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Stable Lagrange points of large planets as possible regions where WIMPs could be sought

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Abstract: *In this Letter we show that the physical properties of the Lagrange points of large planets could provide an effective mechanism for trapping dark matter, if dark matter really exists in our Solar System. Certainly, the familiar trapping mechanism of a potential well combined with some dissipative processes is not a good candidate for particles like WIMPs which are supposed to be very slippery. However, in each of the Lagrange regions, L_4 and L_5, of large planets the potential has a maximum which together with the Coriolis force provides an effective trapping mechanism without the need of any kind of friction. This is a purely inertial and gravitational mechanism with no assumptions on other possible interactions. Hence if the density of dark matter is not negligible in this part of the Universe, a direct experiment to be considered is the establishment of a satellite in orbit around one of the stable Lagrange points, L_4 or L_5, of Jupiter.*

1. Introduction

One of the most intriguing results of modern astronomy is that the mass of all visible matter (that is atomic matter as stars, dust, and gas) is too small to explain the observed velocities in galaxies and clusters of galaxies. Consequently, the total gravitational force required to balance the centrifugal forces related to the observed velocities in galaxies is ascribed to the visible matter plus some kind of 'dark matter' which has been a riddle in astronomy for seventy years. It appears now that there is at least ten times as much dark matter as atomic matter and the formation of galaxies and galaxy clusters is totally dominated by the gravity of dark matter [1]. The most striking information comes from gravitational lensing effects. The light rays from distant galaxies are deflected where the space is curved by gravitational influence of dark matter, making shapes of the background galaxies appear distorted. Moreover, observations of very distant objects revealed that the expansion of the Universe is faster now than it was in the past. This effect is ascribed to dark energy which accelerates the expansion. Opinions whether weakly interacting massive particles (WIMPs) or dark matter exists also in our Solar System, vary considerably from author to author. Our aim is not to discuss here the different points of view but to investigate where dark matter, if it exists in our neighbourhood, could be found and hence, how it might be detected by means of an experiment which takes into account only its inertio–gravitational properties and not its yet unknown interactions.

Potential hills and Coriolis forces versus potential wells and dissipation

A familiar mechanism which leads particles to be trapped in certain regions of space or, more generally, a system in a certain range of parameters, is a potential well combined with some dissipative process. In the absence of dissipation an incident particle entering a potential well will leave it with the same energy. However, if this particle is subject to collisions with other particles already existing there, its energy will be dissipated in the far degrees of freedom of the system and so it might eventually be trapped inside. But dissipative processes are obviously not good candidates for trapping particles like WIMPs, which are expected to interact only weakly with the rest of the Universe. If a falling meteorite remains on the surface of the Earth this is due to van der Waals forces depending on another coupling constant, the electric one, while WIMPs will very likely penetrate the Earth crust as neutrinos do.

However, potential wells combined with dissipative processes are not the only trapping mechanisms. For example, when the pivoting point of an inverted pendulum is subject to some specific vertical oscillations, there is a dynamic stability region near its *top position* , as predicted by the theory of the Mathieu-Hill equations ; related phenomena are the *phase-lock loops* in electronics. Similar situations appear in celestial mechanics in the restricted three-body problem [2–5], where particles gather around the Lagrange points L_4 and L_5 of the large planets. The latter are *potential hills*, and in this case the phenomenon responsible for trapping is not any more due to an energy loss as in the case of the potential well, but to the Coriolis force, which hinders the descent of the particles making them to spin around the hill. See the discussion at the end of Section 2, where we artificially altered the form of the Coriolis term to better understand how the mechanism works.

The above trapping effect is in some way similar to the spiraling under the influence of a magnetic field of a charged particle inside a Penning trap . Both Coriolis and Lorentz forces are proportional and perpendicular to the velocity. Hence lowering the velocity by means of some additional dissipative processes may have adverse effects.

Are there large regions in our Solar System without WIMPs ?

Before trying to see whether the Lagrange points could provide an effective mechanism for trapping dark matter, let us first try to understand why this invisible but gravitationally active material, if it exists, has not been detected earlier in our Solar System.* A possible answer is that most of the interplanetary WIMPs might have been scattered away by sling effects in the last four and a half billion years, following the fate of the dust and other small objects contained in a sphere of radius approximately equal to that of Neptune's orbit. To explain this idea, consider the extreme case in which a particle is moving with a velocity $\mathbf{v_1}$ along a circular orbit e.g. in the opposite direction of Jupiter. Let us suppose that this encounter is quite close, so that this particle will be scattered back along the second branch of a narrow hyperbola. If the particle is moving on an orbit similar to that of Jupiter, its initial velocity is $v_1 = |\mathbf{v_1}|$, where $\mathbf{v_1} \approx -\mathbf{v_J}$, the velocity of Jupiter, and it will be scattered back with a velocity $v_1 + 2v_J$. This corresponds to a centrifugal force approximately *9 times* larger than that required to remain on Jupiter's orbit.

This example is certainly a limiting case and it is discussed here only for illustrative purposes, but similar processes take place for a large class of orbits and incidence angles. For example, if the incident particle moves on a great circle orthogonal to the ecliptic with a velocity $v_1 \approx v_J$, $\mathbf{v_1} \perp \mathbf{v_J}$, before being scattered along the forward direction of the planet with a velocity $v_J + \sqrt{v_1^2 + v_J^2}$, the centrifugal force acting on the scattered particle will be approximatively $(1+\sqrt{2})^2 \approx 5.82$ times larger than that required to remain on Jupiter's orbit. Consequently, this particle will also be scattered on higher orbits. Even if the particles are not scattered outside the Solar System their density will diminish considerably in the region of large planets.

The Lagrange points of large planets as possible regions where WIMPs might exist

Subjects related to the Architecture of the Solar System have been studied in the past, e.g. in the classical book of Öpick [7], or in the more recent papers [8–10]. Our statement is that, if WIMPs existed in the past in our Solar System, we may find them nowadays *only there* where the dynamical laws allow them to be. Such locations might be of course the interior of the large bodies of the Solar System, but also around the Lagrange points L_4 and L_5 of the large planets, where otherwise much dust and many asteroids have accumulated over time. Moreover, in contrast to the interior of the celestial bodies, they have the advantage of being experimentally accessible. Thus a spacecraft might be sent towards the Trojan asteroids region of Jupiter for example, to detect first how much visible matter is there. Then, from this spacecraft, a second small one could be launched to an orbit perpendicular to Jupiter's one, around the Lagrange region. From the characteristics of its trajectory it can be found easily how much

*It is probably worthwhile to mention here an astounding observation related to the unexpected slowing down of the Pioneer 10 and Pioneer 11 spacecrafts [6], which are now traveling far beyond Neptune. Among the plausible explanations, there is the possible pull back due to dark matter. It is interesting to notice that this seems to be in agreement with the above discussion that the sky has been already cleaned in the region of the large planets.

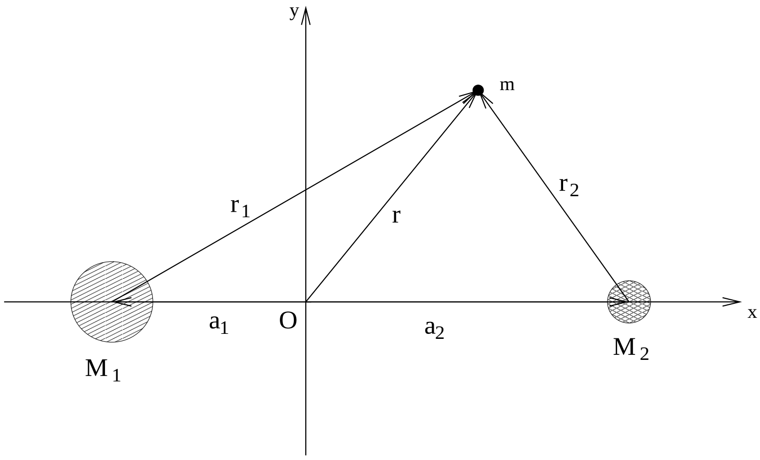


Figure 1: *The restricted three-body problem*

matter, visible and invisible, is present there. Since the parameters of a trajectory can be measured very accurately, the benefit of this procedure is that the overall precision of the amount of the invisible matter – if any – *depends only* on the precision with which the total mass of the *visible* matter there can be determined. An obvious advantage of such an experiment is that it is based on purely inertial and gravitational measurements, with no assumptions on other interactions, weak or otherwise, of this hypothetical matter.

2. Some preliminary considerations: the restricted three-body problem and its osculating quadratic approximation

The three-body problem is, in general, not analytically integrable. This problem, together with the extension to more bodies, was the initial motivation for Poincaré in his general considerations for dynamical systems. However in the last years there has been considerable breakthroughs, as the remarkable paper of Chenciner and Montgomery [11] who were able to give an *exact* solution in the special case of three equal masses.

The three-body problem has nine degrees of freedom and so gives rise to a system of differential ordinary equations of order eighteen. By working in the center of mass system and making use of the conservation of energy and conservation of angular momentum we can reduce the order to eight. Furthermore, restricting our consideration to motions which are planar, the order can be reduced to four. This is the best that can be done in general. Even after all these restrictions the system is not integrable, i.e. it does not possess a complete set of global isolated conserved quantities. Therefore, the problem is still extremely complicated and has kept mathematicians busy for several hundred years [2].

Although in this Letter we solve numerically the *planar restricted three-body* problem in its full extent, let us first review briefly its quadratic approximation [2–5]. Two bodies M_1 and M_2 are moving in circular orbits of radii a_1 and a_2 about their center of mass O. The *restricted three-body problem* concerns the motion of a third small mass m ($\ll M_1$, M_2) in their gravitational field. Assuming that the third body is moving in the plane of the first two (see Figure 1), the total force exerted on m will be:

$$\mathbf{F} = -\frac{GM_1 m}{|\mathbf{r}-\mathbf{a_1}|^3}(\mathbf{r}-\mathbf{a_1}) - \frac{GM_2 m}{|\mathbf{r}-\mathbf{a_2}|^3}(\mathbf{r}-\mathbf{a_2})\,. \tag{1}$$

Let us use a rotating frame of reference with origin at the centre of mass O, in which the two large masses hold the fixed positions $\mathbf{a_1} = -\mathbf{a_2}\, M_2/M_1$. The angular velocity ω is determined by Kepler's law:

$$\omega^2 a^3 = G(M_1 + M_2)\,, \quad a = a_1 + a_2\,. \tag{2}$$

The effective force in this rotating frame is:

$$\mathbf{F}_\omega = \mathbf{F} - 2m(\boldsymbol{\omega} \times \dot{\mathbf{r}}) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}). \tag{3}$$

Choosing a set of cartesian coordinates with origin at O we can write:

$$\boldsymbol{\omega} = \omega\mathbf{k}\,, \quad \mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}\,, \quad \mathbf{a_1} = -a_1\mathbf{i}\,, \quad \mathbf{a_2} = a_2\mathbf{i}\,.$$

The equations of motion for the third small body are:

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} &= -\frac{\partial V}{\partial x}\,, \\ \ddot{y} + 2\omega\dot{x} &= -\frac{\partial V}{\partial y}\,. \end{aligned} \tag{4}$$

where $r_1 = \sqrt{(x+a_1)^2 + y^2}$, $r_2 = \sqrt{(x-a_2)^2 + y^2}$, and V is the effective potential in the rotating frame:

$$V = -\frac{1}{2}\omega^2(x^2+y^2) - \frac{GM_1}{r_1} - \frac{GM_2}{r_2} + \mathfrak{C}\,. \tag{5}$$

The constant $\mathfrak{C}$ is chosen so that the maximum value V_{max} is zero and is attained when $r_1 = r_2 = a$. For the Sun-Jupiter system $\mathfrak{C} \simeq 2.5541 \times 10^7$ daMKS. Further $\omega \approx 2\pi/11.83$ years^{-1}, $a \approx 778.33 \times 10^6 km$ and $M_1/M_2 \approx 1047$.

By multiplying equations (4) by $\dot{x}$ and $\dot{y}$, respectively, and adding, we find the following integral of motion (Jacobi's constant):

$$V + \frac{1}{2}(\dot{x}^2 + \dot{y}^2) = \mathcal{C} = const\,, \tag{6}$$

which is essentially the energy (the Hamiltonian). No other integral of motion for this system is known. For a given value of $\mathcal{C}$, we obtain

$$\dot{x}^2 + \dot{y}^2 = 2(\mathcal{C} - V) > 0\,. \tag{7}$$

This shows that the motion is restricted to the region bounded by the curve $V = \mathcal{C}$, known as the Hill curve [12, 13], which corresponds to zero velocity $\dot{x} = \dot{y} = 0$. Since according to our definition (5) of $\mathfrak{C}$ the effective potential V is never positive, the equipotential curves (the Hill curves) correspond to negative values of $\mathcal{C}$. See Figures 2, 3 and 4 for the Sun-Jupiter system, where, to have a closer insight into the problem, we have labeled the effective potential curves in daMKS units (in deca m^2/sec^2), since on Earth this corresponds to a difference of approximately one metre between level curves. For $\mathcal{C} = 0$ the Hill curve degenerates into two isolated points, usually denoted by L_4 and L_5, which form the apices of two equilateral triangles having their other vertices at the large masses:

$$L_4: \quad (x_{L_4}\,, y_{L_4}) = (\mu\frac{a}{2}\,, \frac{\sqrt{3}}{2}a) \quad L_5: \quad (x_{L_5}\,, y_{L_5}) = (\mu\frac{a}{2}\,, -\frac{\sqrt{3}}{2}a)\,, \tag{8}$$

where $\mu = (M_1 - M_2)/(M_1 + M_2)$.

In order to investigate stability around these Lagrange points, we can expand the effective potential (5) in a Taylor series, for instance about its maximum at (x_{L_4}, y_{L_4}), and then solve the equations of motion (4) for small departures $\tilde{x}$, $\tilde{y}$ from the equilibrium point

$$x = x_{L_4} + \tilde{x}\,, \quad y = y_{L_4} + \tilde{y}\,, \tag{9}$$

and retain only quadratic terms:

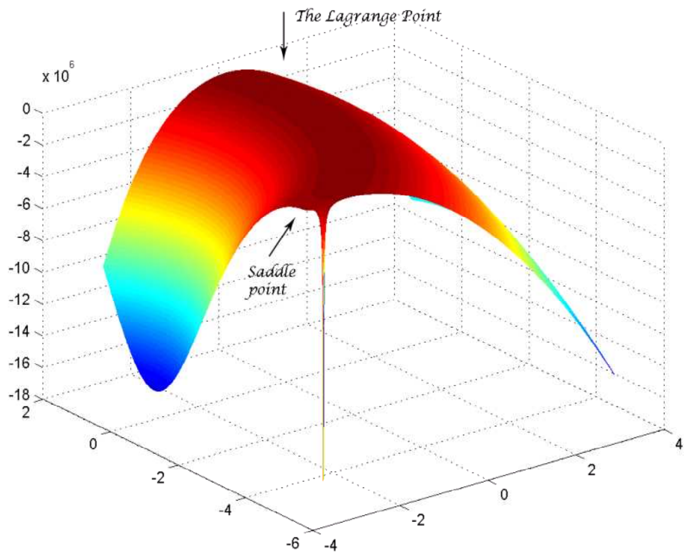


Figure 2: *Three-dimensional plot of the effective potential for the Sun-Jupiter system.*

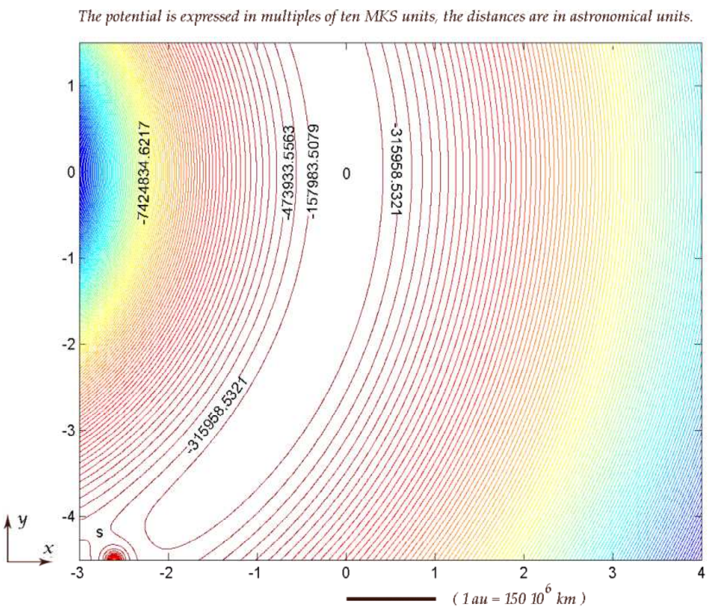


Figure 3: *Equipotential curves (the Hill curves) for the Sun-Jupiter system.*

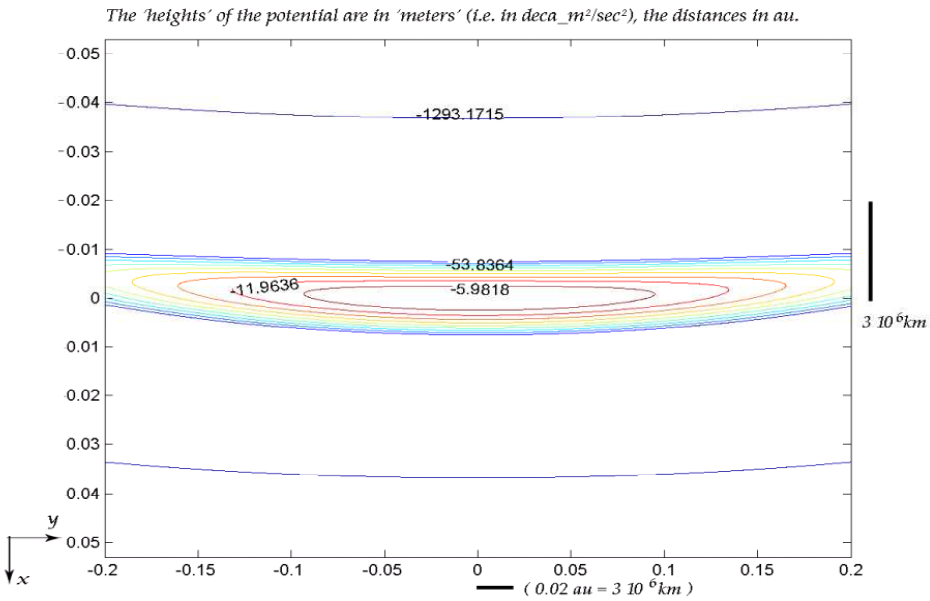


Figure 4: *Equipotential curves around the Lagrange point L_4 for the Sun-Jupiter system. The scale in the x direction was enlarged by a factor of three to make the equipotential curves easily visible.*

$$\frac{d}{dt}\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ v_x\\ v_y\end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{3}{4}\omega^2 & \frac{3\sqrt{3}}{4}\mu\omega^2 & 0 & 2\omega\\ \frac{3\sqrt{3}}{4}\mu\omega^2 & \frac{9}{4}\omega^2 & -2\omega & 0\end{pmatrix}\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ v_x\\ v_y\end{pmatrix}. \tag{10}$$

These equations can be further simplified if we perform the change of variables $\tilde{t} = \omega t$ (if we measure the time in units corresponding to a radian , i.e. in 1.8875 years in the case of Jupiter). Redefining the velocities accordingly, $\tilde{v}_{x,y} = v_{x,y}/\omega$, we obtain

$$\frac{d}{d\tilde{t}}\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{v}_x\\ \tilde{v}_y\end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{3}{4} & \frac{3\sqrt{3}}{4}\mu & 0 & 2\\ \frac{3\sqrt{3}}{4}\mu & \frac{9}{4} & -2 & 0\end{pmatrix}\begin{pmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{v}_x\\ \tilde{v}_y\end{pmatrix}. \tag{11}$$

The motion around L_4 will be stable according to Laplace's stability criterion if the four eigenvalues $\lambda = \pm i\frac{1}{2}\sqrt{2 \pm \sqrt{27\mu^2 - 23}}$ of the evolution matrix are pure imaginary and this will be true if

$$\sqrt{27\mu^2 - 23} \leq 2 \quad \text{and} \quad \mu^2 \equiv ((\mathrm{M}_1 - \mathrm{M}_2)/(\mathrm{M}_1 + \mathrm{M}_2))^2 > 23/27\,. \tag{12}$$

The first condition is always satisfied since μ is smaller than one, while the second requires

$$M_1/M_2 > (25 + \sqrt{621})/2 \approx 24.9599\,. \tag{13}$$

The stability condition (13) is satisfied for all the planets of our Solar System; for instance in the case of Sun and Jupiter $M_1/M_2 \approx 1047$. The regions around these points are occupied by the Trojan asteroids whose orbital periods are the same as Jupiter's, 11.86 years. The periods of small oscillations about these equilibrium points are given by:

$$\Omega^2 = -\omega^2\lambda^2 = \frac{1}{2}\omega^2\left(1 \pm \sqrt{1 - \frac{27M_1M_2}{(M_1+M_2)^2}}\right), \tag{14}$$

which yields 11.90 years and 147.4 years, respectively. More detailed studies of the motion and the stability of the motion around the Lagrange points L_4 and L_5, in general, or for the Sun-Jupiter system, in particular, can be found in [14–18].

To have an intuitive picture, the particles gather around the maxima L_4 and L_5 as clouds gather around high mountains since the Coriolis force acts perpendicularly to their descent movement and makes them spin. The role of the Coriolis force can be investigated directly by *artificially* weakening it by a factor $\xi < 1$. The eigenvalues of this artificial evolution matrix become

$$\lambda_\xi = \pm\frac{\omega}{2}\sqrt{6 - 8\xi^2 \pm \sqrt{64\xi^4 - 96\xi^2 + 9 + 27\mu^2}}\,. \tag{15}$$

One can check that independently of the value of μ the eigenvalues (15) lose their property of being pure imaginary if the Coriolis force is weakened by a factor $\xi < \sqrt{3}/2$, which of course is quite different from its true value $\xi \equiv 1$.

3. An academic example: a sole potential hill.

Many of the features of the restricted three-body problem are missing in its osculating quadratic approximation, where the potential is not bounded below and moreover does not follow the curvature of the trajectory of the planet. In this section we present a 'Gedankenbeispiel' half way toward reality which shows how particles can be trapped around potential maxima due to the Coriolis force. This example is only illustrative and has a role similar to that of the potential box in Quantum Mechanics.

We are especially interested in the metastable states, i.e. non-equilibrium states which persists for some period of time, since we would like to understand how WIMPs might accumulate in these regions. Hence, let us consider the effective potential given in equation (5) but artificially truncated below in order to obtain potential hills of finite height such as the example shown in Figure 5(a). The system of differential equations (4) corresponding to this potential is then solved using a Runge-Kutta method subject to initial conditions yet to be determined.

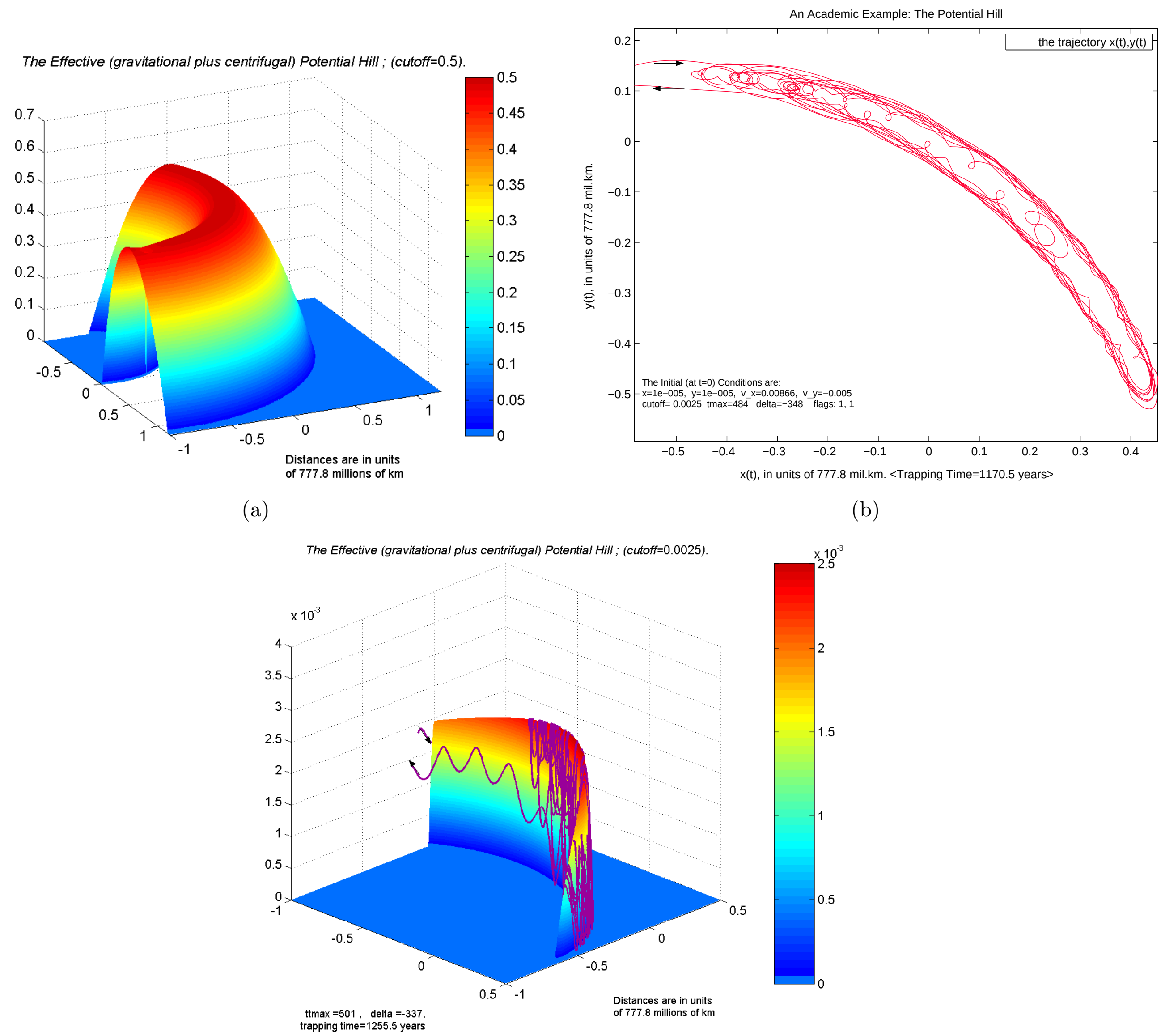
First, since we are merely interested in metastable trajectories and not in trajectories confined to some given region of the space, we choose the initial positions and velocities so that the constant $\mathcal{C}$ in equation (6) should be small but positive. This is because, according to the discussion which follows equation (7), positive values of $\mathcal{C}$ correspond to complex valued Hill curves.

Secondly, to ensure that the trajectory passes near a Lagrange point we chose the initial positions in the neighbourhood of L_4, for example, while the direction of the initial velocity is taken close to the direction of the velocity component of the eigenvectors of the evolution matrix from equation (11). Then the differential equations are solved forward and backward in time.

We preferred to work with neighbourhoods of the eigenvalues instead with their exact values in order to have a relevant phase space of non-vanishing measure. Many metastable trajectories were found in this way and one of them is depicted in Figure 5(b). In Figure 5(c) the trajectory and the potential are represented on the same graph in order to better visualize the state of havoc of the trajectory as well as the energy variations along it. The undulating shapes of the trajectory in the ingoing and outgoing regions are direct consequence of the Coriolis force.

4. Metastable trajectories for the exact potential

In this Section the system of differential equations (4) is solved for the exact effective potential (5) using again a Runge-Kutta method. The initial conditions were chosen as explained previously. Two types of metastable trajectories were found: some which are concentrated around a Lagrange point similar to that presented in Figures 5(b) and 5(c), and others which pass from the Lagrange point L_4 to L_5 similar to the trajectory of the Earth's second natural satellite, Cruithne, (see Figure 6).



(a) (b)

(c)

Figure 5: *An academic example: the movement of a particle around a potential hill emerging in a limited region around a Lagrange point L_4.*

The trajectories in the region of the Lagrange points are usually described as being chaotic. We prefer the term 'almost chaotic' †), since in spite of the cramped trajectories in these regions, the solutions $x(t)$ and $y(t)$, of the system of differential equations (4) are reproducible curves of class C^2. Moreover, unlike what happens in the case of chaos, the trajectories are not recurrent since they eventually leave the entanglement region and continue far away.

The characteristics of these entangled trajectories can be analyzed by examining the Poincaré Sections, i.e. the intersections of the curves from the phase portrait with a suitably chosen transversal manifold [2,4]. We find here several stable (elliptical) critical points alternating with hyperbolic (unstable) ones. The hyperbolic critical points are the loci of intricate entanglements where the system may spend much time.

These 'pandemonia' are welcome in our case, since they lead to the increase of the bulk density of

†L. A. Smith [19] would probably call such a region a *pandemonium*.

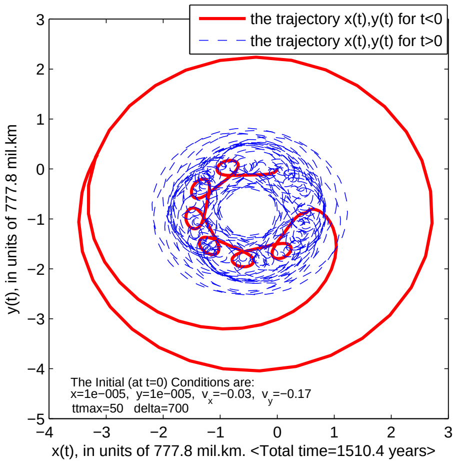


Figure 6: *A metastable trajectory for the Sun-Jupiter system encompassing the Lagrange points L_4 and L_5. Since we are in a rotating frame, free particles, both incident and outgoing, move along spirals.*

particles there. The Arnol'd diffusion zones [20], which can appear for small nonzero incidence angles around the planar movement can have the same effect. ‡

Our main aim in this paper was not to investigate the characteristics of all possible metastable trajectories, but to show that the physical features of the Lagrange points of large planets provide an effective mechanism for trapping dark matter, without the intervention of any dissipative process. The depletion/accretion of usual matter in the Trojan satellite region is treated strictly as a three body problem and does not take into account the frictions or any other many body interactions. Consequently, the accumulation of the invisible, but gravitationally active matter, should be similar as can be seen from direct calculations.

It is interesting to note that according to Monte Carlo simulations [10, 21] the time necessary for the accretion of ordinary dust and matter in the Trojan region is of the order of a hundred thousands years, i.e. much shorter than one would expect for phenomena on the astronomical scale.

5. Conclusion

While meteorites and other space debris can be found in Antarctica or in some stony deserts, since WIMPs, if they exist, will penetrate Earth's crust like neutrinos do, they might be found in the interior of Earth which is not easily accessible to experiments but also in the nonlinear dynamical nodes of our Solar System, such as in the Lagrange points L_4 and L_5 of the large planets. As discussed in the Introduction, the precision with which a small space probe can determine the amount of the invisible matter present in these regions is essentially given by the accuracy of determining the amount of the visible matter existent there. Hence, it might be worthwhile to land on one of the large Trojan asteroids of Jupiter, Agamemnon, Achilles or Hektor, to have a better determination of their density and, consequently, of the total visible mass. Moreover, since each of these Lagrange regions is quite narrow — see Figures 4 and 5(b) — we expect that it should not be too difficult to establish a satellite in orbit around it, to determine the total amount of mass, visible *and* invisible, present there.

‡The Arnol'd diffusion zones are meaningful in problems with three or more degrees of freedom while the planar three body problem has only two degrees of freedom. However, even small nonzero angles of incidence of particles make the problem three dimensional and in this case the Arnol'd zones are bounded by semi-permeable surfaces similar to Cantori-like sets. These may significantly obstruct the diffusion of the trajectories and thus enhance the time spent there by the system. This might lead to an increase of the bulk density of particles near the plane $z = 0$, thus emphasizing the practical importance of the planar three body problem.

Acknowledgements

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References

[1] E. Høg, *The depth of the heavens: Belief and Knowledge during 2500 years*, Europhysicsnews **35** (3), 76-80, 2004.

[2] T. W. Kibble and F. H. Berkshire, *Classical Mechanics*, Imperial College Press (5th Edition), 2004.

[3] T. Padmanabhan. *Theoretical Astrophysics: Astrophysical Processes*, Vol. 1, Cambridge University Press, 2000.

[4] J. V. Jorge and E. J. Saletan, *Classical Dynamics, a Contemporary Approach*, Cambridge University Press, 1998.

[5] V. Szebehely, *Theory of Orbits*, Academic Press, New York, 1967.

[6] H. Dittus and C. Lämmerzahl, *Die Pioneer Anomalie. Das größte von Menschen durchgeführte Experiment und sein Rätsel*, Gravitationphysik, Physik Journal **5**, 25-31, 2006.

[7] E. J. Öpik, *Interplanetary Encounters*, Elsevier, New York, 1976.

[8] J. Petit, A. Morbidelli and J. Chambers, *The primordial excitation and clearing of the asteroid belt*, Icarus **153**, 338-347, 2001.

[9] M. Fouchard, Ch. Froechlé and G. B. Valsecchi, *Is the dynamics of Jupiter family comets amenable to Monte Carlo modeling?*, Mon. Not. R. Astron. Soc. **344**, 1283-1295, 2003.

[10] A. Morbidelli, H. F. Levison, K. Tsiganis and R. Gomes, *Chaotic capture of Jupiter's Trojan asteroids in the early Solar System*, Nature **435**, 462-465, 2005.

[11] A. Chenciner and R. Montgomery, *A remakable periodic solution of the three body problem in the case of equal masses*, Annals of Mathematics, **152**, 881-901, 2000.

[12] C. Marchal and D. G. Saari, *Hill regions for the general three-body problem*, Cel. Mech. Dyn. Astron. **12**(2), 115-129, 1975.

[13] C. Marchal and G. Bozis, *Hill stability and distance curves for the general three-body problem*, Cel. Mech. Dyn. Astron. **26**(3), 311-333, 1982.

[14] C. L. Siegel and J. K. Moser, *Lectures on Celestial Mechanics*, Springer-Verlag, Berlin, 1995.

[15] J. Roels, *Sur de nouvelles séries pour le problème de masses critiques de Routh dans le problème restreint plan de trois corps*, Cel. Mech. Dyn. Astron. **12**(3), 327-336, 1975.

[16] A. Deprit and A. Deprit-Bartholome, *Stability of the triangular Lagrangian points*, Astron. J. **72**, 173-179, 1967.

[17] A. Deprit and J. Henrard, *The Trojan manifold - Survey and conjectures*, in Periodic Orbits Stability and Resonances ed. Giacaglia, G. E. O., D. Reidel Publishing Co., Dordrecht-Holland, 1-18, 1970.

[18] J. Henrard, *The web of periodic orbits at L_4*, Cel. Mech. Dyn. Astron. **83**(1-4), 291-302, 2002.

[19] L. A Smith, *Chaos: A Very Short Introduction*, Oxford University Press, 2007.

[20] V.I. Arnol'd, *Instability of Dynamical Systems*, Sov.Math.Dokl. **5**, 581,1964.

[21] A. Morbidelli (private communication), 2007.