Thermodynamic Assessment and Multi-Objective Optimization of Performance of Irreversible Dual-Miller Cycle

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Abstract: In this study, a new series of assessments and evaluations of the Dual-Miller cycle is performed. Furthermore, the specified output power and the thermal performance associated with the engine are determined. Besides, multi-objective optimization of thermal efficiency, ecological coefficient of performance (ECOP) and ecological function ($E_{un}$) by means of NSGA-II technique and thermodynamic analysis are presented. The Pareto optimal frontier obtaining the best optimum solution is identified by fuzzy Bellman-Zadeh, Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP), and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) decision-making techniques. Based on the results, performances of dual-Miller cycles and their optimization are improved. For the results of the condition that ($n < k$) the best point has been LINMAP answer. The thermal efficiency for this point has been 0.5388. In addition, ECOP and $E_{un}$ have been 1.6899 and 279.221, respectively. For the results of the condition that ($n > k$) the best point has been LINMAP and TOPSIS answer. The thermal efficiency for this point has been 0.5385. Also, ECOP and $E_{un}$ have been 1.6875 and 279.7315, respectively. Furthermore, the errors are examined through comparison of the average and maximum errors of the two scenarios.

Keywords: Dual-Miller cycle; thermodynamic analysis; power; ecological coefficient of performance; thermal efficiency; energy; entropy generation; multi-objective optimization (MOO); multi-criteria decision making (MCDM); soft computing; genetic algorithm; finite-time thermoeconomic (FTT)
1. Introduction

There are different approaches applied to evaluate the energy systems [1–4]. Finite-time thermoeconomic (FTT) optimization is among the most appropriate approaches applicable for evaluating the operation quality of internal combustion engine cycles (ICEC) [5–13]. In recent studies of thermodynamic systems [14–23], comprehensive investigations have been carried out [24]. Entropy optimization [25–29], $E_{\text{un}}$ criterion [30–34], and ecological coefficient of performance (ECOP) criterion [35–40] are some of the recent various optimization objectives in ICEC analysis. Generally, entropy reduction is not equal to enhancing thermal efficiency or maximum power generation. Under certain circumstances, minimizing entropy generation leads to the highest power generation [41]. Blank et al. [42] investigated the efficiency of an endoreversible air standard dual cycle considering the system heat loss. Chen and colleagues [43] investigated the air standard dual cycle considering the friction and heat loss. Univ and colleagues [44] studied the performance optimization of an irreversible air standard dual cycle taking the impact of internal irreversibility and heat loss into account. Ghatak et al. [45] investigated the efficiency of an endoreversible air standard dual cycle considering the fluid thermal characteristics. Wang et al. [46] carried out a performance analysis considering the finite-time element and internal loss. Ge and colleagues [47] ran a thermodynamic analysis of the irreversible air standard dual cycle. In the thermodynamic assessment and optimization of air standard Miller cycles, Al-Sarkhi and colleagues [48] optimized the power density of a reversible cycle. Chen and colleagues [49,50] assessed the efficiency of an irreversible air standard Miller cycle considering the thermal properties of the working fluid and friction and heat loss of the system. Lin and colleagues [51] conducted an optimization for an irreversible air standard Miller system. Due to the thermodynamic evaluation of single cycles, Gonca et al. and other researchers [52–58] and Gonca [59] analyzed irreversible Dual-Miller cycles taking power and thermal efficiency into account. Ust and colleagues [60] conducted an exergy optimization for an irreversible Dual-Miller cycle. Gonca and colleagues [61,62] analyzed the ECOP of an irreversible Dual-Miller cycle. Wu and colleagues [63,64] investigated the efficiency of an irreversible Dual-Miller cycle considering linear [63] and nonlinear [64] thermal properties of working fluid. Huleihil et al. [65] presented and evaluated a reversible air-standard Otto model through polytropic processes. Gong and colleagues [66] performed a performance optimization of an endoreversible Lenoir cycle, considering heat losses and polytropic processes. Also, Xiong and colleagues [67] conducted a performance optimization of an endoreversible air standard Otto cycle considering heat losses and polytropic processes. Zhang and colleagues [68] designed and evaluated an irreversible universal cycle model, considering heat and friction losses, polytropic stages, and thermal properties of the working fluid.

Multi-objective optimization (MOO) is a valuable method to model various real-life problems simultaneously [69–71]. Answering a multi-objective optimization question needs simultaneous substantiation of various objectives. Consequently, evolutionary algorithms presented and advanced to answer multi-objective problems applying various methods [72]. A proper approach to find a solution to a multi-objective problem is to examine a group of routes, each satisfies the objectives at an acceptable level and do not interfere with other routes [73]. MOO problems generally represent a practicably numerous collection of routes named frontier of Pareto, which examined vectors that show the possible primary connections in the whole area of the objective function. New studies indicate that MOOs for different energy systems applied in various engineering problems [74–109].

In this study, the performance of the Irreversible Dual-Miller Cycle is studied. Besides, the effect of critical parameters on the performance of the Dual-Miller cycle is investigated. Key parameters that presented include $\epsilon$, $\rho$, and the $n$. The effects of these parameters on the power, efficiency, ECOP, and $E_{\text{un}}$ of the system are evaluated. Then and MOO calculation is performed to obtain the best point of performance of the Dual-Miller Cycle.
2. Dual-Miller Cycle through a Polytropic Stage

In order to accurately model the operation of energy systems, it is necessary to consider all of the factors influencing their performance [110–112]. Figure 1 represents the diagram of an air standard Dual-Miller cycle. To increase the accuracy of performance assessment, the polytropic process replaces by the reversible adiabatic stage, which is impractical to attain in the improved Dual-Miller cycle [64]. As depicted in Figure 1, cycle 1-2-3-4-5-6-1 represents the condition in which \( n = k \). In the Dual-Miller cycle, the basic concept is the same as the Atkinson cycle in which the intake valves are kept open in part of the compression process. However, in Miller cycles, heat rejection is more. Consequently, the intake process can be replaced with a two-step heat rejection process: step one fresh air admittance as a constant volume process and step two partial compression while the valves are still open as a constant pressure compression. The combustion process is as well a two-step process in which the ignition first rises the pressure, then expansion begins with valves partially open, then the valves will be closed up to the end of the complete expansion. The process 1-2 is a reversible compression process corresponding to 1′-2 that is the irreversible compression. Heat addition occurs in two steps: processes 2-3 and 3-4 reversible or 3-4′ irreversible that lead to higher pressure and temperature. Exhaust process or expansion occurs in 4-5 or 4′-5. The intake process is replaced by a two-step heat rejection that can be seen in processes 5-6 and 6-1 or 6-1′. As can be seen in the figure, for each value of \( n \) a different actual compression/expansion process is defined. For \( n < k \) the endpoint of compression and expansion 2 and 5 are taken as the common final point between isentropic and actual process and the beginning point 1′ and 4′ vary between the two processes conversely for \( n > k \) the beginning points of each process are the common references and the endpoints vary in ideal and actual processes. In both cases, the effective area encompassed by the whole process for the actual cycle must be less than the ideal cycle, and this is the reason why \( \Delta s < 0 \) can be seen in both cases that don’t violate second law at all.

![Figure 1](image-url)
2.1. Ideal Air Standard Dual-Miller Cycle

In ideal gas systems (\( n \) equal to \( k \)), the state elements of each stage can be practically obtained, by the ideal gas state equation. According to the first law of thermodynamics, the heat transfer rates and power generation of the cycle can be determined. Equation (1) presents \( \varepsilon, \lambda, \rho, \) and \( r_M \), respectively:

\[
\varepsilon = \frac{V_1}{V_2}, \lambda = \frac{P_3}{P_2}, \rho = \frac{V_4}{V_3}, r_M = \frac{V_6}{V_1}
\] (1)

The primary thermodynamic equations of each stage defined as:

\[
T_2 = T_1 \varepsilon^{k-1}
\] (2)

\[
T_3 = T_2 \lambda
\] (3)

\[
T_4 = T_3 \rho
\] (4)

\[
T_5 = T_4 \left( \frac{\rho}{\varepsilon \cdot r_M} \right)^{k-1}
\] (5)

\[
T_6 = T_5 \left( \frac{r_M^k}{\lambda \cdot \rho^k} \right)
\] (6)

\[
T_1 = \frac{T_6}{r_M}
\] (7)

The total heat absorption and heat rejection are as follows:

\[
Q_{in} = Q_{23} + Q_{34} = \dot{m} \left( C_v (T_3 - T_2) + C_p (T_4 - T_3) \right)
\] (8)

\[
Q_{out} = Q_{56} + Q_{61} = \dot{m} \left( C_v (T_5 - T_6) + C_p (T_6 - T_1) \right)
\] (9)

A desirable reversible air standard Dual-Miller cycle, the heat transfer impact is not considered, while it considered for an actual DMC. This waste is considered relevant to the temperature difference of the cylinder wall and the functioning fluid as follows \([102,103]\)

\[
Q_l = \frac{B}{2} (T_2 + T_4 - 2T_0)
\] (10)

\( B \) is the heat transfer coefficient, and \( T_0 \) is ambient temperature. The power production and performance are calculated as:

\[
P = Q_{in} - Q_{out}
\] (11)

\[
\eta = \frac{P}{Q_{in} + Q_l}
\] (12)

2.2. Air Standard Dual-Miller Cycle (\( n < k \))

As it is depicted in Figure 1, \( T_1 \) in a Dual-Miller cycle (\( n \) less than the \( k \)) is higher than the \( T_1 \) in the air standard Dual-Miller cycle. In order to keep \( T_2 \) fixed through the compression stage, heat is extracted through the polytropic stage 1’–2. Considering heat waste through the heat input stage, \( T_{4'} \) is lower than \( T_4 \). In order to keep \( T_5 \) fixed, heat should be increased through the polytropic stage 4’–5.
The equations of each stage are defined as:

\[ T_2 = T_1' \epsilon^{\frac{k-1}{n-1}} \] (13)

\[ T_5 = T_4' \left( \frac{\rho}{\epsilon \cdot \gamma M} \right)^{\frac{k-1}{n-1}} \] (14)

The heat transfer rate of polytropic stage 1’–2, is defined as follows:

\[ Q_{1'2} = m C_v \left( \frac{k-n}{n-1} \right) (T_2 - T_1') \] (15)

The heat transfer rate of stages 2–3 and 3–4’, are defined as follows:

\[ Q_{23} = m C_v (T_3 - T_2) \] (16)

\[ Q_{34'} = m C_p (T_4' - T_3) \] (17)

The heat transfer rate of stage 4’–5, is defined as follows:

\[ Q_{4'5} = m C_v \left( \frac{k-n}{n-1} \right) (T_4' - T_5) \] (18)

The heat transfer rate of stages 5–6 and 6–1’, are defined as follows:

\[ Q_{56} = m C_v (T_5 - T_6) \] (19)

\[ Q_{61'} = m C_p (T_6 - T_1') \] (20)

The heat input of the cycle is

\[ Q_{in1} = Q_{23} + Q_{34'} + Q_{4'5} \] (21)

The heat output of the cycle is

\[ Q_{out1} = Q_{1'2} + Q_{56} + Q_{61'} \] (22)

In a desirable air standard Dual-Miller cycle, the ratio of the highest temperature to the lowest temperature is defined as follows:

\[ \frac{T_4}{T_1} = \rho \cdot \lambda \cdot \epsilon^{k-1} \] (23)

As stated by [102,103,109], the heat waste ratio is defined as follows:

\[ Q_{l1} = \frac{B}{2} (T_2 + T_4' - 2T_0) \] (24)

As a result, the generated power and the first law efficiency of the system are defined as follows:

\[ P_1 = Q_{in1} - Q_{out1} \] (25)

\[ \eta_1 = \frac{P_1}{Q_{in}} = \frac{P_1}{Q_{23} + Q_{34'} + Q_{4'5} + Q_{l1}} \] (26)

Considering the assumption in [104], the exhaust gas recirculation due to the heat transfer loss is determined as follows:

\[ \sigma_{q1} = \frac{B(T_2 + T_4' - 2T_0)}{2T_0} \] (27)
The exhaust gas recirculation due to the working fluid heat rejection is defined as [105]:

\[
\sigma_{pq1} = m \left( \int_{T_0}^{T_0} C_p \left( \frac{1}{T_0} - \frac{1}{T} \right) dT + \int_{T_0}^{T_5} C_v \left( \frac{1}{T_0} - \frac{1}{T} \right) dT + \int_{T_0}^{T_2} C_v \left( \frac{k-n}{n-1} \right) \left( \frac{1}{T_0} - \frac{1}{T} \right) dT \right)
\]  

(28)

As a result, the total entropy generation (\(\sigma_{un1}\)) of the system is defined as follows:

\[
\sigma_{un1} = \sigma_{q1} + \sigma_{pq1}
\]

(29)

According to refs. [30–34], ECOP of the cycle is defined as follows:

\[
\text{ECOP} = \frac{P_1}{T_0 \sigma_{un1}}
\]

(30)

According to references [30–34], \(E_{un}\) is defined as follows:

\[
E_{un1} = P_1 - T_0 \sigma_{un1}
\]

(31)

2.3. Air Standard Dual-Miller Cycle (\(n > k\))

As depicted in Figure 2, \(T_2\) the highest temperature of the adiabatic stage 1–2 is less than that of the polytropic stage, due to the heat waste through the isochoric stage 2–3 (\(n\) higher than the \(k\)). Thus, more heat applied through the polytropic stage 1–2. On the other hand, heat is extracted through the stage 4–5 taking the heat waste through the adiabatic stage 4–5 is greater than that of the polytropic stage 4–5 taking the heat waste through the isobaric stage 3–4, into account.

The equations of polytropic stages are defined as follows:

\[
T_{2'} = T_1 \cdot e^{n-1}
\]

(32)

\[
T_{5'} = T_4 \left( \frac{\rho}{\varepsilon \cdot r_M} \right)^{n-1}
\]

(33)

For polytropic stage 1–2', the heat transfer ratio is defined as:

\[
Q_{12'} = \dot{m}C_v \left( \frac{n-k}{n-1} \right) (T_{2'} - T_1)
\]

(34)

For stages 2'–3 and 3–4, heat transfer rates are defined as:

\[
Q_{23} = \dot{m}C_v (T_3 - T_{2'})
\]

(35)

\[
Q_{34} = \dot{m}C_v (T_4 - T_3)
\]

(36)

For stage 4–5', the heat transfer rate is defined as:

\[
Q_{45'} = \dot{m}C_v \left( \frac{n-k}{n-1} \right) (T_4 - T_{5'})
\]

(37)

For stages 5'–6 and 6–1', heat transfer rates are defined as:

\[
Q_{56} = \dot{m}C_v (T_{5'} - T_6)
\]

(38)

\[
Q_{61} = \dot{m}C_v (T_6 - T_1)
\]

(39)

The net heat input ratio is calculated as:

\[
Q_{in2} = Q_{12'} + Q_{23} + Q_{34}
\]

(40)
The net heat output ratio is calculated as:

\[ Q_{\text{out}2} = Q_{45'} + Q_{56} + Q_{61} \]  

(41)

The heat waste ratio is calculated as [102,103]:

\[ Q_{12} = \frac{B}{2}(T_{2'} + T_4 - 2T_0) \]  

(42)

The power generation and the first law efficiency of the system are defined as follows:

\[ p_2 = Q_{\text{in}2} - Q_{\text{out}2} \]  

(43)

\[ \eta_2 = \frac{P_2}{Q_{\text{in}}} = \frac{P_2}{Q_{12'} + Q_{23'} + Q_{34} + Q_{12}} \]  

(44)

The exhaust gas recirculation of the heat transfer loss is calculated as follows [104]:

\[ \sigma_{q2} = \frac{B(T_{2'} + T_4 - 2T_0)}{2T_0} \]  

(45)

The exhaust gas recirculation based on the functioning fluid heat rejection is as follows [105]:

\[ \sigma_{pq2} = m\int_{T_{1'}}^{T_6} C_p\left(\frac{1}{T_0} - \frac{1}{T}\right)dT + \int_{T_6}^{T_{1'}} C_v\left(\frac{1}{T_0} - \frac{1}{T}\right)dT + \int_{T_4}^{T_{1'}} C_v\left(\frac{n-k}{n-1}\right)(\frac{1}{T_0} - \frac{1}{T})dT \]  

(46)

The total exhaust gas recirculation of the system is defined as follows:

\[ \sigma_{un2} = \sigma_{q2} + \sigma_{pq2} \]  

(47)

According to refs. [30–34], ECOP of the cycle is defined as follows:

\[ \text{ECOP} = \frac{P_2}{T_0\sigma_{un2}} \]  

(48)

The \( E_{un} \) is defined as follows:

\[ E_{un2} = P_2 - T_0\sigma_{un2} \]  

(49)
Figure 2. Impact of $n$ ($n < k$) on $P_1$–$\varepsilon$ (a), $\eta_1$–$\varepsilon$ (b) and $P_1$–$\eta_1$ (c) relations.
3. Optimization Development: Evolutionary Algorithm

Genetic Algorithm

Genetic algorithms provide the best suitable answer of the studied system employing a repetitious and random exploration approach and duplicate it by simple basics of biological evolution [72]. The individual that is a possible solution to the optimization case [73] presents the values of the decision elements. More explanations about Genetic algorithms and its function is available in References [72,73].

4. Results and Discussions

4.1. Performance Analyses for the Condition n Less than the k

Figure 2 depicts the impact of \( n \) on the performance relations among power, efficiency and compression ratio. It is evident that as \( \epsilon \) increases, \( P_1 \) and \( \eta_1 \) initially increase and finally decrease. It should be noted that that \( P_{1,\text{max}} \) and \( \eta_{1,\text{max}} \) do not take place at the same time. On the other hand, \( P_{1,\text{max}} \) and \( \eta_{1,\text{max}} \) increase by the enhancement of \( n \). Furthermore, the efficiency at maximum power rises by the enhancement of \( n \).

Figure 3 illustrates the impact of \( \rho \) on the relationship between \( P_1 \) and \( \epsilon \) at \( n = 1.2 \). As depicted in Figure 3, the output power has an early rise to the pick, and gradually falls at the various values of the compression ratio (\( \epsilon \)) and the cut-off ratio (\( \rho \)). However, at the constant value of the compression ratio (\( \epsilon \)), as the value of the cut-off ratio (\( \rho \)) rises the output power also follows the rise.

![Figure 3. Effect of \( \rho \) on \( P_1 \) versus \( \epsilon \) (\( n = 1.2 \)).](image)

Figure 4 depicts the \( E_{\text{un}} \) changes against \( P_1 \) and \( \eta_1 \) relations at various \( n \). It is obvious that \( n \) has a direct relationship with \( P_1 \) and \( \eta_1 \). The maximum \( E_{\text{un}} \) point is adjacent to \( P_{1,\text{max}} \) and \( \eta_{1,\text{max}} \). In other words, the optimum values of \( P_1 \) and \( \eta_1 \) could be achieved when \( E_{\text{un}} \) is optimized.
As shown in Figure 5a. As the compression ratio increases, with a very steep gradient, ECOP first increases to its maximum point and then begins to decrease. Also, in a constant compression, the ECOP increases with the increase of the \( n \). Figure 5b,c shows that the maximum value of the coefficient of performance for various \( n \) will occur at almost the maximum power and maximum thermal efficiency.
Figure 5. Impact of $n$ ($n < k$) on ecological coefficient of performance (ECOP)–$\varepsilon$ (a), ECOP–$P_1$ (b) and ECOP–$\eta_1$ (c).
4.2. Performance Evaluation at the Condition of n higher than the k

Figure 6 depicts the impact of \( n \) on the performance relations among power, efficiency and compression ratio. It is obvious that when \( \varepsilon \) increases, \( P_2 \) and \( \eta_2 \) initially increase and finally decrease. Enhancing \( n \) leads to a slow reduction of \( P_2 \) and \( \eta_2 \). It should be noted that \( P_{2, \text{max}} \) and \( \eta_{2, \text{max}} \) do not take place at the same value of epsilon. Hence, the \( \eta_2 \) at \( P_{2, \text{max}} \) reduces by enhancement of \( n \).

Figure 6. Impact of \( n \) (\( n > k \)) on \( P_2 - \varepsilon \) (a), \( \eta_2 - \varepsilon \) (b) and \( P_2 - \eta_2 \) (c).
Figure 7 illustrates the impact of $\rho$ on the relationship between $P_2$ and $\varepsilon$ at $n = 1.6$. As depicted in Figure 7, the output power at the beginning rises to its maximum then gradually decreases as the compression ratio ($\varepsilon$) rises at the various cut-off ratio ($\rho$). However, at the constant compression ratio ($\varepsilon$), when the cut-off ratio ($\rho$) rises the value of the output power declines.

Figure 7. Effect of $\rho$ on $P_2$ against $\varepsilon$ ($n = 1.6$).

Figure 8 presents the $E_{\text{un}}$ impact on $P_2$ and $\eta_2$ at various $n$. It is obvious that increasing $n$ leads to $E_{\text{un}}$, and $E_{\text{un}}$ reduction. The $E_{\text{un, max}}$ is adjacent to $P_{2, \text{ max}}$ and $\eta_{2, \text{ max}}$. In other words, optimum values of $P_2$ and $\eta_2$ could be achieved when $E_{\text{un}}$ is optimized.

Figure 8. Cont.
As shown in Figure 9a. As the compression ratio increases, with a very steep gradient, the ECOP first increases to its maximum point and then begins to decrease. In addition, in a constant compression, then ECOP decreases with the increase of the $n (n > k)$. Figure 9b,c shows that the maximum value of the coefficient of performance for various $n (n > k)$ will occur at almost the maximum power and maximum thermal efficiency.

Figure 8. Impact of $n (n > k)$ on $E_{un–P2}$ (a) and $E_{un–\eta 2}$ (b).

Figure 9. Cont.
Figure 9. Effect of $n$ ($n > k$) on ECOP−ε (a), ECOP–$P_2$ (b) and ECOP–$\eta_2$ (c).

4.3. Optimization Results for $n$ Less than the $k$

Three objective functions are utilized in this optimization: $\eta_1$, ECOP and $E_{un1}$, described by Equations (26), (30) and (31), respectively. Also, three decision variables are considered: $\varepsilon$, $\rho$ and $n$.

Although the decision variables might be various in the optimizing plan, they typically need to be fitted in a sensible range. Thus, the objective functions are determined by the limits of decision variables:

$$18 \leq \varepsilon \leq 25$$

(50)
In this study, ECOP and $E_{an1}$ of the dual Miller cycle are maximized concurrently employing MOO by the mean of the NSGA-II approach. The objective functions are illustrated by Equations (26), (30) and (31) and the limitations by Equations (50)–(52).

The decision parameters of optimization are as follows: $\varepsilon, \rho$ and $n$. The Pareto optimal frontier of objective functions (the thermal efficiency, ECOP, and $E_{un1}$) is depicted in Figure 10. All the optimized solutions for three objective functions obtained by NSGA II depicted in Figure 10. Selected points with different decision-making methods are presented, as well.

Table 1 outlines and compares the optimum values associated with the optimization elements utilizing Bellman-Zadeh, LINMAP, and TOPSIS approaches. In Table 1, the optimal solutions are determined for the considered objective functions and parameters used for decision by applying Fuzzy, LINMAP and TOPSIS methods and the outcomes of [102] for $n$ less than $k$ is represented. In order to calculate the results’ deviation from the solutions in ideal and non-ideal conditions, Equations (53) to (55) are used.

$$d_+ = \sqrt{\left(\eta_1 - \eta_{1,n}\right)^2 + \left(E_{un1} - E_{un1,n}\right)^2 + \left(ECOP - ECOP_n\right)^2}$$  \hspace{1cm} (53)

$$d_- = \sqrt{\left(\eta_1 - \eta_{1,n,non-ideal}\right)^2 + \left(E_{un1} - E_{un1,n,non-ideal}\right)^2 + \left(ECOP - ECOP_{n,non-ideal}\right)^2}$$  \hspace{1cm} (54)

$$d = \frac{d_+}{d_+ + d_-}$$  \hspace{1cm} (55)

where $\eta_{1,n}$ refers to Euclidian the thermal efficiency, $E_{un1,n}$ is Ecological function and $ECOP_n$ denotes ECP. Moreover, in Table 1, the deviation index (d) of the data in each case is shown.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{pareto_frontier.png}
\caption{Distribution of the Pareto optimal frontier.}
\end{figure}
Table 1. Optimization results ($n < k$).

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Variables</th>
<th>Objective Functions</th>
<th>Deviation Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOPSIS</td>
<td>$\varepsilon$</td>
<td>$\rho$</td>
<td>$n$</td>
</tr>
<tr>
<td></td>
<td>19.4528</td>
<td>1.6852</td>
<td>1.4000</td>
</tr>
<tr>
<td>LINMAP</td>
<td>19.8660</td>
<td>1.6408</td>
<td>1.3999</td>
</tr>
<tr>
<td>Fuzzy</td>
<td>20.0708</td>
<td>1.5424</td>
<td>1.4000</td>
</tr>
<tr>
<td>Ideal Solution</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Non-Ideal Solution</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

As represented in Table 1, in the cases of employing LINMAP, TOPSIS, and Fuzzy, the deviation indexes are 0.1333, 0.0412 and 0.3462, respectively. Based on these obtained values for deviation, it is concluded that employing TOPSIS results in a more appropriate solution, therefore, the final solution is selected according to this method of decision making.

Table 2 demonstrates the comparative analysis of the optimization, utilizing the maximum absolute percentage error (MAAE) (see the first row), and the mean absolute percentage error (MAPE) (see the first row) to calculate the performance of the aforementioned optimization. Every optimization is run 30 times in MATLAB® (9.5, MathWorks, Natick, MA, USA) software to deliver the ultimate results considering FUZZY Bellman-Zadeh, LINMAP, and TOPSIS methods.

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>LINMAP</th>
<th>TOPSIS</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>$\eta_1$</td>
<td>ECOP</td>
<td>$E_{un1}(kW)$</td>
</tr>
<tr>
<td>Max Error %</td>
<td>1.3610</td>
<td>1.4930</td>
<td>1.1620</td>
</tr>
<tr>
<td>Average Error %</td>
<td>0.5550</td>
<td>0.7340</td>
<td>0.4610</td>
</tr>
</tbody>
</table>

4.4. Optimization results for $n$ higher than the $k$

Three objective functions are utilized in this optimization: $\eta_2$, ECOP and $E_{un2}$, described by Equations (44), (48) and (49), respectively. Also, three decision variables are considered: $\varepsilon$, $\rho$ and $n$.

Although the decision variables might be different in the optimizing plan, they typically need to be fitted in a sensible range. Thus, the objective functions are determined by the limits of decision variables:

$$18 \leq \varepsilon \leq 25$$  
$$1.5 \leq \rho \leq 1.8$$  
$$1.4 < n \leq 1.6$$

In this study, $\eta_2$, ECOP and $E_{un2}$ of the dual Miller cycle are maximized concurrently utilizing MOO based on the NSGA-II approach. The objective functions are illustrated by Equations (48)–(50) and limitations by Equations (56)–(58).

The decision parameters of optimization are as follows: $\varepsilon$, $\rho$ and $n$. The Pareto optimal frontier of objective functions (the thermal efficiency, ECOP, and $E_{un2}$) is depicted in Figure 11. All the optimized solutions for three objective functions obtained by NSGA II depicted in Figure 11. Selected points with different decision-making methods are presented, as well.
In this study, $\eta$, ECOP and $E_{un2}$ of the dual Miller cycle are maximized concurrently utilizing MOO based on the NSGA-II approach. The objective functions are illustrated by Equations (48)–(50) and limitations by Equations (56)–(58).

The decision parameters of optimization are as follows: $\varepsilon$, $\rho$ and $n$. The Pareto optimal frontier of objective functions (the thermal efficiency, ECOP, and $E_{un2}$) is depicted in Figure 11. All the optimized solutions for three objective functions obtained by NSGA II depicted in Figure 11. Selected points with different decision-making methods are presented, as well.

Figure 11. Distribution of the Pareto optimal frontier.

Table 3 outlines and compares the optimum values associated with the optimization elements utilizing Bellman-Zadeh, LINMAP, and TOPSIS approaches. In Table 3, the optimal answers to the objective functions and the employed variables for decision making are represented for the executed methods. In order to calculate the deviation from ideal and non-ideal cases, Equations (59) to (61) are used.

\[
d_+ = \sqrt{(\eta_2 - \eta_{2,n})^2 + (E_{un2} - E_{un2,n})^2 + (ECOP - ECOP_n)^2}
\]

\[
d_- = \sqrt{(\eta_2 - \eta_{2,n,non-ideal})^2 + (E_{un2} - E_{un2,n,non-ideal})^2 + (ECOP - ECOP_{n,non-ideal})^2}
\]

\[
d = \frac{d_+}{(d_+ + d_-)}
\]

$\eta_{2,n}$ is Euclidian the thermal efficiency, $E_{un2,n}$ denotes Ecological function and $ECOP_n$ refers to COP. In addition, the indexes of deviation in different considered cases are represented in Table 3.

As shown in Table 3, the indexes of deviation in the cases of employing Fuzzy, LINMAP and TOPSIS equal to 0.3173, 0.0696 and 0.0696, respectively. On the basis of the mentioned obtained values of deviation indexes, the final solution is selected by using LIMAP and TOPSIS for the dual Miller cycle.

Table 4 demonstrates the comparative analysis of the optimization, using MAAE and MAPE to calculate the performance of the aforementioned optimization. Every optimization is run 30 times in MATLAB® software to deliver the ultimate results considering FUZZY Bellman-Zadeh, LINMAP, and TOPSIS methods.
Table 3. Optimization results ($n > k$).

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Variables</th>
<th>Objectives Functions</th>
<th>Deviation Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon$</td>
<td>$\rho$</td>
<td>$n$</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>20.2187</td>
<td>1.6870</td>
<td>1.4000</td>
</tr>
<tr>
<td>LINMAP</td>
<td>20.2187</td>
<td>1.6870</td>
<td>1.4000</td>
</tr>
<tr>
<td>Fuzzy</td>
<td>20.3829</td>
<td>1.5243</td>
<td>1.4000</td>
</tr>
<tr>
<td>[102] For maximum of $\eta_1$</td>
<td>27.8400</td>
<td>2.0000</td>
<td>1.4000</td>
</tr>
<tr>
<td>[102] For maximum of ECOP</td>
<td>15.7800</td>
<td>2.0000</td>
<td>1.4000</td>
</tr>
<tr>
<td>[102] For maximum of $E_{un2}$ (kW)</td>
<td>19.8000</td>
<td>2.0000</td>
<td>1.4000</td>
</tr>
<tr>
<td>Ideal Solution</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Non-Ideal Solution</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4. Comparative analysis presenting MAPE and MAAE.

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>LINMAP</th>
<th>TOPSIS</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>$\eta_2$</td>
<td>ECOP $E_{un2}$ (kW)</td>
<td>$\eta_2$</td>
</tr>
<tr>
<td>Max Error %</td>
<td>0.8130</td>
<td>2.0580</td>
<td>2.0640</td>
</tr>
<tr>
<td>Average Error %</td>
<td>0.5421</td>
<td>0.8870</td>
<td>0.9210</td>
</tr>
</tbody>
</table>

5. Conclusions

A thermodynamic optimization has been carried out to obtain the thermal efficiency, ECOP, and $E_{un2}$ of the Dual-Miller Cycle. The compression ratio, the cut-off ratio, and the polytropic index are examined by the NSGA-II approach. Employing various decision-making methods (LINMAP, TOPSIS and fuzzy), the best optimum answer selected from the Pareto frontier. The study achieved a promising and satisfactory state of operation for Dual-Miller systems. The three methods give closed results (with a relative difference less than 3% on the compression ratio, 5% on cut-off ratio, 2% on the objective function. Two scenarios for optimization presented in this article that for the results of condition that ($n < k$) the best point has been LINMAP answer. The thermal efficiency for this point has been 0.5388. Also, the Ecological Coefficient of performance and Ecological function have been 1.6899 and 279.2210, respectively. For the results of the condition that ($n > k$) the best point has been LINMAP and TOPSIS answer. The thermal efficiency for this point has been 0.5385. Also, the Ecological Coefficient of performance and Ecological function have been 1.6875 and 279.7315, respectively. Finally, with the purpose of error investigation, average and maximum errors of obtained results are computed for every two scenarios. The results may be applicable to optimize the performance of practical Dual-Miller Cycle engines. The work which represents a theoretical investigation will be extended to an experimental investigation for future studies.


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Nomenclature

DMC Dual-Miller cycle

$m$ Mass flow rate (kg/S)

$n$ Polytropic exponent

$k$ The specific heat ratio (adiabatic exponent)

$P$ Power (kW)

$Q$ Heat (kW)

$T$ Temperature (K)

ECOP Ecological Coefficient of Performance

$E_{un}$ Ecological function

$V$ volume

$C_v$ The specific heat at constant volume (kJ/kg.K)

$C_p$ The specific heat at constant pressure (kJ/kg.K)

$\sigma_{un}$ total entropy generation (kW/K)

$\eta$ Efficiency

$r_M$ The Miller cycle ratio of a Dual-Miller cycle

$\rho$ The cut-off ratio

$\lambda$ The pressure ratio

$\epsilon$ The compression ratio

$B$ Heat transfer coefficient

$T_0$ Ambient temperature

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