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New Algorithms for Optimal Reduction of Technical Risks

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The paper features exact algorithms for reduction of technical risk by (i) optimal allocation of resources in the case where the total potential loss from several sources of risk is a sum of the potential losses from the individual sources; (ii) optimal allocation of resources to achieve a maximum reduction of system failure and (iii) making an optimal choice among competing risky prospects. The paper demonstrated that the number of activities in a risky prospect is a key consideration in selecting a risky prospect. In this respect, the maximum expected profit criterion, widely used for making risk decisions is fundamentally flawed, because it does not consider the impact of the number of risk-reward activities in the risky prospects. A popular view, that if a single risk-reward bet is unacceptable then a sequence of independent risk-reward bets is also unacceptable has been analysed and proved incorrect.

Keywords: risk reduction, risk-reward bets, maximum expected profit.

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1. Introduction

In this work, the problem related to achieving a maximum reduction of technical risk has been investigated in two broad aspects: (i) a maximum risk reduction achieved as a result of optimal allocation of risk reduction resources and (ii) a maximum risk reduction achieved as a result of making an optimal choice among risky prospects.

In turn, the optimal allocation of risk-reduction resources has been discussed in two major directions: a) in the case where the total potential loss from several sources of risk is a sum of the potential losses from the individual sources; b) in the common case of several units logically arranged in series and c) in the case where a complex system is built within a fixed budget.

The problem of optimal allocation of limited resources to attain a maximum risk reduction, is an important problem which appears frequently in the budget planning of companies and enterprises and during the design of complex safety-critical systems. Huge amounts of invested resources are often wasted because the resource allocations are usually far from optimal and do not guarantee the maximum possible total risk reduction.

Despite the importance of this problem, few attempts have been made to solve it. Sherali et al. (2008) for example, solved the risk optimization problem on event trees, by using linear programming and assuming that the consequences are a linear function of the level of investment. This assumption however is too restrictive, and often not confirmed by the real-life observations. Often, with increasing the level of investment, the magnitude of the consequences decreases in a non-linear fashion and reaches a saturation region, beyond which, an increase of the level of investment leads to a small decrease in the magnitude of the consequences. A typical example is increasing the level of investment in protecting against the harmful effect from disintegrating rotating parts. After attaining a particular level of protection, a further investment towards strengthening protection has a negligible effect on the magnitude of the consequences.

Richter et al. (1999), solved an optimal resource allocation problem to achieve a maximum prevention from infection. The objective function of the formulated model however was rather simple, involving only two additive terms, corresponding to two independent populations. Furthermore, no details were given about the optimization algorithm.

Mehr and Tumer (2006) solved the optimal budget allocation problem as a portfolio optimization problem, similar to the problem commonly solved in managing investment portfolios. This model however has a narrow application and cannot be used in the important case of individual risk reduction options, where, for each risk-reduction option, the amount of removed risk is well defined by a function or a table.

The optimal risk reduction problem is very important for safety- critical systems, which are widely used in industry, hospitals, construction engineering, and public buildings.

The loss from failure of a safety-critical system can be very high. This circumstance significantly affects their design, which has needs to be a risk-based design. Safety-critical systems must be highly reliable, to guarantee a small risk of failure yet they must be designed at affordable cost, often within a limited budget. During a design of a safety-critical system, at least two options are usually present for each component – to improve the reliability of the component or to introduce redundancy. Both options increase the reliability of the system. The problem is how to allocate the limited risk-reduction resources among the available options so that the risk of system failure is minimized. This problem is considerably complicated if the exact architecture (topology) of the system to be designed is unknown. Unfortunately, the traditional reliability optimization design concentrates on optimal allocation of redundancy or on reliability allocation among the components in a system (Kuo and Prasad, 2000; Kuo et al., 2001). The traditional reliability optimization design operates on a system with fixed architecture. As a result, existing design tools *do not* perform a repeated modification of the system topology and *do not* search in a large space of alternatives in order to determine the system design which combines a maximum possible reliability attained within a specified budget. This is the reason why the existing computational tools *are not capable of supporting the optimal design of safety-critical systems*.

In the complex, safety-critical systems used today, there is a very large number of possibilities for selecting components with different reliabilities and costs, design configurations, cross-bridges and redundancies (e.g. active, standby, k-out-of-n, etc.).

In this huge space of alternatives, identifying the optimum set of alternatives for the components, the optimum system architecture and the necessary redundancies is not a trivial task. Without the right models and tools, design alternatives far from optimal will be selected - either associated with a significant risk of failure or with a significant cost for building the system.

Consequently, one of the objectives of this work, is developing an algorithm and a software tool for minimizing the risk of failure of a safety-critical system, within a specified budget for building the system, in the case where the exact architecture of the system is unknown.

Very often, the optimal allocation of risk-reduction resources faces another problem. For a company or an enterprise for example, it is important to determine how to allocate its budget among a set of sources of risk: accidents, theft, warranty claims, equipment failures, etc., so that a maximum total risk is removed. The potential losses can be reduced by purchasing new, more reliable and safer equipment, investing in personnel training, investing in improved security and control, investing in improved reliability of the components and systems, etc. For each specified risk-reduction option, it is usually known from statistical data, how much risk-reduction effect is achieved for a particular level of investment.

There have been attempts to solve an optimal resource allocation problem, related to maximising the gain in production with a nonlinear-additive objective function and a single linear constraint. This work has been comprehensively reviewed in (Zipkin, 1980). Unfortunately, the presented solutions require severe constraints on the functions describing the gain – *a requirement for linear or monotonically increasing and concave functions*.

Unfortunately, these methods are not suitable for solving the optimal risk reduction problem. Typically, the functions describing the risk removal are non-linear because, in general, with increasing the level of investment, the amount of removed risk per unit investment decreases. In other words, the amount of removed risk enters a saturation region.

A typical example is removing bugs from a large piece of software and removing faults from a large system. After a certain level of investment towards bugs/faults removal, further investment yields very little. If a number of software subroutines in a large programme or a number of electronic blocks composing a large system need to be tested for faults, the optimal distribution of testing resources aimed at achieving a maximum bugs/fault reduction is not a trivial problem. Clearly, concentrating all of the available resources on a single subroutine or on a single electronic block is a poor strategy which yields insufficient overall reduction of the risk of system failure. Similarly, concentrating all of the available resources on a single component (e.g. the least reliable component x), among several components arranged in series is also a poor strategy for reducing the risk of system failure. This is because the reliability of a system composed of components logically arranged in series is limited by the reliability of the least reliable component x. Once its reliability level is brought up to the reliability level of the next least reliable component y, a further improvement of the reliability of component x no longer improves the reliability of the system, which is now limited by the reliability of component y.

Furthermore, the functions describing the amount of removed risk are not even continuous, because they are usually presented in tabular form. Finally, the functions describing the risk removal are not monotonically increasing or concave in general. Indeed, a resource contribution towards a particular risk-reduction option, often does not automatically translate into a risk reduction. Often, the contribution of the allocated resource needs to reach a particular threshold level before a risk reduction follows. Meanwhile, the risk level could remain the same or even increase. Consider for example the case where the risk source is 'erroneous transaction'. Introducing a new computer system in order to reduce the risk of erroneous transactions, could initially be associated with increased potential losses until the staff becomes familiar with the new system. After an initial increase, the losses from erroneous transactions start to decrease. The investment into a new production equipment to reduce the risk of lost production also follows a similar trend. Despite the heavy investment, failures at the start of operation are frequent (early-life failures), until the staff learns to avoid costly operational mistakes.

In this respect, it is important to present an exact solution of the optimal risk reduction problem, without imposing constraints on the risk reduction functions. This is another objective of this paper.

The next direction in the optimal risk reduction is the optimal choice from a number of risky prospects, each containing a set of *risk-reward activities*. This problem is part of an important class of risk decisions made in business, economics, technology, medicine, etc. Currently, the *maximum expected profit criterion* is used for making an optimal choice among risky prospects (Moore 1983; Denardo, 2002). According to this criterion, a rational decision maker compares the expected profits from a number of risky prospects and selects the prospect with the largest expected profit. An expected profit from a risky prospect is obtained by adding the monetary outcomes characterising the prospect multiplied by their probabilities. As it has been demonstrated in this work, *a choice based on maximizing the expected profit is deeply flawed if a small number of risk-reward bets are present in the risky prospects*.

In the past, the expected profit from an infinite number of statistically independent repeated bets has led to the *Petersburg paradox*. Its avoidance was one of the reason for proposing the expected utility theory by D.Bernouli (1738), later developed by von Neumann and O.Morgenstern (1944). The effort towards understanding statistically independent, repeated bets did not stop with the work of D.Bernouli. Statistically independent repeated bets for example, have been at the focus of a paper from Samuelson (1963). In this paper the author brings the following argument, through a story in which he offered to his colleague a good bet (with a positive expected value): 50-50 chance of winning 200 or losing 100. The colleague refused by saying that he would feel the 100 loss more than the 200 gain. He said that he would agree to participate if he was offered to make 100 such bets. Samuelson criticised the reasoning of his colleague and went on to propose and prove a "theorem" which stated that if a single good bet is unacceptable then any finite sequence of such bets is unacceptable too. Samuelson claimed that increasing the number of unacceptable bets does not reduce the risk of a net loss and termed accepting a sequence of individually unacceptable bets 'a fallacy of large numbers'. Samuelson's "theorem" have been reproduced in several related papers (Ross, 1999). This "theorem" sprawned several related papers where researchers have extended Samuelson's condition to assure that they would not allow the 'fallacy of large numbers'. Ross (1999) pointed out that the Samuelson criterion, despite that it does not have a universal application, is strictly valid only for linear, risk-neutral utility functions.

In this paper, it is shown that contrary to the Samuelson's theory, there exist cases where increasing the number of unacceptable bets does reduce the risk of a net loss and this is demonstrated by using Samuelson's own example.

A frequently pointed out weakness of the expected profit criterion is that it assumes too much knowledge necessary to make a decision. The information regarding the likelihood of an event and the consequences associated with the event is rarely available. Here, it is shown that for a limited number of risk-reward activities in the risky prospects, the maximum expected profit criterion *could lead to accepting a decision associated with a large risk of a net loss*. It will be shown that this is true even with a full knowledge related to the likelihood of an event and its consequences, and without the existence of a subjective bias while making a decision.

The case considered in the developments to follow, is where the results from the different outcomes can be adequately measured in monetary terms and the analysis is confined to linear utility functions. For the sake of simplicity, the inadequacy of the maximum expected profit criterion is demonstrated for the simplest case, involving *statistically independent risk-reward bets* in the risky prospects.

It will be demonstrated that the maximum expected profit criterion does not account for the significant impact of the actual number of risk-reward events/bets in a risky prospect. The choice under risk is made as if each compared risky prospect contains a very large number of risk-reward events/bets. Because the maximum expected profit criterion as a decision-making tool is incompatible with the risk of a net loss, the critical dependence of the choice on the number of risk-reward bets in the risky prospects has not been discussed in studies related to ranking risky alternatives (Tobin, 1958; Hansch and Leuy, 1969; Hador and Russel, 1969). This is also true for more recent models related to ranking risky alternatives (Richardson and Outlaw, 2008, Nielsen and Jaffray, 2006; Starmer, 2000). Even in a recent, probably the most comprehensive treatise of the theory of betting (Epstein 2009), no discussion has been provided on the impact of the limited number of risk-reward bets on the choice of a risky prospect. Consequently, another objective of this paper is to show that the number of risk-reward events/bets in a risky prospect has a critical impact on the choice of the risky prospect and cannot be ignored.

2. Optimal risk reduction if the potential loss is a sum of the potential losses from several risk sources

The concept *potential loss* and its properties have been discussed extensively in (Todinov, 2006). This concept incorporates uncertainly associated with the occurrence of a loss-generating event and uncertainty associated with the extent of the consequences, given that the loss-generating event has occurred. The absolute value of the potential loss X from a loss-generating event/risk source will not be greater than a particular value $x \geq 0$, in the following two ways: (i) the loss-generating event will not occur, which means that the loss will be zero and therefore smaller than x, and (ii) the loss-generating event will occur and the loss is smaller than x (the loss is taken with a positive sign here). Consequently, according the total probability theorem, for a particular loss-generating event/bet, the distribution $C(x)$ of the potential loss is given by (Todinov, 2006)

$$P(X \leq x) \equiv C(x) = (1 - p_f)H(x) + p_f C(x \mid f) \tag{1}$$

where $C(x \mid f)$ is the conditional distribution of the loss given the event occurrence, p_f is the probability of occurrence of the loss-generating event and $H(x)$ is the Heaviside step function ($H(x) = 1$ for $x \geq 0$ and $H(x) = 0$ for $x < 0$). Taking expected values from both sides of equation (1), results in an expression regarding the expected value of the potential loss:

$$E[C(x)] = p_f \overline{C}_f \tag{2}$$

where $\overline{C}_f = E[C(x \mid f)]$ is the expected value of the potential loss given event occurrence. From equation (3), it follows that the expected value of the potential loss is numerically equal to the risk of the loss. This is because the expected value of the Heaviside step function is zero, $E[H(x)] = 0$. Equation (2) is in fact the classical risk equation (Henley & Kumamoto, 1981) $K = p_f C$, where the risk K is measured by the product of the probability of failure p_f and the cost C given failure.

Often, an additive relationship exists for the potential loss from several risk sources. Consider for example the following three sources of risk: (i) erroneous transactions associated with potential loss X_1, (ii) theft, associated with a potential loss X_2 and (iii) damage to equipment associated with a potential loss X_3. Clearly, in this common case, for the total potential loss X from the three risk sources, the relationship

$$X = X_1 + X_2 + X_3 \tag{3}$$

holds.

Accordingly, the problem considered in this section is related to optimal allocation of risk-reduction resources among risk sources, the total potential loss from which is a sum of the potential losses from the individual risk sources.

Consider now a general case where m risk sources exist, (not necessarily statistically independent), each of which is a source of potential loss. The potential losses from the separate risk sources are modelled by the random variables X_1, $X_2,..., X_m$. These random variables are *not necessarily statistically independent* because the sources of risks are not necessarily statistically independent. Consider the important case where the potential loss X from all sources of risk is a sum of the potential losses from the individual risk sources.

$$X = X_1 + X_2 + ... + X_m \tag{4}$$

According to a well-known theorem in statistics (DeGroot, 1989), the expected value of a sum of random variables is always equal to the sum of the expected values of the random variables, irrespective of whether the random variables are statistically independent or not:

$$E(X) = E(X_1) + E(X_2) + ... + E(X_m) \tag{5}$$

According to equation (2) however, the expected value of the potential loss X_i from risk source i is equal to the risk associated with the ith source. Therefore, the total risk K, which is equal to the expected value $E(X)$ of the total potential loss from all sources of risk is equal to the sum of the individual risks associated with the separate risk sources, irrespective of whether they are statistically independent or not:

$$K = p_{f1}\overline{C}_{f1} + p_{f2}\overline{C}_{f2} + ... + p_{fm}\overline{C}_{fm} \tag{6}$$

Suppose that for the ith risk reduction option/activity, $\varphi_i(x)$ determines the amount of removed risk (risk reduction) resulting from investing resources of amount x ($0 \le x \le B$) towards the reduction of the ith risk. For all 'i' $\varphi_i(0) = 0$. The problem is now expressed in terms of total removed risk $\varphi(x)$. Again, the amount of the total removed risk $\varphi(x)$ is equal to the sum of the amounts of risk $\varphi_i(x)$ (i=1,2,...,m) removed from each source, irrespective of whether the sources of risk are statistically independent or not.

The problem now consists of determining a vector of values **x*** = ($x_1^*, x_2^*,..., x_n^*$), which corresponds to the global maximum

$$\max \sum_{i=1}^{m} \varphi_i(x_i) \tag{7}$$

defined in the domain $0 \le x_i \le B$, (i=1,2,...,m), with a budget constraint

$$\sum_{i=1}^{m} x_i \le B \tag{8}$$

and additional constraints

$$0 \le x_i \le B, \ (i=1,2,...,m) \tag{9}$$

Solving the optimal risk-reduction problem (7-9) requires specifying the amount of removed risk $\varphi_i(x)$ associated with any level of investment x, for each risk source.

Here, a solution of the optimal risk-reduction problem is presented, by not putting any constraints on the risk reduction functions $\varphi_i(x)$ and not requiring these functions to be linear or monotonically increasing or concave.

Although, one of the first applications of the dynamic programming created by Bellman (1957) was for optimal resource allocation, to the best of our knowledge, dynamic programming algorithms have not yet been used for optimal reduction of the risk from multiple sources. Here, an exact dynamic programming algorithm for solving this risk optimization problem is proposed, which does not impose any constraints on the risk-reduction functions $\varphi_i(x)$.

First, a discrete step is selected – a quantum q of allocated resources ($B = nq$). As a result, the resources allocated to the separate risk sources are a multiple of the quantum of resources. The algorithm starts from the last step, assuming that the resource has already been allocated among the m-1 sources of risk and only a single risk source remains (the mth risk source). Suppose that the last step has been reached with amount of resources t ($0 \le t \le B$). Then, the only possibility at the last step is to allocate all of the remaining resources on the last risk source and get a risk reduction $\varphi_m(t)$. By specifying the whole possible spectrum of resources $t = 0, q, 2q, ..., nq$ $0 \le t \le B$, the optimum values $opt_m(0) = \varphi_m(0)$, $opt_m(q) = \varphi_m(q)$, $opt_m(2q) = \varphi_m(2q), ..., opt_m(nq) = \varphi_m(nq)$ are obtained. For each t, $0 \le t \le B$, the values of $x_m(t)$ and $opt_m(t)$ characterising the last source ($t = 0, q, 2q, ..., nq$) are recorded. The quantity $x_m(t)$ is the optimum resource allocation to source m if the amount of remaining resources is t. For the last risk source m, $x_m(0) = 0$, $x_m(q) = q, ..., x_m(nq) = nq$. The quantity $opt_m(t)$ is the maximum amount of removed risk for remaining resources t.

The last step has therefore been accomplished. Now suppose that the resource has already been allocated among the m-2 risk sources and the amount of remaining resources to be allocated is t. If the amount of allocated resources on the m-1st risk source is x, the amount of allocated resources on the mth risk source will be t-x. A value x ($x \le t$, $0 \le t \le B$) needs to be found, which maximizes the next expression:

$$opt_{m-1}(t) = \max_{x \le t} \{\varphi_{m-1}(x) + opt_m(t - x)\} \tag{10}$$

The amount of allocated resource x on the m-1st risk source varies between 0 and t. The remaining resource t with which the m-1st risk source has been reached varies between 0 and the size of the budget B. The value $x*$, which corresponds to the maximum and the maximum are recorded against the current value of the remaining resource t. As a result, $x_{m-1}(t) = x*$ is obtained for each possible value of the remaining resource t. In this way, the next two columns $x_{m-1}(t)$ and $opt_{m-1}(t)$ of the dynamic table are obtained.

If the i-th source has been reached with remaining resources t, an amount x ($x \le t$, $0 \le t \le B$) must be found, which maximizes

$$opt_i(t) = \max_{x \le t} \varphi_i(x) + opt_{i+1}(t - x)\} \tag{11}$$

where $\varphi_i(x)$ is the risk reduction obtained from allocating x amount of resources on the ith source and $opt_{i+1}(t - x)$ is the maximum amount of removed risk from all sources with indices greater than i, for the amount of remaining resources t-x.

The values of the $opt_{i+1}(t - x)$ however are already known, because the i+1^{st} step has already been optimized. For each t, $0 \le t \le B$, the amount of x varies between 0 and t, and the value $x*$, corresponding to the maximum and the maximum are recorded against the current value of the resource t. As a result, $x_i(t) = x*$ is obtained for each possible value of the remaining resource. In this way, the columns $x_i(t)$, $opt_i(t)$ of the dynamic table are obtained. Continuing in this fashion, the first risk source is reached. For the first risk source, there is no need to vary the value t because t is equal to the initial budget B (t=B). As a result, a value x_1^* ($x_1^* \le B$) needs to be found, such that it maximizes

$$\max_{x \le B} \{\varphi_1(x) + opt_2(B - x)\} \tag{12}$$

After allocating x_1^* risk-reduction resources on the first risk source, the remaining resources for the second source of risk are $t = B - x_1^*$. The maximum x_2^* corresponding to resources

$t = B - x_1^*$ can be found in the dynamic table; it is $x_2^* = x_2(B - x_1^*)$. Continuing in this fashion, the optimal allocations of risk reduction resources on all sources of risk will be made.

The proposed method will be illustrated by the following numerical example. Suppose that five sources of risk are present, characterised by risk reduction functions $\varphi_1(x) = 1.2 \times x^{0.37}$, $\varphi_2(x) = 40 \times (1 - \exp(-0.5x))$, $\varphi_3(x) = 25 \times (1 - \exp(-0.6x))$, $\varphi_4(x) = 2.4 \times x^{0.22}$ and $\varphi_5(x) = 20 \times (1 - \exp(-0.7x))$ approximating the reduction of risk from investing amount of x in reducing the risk from the separate risk sources. Suppose that the initial budget is 50 units. A *C*++ programme implementing the proposed algorithm yields the following allocation: 13,11,9,10,7 for the separate sources which reduces the total risk by 91.6 units.

In order to verify the proposed algorithm, a 'brute-force' Monte Carlo simulation algorithm has also been developed for optimal budget allocation. The simulation algorithm consists of the following.

Four random, uniformly distributed 'splitting points' were generated within the specified budget of *B*=50. They were ordered in ascending order p1,p2,p3 and p4 (0≤pi≤50). The parts of the budget allocated to the 5 sources of risk were p1-0 for the first risk source, p2-p1 for the second risk source, p3-p2 for the third risk source, p4-p3 for the fourth risk source and finally 50-p4 for the fifth risk source. Then, the total risk reduction characterising this budget allocation was calculated. This process was repeated hundreds of thousands of times and the maximum risk reduction was determined for all simulation trials. The budget allocation corresponding to the maximum risk reduction served as an approximation of the optimal budget allocation. The results from 1000000 simulation trials confirmed the optimal allocation 13,11,9,10,7 obtained from the dynamic programming algorithm.

3. Optimal allocation of risk-reduction resources to achieve a maximum reduction of the risk of failure of safety-critical systems

3.1 Optimal allocation of risk-reduction resources to systems with reliability networks in series

A very important special case in optimal allocation of risk-reduction resources is the case where the successful operation depends on the failure-free operation of each of several (*n*) units (e.g. components, pieces of equipment, pieces of software, etc.). In this case, the *n* units are logically arranged in series. Failure of any unit causes a system failure with expected cost $\overline{C}$. Each unit *i* is characterised by a reliability R_i. The risk-reduction resources can be distributed for increasing the reliabilities of the separate units. Increasing the amount of resources allocated for the reliability improvement of a particular unit does not necessarily translate into optimal improvement of the reliability of the system. Let *m1* be the index of the unit characterised by the smallest reliability and let R_{m1} be the reliability of this unit. The system reliability can be presented as

$$R_{sys} = R_1 \times R_2 \times \ldots \times R_{m1} \times \ldots \times R_n \tag{13}$$

and because $0 \le R_i \le 1$ holds for all $i = 1,2,\ldots,n$, the relationship $R_{sys} \le R_{m1}$ holds too. In words, the reliability of the system *is always limited by the unit with the smallest reliability*. The only way to increase the reliability of the system R_{sys} is to increase the reliability R_{m1} of the unit with index *m1*. The optimal allocation of resources then follows the following algorithm. Risk reduction resources are allocated on improving the reliability of unit *m1*, until its reliability becomes equal to the reliability R_{m2} of unit *m2*, characterised by the next smallest reliability in the system. Now, improving the reliability of unit *m1* is no longer beneficial because the system reliability R_{sys} will be limited by the reliability R_{m2} of unit *m2*, which will be now the unit with the smallest reliability. The system reliability R_{sys} will increase beyond R_{m2} only if resources are allocated to simultaneously increasing the reliabilities of units *m1* and *m2*, to the same level for both units. This process continues until the reliability level R_{m3} of unit *m3* characterised by the third smallest reliability is reached and so on.

Suppose that the only risk reduction option associated with an unit arranged in series is 'testing the unit for critical faults'. Each inspection costs a fixed amount *u*. Failure of an unit is caused by a critical fault and let p_i denote the probability that a critical fault will be present in unit *i*. The critical fault will certainly cause failure of the unit (and a system failure) if it goes unnoticed during testing/inspection of the unit. Because of imperfections, each inspection of unit *i* is associated with a probability q_i that the critical fault will be detected. Consequently, given that a critical fault is present in unit *i*, the probability of missing it after *n* independent inspections is $(1-q_i)^n$. The probability that a critical fault will be present in the *i*th unit after *n* inspections is $p_i(1-q_i)^n$, which is the product of the probability that the fault will be present and the probability that it will be missed by all inspections. The probability of failure of unit *i* after *n* inspections is then $p_i(1-q_i)^n$. Allocation of resources for inspections should be done initially on the unit characterised by the largest probability of failure until its probability of failure is reduced to the next largest probability of failure. The allocation of resources then continues on simultaneously reducing the probability of failure of the two units and so on.

3.2 Optimal allocation of risk-reduction resources to systems with complex reliability networks

A number (*n*) of possible options/activities are available, and each of the *n* non-negative variables $x_1, x_2, ..., x_n$, ($0 \le x_i \le a_i$) corresponds to the levels of the separate investments. If $x_i = 0$, the *i*-th option has not been not implemented; a value $x_i = t > 0$ means that an investment at a level *t* has been made in the *i*th option. The maximum investment levels which can be assigned to the separate options are given by a_i. A function $K(x_1, x_2, ..., x_n)$ regarding the risk of system failure is also defined, expressing the total risk as a function of the levels of investment in the possible options. Let the level of investment in the *i*th option be $x_i > 0$ if the option has been implemented and $x_i = 0$, if the option has not been implemented. The problem consists of determining a vector of values **x***=($x_1^*, x_2^*, ..., x_n^*$), which corresponds to the global minimum

$$\min K(x_1, x_2, ..., x_n) \tag{14}$$

of the risk of failure $K(x_1, x_2, ..., x_n)$, defined in the domain $0 \le x_i \le a_i$, (*i*=1,2,...,*n*), with a budget constraint

$$\sum_{i=1}^{n} x_i \le B \tag{15}$$

and option investment constraints

$$0 \le x_i \le a_i, \ (i=1,2,...,n) \tag{16}$$

For the sake of simplicity, the risk of failure will be measured by the probability of system failure. Essentially, this assumes ($C = 1$) for the consequences of failure, irrespective of what combination of component failures has caused the system failure. Consequently, the total risk function $K(x_1, x_2, ..., x_n)$ in equation (14) can be replaced by the probability of system failure $p_f(x_1, x_2, ..., x_n)$.

The goal is to minimize the probability of system failure within a fixed budget *B*. The system is modelled by its reliability network, representing the logical arrangement of the components. The system reliability is equal to the probability of existence of a path through working components in the reliability network, from the start node to a set of end nodes, after a specified period of operation of the system.

The idea behind the algorithm will be illustrated by an example.

Consider a basic safety critical block (Fig.1a) consisting of two components '*A*' and '*B*', logically arranged in series (for example, component *A* could be an electronic circuit and component *B* could be an electro-mechanical device controlled by the electronic circuit. Suppose, that the possible ways for reducing the risk of failure of the safety-critical block are: introducing up to three safety-critical blocks working in parallel, introducing bridges '*p*' and including active redundancy for each component. The full-complexity reliability network is shown in Fig.1b, where the slots marked by numbers 1,2,...,8, are in fact the available options $x_1, x_2, ..., x_8$. For each option, three possible levels exist: '1'- a single component in the slot, '2'- two components working in parallel and '0'- no component in the slot. The option levels constrains are therefore

$$0 \le x_i \le 2, \ (i=1,2,...,8)$$

where x_i can take only one of the discrete values '0', '1' or '2'.

By varying the levels of the options, *all possible reliability networks, locked in the full-complexity network, can be obtained*. Thus, the size of the space of all possible combinations of option levels for the full network in Fig.1b is 3^8. For larger networks, the size of the space of alternatives increases exponentially. Despite that not all of these alternatives correspond to feasible networks, the number of feasible networks within the total space of possible options is still a very large number.

For the sake of simplicity, suppose that the separate components in the slots of the full-complexity reliability network are characterized by negative exponential time to failure distributions $F_i(t) = 1 - \exp(-t/\eta_i)$, where the mean times to failure are $\eta_A = 3$ years, $\eta_B = 7$ years and $\eta_p = 2$ years. The reliabilities characterising the separate components, for a specified time interval (0,*t*) are therefore given by $R_i(t) = \exp(-t/\eta_i)$.

The specified time of operation for the safety critical block is $t = 2$ years. The costs of the single components are as follows: $c_A = \$130$, $c_B = \$390$ and $c_p = \$5$.

If a particular slot '*i*' contains a single component, the reliability of the slot is $R_i(t) = \exp(-t/\eta_i)$ and the associated cost of the option $c_i(1)$ is equal to the cost of the component $c_i(1) = c_{comp}$. If slot '*i*' contains two components working in parallel, the reliability of the slot is $1 - [1 - \exp(-t/\eta_i)]^2$ and the cost of the option becomes $c_i(2) = 2c_{comp}$. If no components are present in a particular slot, the reliability of the slot becomes zero and the cost of the option is also zero $c_i(0) = 0$. The available budget for implementing different risk reduction options is $B = \$1000$.

In order to optimize the topology of the safety-critical system, the central idea is to start with the reliability network with full complexity, including all possible bridges and redundancies. Different combinations of components (edges) from the full-complexity reliability network are then pruned by a *branch and bound* algorithm. This process permits prospective reliability networks embedded in the full-complexity reliability network to be explored. After pruning an edge, from the tree of the edges to be pruned, the risk of system failure of the network within $t = 2$ years is calculated by using a system reliability algorithm.

Here, we must point out that the reliability network (the structure function) of the optimal system *is not known in advance*. Only the reliability network (the structure function) of the full-complexity safety-critical system is known. The algorithm eliminates the unnecessary components from the full-complexity reliability network of the safety-critical system to reach the optimal reliability network (system architecture) locked in the full-complexity reliability network.

The topology optimization of the reliability network can also be done by an exact, full exhaustive search recursive algorithm. This solution however is suitable only for networks with a relatively small size. The topology optimization algorithm, based on the branch and bound search, is superior because it does not require all possible reliability networks locked in the full-complexity reliability network to be explored.

If the calculated probability of system failure is greater than the achieved so far minimum probability of system failure (corresponding to a reliability network which satisfies the budget constraint), further pruning of components is not performed. Here again, in order to stop further branching and unnecessary exploration of the space of alternative reliability networks, we use the following fact (not proved here). *By removing a component, from any slot of the full-complexity network, the probability of system failure cannot be reduced*. This is true because *the function giving the probability of system failure is monotonically decreasing in the reliabilities of the components in the slots*. We use this fact to construct an algorithm whose running time is superior to the running time of an algorithm based on a full exhaustive search.

If, for several pruned components, the probability of system failure is larger than the achieved so far minimum, there is no need for pruning other components from the reliability network in order to satisfy the budget constraint. Further pruning will only result in even larger probability of system failure. This permits the search to be continued, without having to descend on the sub-tree of pruned edges where the absolute minimum of the probability of system failure cannot possibly be found. The tree of alternatives that follows the removal of a component leads to reliability networks with a larger probability of system failure than the achieved so far minimum.

Suppose that, as a result of the pruning, the resultant reliability network satisfies the budget constraint and the probability of system failure is smaller than the achieved so far global minimum (record). In this case, the current record is replaced by the calculated probability of system failure characterizing the current system.

Take in Figure 1.

Take in Figure 2

Applying the topology optimization procedure to the network in Fig.1b, yields an optimum vector (1,2,1,2,2,0,1,0) for the levels of the available options.

This means that in the optimal network topology, there are single components in slot 1, slot 3 and slot 7, two components logically arranged in parallel in slot 2, slot 4 and slot 5 and no components in slots 6 and 8. The optimal reliability network is shown in Fig.2 with a probability of system failure within $t=2$ years equal to 0.34. The cost of the system is within the specified budget of B=\$1000. From the optimal reliability network, we can then reconstruct the optimal system architecture.

The results from the branch and bound algorithm have been verified with a result obtained from an independent recursive exhaustive search algorithm. The optimum vector (1,2,1,2,2,0,1,0) for the levels of the available options has also been obtained by the alternative method. This essentially verifies the proposed algorithm.

The presented method creates the attractive opportunity for increasing the reliability of common safety critical systems without increasing the cost of current designs.

Alternatively, the method can also be used for exploring whether a particular level of reliability can be achieved at a reduced cost for building the system. This creates the attractive opportunity for increasing the competitiveness of companies designing and manufacturing safety-critical systems.

4. Reducing technical risk by an optimal choice of a risky prospect

Companies and entrepreneurs often make decisions under risk. Investing in an activity whose outcome is uncertain is a commonly made decision. Such is for example the drilling for oil, the advertising campaigns for particular products on particular markets. The commonly used method for selection among risky prospects is the maximum expected profit criterion. According to the maximum expected profit criterion, the activity characterised by the largest expected profit is selected. Such a decision however, can be justified if the choice is associated with a large number of risk-reward bets, with high expected profit. Often, the opportunities for sequential bets are limited. A common cause for the bankruptcy of small companies is their inability to sustain financially the losses from several unsuccessful investments, despite that each investment may be characterised by a positive expected gain. Drilling for oil for example, or running a large advertising campaign may be associated with a large expected profit/payoff and a large probability of success, but if unsuccessful, they can also eat up all of the resources and bankrupt a small company. In contrast, large companies can sustain losses from a number of unsuccessful oil drillings or advertising campaigns and still be profitable in the long run. In short, small companies, because of their limited resources, are more likely to face the large risk associated with a small number of sequential bets. This is an example where the blind adherence to the maximum expected profit criterion has been and has remained a source of heavy losses.

The inadequacy of the expected profit criterion as a basis for making a choice between risky prospects containing a limited number of risk-reward events/bets activities will be demonstrated in the simplest possible case, where the risk-reward bets in the risky prospects are statistically independent.

Risk-reward events/bets can materialise as benefit or loss. An investment in a particular enterprise is a typical example of a risk-reward bet. A successful investment is associated with returns (benefits) while an unsuccessful investment is associated with losses. Usually, for risk-reward bets, the larger the magnitude of the potential loss, the larger is the magnitude of the potential benefit.

The importance of considering benefit in parallel with the loss has already been stressed in the literature (e.g. Hillson, 2002, Chapman, 2003). Hillson (2002) for example, proposed an integrated qualitative risk management approach for responding to identified threads and opportunities. An example of existing framework for dealing with opportunity and failure events is the double probability-impact matrix for opportunities and threats proposed by Hillson (2002). Accordingly, expected potential reward R can be defined, as a product of the probability p_s that a risk-reward event will materialise as 'success' and the benefit $\overline{B}$ given success:

$$R = p_s \overline{B} \tag{17}$$

In our opinion, it is not beneficial to treat the potential losses and the potential benefits as risk. Reserving the term risk for the potential loss only, provides a better analysis structure for risk reduction and profit increase. Separating potential benefit from potential loss focuses attention on eliminating hazards and creating opportunity events as a way of increasing the potential profit.

Suppose that $0 \le p_s \le 1$ is the probability that the risk-reward event/bet will be a 'success' and will bring benefits characterised by the conditional cumulative distribution $B_s(x \mid s)$ (*given* that the risk-reward event has materialised as success). Correspondingly, $0 \le p_f = 1 - p_s$ is the probability that the risk-reward event/bet will generate a loss, associated with a conditional cumulative distribution function $C_f(x \mid f)$ (given that the risk-reward event has materialised as a loss).

The expected values of the benefit and the loss given that the risk-reward event has materialised are denoted by $\overline{B}_s$ and $\overline{C}_f$, respectively.

The expected profit $\overline{G}$ from a risk-reward event/bet is given by:

$$\overline{G} = p_s \times \overline{B}_s + p_f \times \overline{C}_f \tag{18}$$

where p_s is the probability of a beneficial outcome with magnitude $\overline{B}_s$ of the expected benefits and $p_f = 1 - p_s$ is the probability of a loss with expected magnitude $\overline{C}_f$ (the loss $\overline{C}_f$ is taken with a negative sign).

An example of such a risk-reward event/bet has already been given with drilling for oil on a particular spot. Suppose that the geological analysis suggests that the probability of recovering oil by drilling on a particular spot is p_s. If oil is recovered, the benefit (after covering the cost of drilling) will be $\overline{B}_s$. With probability $p_f = 1 - p_s$ however, oil will not be recovered and a loss of magnitude C_f (the cost of drilling) will be incurred. Drilling an oil well is essentially a risk-reward bet.

A risky prospect may contain a number of risk-reward bets. The expected profit G_A from a risky prospect *A*, containing *M* risk-reward bets is given by:

$$\overline{G}_A = \sum_{i=1}^{M} \overline{G}_i \tag{19}$$

where G_i is the expected profit characterising the *i*th risk-reward bet in the risky prospect. Risk-reward bets with a positive expected potential profit ($\overline{G} > 0$) will be referred to as *risk-reward opportunities* or *opportunity bets*, while risk-reward bets with a negative potential expected profit ($\overline{G} < 0$) will be referred to as *risk-reward gambles*. Note that the concept 'opportunity bet' is the same as the concept 'good bet' used in Samuelson (1963).

Consider initially risk-reward opportunities only. The decision to be made is whether to invest in a risk-reward opportunity or not and which risk-reward opportunity should be preferred. The potential profit *G* from an investment is a random variable following a Bernoulli distribution with parameter p_s. For constant values of the benefit given success $\overline{B}_s$ and the loss given failure $\overline{C}_f$, the probability distribution of the potential profit *G* is given by $P(G = \overline{B}_s) = p_s$ and $P(G = \overline{C}_f) = p_f$. The expected value of the potential profit is given by equation (18). Since $E(G^2) = p_s \times \overline{B}_s^{\,2} + p_f \times \overline{C}_f^{\,2}$, for the variance of the potential profit we get

$$Var(G) = E(G^2) - [E(G)]^2 = p_s p_f (\overline{B}_s - \overline{C}_f)^2 \tag{20}$$

For a large number of risk reward bets, the expected profit is approximated well by equation (18). In short, for risky prospects all containing a large number of statistically independent risk-reward bets, choosing the prospect characterised by the maximum expected profit is a sound decision making criterion. For a risky prospect containing a limited number of risk-reward bets however, despite the existence of a large expected profit, the risk of a net loss can be significant.

As a simple illustrating example, consider two risky prospects *A* and *B*, containing single risk-reward bets. The risky prospect *A* contains a single risk reward bet with probability of success $p_{sA} = 0.01$, benefit, given success $\overline{B}_{sA} = 200000$, probability of failure $p_{fA} = 0.99$ and loss given failure $\overline{C}_{fA} = -1000$. The expected profit from risky prospect *A* is $\overline{G}_A = 0.01 \times 200000 - 0.99 \times 1000 = 1010$; the probability of a loss is 99% and the risk of a net loss is $K_A = p_{fA} \times C_{fA} = -990$.

The risky prospect *B* contains a single bet with a probability of success $p_{sB} = 0.99$, benefit given success $\overline{B}_{sB} = 1000$, probability of failure $p_{fB} = 0.01$ and loss given failure $\overline{C}_{fB} = -1000$. The expected profit from risky prospect *B* is $\overline{G}_B = 0.99 \times 1000 - 0.01 \times 1000 = 980$, the probability of a loss is 1% and the risk of a loss is $K_B = p_{fB} \times C_{fB} = -10$.

Because the expected profit characterising risky prospect *A* is larger, according to the maximum expected profit criterion, risky prospect *A* should be selected, despite the 99 times larger risk of a net loss. In other words, the maximum expected profit criterion does not reflect the risk of a net loss while selecting among one-time opportunities.

Consider now two risky prospects, containing again single risk-reward bets. Risky prospect *A* contains a single risk-reward bet with a probability of success $p_{sA} = 0.2$, benefit given success $\overline{B}_{sA} = 2000$, probability of failure $p_{fA} = 0.8$ and loss given failure $\overline{C}_{fA} = -125$. The expected profit from risky prospect *A* is $\overline{G}_A = 0.2 \times 2000 - 0.8 \times 125 = 300$, the probability of a loss is 80%, the magnitude of the loss is -125.

Risky prospect *B* contains a single risk-reward bet with probability of success $p_{sB} = 0.2$, benefit given success $\overline{B}_{sB} = 20000$, probability of failure $p_{fB} = 0.8$ and loss given failure $\overline{C}_{fB} = -4625$. The expected profit from risky prospect *B* is $\overline{G}_A = 0.2 \times 2000 - 0.8 \times 4625 = 300$, the probability of a loss is again 0.8, the magnitude of the loss is -4625.

In this example, the maximum expected profit criterion does not distinguish between risky prospects with the same expected profit, probability of success and probability of a loss but with different magnitudes of the loss.

In both examples, the risk of a net loss from the risky prospects has not been revealed because in the expected profit (see equation (18)), the expected potential loss (the risk) $p_f \overline{C}_f$ has been aggregated with the expected potential reward $p_s \overline{B}_s$.

The next example involves two risky prospects containing a different number of risk-reward bets. The first risky prospect contains a single risk-reward bet with parameters: $p_s = 0.3$, $\overline{B}_s = 300$, $p_f = 0.7$, $\overline{C}_f =$ -90 and its expected profit is $E(G) = 0.3 \times 300 - 0.7 \times 90 = 27$. The probability of a net loss from this risky prospect is 70%. The risk of a net loss is $-0.7 \times 90 = -63$.

The second risky prospect contains three risk-reward bets with the same probability of success and failure but with three times smaller magnitudes for the benefit given success and the loss given failure: $p_s = 0.3$, $\overline{B}_s = 300/3 = 100$, $p_f = 0.7$, $\overline{C}_f = -90/3 = -30$. The expected profit from the risky prospect containing the three split bets is $E(G_{123}) = 3 \times (0.3 \times 100 - 0.7 \times 30) = 27$. Because a net loss from the second risky prospect can be generated only if a loss is generated from every single bet, the probability of a net loss from the second risky prospect is $p_{f,123} = 0.7^3 \approx 0.34$. The risk of a net loss is $-0.34 \times 90 = -30.6$.

The maximum expected profit criterion does not distinguish between these two risky prospects, characterized by the same expected profit. This is despite that the probability of a net loss and the risk of a net loss from the second risky prospect are clearly much smaller compared to the probability of a net loss and the risk of a net loss from the first risky prospect. Clearly, the second risky prospect is to be preferred to the first risky prospect. The real problem from applying the maximum expected profit criterion becomes apparent if the three opportunity bets from the second risky prospect are characterised by a slightly smaller benefit given success $\overline{B}_s = 300/3 - 1 = 99$. In this case, the expected profit characterising the second risky prospect will be slightly smaller

$$E(G_{123}) = 3 \times (0.3 \times 99 - 0.7 \times 30) = 26.1$$

than the expected profit from the first risky prospect. The probability of a net loss $p_{f,123} = 0.7^3 \approx 0.34$ for the second risky prospect remains the same. Adherence to the maximum expected profit criterion only, will favour the selection of the first risky prospect, characterised by a marginally larger expected profit of 27, the significantly larger probability of a net loss of 70% and more than twice magnitude -63 of the risk!

Again, the significant risk of a net loss associated with one of the risky prospects has not been revealed by the maximum expected profit criterion.

The last example shows how the risk associated with a risk reward bet can be reduced significantly if the risk reward bet is split into several risk reward bets characterised by the same probability of success and failure as the original bet but with proportionally smaller benefit and loss. Indeed, consider a risky prospect containing a single risk-reward bet, characterised by a probability of success p_s, benefit given success $\overline{B}_s$, probability of failure p_f and loss given failure $\overline{C}_f$. This risk reward bet can be split into *m* risk-reward sub-bets, each characterised with the same probability of success and failure p_s and p_f and with *m* times smaller expected benefit and loss $\overline{B}_s / m$ and $\overline{C}_f / m$. The expected profit from the *m* risk-reward bets:

$$E(G_{1,\ldots,m}) = m \times (p_s \overline{B}_s / m + p_f \overline{C}_f / m) = E(G) \tag{21}$$

is equal to the expected profit from the original bet. Considering equation (20), the variance

$$V(G_{1,\ldots,m}) = \sum_{i=1}^{m} V_i = \sum_{i=1}^{m} p_s p_f (\overline{B}_s / m - \overline{C}_f / m)^2 = \frac{1}{m} p_s p_f (\overline{B}_s - \overline{C}_f)^2 \tag{22}$$

of the profit from the risky prospect with *m* risk-reward bets is *m* times smaller than the variance $p_s p_f (\overline{B}_s - \overline{C}_f)^2$ of the profit characterising the initial risk-reward bet.

Consider another example, of two competing risky prospects each containing a single loss-generating event. The probability of a loss is the same for both risk events while the loss distributions are different. The expected losses associated with the events are $\mu_1 < \mu_2$. The variance of the losses V_1 characterising the first event is large, whereas the variance of the losses V_2 characterising the second event is small. Contrary to the maximum expected value criterion, often the second bet, with the larger expected value μ_2 of the loss, will be the preferable bet. This is because of the small variance of the losses and the high likelihood that the magnitude of the losses will be close to the mean μ_2. For the first bet, because of the large variance, the amount of the losses may significantly exceed the mean μ_1.
In other words, in the case of a single risk-reward bet in a risky prospect, the loss distribution is sampled *only once* and there is no guarantee that the loss will be close to its mean.

The variance of the loss however, still does not provide a good measure of the risk of exceeding a particular critical value for skewed distributions. Take as an example two very different skewed distributions, characterized by the same mean μ and variance σ^2 (Fig.3). One of the distributions (distribution *a*) has a large upper tail of the loss, while the other distribution (distribution *b*) has a large lower tail. Distribution *a* in Fig.3 is less desirable compared to distribution *b* in Fig.3, because distribution *a* is associated with a larger risk that the loss will exceed a particular large quantity.

Take in Figure 3

In summary, the maximum expected profit criterion is fundamentally flawed, because it does not reflect the impact of the number of risk-reward activities in the risky prospects. *The number of the risk-reward activities should be a key consideration in selecting a risky prospect.*

5. Risk of a net loss and expected potential reward associated with a limited-number of statistically independent risk-reward bets in a risky prospect

Suppose that a finite number *n* of risk-reward opportunities of the same type are present. A series of *n* statistically independent risk-reward bets, characterised by the same probability of success in each trial is a Binomial experiment, where the number of successful outcomes is modelled by the binomial distribution.

The risk of a net loss from *n* risk-reward bets can be derived by the following probabilistic argument.

Let x denote the number of bets which materialise as 'benefit' among *n* bets, *x=0,1,…n*. Correspondingly, *n-x* will be the number of bets which materialise as losses. The probability of a net loss equals the probability that the sum of the benefits from *x* benefit-generating bets will be smaller than the sum of the losses from *n-x* loss-generating bets.

Let $\overline{B}_s$ be the expected value of the benefit given a successful bet and $\overline{C}_f$ be the expected value of the loss given a loss-generating bet. The probability that the sum of the benefits from *x* benefit-generating bets will be smaller than the sum of the losses from *n-k* loss-generating bets is equal to the probability that the number of benefit-generating bets *x* does not exceed k, the largest integer satisfying the inequality

$$k \times \overline{B}_s < (n-k) \times |\overline{C}_f| \tag{23}$$

Condition (23) is equivalent to $k = \frac{n|\overline{C}_f|}{\overline{B}_s + |\overline{C}_f|} - 1$, if $n|\overline{C}_f|/[\overline{B}_s + |\overline{C}_f|]$ is an integer number. Otherwise condition (23) is equivalent to $k = \left[\frac{n|\overline{C}_f|}{\overline{B}_s + |\overline{C}_f|}\right]$ where $\left[\frac{n|\overline{C}_f|}{\overline{B}_s + |\overline{C}_f|}\right]$ is the largest integer not exceeding the ratio $n|\overline{C}_f|/[\overline{B}_s + |\overline{C}_f|]$ if $n|\overline{C}_f|/[\overline{B}_s + |\overline{C}_f|]$. The probability that there will be a net loss then becomes

$$P(net\ loss) = \sum_{x=0}^{k} \frac{n!}{x!(n-x)!} p_s^x (1-p_s)^{n-x}, \tag{24}$$

The risk of a net loss is equal to the expected value of the potential loss. This can be determined by adding the probability of zero benefit generating bets times the losses $n|\overline{C}_f|$ from n loss-generating bets plus the probability of a single benefit-generating bet and *n-1* loss-generating bets times $(n-1)\overline{C}_f + \overline{B}_s$ and so on,…,plus the probability of k benefit-generating bets and *n-k* loss-generating bets times the net loss $(n-k)\overline{C}_f + k\overline{B}_s$ from *n-k* loss-generating bets and k benefit-generating bets. As a result, the risk of a net loss K becomes:

$$K = \sum_{x=0}^{k} \frac{n!}{x!(n-x)!} p_s^x (1-p_s)^{n-x} \times [(n-x)\overline{C}_f + x\overline{B}_s], \tag{25}$$

The expected potential reward can be determined by adding the probability of *k+1* benefit generating bets times the benefits $(k+1)\overline{B}_s + (n-k-1)\overline{C}_f$ from *k+1* benefit-generating bets and (*n-k-1*) loss-generating bets plus, and so on,…,plus the probability of n benefit generating bets times the benefit $n\overline{B}_s$ from them. As a result, the expected potential reward from n risk-reward bets becomes:

$$R = \sum_{x=k+1}^{n} \frac{n!}{x!(n-x)!} p_s^x (1-p_s)^{n-x} \times [(n-x)\overline{C}_f + x\overline{B}_s], \tag{26}$$

Equations (24, (25) and (26) have been verified by a Monte Carlo simulation. Thus, for $\overline{B}_s = 290$, $\overline{C}_f = -100$, $p_s = 0.3$, $p_f = 0.7$ and 12 opportunity bets, equation (24) gives 0.49 for the probability of a net loss, equation (25) gives $K = -152$ for the risk of a net loss and equation (26) gives $R = 356$ for the expected potential reward. These results have been confirmed by the empirical values ($P(net\_loss) = 0.49$, $K = -152$ and $R = 356$) obtained on the basis of 1000000 simulation trials.

The expected profit from n bets can be obtained by using equation (5) stating that the expected value of a sum of random variables is the sum of the expected values of the random variables. Since the expected profit from a single bet is $\overline{G} = p_s \times \overline{B}_s + p_f \times \overline{C}_f$, the expected profit from a sequence of n bets is

$$E(X_1 + ... + X_n) = np_s \mathrm{B}_s + n(1-p_s)\overline{C}_f \tag{27}$$

In other words, with increasing the number of opportunity bets, the expected profit increases proportionally to the number of bets in the sequence. For statistically independent variables, the variance of their sum is equal to the sum of the variances of the random variables (DeGroot, 1989).

$$V(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} V(X_i) \tag{28}$$

Because the variance of the profit from a single bet is given by equation (20), the variance of the profit from n statistically independent bets is given by

$$V(G) = np_s(1-p_s)(\overline{B}_s - \overline{C}_f)^2 \quad (29)$$

In other words, the variance of the profit from a sequence of a number of statistically independent bets increases proportionally to the number of bets in the sequence.

In the case of a very large number *n* of bets, the conditions for the validity of the central limit theorem are fulfilled and a Gaussian distribution with mean

$$\mu = n[p_s\overline{B}_s + (1-p_s)\overline{C}_f] \quad (30)$$

and standard deviation

$$\sigma = (\overline{B}_s - \overline{C}_f)\sqrt{np_s(1-p_s)} \quad (31)$$

can be used for approximating the distribution of the potential profit. The probability of a net loss will be

$$P(net\ loss) = \Phi\left(\frac{0-\mu}{\sigma}\right) \quad (32)$$

where $\Phi(\bullet)$ is the cumulative distribution of the standard normal distribution with mean '0' and standard deviation '1'. Correspondingly, the probability of a net profit will be given by

$$P(net\ profit) = 1-\Phi\left(\frac{0-\mu}{\sigma}\right) \quad (33)$$

From the distribution, the probability that the net loss will exceed any specified quantity can be determined. We need to point out however, that approximations (32) and (33) are valid *only for a very large number of bets in the risky prospect* and do not hold for a limited number of bets. In the case of a limited number of bets in the risky prospect, equations (24) and (25) should be used.

5.1. Samuelson's sequence of good bets revisited

Following the Samuelson's paper (Samuelson, 1963), the same proposed 'good bet' will be used. Probability $p_s = 0.5$ of winning 200 ($\overline{B}_s = 200$) and probability $p_f = 0.5$ of losing 100 ($\overline{B}_s = 200$). Table 1, lists the results from the calculations using equations (24), (25) and (31). These calculations have also been confirmed by a Monte Carlo simulation.

Table 1. Expected profit and risk of a net loss with increasing the number of good bets.

Number of good bets	Expected profit	Standard deviation	Risk of a net loss	Probability of a net loss
1	50	150	-50	0.5
10	500	474.3	-37.1	0.17
20	1000	670.8	-20	0.057
30	1500	821.6	-9.9	0.049
50	2500	1060.7	-3.1	0.0077
80	4000	1341.6	-0.5	0.0012
90	4500	1423	-0.26	0.0010
100	5000	1500	-0.15	0.00044
130	6500	1710.3	-0.025	0.00007
150	7500	1837	-0.007	0.00003

As can be verified from the table, despite that with increasing the number of good bets, the variance of the net profit increases, the risk of a net loss has decreased significantly. In other words, despite that selecting an individual bet is not beneficial because of the high probability of a loss, selecting repeated bets is beneficial, because a longer sequence of repeated bets is characterised by an increased expected profit, a small probability of a net loss and a small risk of a net loss. Contrary to the view expressed by Samuelson which has also been adopted in several related papers (e.g. Ross, 1999), repeating the unacceptable bet reduced significantly the risk of a net loss.

Despite the circumstance that with increasing the number of opportunity bets, the variance of the profit increases significantly (see Table 1), the risk in fact decreases. This seeming anomaly has led some researchers to conclude that because of the increased variance of the profit, the risk will also increase. The analysis shows that the variance of the profit can increase without a simultaneous increase of the risk of a loss. In this case, the commonly accepted rule that a larger variance means a larger risk does not hold. In the case of repeated bets, a larger variance actually coexists with a smaller risk and the variance of the profit cannot serve as a risk measure.

In summary, *the popular view started with the Samuelson's paper (1963), that if a single bet is unacceptable then a sequence of such bets is also unacceptable is incorrect*.

We need to point out that the Parondo-type games discussed in (Epstein 2009 and Astumian, 2001) cannot serve as an example contradicting the Samuelson theory. Parondo-type games deal with risk-reward bets which are unattractive individually because of the negative expected profit characterising each bet. When combined in a sequence, the resultant sequence of bets can be made with a positive expected profit with a very high probability (Astumian, 2001). The problem is in the fact that the Parondo-type games are not independent. Commonly, one of the games produces few outcomes which are associated with a high payoff if the second game is played next. When played in a sequence, the net profit from the composition of games is positive.

6. Variation of the risk of a net loss associated with a small number of opportunity bets

For a large number n of opportunity bets, the expected value of the net profit is approximated well by $\overline{G} \approx p_s \overline{B}_s - p_f | \overline{C}_f |$. According to the definition of an opportunity bet, $p_s \overline{B}_s - p_f | \overline{C}_f | > 0$ and consequently, with increasing the number of opportunity bets, the probability of a negative net profit (net loss) approaches zero.

It seems that with increasing the number of opportunity bets, the probability of a net loss always decreases. There is a common belief that increasing the number of opportunity bets is always beneficial because this increases the exposure to successful outcomes. Interestingly, this conventional belief is not necessarily correct. Here we show that multiple opportunity bets (characterised with a positive expected gain) can in fact be associated with a larger risk of a net loss than a single opportunity bet).

This is shown in Figure 4, where the parameters characterising the risk-reward bets are $p_s = 0.3$, $p_f = 0.7$, and $\overline{C}_f = -100$. Different values $E(G)$ of the expected profit were obtained by varying the expected benefit $\overline{B}_s$. Thus, for $\overline{B}_s = 299$, $E(G) = 19.7$, for $\overline{B}_s = 493$, $E(G) = 77.9$ and for $\overline{B}_s = 247$, $E(G) = 4.1$. These correspond to the three curves in Figure 4. Increasing the number of opportunity bets is associated with a decrease of the absolute value of the risk of a net loss. The decrease however, does not occur monotonically as indicated by all three graphs. The analysis shows that the third curve, corresponding to

$E(G) = 4.1$ also tends to zero absolute value of the risk of a net loss but after thousands of opportunity bets.

Take in Figure 4

While for a large value of the expected gain, the absolute value of the risk of a net loss is quickly reduced to zero, for small values of the expected gain, the risk of a net loss may actually increase with increasing the number of opportunity bets as indicated by the graph $E(G) = 4.1$ and $E(G) = 19.7$ in Fig.4. As a result, increasing the number of opportunity bets characterised by a small expected gain, may have the opposite effect on the risk of a net loss!

7. Distribution of the potential profit from a limited number of risk-reward activities

Calculating solely the probability of a net loss is not a reliable criterion for selecting risky alternatives. This is because a high probability of a net loss may co-exist with a low magnitude of the loss and a low probability of a net loss may co-exist with a high magnitude of the loss. Ranking the risky prospects correctly and making risk decisions requires evaluating the risk of a net loss, which incorporates both the probability and the magnitude of the net loss.

The widely adopted in the financial literature *VaR* reliability measure specifies a threshold amount of losses, the probability of exceeding which is a known quantity. This reliability measure however, provides no insight on the extent of the losses beyond the specified threshold value.

This was the reason for the *CVaR* reliability measure (Pflug 2000; Rockafellar and Uryasev, 2002). For continuous loss distributions, the *CVaR* at a given confidence level is the expected loss given that the loss is greater than the *VaR* at that level (Rockafellar and Uryasev, 2002). This reliability measure is also insufficient to describe the behaviour of the potential losses. The drawbacks of risk measures based on expected values can be readily demonstrated by constructing two distributions of the potential loss, characterised by the same expected value of the potential loss and different variances of the losses in the tails. Because the *CVaR* reliability measure is based on an expected value, it suffers from the same drawback - examples of potential loss distributions can be constructed that have the same *CVaR* value and different potential loss distributions in the tails. As a result, neither VaR nor *CVaR* reliability measure fully captures the risk of large losses significantly deviating from the mean.

The distribution of the potential loss and the distribution of the potential benefit can be combined by evaluating the *distribution of the potential profit*. In the case of a limited number of risk-reward bets, ranking the risky prospects and making risk decisions is facilitated by evaluating the distribution of the potential profit.

Consider a single risk-reward event, whose probability of occurring is p_{oc}. Once the risk-reward event has occurred, it materialises as success with probability p_s and as a loss, with probability p_f ($p_s + p_f = 1$). The potential profit *X* is smaller than a particular value '*x*' (*x* can be also be negative) in the following mutually exclusive cases: (i) the risk-reward event does not occur and the profit is smaller than *x*, (ii) the risk-reward event materialises as 'success' and the benefit given success is smaller than *x* and (iii) the risk-reward event materialises as a loss and the loss given failure is smaller than *x* (the loss is with a negative sign). Consequently, according the total probability theorem, for a single risk reward event, the distribution of the potential profit $G(x)$ is given by

$$P(X \le x) \equiv G(x) = (1 - p_0)H(x) + p_0 p_s B(x \mid s) + p_0 p_f C(x \mid f) \quad (34)$$

where $B(x \mid s)$ is the distribution of the benefit given success and $C(x \mid f)$ is the distribution of the loss given failure and $H(x)$ is the Heaviside step function ($H(x) = 1$ for $x \ge 0$ and $H(x) = 0$ for $x < 0$). The potential profit *is a mixture of three distributions*: the distribution of the benefit given success $B(x \mid s)$, the distribution of the loss given failure $C(x \mid f)$ and the Heaviside step function. For the expected value of the potential profit:

$$\overline{G} = p_0 p_s \overline{B}_s + p_0 p_f \overline{C}_f \quad (35)$$

is valid.

For a risky prospect containing multiple risk-reward events, if no analytical solution exists, building the distribution of the potential profit and evaluating the risk of a net loss and the expected potential reward can be made by a Monte Carlo simulation, whose algorithm in pseudo-code is described in Appendix A.

Take in Figure 5

In Fig.5, the cumulative distribution of the potential profit has been built for risk-reward events following a homogeneous Poisson process with density 0.8 year^{-1} in the interval (0,2) years. The benefit given success follows a uniform distribution in the interval 0,320 $B_s(x \mid s) = U(0{,}320)$ and the loss given failure follows a uniform distribution in the interval (-50,0); $C_f(x \mid f) = U(-50{,}0)$. The empirical risk of a net loss is -23.9, the expected potential reward is 47.9 and the expected potential profit is 19.3. It is interesting to note the jump of the net profit dependence at zero. This is caused by all outcomes for which no risk-reward events occurred in the specified finite time interval. The expected profit in this case is zero.

Take in Figure 6

Figure 6 gives the distribution of the potential profit from five opportunity bets with parameters $p_s = 0.4$, $\overline{B}_s = 450$, $p_f = 0.6$, $\overline{C}_f = -180$. The empirical risk of a net loss is -140; the empirical expected potential reward is 499.6. Figure 7 gives the distribution of the potential profit if the number of opportunity bets is increased to 30. The risk of a net loss has decreased to -75.3 while the expected potential benefit has increased to 2237.2.

Take in Figure 7

In each particular case, a simulation is required to reveal the risk of a net loss. Simulation is also necessary for risk-reward bets characterised by different probabilities of success or by a complex distribution of the loss given failure or the benefit given success.

The results from the simulation have been validated by an alternative method for calculating the expected value of the total profit based on analytical reasoning. The expected value of the total profit can be calculated as a sum of the expected value of the total benefit from risk-reward bets which materialise as success and the expected value of the total loss from risk-reward bets which materialise as a loss.

The total benefit T_B from N_s risk reward bets which materialise as success is $T_B = N_s \overline{B}_s$. Correspondingly, the total loss T_L is $T_L = N_f \overline{C}_f$. The total profit T_P is therefore given by

$$T_P = N_s \overline{B}_s + N_f \overline{C}_f \quad (36)$$

Taking expected values from both sides of equation (36) results in

$$\overline{T}_P = \overline{N}_s \overline{B}_s + \overline{N}_f \overline{C}_f \quad (37)$$

for the expected value of the total profit. The expected number of the risk reward bets $\overline{N}_s$ which materialise as success is

$$\overline{N}_s = \rho \times a \times p_s \quad (38)$$

where ρ is the density of the homogeneous Poisson process modelling the occurrence of the risk-reward bets; a is the length of the time interval and p_s is the probability that a given risk-reward bet will materialize as 'success'. Similarly, the expected number of the risk reward bets which materialise as a loss is

$$\overline{N}_s = \rho \times a \times p_f \quad (39)$$

Substituting equations (39) and (38) in equation (37) results in

$$\overline{T}_P = \rho a(p_s \overline{B}_s + p_f \overline{C}_f) \quad (40)$$

In the first example, $B_s(x \mid s) = U(0,320)$ and $C_f(x \mid f) = U(-50,0)$. Therefore, $\overline{B}_s = 160$ and $\overline{C}_f = -25$. Substituting these values and the also the values $\rho = 0.8$, $a = 2$ years, $p_s = 0.2$, $p_f = 0.8$, $\overline{B}_s$ in equation (40), results in $\overline{T}_P = 19.2$ which confirms the empirical result of 19.3 obtained by the simulation algorithm.

8. Conclusions

● If the total potential loss from several risk sources is a sum of the potential losses from the individual sources, the total risk from the risk sources is a sum of the individual risks from the separate sources irrespective of whether the risk sources are statistically independent or not. In this case, an exact algorithm has been proposed for optimal allocation of limited risk reduction resources to achieve a maximum overall risk reduction. The algorithm is general and does not put any constraints on the functions describing the risk reduction.

● An exact algorithm has been proposed for optimal allocation of a fixed budget to achieve a maximum reduction of the risk of failure of a safety-critical system whose exact architecture is not known in advance. The algorithm eliminates unnecessary components from the full-complexity safety-critical system to reach the optimal architecture locked in the full-complexity architecture.

● An exact algorithm has been proposed for optimal allocation of limited risk reduction resources among units logically arranged in series, in order to attain the smallest possible risk of system failure.

● The number of activities in a risky prospect should be a key consideration in selecting the risky prospect.

● The maximum expected profit criterion, widely used for making risk decisions, is flawed, because it does not consider the impact of the number of risk-reward events/bets in the risky prospects.

● A quantitative framework has been developed for making an optimal choice among risky prospects containing a limited number of statistically independent risk-reward activities. The quantitative framework is based on a simple software tool for building the cumulative distribution of the potential profit by simulation.

● A popular view, that if a single bet is unacceptable then a sequence of such bets is also unacceptable has been analysed and proved to be incorrect.

● A decision strategy for making a choice among risky prospects containing a limited number of risk-reward activities is fundamentally different from a decision strategy for making an optimal choice among risky prospects containing a large number of risk-reward activities.

● The commonly accepted rule that a larger variance is associated with a larger risk does not always hold. In the case of repeated opportunity bets, a larger variance actually coexists with a smaller risk. Consequently, the variance of the net profit cannot serve as a risk measure.

● The risk of a net loss does not decrease monotonically with increasing the number of opportunity bets. For a limited number of opportunity bets in a risky prospect and a small expected profit characterising a single opportunity bet, increasing the number of opportunity bets may increase the risk of a net loss.

● Analytical expressions have been derived for the risk of a net loss associated with a finite number of statistically independent identical risk reward activities in a risky prospect.

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Appendix A. A Monte Carlo simulation algorithm for building the cumulative distribution of the potential profit

```
function Benefit_given_success(k);  //Samples the conditional distribution of the benefits
    given success of the k-th risk-reward bet and returns a random value
function Loss_given_failure(k);  //Samples the conditional distribution of the loss given
    failure of the k-th risk-reward bet and returns a random value

ps[Number_of_bets];     //Contains the probabilities of success
                        characterising the
                        risk-reward bets in the risky prospect;
Net_revenue = 0;        //where the net revenue will be accumulated
Sum_net_Benefit = 0;    //where only the net benefit will be accumulated
Sum_net_Loss = 0;       //where only the net loss will be accumulated

  for i=1 to Number_of_trials do
{
  for j=1 to Number_of_bets do
   {
   t = generate_random_number(); // Generates a random number
                                  uniformly distributed in the interval (0,1)
   if (t < ps[j]) then
          Net_revenue = Net_Revenue + Benefit_given_success(j);
                 else Net_revenue = Net_Revenue + Loss_given_failure(j);
   }

  distr_pot_profit[i] = Net_revenue;
  if (Net_revenue > 0) then
          Sum_net_Benefit = Sum_net_Benefit+Net_revenue;
          else Sum_net_Loss = Sum_net_Loss+Net_Revenue;
}
 Risk_of_net_loss = Sum_net_Loss / Number_of_trials;
```

Potential_expected_reward = Sum_net_Benefit / Number_of_trials;

Sort the array distr_pot_profit[] ***in ascending order***.

For a number of risk-reward bets 'Number_of_bets'in a risky prospect, each characterised by different probabilities of success and failure, determining the risk of a net loss consists of the following steps. A simulation loop with control variable *i* is entered first, within which a nested loop with control variable *j* is entered, scanning through all risk-reward bets in the risky prospect. In the nested loop, a random variable *t* following Bernoulli distribution is simulated, by generating a uniformly distributed random number in the interval (0,1) and comparing it with the probability of success ps[j] of the scanned risk-reward bet. This random variable simulates success or failure outcome from the separate risk-reward bets. If success is simulated (*t* <= ps[*j*]), the distribution of the benefits given success is sampled by calling the function Benefits_given_success(*j*); if failure is simulated (*t* > ps[*j*]), the distribution of the loss given failure is sampled by calling the function Loss_given_failure(*j*). For all bets/events in the risky prospect, the sampled quantity is accumulated in the variable 'Net_revenue' which contains the net revenue (profit) from the risk-reward bets in the current simulation trial. The magnitude of the profit characterising the current simulation trial is also stored in the array 'distr_pot_profit[]'. At the end of the simulation, the elements of the array 'distr_pot_profit[]' are sorted in ascending order by using the *Quicksort algorithm* [Cormen et al. 2001]. The empirical cumulative distribution of the potential profit is built by plotting the sorted values of the cumulative array versus the probability rank estimates, i=1,2,…Number_of_trials.

During each simulation trial, after obtaining the net revenue (profit) in the variable 'Net_revenue', its sign is checked. If the sign is negative, the net revenue is accumulated in the variable 'Sum_net_Loss'. Correspondingly, if the sign of the net profit in 'Net_revenue' is positive, the net benefit is accumulated in the variable 'Sum_net_Benefit'. The risk of a net loss and the potential expected reward are obtained at the end of the simulation trials by dividing the variables 'Sum_net_Loss' and 'Sum_net_Benefit' to the number of simulation trials.

Figures

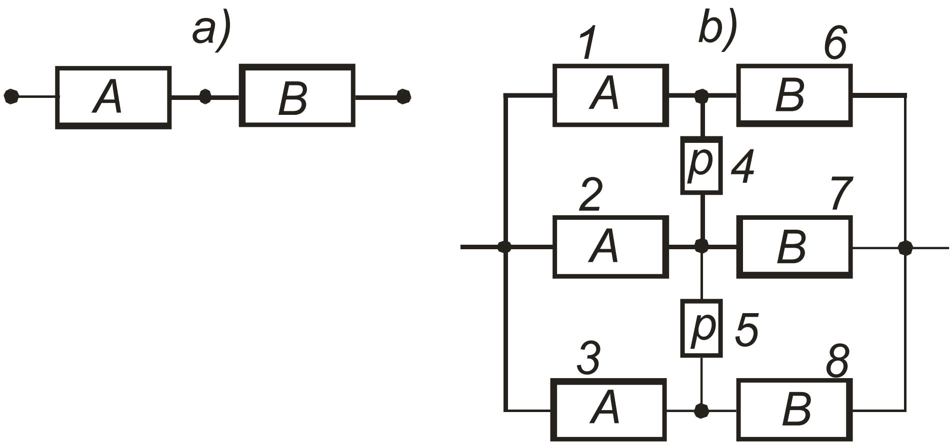

Figure 1. An example (a) of the reliability network of a basic safety-critical system and (b) the full-complexity reliability network.

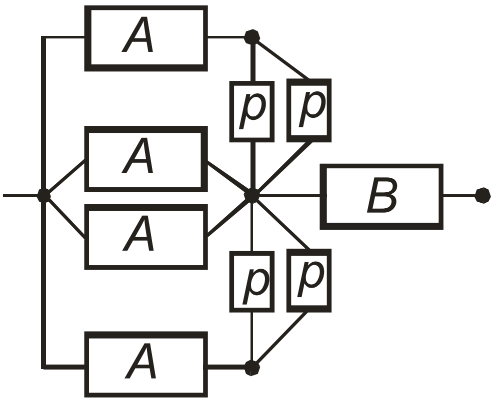

Figure 2. The reliability network after the topology optimisation based on the full-complexity network in Fig.1b.

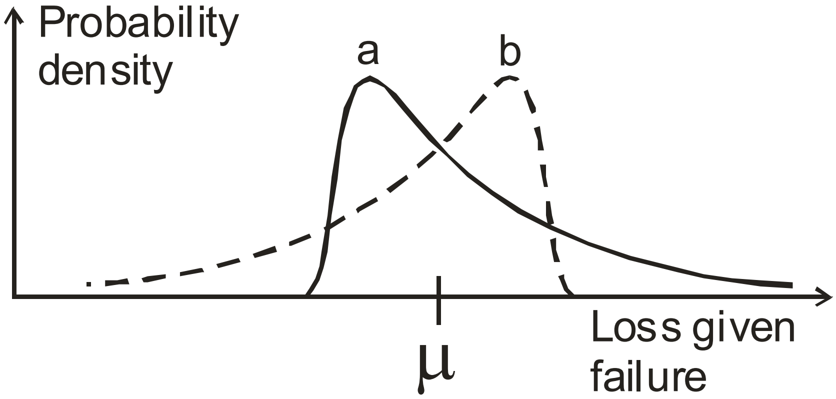

Figure 3. Two distributions of the loss with the same mean μ and variance σ.

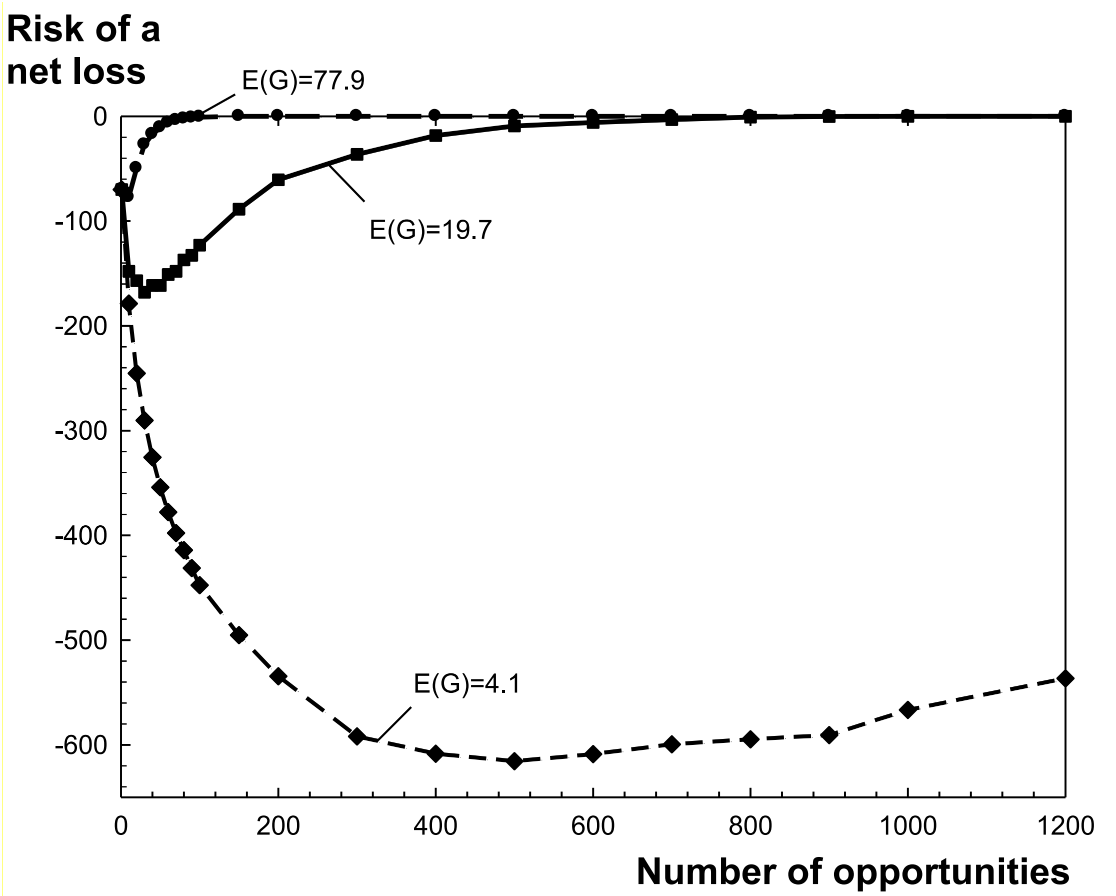

Figure 4. Increasing the number of opportunity bets may not necessarily result in a reduction of the risk of a net loss. The dependences are not necessarily monotonic.

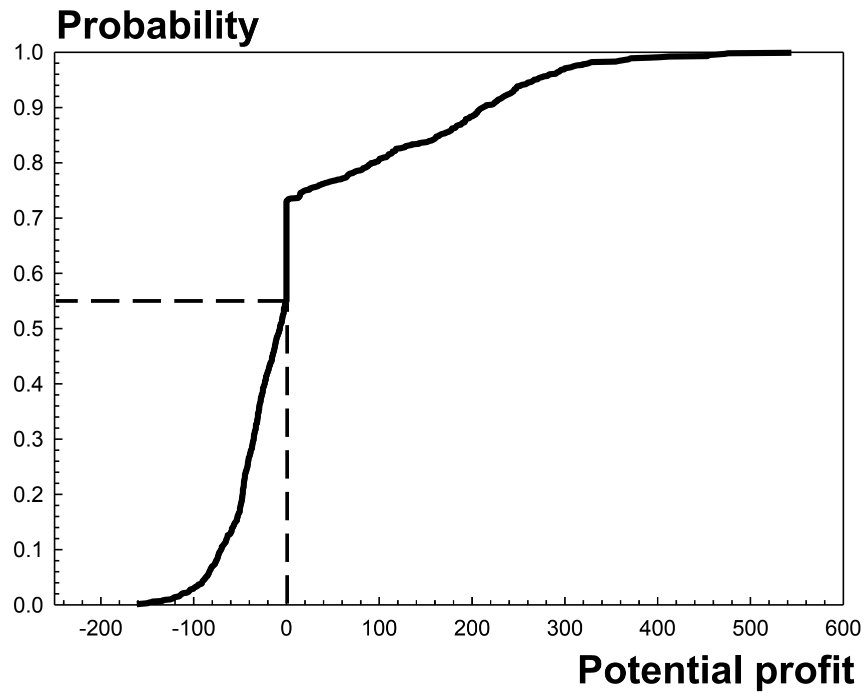

Figure 5. Distribution of the potential profit from risk-reward events following a homogeneous Poisson process in a specified time interval (*a=2* years).

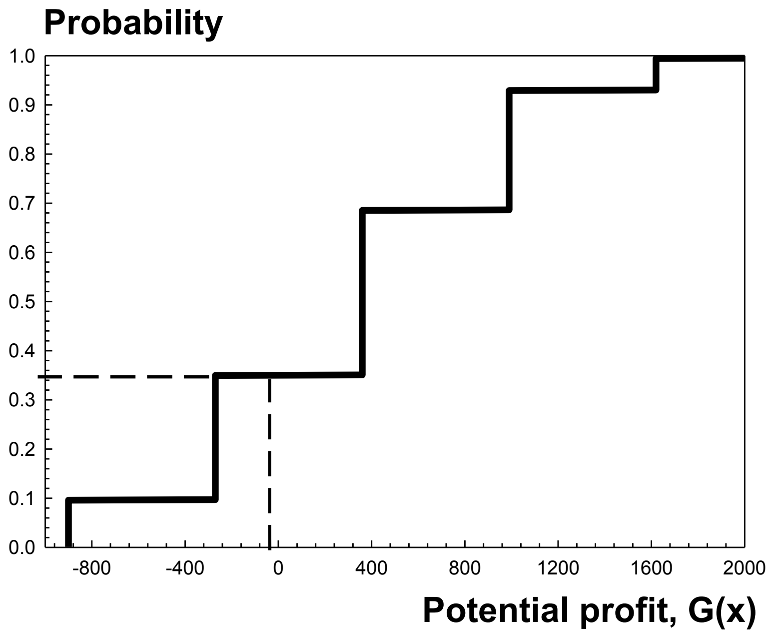

Figure 6. Distribution of the potential profit from five opportunity bets.

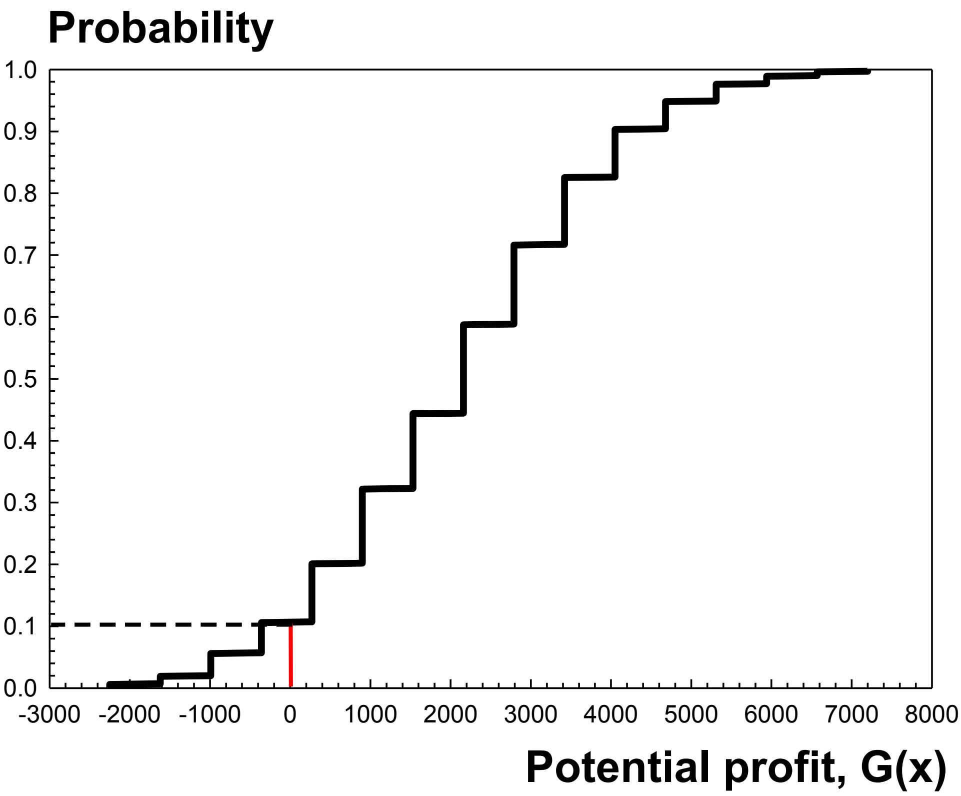

Figure 7. Distribution of the potential profit from thirty opportunity bets.