065	0.917	117.217
0.914	70.9829	0.919	117.459
0.915	71.1442	0.92	117.666
0.917	71.3256	0.921	117.903
0.918	71.4667	0.923	118.135
0.92	71.6583	0.925	118.387
0.921	71.8246	0.926	118.593
0.923	72.001	0.927	118.815
0.926	72.3639	0.929	119.052
0.927	72.5605	0.93	119.294
0.929	72.7268	0.933	119.768

0.93	72.873	0.935	119.995
0.931	73.0696	0.936	120.221
0.933	73.2712	0.937	120.458
0.934	73.4728	0.939	120.7
0.936	73.6442	0.942	121.159
0.937	73.8054	0.943	121.386
0.939	74.0171	0.945	121.623
0.94	74.1784	0.946	121.859
0.943	74.5312	0.948	122.076
0.944	74.7177	0.949	122.313
0.946	74.8891	0.95	122.565
0.947	75.0605	0.952	122.807
0.949	75.2823	0.953	123.034
0.95	75.4436	0.955	123.261
0.952	75.625	0.956	123.503
0.953	75.8014	0.959	123.936
0.955	75.998	0.96	124.153
0.958	76.3458	0.962	124.395
0.959	76.5272	0.963	124.611
0.96	76.6885	0.965	124.879
0.962	76.87	0.966	125.126
0.963	77.0615	0.968	125.357
0.965	77.2278	0.969	125.584
0.966	77.4345	0.971	125.816
0.968	77.626	0.974	126.265
0.969	77.8125	0.975	126.502
0.971	78.0091	0.976	126.733
0.972	78.1804	0.978	126.99
0.973	78.372	0.981	127.479
0.975	78.5232	0.982	127.716
0.976	78.7198	0.984	127.943
0.978	78.9113	0.985	128.19
0.981	79.2944	0.987	128.437
0.982	79.4859	0.988	128.644
0.983	79.6724	0.989	128.891
0.985	79.8538	0.991	129.128
0.987	80.0504	0.992	129.334
0.988	80.247	0.994	129.571
0.989	80.4234	0.997	130.08
0.991	80.6149	0.998	130.297
0.992	80.8266	1	130.534
0.994	81.0181	1.001	130.766
0.997	81.381	1.002	130.987
0.998	81.5927	1.004	131.194
1	81.7893	1.005	131.461
1.001	81.9708	1.007	131.693
1.003	82.1623	1.008	131.93
1.004	82.3488	1.01	132.167
1.005	82.5353	1.013	132.661
1.007	82.7319	1.014	132.878
1.008	82.9183	1.016	133.109
1.01	83.0998	1.017	133.366
1.011	83.3115	1.018	133.603
1.013	83.5232	1.02	133.85
1.014	83.7046	1.021	134.087
1.016	83.9012	1.023	134.359
1.017	84.0827	1.024	134.586
1.018	84.2792	1.026	134.848
1.02	84.4556	1.028	135.322
1.021	84.6724	1.03	135.569
1.023	84.8538	1.031	135.791
1.024	85.0706	1.033	136.003
1.026	85.252	1.034	136.275
1.027	85.4788	1.036	136.537
1.028	85.6754	1.039	136.985
1.03	85.8569	1.04	137.232
1.031	86.0433	1.041	137.479

1.033	86.245	1.045	137.983
1.034	86.4617	1.046	138.22
1.036	86.6734	1.047	138.447
1.037	86.8952	1.049	138.659
1.039	87.0867	1.05	138.911
1.04	87.2984	1.051	139.183
1.042	87.4849	1.053	139.415
1.045	87.8982	1.055	139.662
1.046	88.1048	1.056	139.904
1.047	88.3014	1.057	140.166
1.049	88.4728	1.06	140.609
1.05	88.6845	1.062	140.866
1.053	89.0675	1.063	141.103
1.055	89.2591	1.065	141.345
1.056	89.4556	1.066	141.592
1.057	89.6825	1.068	141.849
1.059	89.8942	1.069	142.111
1.062	90.2974	1.07	142.343
1.063	90.4788	1.072	142.575
1.065	90.6804	1.073	142.812
1.066	90.8972	1.075	143.049
1.069	91.2954	1.078	143.498
1.07	91.497	1.079	143.755
1.072	91.6935	1.08	143.981
1.073	91.9002	1.082	144.198
1.075	92.0817	1.083	144.455
1.076	92.2833	1.086	144.979
1.078	92.505	1.088	145.231
1.08	92.8881	1.089	145.499
1.082	93.0796	1.091	145.746
1.084	93.2964	1.093	146.194
1.086	93.6996	1.095	146.446
1.088	93.8962	1.096	146.708
1.09	94.0978	1.098	146.97
1.091	94.2994	1.099	147.232
1.092	94.496	1.101	147.474
1.095	94.879	1.103	147.933
1.096	95.0655	1.105	148.18
1.098	95.252	1.107	148.452
1.099	95.4788	1.109	148.911
1.102	95.8921	1.111	149.158
1.104	96.0837	1.112	149.415
1.105	96.2954	1.114	149.657
1.108	96.7036	1.115	149.914
1.11	96.88	1.117	150.186
1.111	97.0817	1.118	150.423
1.113	97.2681	1.119	150.65
1.114	97.4748	1.121	150.897
1.115	97.7016	1.122	151.139
1.117	97.9284	1.125	151.633
1.118	98.1099	1.127	151.839
1.12	98.3014	1.128	152.106
1.121	98.4879	1.13	152.374
1.124	98.9163	1.131	152.646
1.125	99.1431	1.132	152.873
1.127	99.3448	1.134	153.14
1.129	99.5716	1.137	153.598
1.13	99.7833	1.138	153.825
1.131	100	1.14	154.072
1.133	100.1764	1.141	154.319
1.134	100.3931	1.143	154.581
1.136	100.5696	1.144	154.838
1.137	100.7712	1.146	155.121
1.139	100.9778	1.147	155.352
1.14	101.1895	1.148	155.579
1.141	101.3861	1.15	155.821
1.143	101.5726	1.151	156.068

1.144	101.7692	1.153	156.34
1.146	101.9909	1.154	156.577
1.148	102.4446	1.157	157.096
1.15	102.6462	1.159	157.353
1.152	102.8427	1.16	157.585
1.153	103.0141	1.161	157.842
1.154	103.2208	1.163	158.074
1.156	103.4425	1.164	158.351
1.157	103.6391	1.166	158.603
1.159	103.8105	1.167	158.825
1.16	104.0071	1.169	159.087
1.162	104.2288	1.17	159.339
1.163	104.4304	1.173	159.843
1.164	104.6522	1.174	160.08
1.166	104.8589	1.176	160.347
1.167	105.0504	1.177	160.625
1.169	105.2722	1.179	160.902
1.17	105.4788	1.18	161.149
1.173	105.9224	1.182	161.396
1.175	106.124	1.183	161.628
1.176	106.3357	1.184	161.905
1.178	106.5524	1.186	162.162
1.179	106.7792	1.189	162.686
1.182	107.1573	1.19	162.928
1.183	107.3689	1.192	163.17
1.185	107.5806	1.193	163.422
1.188	107.999	1.194	163.699
1.189	108.2157	1.196	163.966
1.191	108.4375	1.198	164.198
1.192	108.6593	1.199	164.44
1.193	108.8558	1.2	164.707
1.195	109.1028	1.202	164.974
1.197	109.3397	1.205	165.514
1.199	109.7127	1.206	165.791
1.201	109.9345	1.208	166.063
1.204	110.3931	1.209	166.3
1.205	110.63	1.212	166.799
1.207	110.8518	1.213	167.036
1.208	111.0635	1.215	167.268
1.209	111.3004	1.216	167.53
1.211	111.5171	1.218	167.787
1.212	111.7339	1.219	168.069
1.214	111.9607	1.222	168.588
1.215	112.1724	1.223	168.855
1.216	112.4042	1.225	169.107
1.218	112.6411	1.228	169.556
1.221	113.0796	1.229	169.823
1.222	113.2964	1.231	170.085
1.224	113.5131	1.232	170.327
1.225	113.75	1.234	170.614
1.227	113.9768	1.235	170.902
1.228	114.1885	1.237	171.134
1.23	114.4153	1.238	171.36
1.231	114.622	1.239	171.633
1.233	114.8236	1.241	171.875
1.234	115.0554	1.244	172.434
1.235	115.3075	1.245	172.711
1.237	115.5242	1.247	172.963
1.238	115.7561	1.248	173.256
1.24	115.9879	1.25	173.518
1.241	116.2147	1.251	173.78
1.243	116.4264	1.252	174.022
1.244	116.6683	1.254	174.274
1.245	116.9103	1.256	174.531
1.247	117.1421	1.257	174.788
1.248	117.374	1.26	175.277
1.25	117.6058	1.261	175.539

1.253	118.0393	1.263	175.806
1.254	118.256	1.264	176.053
1.256	118.4829	1.265	176.335
1.257	118.7399	1.267	176.582
1.258	118.9718	1.27	177.101
1.26	119.1986	1.271	177.338
1.261	119.4456	1.273	177.595
1.264	119.9093	1.274	177.878
1.266	120.1562	1.275	178.145
1.267	120.3881	1.277	178.417
1.269	120.6149	1.279	178.694
1.27	120.8417	1.28	178.971
1.272	121.0433	1.281	179.208
1.273	121.2651	1.284	179.732
1.276	121.749	1.286	179.984
1.277	121.9607	1.287	180.262
1.279	122.2026	1.289	180.509
1.28	122.4395	1.29	180.796
1.282	122.6563	1.293	181.3
1.283	122.8931	1.294	181.572
1.284	123.1502	1.296	181.859
1.286	123.3468	1.297	182.116
1.287	123.5736	1.299	182.368
1.289	123.7954	1.3	182.631
1.292	124.2742	1.302	182.883
1.293	124.496	1.303	183.135
1.295	124.748	1.304	183.387
1.296	124.995	1.306	183.639
1.297	125.2167	1.309	184.203
1.299	125.4335	1.31	184.46
1.301	125.6704	1.312	184.712
1.303	126.1593	1.313	185.015
1.305	126.381	1.314	185.277
1.308	126.87	1.316	185.488
1.309	127.0968	1.318	185.786
1.311	127.3438	1.319	186.038
1.312	127.5605	1.32	186.305
1.313	127.8075	1.322	186.567
1.315	128.0544	1.323	186.87
1.318	128.4778	1.325	187.137
1.319	128.7046	1.326	187.363
1.321	128.9264	1.328	187.641
1.322	129.1482	1.329	187.923
1.325	129.6774	1.331	188.17
1.326	129.9042	1.332	188.422
1.328	130.1462	1.333	188.689
1.329	130.4032	1.335	188.971
1.331	130.62	1.336	189.244
1.332	130.8317	1.338	189.521
1.334	131.0685	1.341	190.07
1.335	131.3206	1.342	190.327
1.336	131.5726	1.343	190.609
1.34	132.0313	1.345	190.877
1.341	132.2732	1.347	191.134
1.342	132.5101	1.348	191.411
1.344	132.7671	1.349	191.688
1.347	133.2258	1.351	191.92
1.348	133.4577	1.352	192.242
1.35	133.7046	1.354	192.515
1.351	133.9567	1.356	193.069
1.352	134.2036	1.358	193.331
1.354	134.4557	1.359	193.598
1.357	134.9698	1.361	193.855
1.358	135.2016	1.362	194.133
1.36	135.4183	1.364	194.42
1.363	135.9073	1.365	194.697
1.364	136.1643	1.367	194.979

1.365	136.376	1.368	195.262
1.367	136.6431	1.37	195.544
1.369	136.9153	1.371	195.791
1.37	137.1925	1.374	196.33
1.371	137.4093	1.375	196.612
1.373	137.6714	1.377	196.885
1.374	137.9335	1.38	197.449
1.376	138.1704	1.381	197.736
1.377	138.4123	1.382	198.024
1.379	138.6542	1.384	198.321
1.38	138.9113	1.386	198.583
1.381	139.1784	1.387	198.86
1.383	139.4254	1.388	199.107
1.385	139.7077	1.39	199.38
1.386	139.9496	1.391	199.637
1.387	140.1865	1.393	199.924
1.389	140.4284	1.396	200.473
1.39	140.6855	1.397	200.766
1.392	140.9325	1.399	201.053
1.393	141.1946	1.4	201.325
1.396	141.7036	1.401	201.617
1.397	141.9456	1.403	201.89
1.399	142.1976	1.404	202.122
1.4	142.4546	1.406	202.374
1.402	142.7117	1.407	202.661
1.403	142.9688	1.409	202.973
1.405	143.2208	1.41	203.22
1.406	143.4677	1.411	203.508
1.408	143.6946	1.413	203.775
1.409	143.9516	1.414	204.052
1.412	144.4456	1.416	204.309
1.413	144.6925	1.417	204.571
1.415	144.9597	1.419	204.869
1.416	145.2167	1.42	205.136
1.418	145.4738	1.422	205.403
1.419	145.7157	1.423	205.7
1.422	146.2147	1.424	205.987
1.423	146.4919	1.427	206.552
1.425	146.7691	1.429	206.844
1.426	147.0061	1.43	207.116
1.428	147.248	1.432	207.384
1.429	147.5	1.433	207.671
1.431	147.7319	1.436	208.175
1.432	147.9839	1.438	208.447
1.434	148.246	1.439	208.714
1.435	148.5081	1.44	209.002
1.437	148.7349	1.443	209.611
1.438	148.9819	1.445	209.919
1.439	149.2389	1.446	210.186
1.441	149.5111	1.448	210.458
1.444	150.0353	1.449	210.7
1.445	150.2873	1.451	210.957
1.447	150.5393	1.452	211.255
1.448	150.8165	1.453	211.532
1.45	151.0484	1.455	211.814
1.451	151.3054	1.456	212.096
1.453	151.5373	1.459	212.671
1.454	151.8044	1.461	212.943
1.455	152.0413	1.462	213.246
1.457	152.3185	1.463	213.533
1.458	152.5958	1.465	213.795
1.46	152.8427	1.467	214.077
1.461	153.0998	1.468	214.349
1.463	153.3619	1.469	214.627
1.464	153.624	1.471	214.909
1.467	154.1079	1.472	215.231
1.468	154.3599	1.474	215.519

1.47	154.622	1.475	215.806
1.471	154.879	1.477	216.063
1.473	155.1663	1.478	216.345
1.474	155.4536	1.479	216.592
1.476	155.6956	1.481	216.875
1.477	155.9224	1.482	217.147
1.478	156.1441	1.484	217.409
1.48	156.4214	1.485	217.696
1.483	156.9607	1.487	218.009
1.484	157.2379	1.488	218.306
1.486	157.4899	1.49	218.573
1.487	157.7671	1.492	219.112
1.489	158.0242	1.494	219.405
1.49	158.2762	1.495	219.667
1.492	158.5232	1.497	219.969
1.494	159.0323	1.498	220.252
1.496	159.2691	1.5	220.539
1.497	159.5464	1.501	220.826
1.5	160.0454	1.502	221.119
1.502	160.3024	1.504	221.391
1.503	160.5847	1.507	221.95
1.505	160.8619	1.508	222.242
1.506	161.1391	1.51	222.52
1.507	161.3911	1.511	222.812
1.509	161.6331	1.513	223.109
1.511	161.8901	1.515	223.669
1.512	162.1472	1.517	223.971
1.513	162.4345	1.519	224.254
1.516	162.994	1.52	224.551
1.517	163.251	1.523	225.131
1.519	163.5232	1.524	225.388
1.521	163.7954	1.526	225.65
1.522	164.0524	1.527	225.907
1.523	164.3145	1.528	226.189
1.526	164.874	1.53	226.466
1.528	165.1361	1.531	226.728
1.529	165.4133	1.533	227.031
1.531	165.6754	1.534	227.343
1.534	166.2147	1.536	227.626
1.535	166.4919	1.537	227.908
1.537	166.749	1.54	228.452
1.538	166.996	1.542	228.755
1.539	167.2631	1.543	229.057
1.541	167.5151	1.544	229.359
1.542	167.7823	1.546	229.647
1.544	168.0242	1.549	230.191
1.545	168.2611	1.55	230.494
1.548	168.8407	1.552	230.796
1.55	169.1028	1.554	231.386
1.551	169.37	1.556	231.663
1.552	169.6724	1.557	231.945
1.555	170.1814	1.559	232.258
1.557	170.4335	1.56	232.56
1.558	170.7157	1.562	232.873
1.56	170.998	1.563	233.185
1.561	171.2752	1.566	233.739
1.562	171.5625	1.568	234.012
1.564	171.8548	1.569	234.274
1.566	172.122	1.572	234.823
1.567	172.3841	1.573	235.09
1.568	172.6865	1.575	235.383
1.571	173.2056	1.576	235.71
1.573	173.4778	1.577	236.023
1.574	173.7802	1.579	236.285
1.576	174.0726	1.581	236.597
1.577	174.3397	1.582	236.88
1.579	174.6371	1.583	237.147

1.58	174.9093	1.585	237.399
1.581	175.1815	1.588	237.983
1.583	175.4536	1.589	238.276
1.584	175.7157	1.591	238.583
1.586	175.9677	1.592	238.891
1.589	176.4919	1.594	239.153
1.59	176.7389	1.595	239.425
1.591	177.0464	1.596	239.712
1.594	177.6159	1.598	239.995
1.596	177.873	1.599	240.282
1.597	178.1603	1.601	240.579
1.599	178.4073	1.602	240.887
1.6	178.6643	1.604	241.174
1.602	178.9718	1.605	241.481
1.603	179.254	1.607	241.779
1.605	179.5312	1.608	242.076
1.606	179.8085	1.61	242.348
1.607	180.0958	1.611	242.615
1.609	180.378	1.612	242.918
1.61	180.625	1.614	243.195
1.612	180.8921	1.615	243.472
1.613	181.1744	1.617	243.76
1.615	181.4667	1.618	244.062
1.616	181.7339	1.621	244.642
1.618	182.001	1.622	244.949
1.619	182.3135	1.624	245.241
1.62	182.5857	1.625	245.544
1.622	182.8679	1.627	245.821
1.624	183.1552	1.628	246.103
1.626	183.7198	1.63	246.381
1.628	183.9617	1.631	246.688
1.629	184.2187	1.633	246.996
1.631	184.5212	1.634	247.268
1.632	184.7782	1.637	247.857
1.634	185.0302	1.638	248.155
1.635	185.3528	1.64	248.432
1.638	185.9073	1.641	248.75
1.639	186.1593	1.643	249.062
1.642	186.7339	1.644	249.334
1.644	186.9758	1.645	249.637
1.645	187.248	1.647	249.924
1.647	187.5454	1.649	250.226
1.648	187.8377	1.65	250.509
1.65	188.1099	1.653	251.139
1.651	188.3871	1.654	251.436
1.652	188.6593	1.656	251.718
1.654	188.9214	1.657	251.985
1.655	189.2036	1.659	252.268
1.657	189.4859	1.66	252.555
1.658	189.7782	1.662	252.857
1.659	190.0353	1.663	253.145
1.661	190.3327	1.664	253.452
1.663	190.62	1.667	254.067
1.664	190.8972	1.669	254.364
1.667	191.4567	1.67	254.692
1.669	191.7339	1.672	254.989
1.67	192.0161	1.674	255.534
1.671	192.3135	1.676	255.846
1.673	192.5857	1.677	256.124
1.674	192.8427	1.679	256.431
1.677	193.3518	1.68	256.774
1.678	193.6139	1.682	257.061
1.68	193.9113	1.685	257.641
1.683	194.506	1.686	257.958
1.684	194.7782	1.687	258.256
1.686	195.0655	1.689	258.533
1.687	195.3427	1.69	258.84

1.689	195.5847	1.692	259.153
1.69	195.8468	1.693	259.45
1.692	196.129	1.695	259.753
1.693	196.4365	1.696	260.08
1.694	196.7087	1.698	260.378
1.696	196.9809	1.699	260.705
1.699	197.5353	1.701	260.992
1.7	197.8276	1.702	261.27
1.702	198.1149	1.703	261.572
1.703	198.4173	1.706	262.132
1.706	198.9768	1.708	262.439
1.708	199.2692	1.709	262.736
1.709	199.5514	1.711	263.044
1.71	199.8387	1.712	263.356
1.712	200.1613	1.714	263.644
1.714	200.4788	1.715	263.936
1.716	201.0081	1.716	264.208
1.718	201.3004	1.718	264.496
1.719	201.5726	1.719	264.823
1.722	202.122	1.721	265.141
1.724	202.3891	1.724	265.771
1.725	202.6915	1.725	266.083
1.726	203.0292	1.726	266.36
1.728	203.3014	1.728	266.648
1.729	203.5887	1.73	266.95
1.731	203.8609	1.731	267.242
1.732	204.123	1.732	267.555
1.734	204.4002	1.734	267.852
1.735	204.6976	1.735	268.155
1.738	205.2823	1.738	268.765
1.739	205.5645	1.74	269.087
1.741	205.8619	1.741	269.41
1.742	206.1744	1.742	269.702
1.744	206.4163	1.744	269.969
1.745	206.6835	1.746	270.267
1.747	206.9758	1.747	270.549
1.748	207.2732	1.748	270.821
1.749	207.5756	1.75	271.134
1.751	207.8579	1.751	271.436
1.753	208.1452	1.753	271.723
1.755	208.7097	1.756	272.368
1.757	208.9869	1.757	272.656
1.758	209.2792	1.758	272.943
1.76	209.5817	1.76	273.235
1.761	209.8891	1.761	273.533
1.763	210.1613	1.764	274.097
1.764	210.4284	1.765	274.41
1.765	210.7056	1.767	274.722
1.767	210.9778	1.769	275.02
1.77	211.5322	1.771	275.65
1.771	211.8498	1.773	275.932
1.773	212.1472	1.774	276.194
1.774	212.4345	1.776	276.461
1.776	212.7268	1.777	276.754
1.777	213.0091	1.779	277.066
1.778	213.2813	1.78	277.369
1.78	213.5786	1.782	277.681
1.782	213.881	1.783	277.993
1.783	214.1784	1.785	278.311
1.784	214.4758	1.786	278.613
1.786	214.7631	1.787	278.881
1.788	215.3125	1.789	279.168
1.79	215.6048	1.79	279.48
1.792	215.9123	1.792	279.773
1.794	216.4667	1.793	280.045
1.796	216.7641	1.794	280.337
1.798	217.0615	1.796	280.64

1.799	217.3488	1.798	280.922
1.8	217.631	1.799	281.229
1.802	217.9385	1.8	281.552
1.803	218.246	1.802	281.854
1.806	218.8105	1.803	282.162
1.808	219.0726	1.805	282.469
1.809	219.3347	1.806	282.741
1.81	219.5968	1.808	283.024
1.812	219.8841	1.81	283.588
1.813	220.2117	1.812	283.916
1.815	220.504	1.813	284.203
1.816	220.8266	1.815	284.511
1.818	221.1089	1.816	284.798
1.819	221.4012	1.818	285.09
1.821	221.6835	1.819	285.408
1.822	221.9557	1.822	285.987
1.823	222.2329	1.823	286.3
1.825	222.5101	1.825	286.577
1.828	223.0847	1.827	286.87
1.829	223.3669	1.828	287.187
1.831	223.6946	1.829	287.535
1.833	224.1986	1.831	287.862
1.835	224.496	1.832	288.15
1.837	224.7833	1.834	288.432
1.838	225.0907	1.835	288.745
1.839	225.378	1.837	289.007
1.841	225.6855	1.838	289.274
1.842	225.9778	1.839	289.591
1.845	226.5827	1.841	289.879
1.847	226.88	1.844	290.509
1.848	227.1623	1.845	290.811
1.851	227.7167	1.847	291.113
1.853	228.0191	1.848	291.401
1.854	228.3014	1.85	291.673
1.855	228.5685	1.851	291.96
1.857	228.8558	1.852	292.263
1.858	229.1532	1.854	292.56
1.86	229.4456	1.855	292.862
1.861	229.7682	1.858	293.487
1.863	230.0605	1.86	293.8
1.864	230.3427	1.861	294.112
1.867	230.8972	1.862	294.39
1.868	231.1895	1.864	294.687
1.87	231.5222	1.866	295.01
1.871	231.8095	1.867	295.292
1.873	232.1018	1.868	295.604
1.874	232.3992	1.87	295.927
1.876	232.7167	1.871	296.219
1.877	232.9889	1.873	296.512
1.878	233.2762	1.876	297.122
1.882	233.8861	1.877	297.414
1.883	234.1683	1.878	297.731
1.884	234.4758	1.88	298.029
1.886	234.7631	1.881	298.321
1.887	235.0706	1.883	298.588
1.89	235.6401	1.884	298.886
1.892	235.9728	1.886	299.188
1.893	236.2752	1.887	299.485
1.895	236.5776	1.89	300.08
1.896	236.8548	1.891	300.398
1.899	237.4294	1.893	300.69
1.9	237.7319	1.895	300.977
1.902	238.004	1.896	301.239
1.903	238.2863	1.897	301.547
1.905	238.6089	1.899	301.875
1.906	238.9113	1.9	302.182
1.908	239.1936	1.902	302.495

1.909	239.496	1.903	302.802
1.911	239.8034	1.905	303.094
1.912	240.0756	1.906	303.377
1.913	240.3377	1.907	303.664
1.915	240.6552	1.909	303.966
1.916	240.9627	1.91	304.264
1.918	241.2651	1.912	304.561
1.919	241.5776	1.913	304.833
1.921	241.8901	1.915	305.126
1.923	242.4597	1.916	305.448
1.925	242.7772	1.919	306.093
1.927	243.0544	1.921	306.396
1.928	243.3367	1.922	306.698
1.929	243.6441	1.923	306.99
1.931	243.9415	1.925	307.273
1.932	244.254	1.926	307.545
1.934	244.5464	1.928	307.807
1.935	244.8589	1.93	308.361
1.937	245.1714	1.932	308.694
1.938	245.4637	1.934	309.007
1.94	245.7712	1.935	309.304
1.941	246.0484	1.936	309.591
1.943	246.3508	1.938	309.899
1.945	246.9052	1.941	310.473
1.947	247.1875	1.942	310.751
1.948	247.5151	1.944	311.068
1.95	247.8125	1.947	311.678
1.951	248.1351	1.948	311.985
1.953	248.4224	1.95	312.328
1.954	248.6946	1.951	312.61
1.956	248.9768	1.952	312.852
1.957	249.2641	1.955	313.437
1.958	249.5867	1.957	313.75
1.96	249.8841	1.958	314.047
1.963	250.4889	1.959	314.339
1.964	250.8216	1.963	314.984
1.966	251.0938	1.964	315.292
1.967	251.381	1.965	315.574
1.968	251.6532	1.967	315.861
1.97	251.9556	1.968	316.144
1.971	252.2782	1.97	316.421
1.973	252.5605	1.971	316.698
1.974	252.8528	1.973	316.98
1.976	253.1905	1.974	317.242
1.977	253.4829	1.975	317.52
1.979	253.7702	1.977	317.822
1.98	254.0726	1.98	318.442
1.982	254.3901	1.981	318.719
1.983	254.6673	1.983	319.002
1.985	254.9395	1.984	319.289
1.986	255.2066	1.986	319.561
1.987	255.4587	1.987	319.848
1.989	255.7611	1.988	320.161
1.99	256.0534	1.99	320.473
1.992	256.3155	1.991	320.756
1.993	256.6381	1.994	321.34
1.996	257.2228	1.996	321.633
1.998	257.5302	1.997	321.935
1.999	257.8276	1.998	322.237
2.002	258.372	2.001	322.827
2.003	258.6895	2.003	323.114
2.005	258.9667	2.004	323.437
2.006	259.2742	2.006	323.719
2.008	259.5766	2.007	324.017
2.009	259.8941	2.009	324.314
2.011	260.1663	2.01	324.647
2.013	260.756	2.012	324.934

2.015	261.0635	2.013	325.241
2.018	261.6431	2.015	325.524
2.019	261.9506	2.016	325.786
2.021	262.2581	2.017	326.098
2.022	262.5202	2.019	326.401
2.024	262.8226	2.02	326.653
2.025	263.1603	2.022	327.006
2.027	263.4476	2.023	327.328
2.028	263.75	2.025	327.605
2.03	264.0726	2.027	328.19
2.031	264.3397	2.029	328.503
2.034	264.874	2.03	328.765
2.035	265.1613	2.033	329.329
2.036	265.4939	2.035	329.627
2.038	265.8165	2.036	329.934
2.04	266.124	2.038	330.267
2.041	266.4315	2.039	330.564
2.042	266.7339	2.04	330.841
2.044	267.0262	2.042	331.118
2.045	267.3085	2.043	331.421
2.047	267.6058	2.045	331.718
2.05	268.1855	2.046	332.011
2.051	268.5181	2.049	332.6
2.053	268.8206	2.05	332.898
2.054	269.1179	2.052	333.205
2.057	269.6875	2.054	333.513
2.058	270.0353	2.055	333.81
2.06	270.3427	2.056	334.117
2.061	270.6452	2.06	334.692
2.063	270.9476	2.061	334.994
2.064	271.2198	2.062	335.262
2.067	271.8196	2.064	335.524
2.069	272.1421	2.065	335.811
2.07	272.5	2.066	336.113
2.073	273.0696	2.068	336.466
2.075	273.3468	2.069	336.738
2.076	273.6341	2.071	337.021
2.077	273.9214	2.074	337.575
2.079	274.2339	2.075	337.827
2.08	274.5262	2.077	338.14
2.081	274.8387	2.078	338.467
2.083	275.1714	2.081	339.052
2.085	275.499	2.083	339.364
2.086	275.7964	2.084	339.657
2.089	276.3558	2.085	339.899
2.09	276.6583	2.087	340.186
2.092	276.9405	2.088	340.488
2.093	277.2581	2.09	340.776
2.094	277.5504	2.091	341.088
2.096	277.878	2.093	341.396
2.098	278.1855	2.094	341.683
2.099	278.498	2.097	342.283
2.1	278.7853	2.099	342.61
2.102	279.0676	2.1	342.898
2.103	279.3952	2.101	343.185
2.106	279.9798	2.103	343.467
2.108	280.2923	2.104	343.714
2.109	280.625	2.106	344.002
2.11	280.9123	2.107	344.294
2.112	281.1946	2.108	344.566
2.114	281.5071	2.11	344.874
2.115	281.8246	2.111	345.176
2.116	282.122	2.114	345.746
2.118	282.4244	2.116	346.023
2.119	282.7218	2.117	346.32
2.121	283.0091	2.118	346.572
2.122	283.2762	2.12	346.87

2.124	283.5635	2.122	347.147
2.125	283.876	2.124	347.736
2.126	284.1734	2.126	348.029
2.129	284.7933	2.127	348.316
2.131	285.0907	2.129	348.588
2.132	285.378	2.13	348.871
2.134	285.6754	2.132	349.143
2.135	285.9728	2.133	349.44
2.137	286.3004	2.135	349.712
2.138	286.6079	2.136	350.01
2.139	286.9053	2.137	350.327
2.141	287.2329	2.139	350.625
2.143	287.5252	2.14	350.887
2.145	288.0947	2.142	351.184
2.147	288.3972	2.143	351.497
2.148	288.7046	2.145	351.774
2.15	289.0222	2.146	352.066
2.151	289.3044	2.148	352.328
2.153	289.622	2.149	352.59
2.154	289.9042	2.151	352.852
2.155	290.1966	2.153	353.392
2.157	290.5191	2.155	353.679
2.159	290.8266	2.156	354.007
2.161	291.4617	2.158	354.324
2.163	291.7389	2.159	354.616
2.164	292.0363	2.161	354.899
2.166	292.3085	2.162	355.186
2.167	292.5907	2.163	355.448
2.168	292.8679	2.165	355.69
2.17	293.1905	2.166	355.982
2.171	293.5081	2.169	356.587
2.173	293.8256	2.171	356.9
2.174	294.1381	2.172	357.182
2.176	294.4304	2.174	357.454
2.179	295.0252	2.175	357.736
2.18	295.3075	2.176	358.049
2.182	295.5998	2.178	358.341
2.183	295.9123	2.179	358.608
2.184	296.2147	2.181	358.876
2.186	296.5171	2.182	359.158
2.188	296.8498	2.185	359.753
2.189	297.1825	2.187	360.07
2.19	297.4395	2.188	360.403
2.193	298.0192	2.19	360.68
2.194	298.3316	2.191	360.942
2.196	298.624	2.192	361.219
2.197	298.9264	2.194	361.481
2.2	299.5363	2.195	361.749
2.202	299.8689	2.197	362.036
2.203	300.1865	2.198	362.328
2.205	300.4939	2.201	362.933
2.206	300.7712	2.203	363.261
2.208	301.0686	2.204	363.503
2.209	301.3558	2.205	363.77
2.211	301.628	2.207	364.037
2.212	301.9254	2.208	364.304
2.213	302.2177	2.21	364.586
2.216	302.8377	2.211	364.884
2.218	303.1603	2.213	365.171
2.219	303.4829	2.214	365.448
2.222	304.0928	2.217	366.053
2.223	304.3649	2.219	366.315
2.225	304.6371	2.22	366.587
2.226	304.9496	2.221	366.875
2.228	305.2621	2.223	367.162
2.229	305.5746	2.224	367.424
2.232	306.1693	2.226	367.716

2.234	306.4768	2.227	368.029
2.235	306.754	2.229	368.311
2.236	307.0565	2.23	368.558
2.238	307.374	2.231	368.876
2.239	307.6814	2.233	369.173
2.241	307.999	2.234	369.45
2.244	308.619	2.236	369.748
2.245	308.9315	2.237	370.02
2.248	309.5212	2.24	370.569
2.25	309.879	2.242	370.846
2.251	310.1764	2.243	371.119
2.252	310.4688	2.244	371.386
2.254	310.756	2.246	371.673
2.255	311.0585	2.248	371.98
2.257	311.381	2.249	372.268
2.258	311.6532	2.25	372.54
2.26	311.9708	2.252	372.812
2.261	312.3185	2.253	373.064
2.263	312.6411	2.256	373.629
2.266	313.2409	2.258	373.921
2.267	313.5686	2.259	374.203
2.268	313.8558	2.26	374.49
2.27	314.1179	2.262	374.748
2.271	314.4204	2.263	375.02
2.273	314.7278	2.265	375.302
2.274	315.0353	2.266	375.579
2.275	315.3377	2.268	375.866
2.277	315.6804	2.269	376.129
2.279	315.9929	2.271	376.401
2.28	316.2702	2.273	376.955
2.281	316.5524	2.275	377.258
2.283	316.875	2.276	377.545
2.284	317.1421	2.278	377.837
2.286	317.4244	2.279	378.104
2.287	317.7369	2.281	378.392
2.289	318.0645	2.282	378.654
2.29	318.3972	2.284	378.891
2.291	318.6744	2.285	379.153
2.293	319.002	2.287	379.41
2.295	319.3296	2.289	379.979
2.296	319.6371	2.291	380.277
2.297	319.9244	2.292	380.554
2.299	320.2066	2.294	380.841
2.3	320.5343	2.295	381.124
2.302	320.8165	2.297	381.386
2.304	321.3911	2.298	381.623
2.306	321.7187	2.299	381.875
2.308	322.0464	2.301	382.147
2.309	322.3337	2.302	382.444
2.31	322.6411	2.305	383.029
2.312	322.9083	2.307	383.346
2.313	323.1855	2.308	383.608
2.314	323.4778	2.31	383.84
2.316	323.8004	2.311	384.107
2.318	324.1179	2.312	384.4
2.319	324.4355	2.315	384.959
2.32	324.7429	2.317	385.236
2.322	325.0706	2.318	385.524
2.324	325.3579	2.321	386.134
2.325	325.6199	2.323	386.355
2.326	325.9123	2.324	386.602
2.328	326.2298	2.326	386.885
2.329	326.5272	2.327	387.142
2.331	326.8347	2.328	387.429
2.332	327.1371	2.33	387.696
2.334	327.4546	2.331	387.923
2.336	328.0847	2.333	388.2

2.338	328.4022	2.334	388.487
2.34	328.7197	2.336	388.785
2.341	328.997	2.337	389.077
2.342	329.2792	2.339	389.334
2.344	329.5464	2.34	389.586
2.345	329.8337	2.341	389.843
2.348	330.4133	2.343	390.1
2.349	330.7309	2.344	390.367
2.351	331.0635	2.346	390.665
2.353	331.371	2.347	390.927
2.354	331.6633	2.349	391.209
2.355	331.9556	2.35	391.507
2.357	332.2682	2.351	391.759
2.358	332.5403	2.355	392.288
2.359	332.8377	2.356	392.585
2.361	333.1452	2.357	392.852
2.363	333.4526	2.36	393.397
2.364	333.7752	2.362	393.674
2.365	334.0524	2.363	393.956
2.367	334.37	2.365	394.208
2.368	334.6774	2.366	394.496
2.37	334.9647	2.368	394.808
2.371	335.2772	2.369	395.08
2.373	335.5544	2.371	395.332
2.376	336.1593	2.372	395.584
2.377	336.4768	2.373	395.831
2.378	336.7843	2.376	396.35
2.38	337.0665	2.378	396.633
2.381	337.3992	2.379	396.935
2.383	337.7016	2.381	397.232
2.384	337.999	2.382	397.479
2.386	338.2913	2.384	397.752
2.387	338.5786	2.385	398.059
2.389	338.8558	2.386	398.316
2.39	339.123	2.388	398.573
2.393	339.6925	2.389	398.815
2.394	340.0303	2.392	399.324
2.396	340.3679	2.394	399.652
2.398	340.6653	2.395	399.944
2.399	340.9778	2.396	400.221
2.4	341.2651	2.398	400.478
2.402	341.5625	2.399	400.72
2.403	341.8448	2.401	401.008
2.404	342.1068	2.402	401.27
2.406	342.4143	2.404	401.517
2.407	342.7016	2.405	401.794
2.409	343.0292	2.407	402.091
2.41	343.3216	2.41	402.671
2.412	343.6341	2.411	402.943
2.413	343.9113	2.412	403.23
2.416	344.5464	2.414	403.508
2.418	344.8538	2.415	403.755
2.419	345.131	2.417	404.007
2.421	345.4234	2.418	404.279
2.422	345.7409	2.419	404.556
2.423	346.0282	2.421	404.803
2.426	346.6784	2.423	405.085
2.428	347.0061	2.424	405.367
2.429	347.2732	2.425	405.645
2.432	347.8528	2.427	405.866
2.434	348.1452	2.428	406.139
2.435	348.4425	2.43	406.416
2.437	348.7147	2.431	406.658
2.438	349.0423	2.433	406.925
2.439	349.3548	2.434	407.207
2.441	349.6925	2.436	407.484
2.442	349.995	2.437	407.741

2.444	350.2571	2.438	408.014
2.445	350.5645	2.441	408.578
2.448	351.1038	2.443	408.83
2.449	351.4062	2.444	409.102
2.451	351.7137	2.446	409.349
2.452	352.001	2.447	409.606
2.454	352.3236	2.448	409.868
2.455	352.6512	2.45	410.146
2.457	352.9486	2.452	410.433
2.458	353.2157	2.453	410.715
2.46	353.503	2.454	411.018
2.461	353.8054	2.456	411.32
2.462	354.1129	2.459	411.839
2.466	354.7228	2.46	412.076
2.467	355.0252	2.462	412.323
2.468	355.2873	2.463	412.595
2.47	355.5897	2.464	412.857
2.471	355.9022	2.466	413.125
2.473	356.1996	2.467	413.392
2.474	356.5171	2.469	413.664
2.476	356.8195	2.47	413.936
2.477	357.1069	2.472	414.193
2.478	357.3891	2.473	414.465
2.48	357.6663	2.475	414.737
2.481	357.9637	2.476	414.989
2.483	358.251	2.477	415.221
2.484	358.5635	2.481	415.766
2.487	359.1835	2.482	416.048
2.489	359.4809	2.483	416.295
2.49	359.7732	2.485	416.577
2.492	360.0605	2.486	416.844
2.493	360.3428	2.487	417.111
2.494	360.6603	2.489	417.338
2.496	360.9577	2.491	417.615
2.499	361.5776	2.492	417.867
2.5	361.8801	2.493	418.114
2.502	362.1623	2.496	418.709
2.503	362.4546	2.498	418.997
2.505	362.7369	2.499	419.269
2.506	363.0494	2.501	419.546
2.507	363.3317	2.502	419.818
2.509	363.6341	2.504	420.085
2.511	363.9365	2.505	420.332
2.512	364.2137	2.506	420.584
2.513	364.5514	2.508	420.826
2.515	364.8387	2.509	421.083
2.516	365.1714	2.512	421.618
2.519	365.7611	2.514	421.92
2.521	366.0635	2.515	422.212
2.522	366.3004	2.516	422.469
2.523	366.5877	2.518	422.726
2.525	366.8599	2.521	423.23
2.526	367.1573	2.522	423.467
2.528	367.4546	2.524	423.719
2.529	367.7822	2.525	423.997
2.531	368.0897	2.528	424.561
2.532	368.372	2.529	424.818
2.534	368.6744	2.531	425.08
2.535	368.9718	2.532	425.337
2.537	369.2137	2.534	425.604
2.538	369.506	2.535	425.872
2.539	369.7984	2.537	426.124
2.541	370.0958	2.538	426.386
2.544	370.7107	2.539	426.658
2.545	371.0232	2.541	426.925
2.547	371.3105	2.543	427.207
2.548	371.5776	2.544	427.505

2.55	371.875	2.545	427.782
2.551	372.1724	2.547	428.054
2.552	372.4546	2.549	428.286
2.554	372.7772	2.55	428.553
2.556	373.0897	2.551	428.805
2.557	373.3871	2.553	429.042
2.56	374.0171	2.554	429.304
2.561	374.3246	2.556	429.546
2.563	374.627	2.557	429.843
2.564	374.879	2.56	430.418
2.566	375.1563	2.561	430.665
2.567	375.4637	2.563	430.922
2.568	375.751	2.564	431.184
2.57	375.998	2.566	431.436
2.571	376.2802	2.567	431.683
2.573	376.5675	2.568	431.96
2.574	376.8851	2.57	432.227
2.576	377.1976	2.572	432.495
2.577	377.4849	2.573	432.767
2.579	377.7067	2.574	433.039
2.58	377.9587	2.577	433.548
2.583	378.503	2.579	433.805
2.584	378.7147	2.58	434.072
2.586	379.0474	2.582	434.339
2.587	379.3599	2.583	434.622
2.589	379.6724	2.584	434.899
2.59	379.9899	2.586	435.176
2.592	380.2873	2.587	435.428
2.595	380.8972	2.589	435.695
2.596	381.1946	2.59	435.942
2.599	381.7792	2.592	436.224
2.601	382.0867	2.593	436.481
2.602	382.3992	2.595	436.723
2.603	382.6814	2.596	436.98
2.605	382.9788	2.597	437.222
2.606	383.2964	2.599	437.479
2.608	383.5786	2.601	437.752
2.609	383.8911	2.602	438.019
2.611	384.1633	2.603	438.281
2.612	384.4355	2.605	438.578
2.613	384.7228	2.606	438.84
2.616	385.3075	2.607	439.092
2.618	385.625	2.611	439.581
2.619	385.9526	2.612	439.838
2.621	386.255	2.613	440.126
2.622	386.5322	2.615	440.408
2.624	386.8397	2.616	440.69
2.625	387.1321	2.618	440.962
2.627	387.3891	2.619	441.244
2.628	387.6563	2.621	441.476
2.631	388.246	2.622	441.744
2.634	388.881	2.625	442.258
2.635	389.1734	2.626	442.5
2.637	389.4456	2.628	442.762
2.64	390.0202	2.629	443.044
2.641	390.3226	2.632	443.593
2.642	390.6048	2.634	443.896
2.644	390.8871	2.635	444.183
2.645	391.1794	2.637	444.425
2.648	391.7742	2.638	444.692
2.65	392.1018	2.64	444.934
2.651	392.3992	2.641	445.141
2.653	392.6966	2.642	445.373
2.656	393.2661	2.644	445.609
2.657	393.5333	2.645	445.882
2.658	393.8307	2.648	446.456
2.66	394.1028	2.65	446.708

2.661	394.3548	2.651	446.965
2.663	394.6875	2.652	447.248
2.664	395.0101	2.654	447.489
2.666	395.3024	2.655	447.701
2.667	395.5847	2.657	447.958
2.669	395.8669	2.658	448.235
2.67	396.119	2.66	448.498
2.671	396.4063	2.661	448.775
2.673	396.6885	2.664	449.359
2.674	397.0161	2.665	449.606
2.676	397.3085	2.667	449.843
2.677	397.631	2.669	450.11
2.679	397.9436	2.67	450.373
2.68	398.1956	2.671	450.625
2.682	398.4778	2.673	450.897
2.683	398.7651	2.676	451.466
2.685	399.0524	2.677	451.743
2.686	399.3448	2.679	452.031
2.687	399.6522	2.681	452.525
2.689	399.9395	2.683	452.762
2.69	400.2268	2.684	453.024
2.692	400.5443	2.686	453.271
2.695	401.1542	2.687	453.518
2.696	401.4516	2.689	453.765
2.698	401.7188	2.69	454.017
2.699	402.001	2.692	454.294
2.701	402.2631	2.693	454.571
2.702	402.495	2.694	454.848
2.703	402.7823	2.697	455.342
2.705	403.0746	2.699	455.574
2.706	403.3468	2.7	455.811
2.708	403.6542	2.703	456.335
2.709	403.9617	2.704	456.602
2.711	404.2188	2.706	456.875
2.712	404.5212	2.708	457.147
2.714	404.7782	2.709	457.404
2.715	405.0454	2.71	457.661
2.717	405.2974	2.712	457.933
2.719	405.8972	2.713	458.195
2.721	406.1895	2.715	458.447
2.722	406.4768	2.716	458.704
2.724	406.7943	2.718	458.992
2.725	407.0615	2.719	459.274
2.727	407.3186	2.72	459.531
2.728	407.5806	2.722	459.783
2.73	407.878	2.724	460.08
2.731	408.1452	2.725	460.378
2.732	408.4324	2.726	460.62
2.734	408.7551	2.728	460.856
2.735	409.0373	2.729	461.093
2.737	409.3347	2.731	461.32
2.738	409.6421	2.732	461.617
2.74	409.9345	2.735	462.127
2.741	410.2268	2.737	462.409
2.743	410.504	2.738	462.716
2.744	410.756	2.739	462.857
2.746	411.0282	2.741	463.135
2.747	411.3054	2.744	463.644
2.748	411.5423	2.745	463.906
2.751	412.0917	2.747	464.163
2.753	412.4143	2.748	464.43
2.754	412.7067	2.751	465.005
2.756	412.9788	2.753	465.302
2.759	413.5333	2.754	465.544
2.76	413.7853	2.755	465.791
2.761	414.0524	2.757	466.058
2.763	414.3447	2.758	466.345

2.764	414.6421	2.76	466.587
2.766	414.9546	2.762	467.116
2.767	415.2369	2.764	467.379
2.769	415.5141	2.765	467.651
2.77	415.7863	2.768	468.22
2.771	416.0584	2.77	468.482
2.773	416.371	2.771	468.724
2.775	416.6532	2.773	469.002
2.776	416.9153	2.774	469.243
2.777	417.1724	2.776	469.475
2.779	417.4647	2.777	469.707
2.78	417.7268	2.78	470.247
2.782	418.0292	2.781	470.519
2.783	418.2914	2.783	470.791
2.785	418.629	2.784	471.048
2.786	418.9012	2.786	471.3
2.788	419.1582	2.787	471.537
2.789	419.4355	2.789	471.779
2.792	419.9698	2.79	472.031
2.793	420.247	2.791	472.308
2.795	420.499	2.793	472.58
2.797	420.8064	2.794	472.847
2.798	421.1089	2.796	473.125
2.799	421.4264	2.797	473.397
2.801	421.6684	2.799	473.629
2.802	421.9254	2.801	474.133
2.804	422.2127	2.803	474.375
2.805	422.4748	2.805	474.662
2.807	422.7419	2.807	475.176
2.808	423.0192	2.809	475.438
2.809	423.3115	2.81	475.72
2.811	423.6089	2.812	476.003
2.813	423.9062	2.813	476.275
2.815	424.4506	2.816	476.789
2.817	424.7228	2.817	477.026
2.818	425.0101	2.819	477.268
2.82	425.3024	2.82	477.5
2.821	425.5897	2.823	478.024
2.822	425.877	2.825	478.316
2.824	426.1744	2.826	478.603
2.825	426.4466	2.828	478.845
2.827	426.7187	2.831	479.375
2.828	427.0262	2.832	479.632
2.831	427.5907	2.834	479.848
2.833	427.8579	2.835	480.08
2.834	428.12	2.836	480.342
2.836	428.4022	2.839	480.892
2.837	428.6643	2.84	481.159
2.838	428.9214	2.842	481.456
2.841	429.4809	2.844	481.698
2.843	429.7631	2.845	481.935
2.844	430.0806	2.846	482.197
2.846	430.3528	2.848	482.444
2.848	430.8821	2.849	482.706
2.85	431.129	2.851	482.973
2.851	431.381	2.852	483.225
2.854	431.9859	2.855	483.775
2.856	432.2782	2.857	484.072
2.857	432.5605	2.858	484.349
2.859	432.8377	2.86	484.601
2.86	433.0847	2.861	484.858
2.862	433.3669	2.862	485.105
2.863	433.6694	2.864	485.332
2.866	434.1684	2.867	485.801
2.867	434.4254	2.868	486.083
2.87	435.0302	2.869	486.365
2.872	435.3125	2.871	486.648

2.873	435.6149	2.873	486.925
2.875	435.9123	2.874	487.172
2.876	436.1744	2.875	487.429
2.878	436.4617	2.877	487.666
2.879	436.6986	2.878	487.913
2.881	436.9355	2.88	488.145
2.882	437.1875	2.881	488.402
2.883	437.4446	2.883	488.669
2.885	437.7167	2.884	488.931
2.888	438.3266	2.885	489.208
2.889	438.5938	2.887	489.475
2.891	438.8861	2.888	489.732
2.893	439.3901	2.89	489.974
2.895	439.6371	2.891	490.247
2.897	439.9194	2.894	490.71
2.898	440.1966	2.896	490.962
2.899	440.4637	2.897	491.229
2.902	441.0584	2.898	491.502
2.904	441.3256	2.9	491.784
2.905	441.5928	2.902	492.091
2.907	441.8397	2.903	492.404
2.91	442.4042	2.906	492.888
2.911	442.6865	2.907	493.13
2.912	442.9738	2.91	493.619
2.914	443.2561	2.912	493.886
2.915	443.5383	2.913	494.123
2.917	443.8306	2.914	494.395
2.919	444.1482	2.916	494.712
2.92	444.375	2.918	494.974
2.921	444.627	2.92	495.433
2.923	444.8791	2.922	495.66
2.924	445.1462	2.923	495.937
2.927	445.6451	2.926	496.441
2.928	445.9123	2.927	496.728
2.93	446.1845	2.929	496.99
2.931	446.4668	2.93	497.263
2.934	447.0716	2.932	497.545
2.936	447.3387	2.935	498.044
2.937	447.6109	2.936	498.306
2.938	447.8428	2.937	498.548
2.94	448.0998	2.939	498.805
2.941	448.3972	2.942	499.349
2.943	448.6744	2.943	499.581
2.944	448.9516	2.945	499.843
2.946	449.2389	2.947	500.126
2.947	449.5161	2.948	500.403
2.949	449.7833	2.949	500.655
2.95	450.0554	2.951	500.922
2.952	450.3578	2.952	501.149
2.953	450.6351	2.953	501.381
2.955	450.9073	2.955	501.607
2.956	451.1643	2.956	501.885
2.957	451.4365	2.959	502.404
2.959	451.7238	2.961	502.661
2.96	451.9909	2.962	502.953
2.962	452.253	2.964	503.23
2.963	452.5353	2.965	503.452
2.966	453.0897	2.966	503.694
2.968	453.3165	2.968	503.951
2.969	453.5786	2.97	504.223
2.971	453.8407	2.971	504.485
2.972	454.118	2.972	504.737
2.973	454.375	2.974	504.974
2.975	454.6371	2.975	505.252
2.977	454.9345	2.977	505.514
2.978	455.1915	2.978	505.73
2.979	455.4637	2.98	506.003

2.981	455.7409	2.981	506.295
2.983	456.2298	2.982	506.547
2.985	456.4718	2.984	506.799
2.986	456.7339	2.985	507.046
2.988	457.0262	2.987	507.293
2.989	457.2833	2.988	507.58
2.991	457.5958	2.99	507.862
2.992	457.873	2.992	508.165
2.994	458.125	2.993	508.422
2.995	458.4123	2.994	508.669
2.997	458.7097	2.996	508.901
2.998	458.9668	2.998	509.359
3	459.2339	3	509.596
3.001	459.4859	3.001	509.879
3.002	459.7681	3.003	510.131
3.005	460.3428	3.004	510.423
3.007	460.625	3.006	510.69
3.008	460.9022	3.007	510.952
3.01	461.1794	3.009	511.224
3.011	461.4768	3.01	511.476
3.013	461.7238	3.011	511.728
3.014	461.9859	3.014	512.172
3.016	462.2228	3.016	512.419
3.017	462.4698	3.017	512.671
3.018	462.7218	3.019	512.938
3.02	463.0091	3.02	513.246
3.021	463.3065	3.022	513.508
3.023	463.6089	3.024	513.876
3.024	463.8609	3.026	514.143
3.026	464.0978	3.027	514.395
3.027	464.3498	3.029	514.662
3.028	464.622	3.03	514.924
3.03	464.8841	3.032	515.206
3.031	465.1664	3.033	515.488
3.033	465.4335	3.035	515.811
3.034	465.7258	3.036	516.088
3.036	466.0181	3.038	516.33
3.037	466.2752	3.039	516.557
3.039	466.5272	3.04	516.799
3.04	466.7238	3.042	517.071
3.041	467.0313	3.043	517.298
3.043	467.2782	3.045	517.54
3.046	467.8226	3.046	517.782
3.047	468.1099	3.048	518.064
3.049	468.372	3.049	518.326
3.05	468.6492	3.05	518.598
3.052	468.9466	3.052	518.865
3.053	469.2086	3.053	519.097
3.056	469.7178	3.055	519.344
3.057	469.9748	3.056	519.581
3.059	470.2268	3.058	519.798
3.062	470.7762	3.059	520.065
3.063	471.0383	3.061	520.327
3.065	471.3256	3.063	520.897
3.066	471.623	3.065	521.139
3.068	471.8599	3.066	521.381
3.071	472.4143	3.068	521.638
3.072	472.631	3.069	521.925
3.073	472.8226	3.071	522.167
3.075	473.1099	3.072	522.419
3.078	473.6542	3.073	522.686
3.079	473.9314	3.075	522.953
3.081	474.2137	3.078	523.457
3.082	474.501	3.079	523.744
3.084	474.7631	3.081	524.032
3.085	475.0151	3.083	524.314
3.086	475.2772	3.084	524.571

3.088	475.5443	3.085	524.818
3.09	475.8317	3.087	525.04
3.091	476.119	3.09	525.534
3.092	476.4012	3.091	525.806
3.095	476.9506	3.092	526.033
3.097	477.2329	3.094	526.32
3.098	477.4899	3.095	526.597
3.101	478.0494	3.097	526.834
3.102	478.2712	3.098	527.076
3.104	478.4929	3.1	527.313
3.105	478.7399	3.101	527.57
3.107	478.9819	3.102	527.772
3.11	479.5766	3.104	528.029
3.111	479.874	3.105	528.266
3.113	480.1461	3.107	528.538
3.114	480.4083	3.108	528.82
3.117	480.9073	3.111	529.339
3.118	481.1391	3.113	529.586
3.12	481.3911	3.114	529.858
3.121	481.6885	3.116	530.115
3.123	481.9607	3.117	530.378
3.124	482.2329	3.118	530.604
3.126	482.5101	3.12	530.861
3.127	482.7621	3.121	531.134
3.129	483.0443	3.123	531.406
3.13	483.3216	3.126	531.996
3.131	483.5988	3.129	532.489
3.134	484.118	3.13	532.772
3.136	484.38	3.131	532.978
3.137	484.6421	3.133	533.23
3.139	484.9345	3.134	533.462
3.14	485.2218	3.136	533.724
3.142	485.5343	3.137	534.002
3.143	485.7964	3.139	534.274
3.145	486.0685	3.14	534.551
3.146	486.3054	3.142	534.828
3.147	486.5423	3.143	535.035
3.149	486.7944	3.145	535.262
3.15	487.0817	3.146	535.519
3.152	487.3186	3.147	535.786
3.153	487.5907	3.15	536.305
3.155	487.8578	3.152	536.577
3.156	488.1502	3.153	536.849
3.158	488.377	3.155	537.106
3.159	488.6341	3.156	537.343
3.16	488.871	3.158	537.59
3.162	489.1179	3.159	537.852
3.163	489.3901	3.16	538.13
3.165	489.6472	3.162	538.366
3.166	489.9093	3.163	538.624
3.168	490.2016	3.165	538.871
3.169	490.4688	3.166	539.138
3.17	490.746	3.168	539.415
3.174	491.2903	3.169	539.712
3.175	491.5524	3.171	540
3.176	491.8095	3.172	540.236
3.178	492.0665	3.174	540.483
3.179	492.3437	3.175	540.675
3.181	492.626	3.176	540.866
3.182	492.8679	3.178	541.154
3.185	493.4274	3.179	541.436
3.186	493.7147	3.182	541.945
3.188	493.9869	3.184	542.227
3.19	494.249	3.185	542.51
3.191	494.5111	3.186	542.747
3.192	494.7631	3.188	542.988
3.194	495	3.189	543.246

3.195	495.2873	3.191	543.493
3.197	495.5494	3.194	544.022
3.2	496.0786	3.195	544.299
3.201	496.376	3.197	544.566
3.203	496.623	3.199	545.055
3.204	496.8952	3.201	545.302
3.207	497.379	3.203	545.564
3.208	497.6462	3.204	545.816
3.21	497.9335	3.205	546.063
3.213	498.4778	3.207	546.31
3.214	498.7701	3.208	546.562
3.215	499.0272	3.21	546.834
3.217	499.2692	3.211	547.081
3.218	499.5313	3.213	547.374
3.22	499.8186	3.215	547.913
3.221	500.0806	3.217	548.17
3.223	500.3175	3.219	548.442
3.224	500.5847	3.221	548.881
3.226	500.8417	3.223	549.128
3.229	501.3962	3.224	549.37
3.23	501.7087	3.226	549.642
3.231	501.9607	3.227	549.914
3.233	502.2228	3.23	550.483
3.235	502.495	3.231	550.725
3.236	502.7117	3.233	550.967
3.239	503.1855	3.234	551.209
3.24	503.4576	3.237	551.688
3.242	503.7349	3.238	551.945
3.244	504.3145	3.24	552.217
3.246	504.5766	3.242	552.495
3.248	504.8437	3.243	552.742
3.249	505.0907	3.244	552.994
3.25	505.3427	3.246	553.256
3.253	505.8216	3.247	553.548
3.255	506.0585	3.249	553.82
3.256	506.3206	3.25	554.062
3.258	506.5927	3.252	554.314
3.26	507.1875	3.255	554.798
3.262	507.4194	3.256	555.04
3.263	507.6059	3.258	555.347
3.265	507.873	3.26	555.917
3.266	508.1199	3.262	556.169
3.267	508.4022	3.263	556.406
3.269	508.6996	3.265	556.638
3.271	508.9668	3.266	556.875
3.272	509.2339	3.267	557.137
3.273	509.506	3.269	557.389
3.275	509.8084	3.271	557.646
3.278	510.3075	3.272	557.928
3.279	510.5746	3.273	558.23
3.281	510.8115	3.276	558.674
3.282	511.0736	3.278	558.901
3.283	511.25	3.281	559.395
3.285	511.4768	3.282	559.637
3.287	511.7742	3.283	559.899
3.289	512.3236	3.286	560.433
3.291	512.6109	3.288	560.695
3.292	512.8679	3.289	560.937
3.294	513.1099	3.291	561.189
3.295	513.3468	3.292	561.461
3.296	513.5938	3.294	561.718
3.298	513.871	3.295	561.95
3.299	514.1633	3.296	562.202
3.301	514.4203	3.298	562.479
3.302	514.6824	3.299	562.736
3.304	514.9748	3.302	563.256
3.305	515.2218	3.304	563.548

3.307	515.4738	3.305	563.805
3.308	515.7661	3.307	564.057
3.31	516.0383	3.308	564.329
3.311	516.3054	3.31	564.551
3.312	516.5474	3.311	564.773
3.314	516.8196	3.312	565.04
3.317	517.3337	3.314	565.302
3.318	517.5957	3.315	565.569
3.32	517.8931	3.317	565.801
3.321	518.1905	3.32	566.34
3.323	518.4577	3.321	566.572
3.324	518.6895	3.323	566.819
3.326	518.9264	3.324	567.066
3.328	519.4304	3.325	567.323
3.33	519.7026	3.327	567.57
3.333	520.2218	3.328	567.812
3.334	520.4889	3.329	568.094
3.336	520.7762	3.331	568.361
3.337	521.0333	3.334	568.896
3.339	521.2954	3.335	569.138
3.34	521.5373	3.337	569.395
3.341	521.7893	3.339	569.647
3.343	522.0564	3.341	570.126
3.344	522.3286	3.343	570.383
3.346	522.6008	3.344	570.645
3.347	522.878	3.346	570.937
3.35	523.4073	3.347	571.229
3.352	523.6693	3.349	571.497
3.353	523.9415	3.35	571.733
3.356	524.496	3.351	571.996
3.357	524.7278	3.353	572.242
3.359	524.9748	3.354	572.474
3.36	525.2671	3.356	572.696
3.362	525.5443	3.357	572.943
3.365	526.124	3.359	573.205
3.366	526.3861	3.36	573.472
3.368	526.6583	3.362	573.755
3.369	526.9254	3.363	574.037
3.372	527.3589	3.365	574.274
3.373	527.6109	3.366	574.541
3.375	527.8831	3.368	574.798
3.376	528.1653	3.369	575.03
3.378	528.2208	3.37	575.267
3.379	528.5282	3.373	575.746
3.382	529.0171	3.374	576.008
3.384	529.2541	3.376	576.275
3.385	529.5161	3.378	576.572
3.388	530.1209	3.379	576.819
3.389	530.4083	3.38	577.046
3.391	530.6805	3.382	577.293
3.392	530.9778	3.385	577.782
3.394	531.25	3.386	578.034
3.395	531.4919	3.388	578.281
3.396	531.7439	3.389	578.538
3.398	532.0262	3.391	578.81
3.4	532.3135	3.392	579.112
3.401	532.5807	3.394	579.38
3.404	533.1048	3.395	579.627
3.405	533.382	3.396	579.848
3.407	533.6442	3.398	579.974
3.408	533.9567	3.399	580.196
3.41	534.2339	3.401	580.423
3.411	534.506	3.402	580.68
3.413	534.7631	3.403	580.937
3.415	535.2117	3.407	581.471
3.417	535.4637	3.408	581.759
3.418	535.746	3.409	582.041

3.421	536.2601	3.412	582.51
3.423	536.5625	3.414	582.767
3.424	536.8246	3.415	583.034
3.426	537.0766	3.417	583.311
3.427	537.3438	3.418	583.558
3.429	537.6008	3.421	584.082
3.43	537.8578	3.422	584.344
3.431	538.0947	3.424	584.601
3.433	538.3669	3.425	584.868
3.434	538.6492	3.428	585.383
3.436	538.9163	3.43	585.614
3.437	539.1835	3.431	585.877
3.439	539.4707	3.432	586.124
3.44	539.7228	3.434	586.391
3.441	539.9798	3.436	586.638
3.443	540.2319	3.438	587.187
3.444	540.4889	3.44	587.469
3.446	540.1462	3.441	587.741
3.447	540.1664	3.444	588.175
3.449	517.253	3.446	588.412
3.451	500.9426	3.447	588.674
3.452	500.5847	3.448	588.906
3.453	500.8921	3.45	589.163
3.455	501.6935	3.453	589.727
3.456	502.4395	3.454	589.969
3.458	503.13	3.456	590.231
3.459	503.8256	3.457	590.478
3.462	505.0957	3.46	590.922
3.463	505.751	3.462	591.199
3.465	506.2853	3.463	591.436
3.466	506.875	3.464	591.703
3.468	507.4849	3.466	591.985
3.469	508.1048	3.467	592.253
3.471	508.6492	3.469	592.479
3.472	509.1986	3.47	592.747
3.474	509.7379	3.472	592.983
3.475	510.2671	3.473	593.235
3.476	510.751	3.475	593.482
3.478	511.2701	3.477	593.961
3.479	511.7843	3.479	594.243
3.481	512.2833	3.481	594.506
3.482	512.7772	3.482	594.793
3.484	513.2409	3.483	595.065
3.485	513.7349	3.491	179.299
3.486	514.2137	3.494	0.085
3.488	514.6875	3.49	0.085
3.489	515.1613	3.49	0.11
3.491	515.62	3.491	0
3.494	516.4919	3.491	0.136
3.495	516.9557	3.493	0.221
3.497	517.4193	3.495	0.105
3.498	517.9083	3.497	0.11
3.501	518.755	3.499	0.236
3.502	519.1431	3.5	0.156
3.504	519.5212	3.502	0.105
3.505	519.9143	3.503	0.211
3.508	520.6905	3.505	0.211
3.51	521.1038	3.506	0.115
3.511	521.5272	3.508	0.171
3.513	521.9254	3.511	0.161
3.516	520.8871	3.512	0.131
3.517	521.003	3.514	0.191
3.519	521.2903	3.515	0.161
3.52	521.6079	3.516	0.11
3.521	521.9254	3.518	0.141
3.523	522.3185	3.519	0.171
3.524	521.5726	3.521	0.1

3.526	517.5655	3.522	0.105
3.527	517.2883	3.525	0.115
3.529	516.9657	3.527	0.075
3.53	515.373	3.528	0.121
3.533	513.372	3.529	0.121
3.534	512.4748	3.531	0.07
3.536	510.0252	3.532	0.09
3.538	456.8548	3.534	0.11
3.548	0	3.535	0.08
3.545	0	3.537	0.055
3.543	0.0907	3.538	0.095
3.543	0.0605	3.539	0.095
3.543	0.0958	3.541	0.05
3.545	0.1714	3.542	0.08
3.547	0.131	3.544	0.085
3.55	0.1915	3.545	0.055
3.551	0.1966	3.548	0.075
3.553	0.1562	3.55	0.065
3.556	0.2117	3.551	0
3.557	0.1764	3.553	0.055

APPENDIX N Results comparison of ANSYS and LS-Dyna

This section presents the comparison of results between the two FE software packages. LS-Dyna and ANSYS finite element analysis was carried out to check the suitability of the two packages for predicting the residual stresses. Identical material properties, fibre volume fraction and thermal loadings were applied between each group. Finite element results between two FE software packages are given in APPENDIX N. It is to be noted that the simulation results presented are only to compare the two finite element software packages. The results from the two packages demonstrated high similarity ranging from 89% to 99%, which proves the effectiveness of LS-Dyna in thermal stress analysis.

The FEA models in Table 1 are using the material properties presented in Chapter 3. Thermal loading applied on the models in group (1) is -507K. For FE models in groups (2) and (3), the thermal loading is -50K.

Table 1 Finite element results of ANSYS and LS-Dyna

Group	LS-Dyna	ANSYS
(1)	GLOBAL Y Direct Stress	MY
(2)	GLOBAL Z Direct Stress	

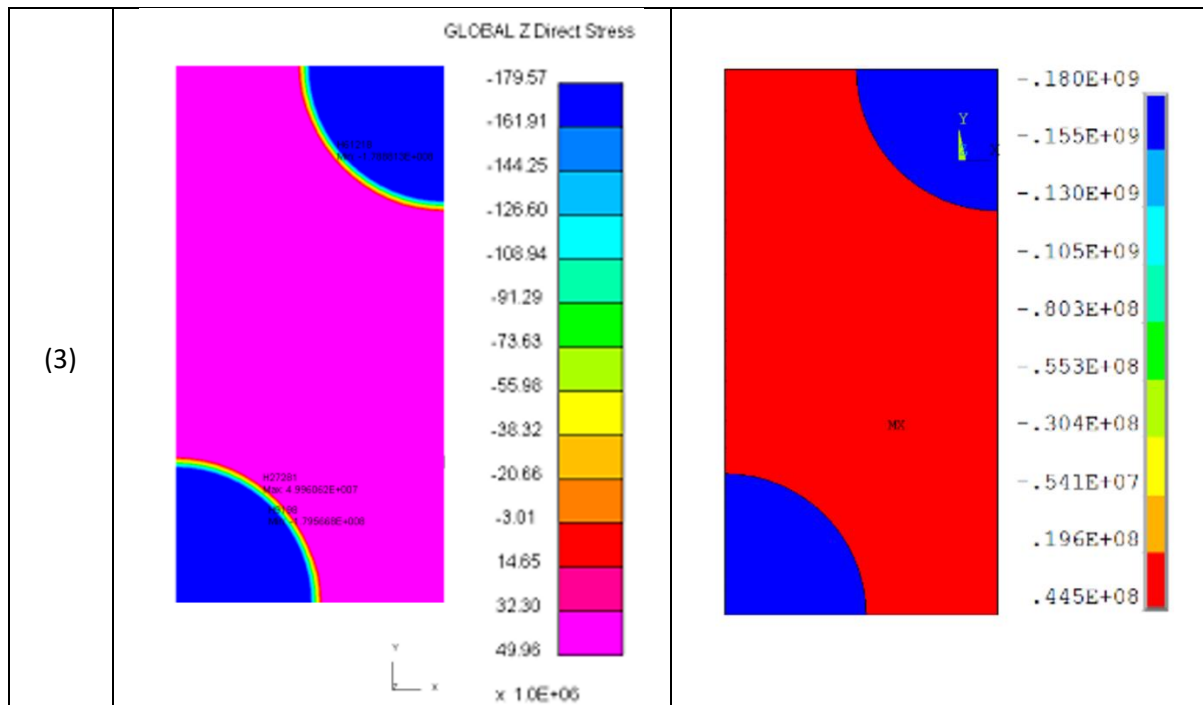


Table 2 Thermal residual stress comparison

Group number	FE package	Axial σ_f (MPa)	Similarity	Axial σ_m (MPa)	Similarity
Group 1	LS-Dyna	-2329	99%	593	97%
	ANSYS	-2300		576	
Group 2	LS-Dyna	-178.94	96%	49.7	94%
	ANSYS	-172		52.6	
Group 3	LS-Dyna	-179.57	99%	49.96	89%
	ANSYS	-180		44.5	

Table 2 presents the residual thermal stress results from the two FE simulations. Three types of microstructure models with different geometries and volume fractions were investigated. Thermal residual stresses were compared, and their ratios are shown in terms of similarity in the table. 3D models with complicated layups utilising elastic-plastic and creep material behaviour take a long time to run so a decision was made to run the models using LS-Dyna. This was due to the availability of multiple research licences allowing models to be run simultaneously and the implementation within LS-Dyna of a suitable material creep model.

The results from the two packages demonstrated high similarity ranging from 89% to 99%, which proves the effectiveness of LS-Dyna in thermal stress analysis. Even though the results

are similar to the FE analysis shown in previous sections, the results should not be compared with the previous thermal stress results due to the different thermal loadings applied.

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