Quotitive division in the bilingual classroom: Exploring structure to support the development of conceptual understanding with primaryaged multilingual learners

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This paper focuses on how exploring the mathematical structure of quotitive division supported multilingual learners to develop conceptual understanding in a focused maths lesson. The pupils learnt mathematics in an immersion bilingual context, with their lessons delivered through the mediums of English and French equally. The extract selected for this paper was recorded as part of a series of 4 lessons, video-recorded, transcribed and then analysed using discourse analysis as part of a larger PhD study. This extract illustrates how the teachers' focus on structure presented pupils with the language required to discuss their mathematical thinking, scaffolding the learning to enable pupils to draw upon their full linguistic repertoire as they solved the quotitive division problem.

Keywords: Multiplicative reasoning, quotitive division, mathematical structure, multilingual learners, bilingual education.

Introduction

Teachers and learners of mathematics are often caught between the two worlds of language and mathematics (Barwell, 2021). In recent years, mathematics education research has shifted perspectives, from viewing maths learning through deficit-orientations towards additive recognition of the resources and competencies of diverse pupil populations (Planas, Morgan & Schütte, 2021). Alongside this, linguistics research into bilingualism has shifted from viewing languages as discrete and separate, towards a more holistic/heteroglossic perspective, in which language is viewed as interconnected and contributing towards a single, universal linguistic system (García & Wei, 2014). This perspective underpins the notion of translanguaging as a pedagogical strategy, building bilingual students' language practices flexibly, in order to develop new understanding and language use (García, Flores & Woodley, 2012). Halliday defines a mathematics register as a "set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (Halliday, 1975, p. 65). O'Halloran (2015) has broadened this definition of register to incorporate elements of mathematical symbolism and drawn mathematical images, since these also contribute significantly to one's ability to communicate in mathematics. Pimm (2021) states that a personal mathematical register may cross languages. Furthermore, mathematical registers of any specific language need to enable the articulation of mathematics orally, in whichever natural language is required, which Pimm describes as "linguifying" mathematical symbols (Pimm, 2021, p.27). Thus, in a classroom, this requires the use of the language of instruction, which presents interesting complexities for those multilingual learners working in a monolingual classroom and those in bilingual education contexts (Pimm, 2021).

The data for this paper is drawn from a PhD study, exploring how translanguaing may support the development of conceptual understanding in multilingual learners who learn mathematics in a bilingual curriculum context. The setting for this study is an 'immersion-style' bilingual school in England, in which pupils learn through the medium of English for the first half of the week and then

through either French, German or Spanish for the remainder of the week. The school is a non-selective, all-through free school, which was first established in 2012. The particular focus for this paper is how exploring mathematical structure may support pupils' understanding of quotitive division, as the pupils draw upon their full linguistic repertoire, in the process of discussing and articulating their mathematical thinking.

Theoretical Framework

This study adopts a sociocultural and social constructivist viewpoint. Language is perceived as holistic/heteroglossic and classroom dialogue supports pupils' conceptual and linguistic development through translanguaging practices, building on Pimm's (2021) notion of linguifying mathematical symbols as discussed above. In this study, this lens is applied to the concept of quotitive division.

The challenge of division

Division was chosen as the focus for the study, as it is often considered the hardest concept of the four calculation operations to grasp, both in terms of pedagogy and psychology (Back, 2011). Understanding division draws heavily upon pupils' understanding of addition, subtraction and multiplication and forms the foundations for other mathematical concepts such as fractions, decimals, ratio and proportion (Back, 2011). As the connections between division and a wide range of other concepts of mathematics are significant, division is considered a fundamental foundation concept for mathematics learning in the primary curriculum (Back, 2011). Multiplicative reasoning is vital for developing understanding of other areas of maths, such as fractions, functions and algebra (Lu & Richardson, 2018). However, the required shift from the 'one to one' understanding of additive reasoning to the 'one to many' required for multiplicative reasoning is 'not trivial' and is often challenging (Siemon et al., 2005, p.6). Since division and multiplication are inverse operations, they are connected in terms of their structures and the underlying reasoning required to solve them. In particular, studies have illustrated how children aged 8-9 years old perform well on multiplicative tasks that require repeated addition but struggle where different multiplicative and divisive structures are required (Squire, Davies & Bryant, 2004). Thus, division was selected as the focus for this paper, due to the opportunities such challenge presents in revealing the classroom discourse in a Y4 class.

The role of structure in developing conceptual understanding

Research demonstrates the importance of understanding the mathematical structure of the operations (Venkat et al., 2019). Kieran (2018) states that structure is one of the big ideas of mathematics, since it can be found everywhere. Although there is no one agreed definition of what structure is (Venkat et al., 2019), it is often used when referring to the relationship between numbers and their associated connections (Kieran, 2018). The basic mathematical structures of division are recognised as partitive/sharing and quotitive/grouping. Sharing/partitive division relates to knowing the number of sets and calculating the number contained within each set. Grouping/quotitive division relates to knowing the number in each set and needing to calculate the number of sets. The latter is often the more challenging, yet least explored, structure of the two.

Methodology

This paper is based on a 6-minute teaching interaction between a teacher and her pupils, collected as part of the larger PhD study utilising case study methodology. The participating teachers co-teach

one Y4 class. They are both qualified teachers, native speakers of their respective languages and experienced in working with multilingual learners. Beth[1] teaches all subjects, including maths, through English on Mondays, Tuesdays and Wednesday mornings. Lauren teaches all subjects, including maths, through French on Wednesday afternoons, Thursdays and Fridays. Both teachers use textbooks and resources in their own languages and the classroom environment includes maths resources in both languages. The pupils in their class have a diverse linguistic profile. All but two pupils have either French or English as a home language and only four were new to the school since Reception. There is wide variation in the French proficiency amongst the pupils. Four maths lessons were video-recorded for the PhD study, two English and two French, all focusing on developing conceptual understanding of division. Audio recordings were made of the pupil-pupil interactions as they worked. All recordings were transcribed (translated into English where necessary) and then analysed using discourse analysis (Walsh, 2011). The researcher then identified what they considered to be mathematically-significant moments where conceptual understanding was explored through the discourse, including teacher-pupil, teaching assistant-pupil and pupil-pupil interactions.

Selected data

This paper focuses on a mathematically-significant moment in the first of the two French medium lessons. In the preceding three lessons, Beth introduced the structure of multiplication and division, introducing the correct English terminology/vocabulary for the component parts of each structure. In English, she demonstrated how the different parts fit together as inverses, e.g. the product in multiplication becoming the dividend in division. The teachers use translanguaing pedagogies where possible, including in this instance where they purposefully connect lesson objectives and related vocabulary. This provides a progressive and connected teaching sequence across the week and facilitates pupils' language development. By working on the same concept and vocabulary in two different languages, pupils are encouraged to use their full linguistic repertoire in all lessons, regardless of the target language, if and when this is beneficial to their conceptual development.

The short extract presented here was selected due to its purposeful use of language, terminology and structure to support pupils in developing a deeper understanding of this structure of division. Lauren selected a problem from her usual textbook to work with the pupils on quotitive division (grouping) within a real-life scenario. The author acknowledges that the use of French in this extract is fragmented and not necessarily grammatically accurate. However, the author wished to represent the interaction as authentically as possible. It may be the case that the French was simplified by Lauren to support those with lower French proficiency.

Lauren read the problem to the class, which involved Grand-mére, Manon and Emeric baking 38 waffles and putting them into bags of 4. They are wondering how many bags of 4 they can fill. This question requires pupils to recognise that this problem has a grouping structure, in order to interpret the problem and then find the answer. The problem poses that Grandma thinks that they can fill 7 bags, Manon thinks 8 bags and Emeric thinks 10 bags. Therefore, the framing of the problem in this way increased its complexity, so that the pupils needed to calculate which of the three characters was correct, rather than just answer a simple 38÷4 calculation.

[1] Pseudonyms used throughout for teachers and pupils

Lauren: Voilà les gaufres. Donc- combien ont-ils fait de gaufres?
Here are the waffles. So- how many waffles did they make?
sev. pupils: Trente-huit
Thirty-eight
Lauren: Trente-huit gaufres. D'accord.
Thirty-eight waffles. Of course.

In this Initiation-Response-Evaluation, (IRE) sequence (Walsh, 2011), Lauren ensured that the pupils understood how many waffles there are, thus confirming the dividend. Lauren then explored with the pupils how many bags of waffles each person thinks that they can create. She presented the components of the calculations on the whiteboard, in a structure that mirrors their learning in their English maths lessons. In this way, she is emphasising the link between the vocabulary, as well as the link between multiplication and division as inverse operations (see figure 1).

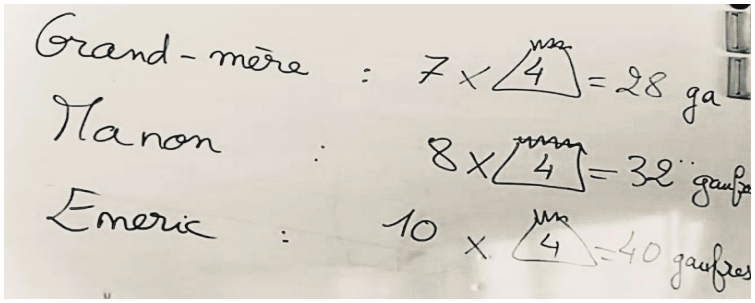

Figure 1: Systematic recording of the three possible solutions to the division problem.

Lauren continued to work with the pupils on the link between the two operations.

Lauren: Alors, ma question est…qui a raison? Grand-mère?
So, my question is…who is right? Grandma?

When no pupils offered an answer to this pivotal question, pupils were encouraged to discuss their ideas, some of whom discussed in French and others in English. This facilitated an informal translanguaging opportunity for the pupils, particularly those less proficient in French. Having already articulated their answers in their chosen language, pupils were then able to provide their output in French.

Pupil 1: Non, parce que nous avons 38 gaufres.
No, because we have 38 waffles.
Lauren: Parce que nous avons 38 gaufres et que Grand-mère en propose 28.
Because we have 38 waffles and grandma offers 28.
Pupil 2: C'est pas si. (Indicating that Grand-mére can't be right)
It's not so.
Lauren: C'est pas si…pourquoi? (Lauren repeated the pupil's language for encouragement)
It's not so… why?

Pupil 2's response provided the ideal opportunity to build on the mathematical structure. Lauren asked the pupils which operation they needed to use, to find the difference between the number of waffles made and how many are present in grandma's answer. This presented another informal translanguaing opportunity, in which pupils discussed in their chosen languages, before coming back together to work on the problem in French.

Lauren: Donc…alors…quelle opération je vais faire…je vais savoir combien de reste?
So…so…what operation am I going to do…will I know how much is left?

Pupil 3: Dix
Ten
Lauren: Dix…comment tu fais trouver dix?
Ten…how do you find ten?

Although the pupil gave the answer correctly, they were unable to explain how they knew that the difference was 10. The teacher wrote the calculation on the board, as shown in figure 2. This prompt suggests a subtraction but one pupil responded with addition, which was unexpected.

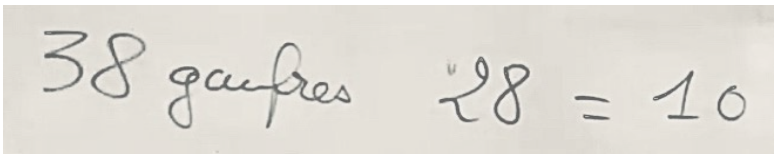

Figure 2: Prompting the pupils to explain how they found the solution of 10

Pupil 4: L'addition
Addition
Lauren: l'addition a quoi?
Addition to what?
Pupil 4: On peut faire 28 plus 10 égale 38?
Can we do 28 add 10 equals 38?

Lauren and the pupils then discussed whether this was an acceptable answer by discussing that addition is the inverse operation of subtraction. Lauren then draws the conversation back to the division question, in order to work with the difference of ten.

Lauren: Avec 10 gaufres, combien de sachets? Morris?
With 10 waffles, how many bags? Morris?
Pupil 2: Deux
Two
Lauren: Deux…d'accord…on peut faire deux sachets …
Two…of course…we can make two bags…
Pupil 5: Si il y a neuf sachets, neuf fois quatre égal 36…
If there are 9 bags, nine times 4 equals 36…
Lauren: D'accord…donc toi….tu proposes… mis sachets de 4 gaufres…tu multiplie par 9 … tu fait 9 fois et tu arrivé a…
Of course…so you…you suggest … bags of 4 waffles… you multiplied by 9…you do 9 times … and you arrived at …
Pupil 2: 36 gaufres…
36 waffles…
Lauren: Est-ce que j'arrive a 38 gaufres?
Did I get to 38 waffles?
Pupil 2: Non- deux
No- two
Lauren: Deux quoi?
Two what?
Pupil 2: Deux sachets?
Two bags?
Lauren: Non, pas de sachets.
No, not bags
Pupil 5: Reste!
Remainder!
Lauren: Reste! Elle est la mot que je cherche! Donc j'ai 38 gaufres…
répartis…partagées…distribuées dans mis sachets et mon reste… 2
Remainder! That's the word I'm looking for! So I have
divided…shared…distributed…38 waffles in my bags and my remainder …2

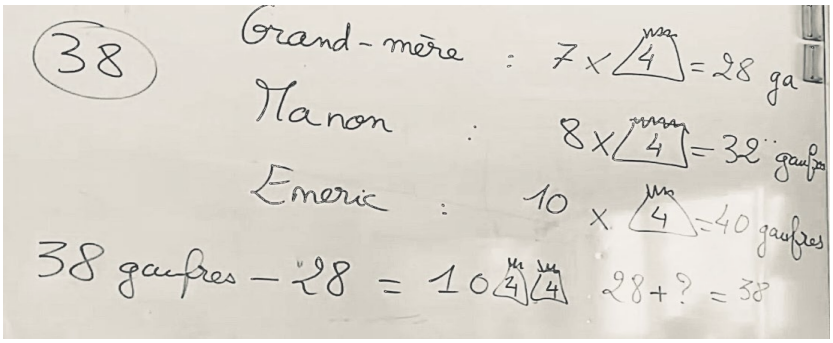

Figure 3: Lauren's systematic recording as the discussion progressed

Figure 3 shows how Lauren built the pictorial representations to illustrate the concept of regrouping into bags of 4. This focused the pupils' thinking on the structure of quotitive division, before they decided what to do with the remaining 2 waffles. Lauren prompted the pupils to recall the term for this, illustrating the importance of using contextualised mathematical terminology in her teaching.

Discussion

Exploring structure

Lauren's choice of question presented multiple ways in which the pupils could work with structure. Firstly, the question involved an understanding of quotitive division (grouping) and required that the pupils understood what the different numbers in the question represented. Furthermore, the way in which the question was phrased required deeper mathematical thinking. Had the question been 'how many bags of 4 waffles can they fill?' the solution would have been more straightforward and would not have provided such rich grounds for reasoning and discussion. Instead, offering a range of different scenarios provided the opportunity for deeper thinking and better connections for the pupils.

Building understanding through working across the linguistic repertoire

The extract presented illustrates how Lauren worked with language to question and prompt the pupils' thinking. Her focus remained on ensuring that the pupils understood the concepts and her use of visual representations supported this. For example, Lauren guided the pupils to make connections not just between division and multiplication as inverse operations, but across to addition and subtraction, which she supported visually in figure 3. This scaffolding may have helped pupils' understanding of the mathematical concepts alongside developing their French language. Facilitating opportunities to translanguage in their peer discussions afforded pupils the opportunity to justify and reason using their stronger language, before returning to the target language in their answer. Exploring the concept and terminology/vocabulary in context in both languages enabled pupils to draw upon their full linguistic repertoire as they worked, ensuring that the focus remained on developing their mathematical understanding, alongside the language development. By working on the same concept in English and French across lessons, pupils were supported to make connections not just within the mathematical areas, but also across the languages. This supported pupils to see language as interconnected and contributing towards their single, universal linguistic system (Garcia & Wei, 2014). Providing the opportunity to articulate their thinking and reasoning in both French and English, whilst working symbolically on structure, their personal mathematical registers are able to develop and the mathematical symbols can become "linguified" (Pimm, 2021, p.27).

Concluding remarks

In this paper, I have presented a short extract, in which pupils appear to have developed their understanding of quotitive division through discussion and articulation of a range of mathematical concepts. Focusing on the mathematical structure enabled the pupils to analyse the problem and to reason and justify. Although this extract shows pupils only working within the medium of French, pupils have the confidence and resilience to translanguage if/when necessary, as the teachers view language use fluidly. The two teachers working with this class have supported pupils' conceptual understanding through the use of specific terminology in both languages of instruction. By connecting this to the mathematical structures, pupils' understanding of the component parts may be enhanced, providing them with the language with which to articulate their ideas. In this way, the teachers have facilitated the integration of the two worlds identified by Barwell (2021).

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