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Hyperelastic polymer material models for robust fatigue performance of automotive LED lamps

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Abstract

The object of this paper is to determine the statistics of parameters of hyperelastic models specific to Polybutylene Terephthalate filled with 30% glass fibre (PBT GF30) and Polymethyl Methacrylate (PMMA) materials used in automotive lamps. The hyperelastic behaviour of both materials, a semi-crystalline and an amorphous, is modelled using appropriate hyperelastic models. The stress-strain curves of the materials were measured under uniaxial tension using a non-contact video gauge. Five samples each were tested to measure the effect of manufacturing variability. The model parameter statistics were determined, the mean value of the model parameters were used to construct average stress-strain behavior, which is then compared to the experimental stresses. Among all the models and their associated parameters studied, the 3-parameter Mooney-Rivlin model provided the most accurate prediction of the behaviour for both materials. The model showed excellent stability and is therefore the most appropriate model to represent variations due to the manufacturing process. The detailed study of the correlation of the model parameters provided a good understanding of how the parameters are related to each other, enabling construction of complete probability distribution functions for further analysis.

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Keywords: Hyperelastic; material models; nonlinear stress-strain; fatigue; manufacturing variations

1. Introduction

The modern automotive LED lamp housings and lenses are constructed of polymers providing a substantial weight saving and design flexibility. The material switch is attributed to the stringent emission legislations as well as functional requirements, allowing the realisation of highly sophisticated and complex lighting designs.

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Traditionally, the lamp assemblies are designed and developed based on accumulated knowledge. The historical information becomes vital as the lamp assemblies are subjected to very harsh loading under accelerated vibration and impact testing, which is of a wider frequency spectrum. The complexity of modern lamp design – the shape and size, the use of alternative material and manufacturing involved, however, have made lamps susceptible to fatigue failure from vibrational and impact loading. There is a drive towards virtual prototyping to address this aspect by using finite element methods. However, the key to robust fatigue analysis of lamp assemblies is the availability of reliable input parameters, such as material models and variability in the behaviour of manufactured constructions. Analysis of mechanical behaviour of polymers for robust fatigue analysis can be a challenging and complex task, as their properties are significantly affected by their molecular structures, environmental condition and the manufacturing process. Under elastic deformation, the stress strain relationship of polymers is notably non-linear, this mean that the Hooke's law does not hold for such materials. The linear isotropic model cannot be used in the analysis and modelling of mechanical behaviour of such materials with hyperelastic characteristics (Serban et al, 2012). Generally, hyperelastic models are used to analyse the mechanical behaviour of hyperelastic materials.

The two most commonly used materials are PBT-GF30 and PMMA. PBT-GF30 is a semi-crystalline thermoplastic material; a class of polymer with a highly ordered molecular structure. It is known to be hard and rigid, and its ability of withstanding dynamic load at wide range of temperature makes it a material of choice for designing mounting brackets and control module casing for automotive lamps. Mostly, lamps that are mounted close to the vehicle exhaust or on an area with extreme temperature are made of PBT-GF30 material. PMMA is a very important material in the design of automotive lamps. The high optical quality, resistance to UV light and weathering, decent stiffness, strength and dimensional stability of PMMA earned it an important place in the design of automotive lamps. It is used in the design of optics and outer lens of automotive lamps. The material is an amorphous thermoplastic; a class of polymer with randomly oriented molecular chains.

The object of this paper is to determine the statistics of parameters of hyperelastic models specific to PBT-GF30 and PMMA materials. To achieve this, three hyperelastic models and their associated parameters are studied: a) Neo-Hookean, b) 2, 3, and 5 parameters Mooney-Rivlin and c) 1, 2, and 3 orders Ogden model. The stress-strain curves of the materials will be measured under uniaxial tension using a non-contact video gauge. Five samples each will be tested to measure the effect of manufacturing variability. The models' stress, which is the first derivative of strain energy density function, will be obtained by fitting the models to the experimental data. The model parameter statistics will be determined; these will be then used to construct models' stresses and compared to the experimental stresses. The correlation between the model parameters will be analysed to have a better understanding of how parameters are related. It is shown that Mooney-Rivlin offers a significant and robust performance in faithfully replicating the stress-strain curves of the materials studied in this paper.

2. Hyperelastic material models

The hyperelastic models can be of phenomenological and micromechanical type. In this study, parameters of three phenomenological hyperelastic models identified from the experimental data. The stress-strain relationship for hyperelastic material is generally obtained from a strain energy density function, which is normally denoted as W; stress is obtained as a first derivative of the strain energy density function and with respect to strain:

$$\sigma_{ij} = \frac{\partial W}{\partial E_{ij}} \tag{1}$$

For incompressible materials, the strain energy density function is dependent on the stretch invariants $I_{1,2}$. The stretch invariants are given by: $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ and $I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2$. The principal stretch ratios ($\lambda_{1,2,3}$) are obtained from the transformation of principal axis, and for uniaxial tension they are:

$$\lambda_1 = \lambda = \frac{L}{L_o}; \quad \lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda}} \tag{2}$$

In the following section, brief details of three models considered are given.

2.1. Neo-Hookean Model

Neo-Hookean model (Treloar, 1943) is molecular theory based; it is the simplest available hyperelastic models. It is known as a special case of Mooney Rivlin model. The model describes the hyperelastic behaviour of material using only one independent material constant. The uniaxial stress for incompressible Neo-Hookean model is given as (Martins et al, 2006):

$$\sigma = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) \tag{3}$$

where C_{10} is a material constants which can be determined from the experimental data.

2.2. Mooney-Rivlin model

Mooney-Rivlin model (Mooney, 2006, Rivlin, 1948) is one of the early hyperelastic models. Its simplicity makes it the most widely used model in the analysis of nonlinear elastic behaviour of material. The order Mooney-Rivlin model can be varied to fit the complex of stress-strain variation. The uniaxial stress expressions for incompressible material for two, three and five parameter Mooney-Rivlin model are given below (Kumar et al, 2016, Nowark, 2008):

2 – Parameters: $\sigma_{2p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right)$ (4)

3 – Parameters: $\sigma_{3p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6C_{11}(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4}$ (5)

5 – Parameters:

$$\sigma_{5p} = 2C_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2C_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6C_{11}\left(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} - \frac{1}{\lambda^4}\right) + 4C_{20}\lambda\left(1 - \frac{1}{\lambda^3}\right)\left(\lambda^2 + \frac{2}{\lambda} - 3\right) + 4C_{02}\left(2\lambda + \frac{1}{\lambda^2} - 3\right)\left(1 - \frac{1}{\lambda^3}\right) \tag{6}$$

The material constants $C_{10}, C_{01}, C_{11}, C_{20},$ and C_{02} are determined from the experimental data.

2.3. Ogden model

The Ogden strain energy for incompressible material is based on principal stretches (Ogden, 1972). The model requires initial values for the calculation of the parameters and the accuracy of the parameters is influenced by the initial values set. The Ogden uniaxial stress for incompressible material is given as:

$$\sigma = \sum_{r=1}^{n} \mu_r \left(\lambda^{\alpha_r} - \lambda^{-\frac{1}{2}\alpha_r}\right) \tag{7}$$

where $\boldsymbol{\mu_r}$ **and** $\boldsymbol{\alpha_r}$ are material constants obtained from fitting experimental data.

3. Experimental

The PBT-GF30 test specimens were standard A1 injection moulded dumb bell tensile specimens supplied by Albis. The dimensions are in line with the test standard (ISO 527-2, 212). For PMMA, dog bone shaped specimens were cut out from optical plates that were injection moulded at Wipac. The dimensions of the narrow parallel sided portion are 80mm x 10mm x 3mm. The tensile testing was performed under room temperature using Instron 5582 tensile test machine. A constant crosshead speed of 1mm/min was used to pull the samples to failure. The stress-strain curve was measured using non-contact video gauge.

4. Experimental results

The stress-strain curves of PBT-GF30 and PMMA are shown in Fig 1. Both materials have no defined yield points. The yield stress, which is also the peak stress, for PBT-GF30 is 119MPa and for PMMA it is 63MPa. The stress-strain curves show nonlinearity up to the yield point (elastic limit). This nonlinear behaviour in the elastic region will result in the variation of elastic properties. The stiffness of both materials will vary and the rate of variation can be significant depending on the level of the strain experienced by the materials when subjected to external loading. In the engineering design, 80% to 90% material yield stress is normally adopted as the safe design stress. Therefore, it is vital that materials of automotive lamp remain elastic at 90% yield stress.

Fig 2 shows the typical behaviour of PBT-GF30 and PMMA materials subjected to load cycles with different stress levels; both materials return to their original position at every load step. The materials remain elastic up to 90% yield stress. The curves show hysteresis at every level of stress with the loop notably increasing with increasing stress; more energy is lost at higher stress loading and unloading cycles.

Since the materials tested do not obey Hooke's law, it is expected that the elastic properties will vary accordingly. Fig 3 shows the variation of the elastic modulus across the strain for both materials which were obtained by applying second order polynomial regression. The figure shows significant decrease in the elastic modulus with increasing strain. The PBT-GF30 and PMMA materials show 63% and 88% decrease in the elastic modulus to the strain corresponding to 90% yield stress. It is very clear that both materials show hyperplastic behaviour, and so modelling them with linear isotropic model would not in any way represent the actual behaviour of the materials. Therefore, to model the behaviour of both materials accurately, hyperplastic models that can give good fitting to both stress-strain curves are required.

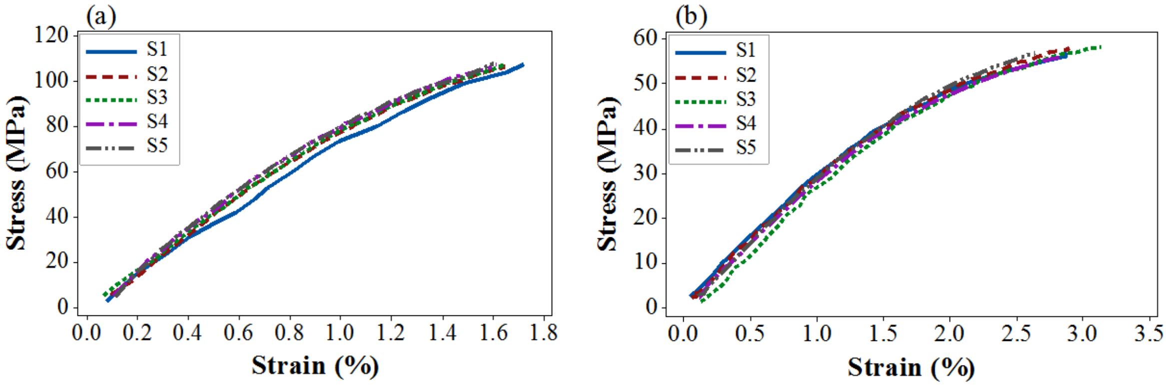

Fig 1: Stress-strain curves. (a) PBT-GF30; (b) PMMA

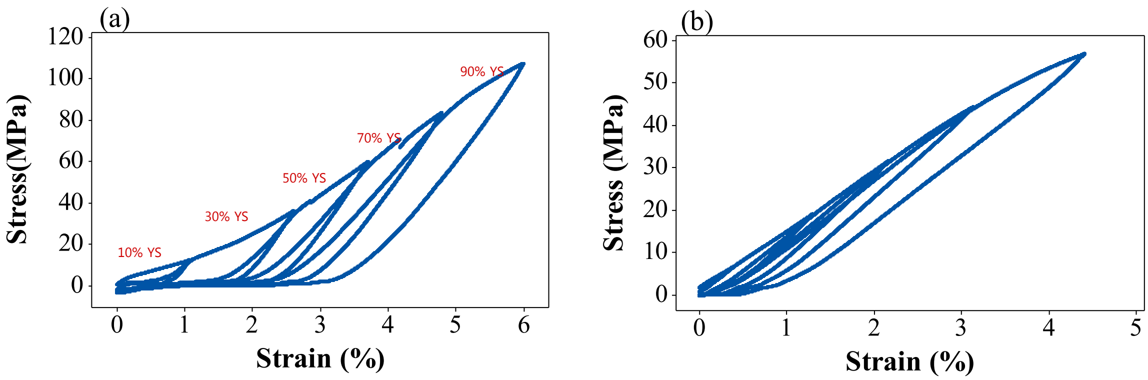

Fig 2: Stress-strain curves of loading and unloading at different stress levels. (a) PBT-GF30; (b) PMMA

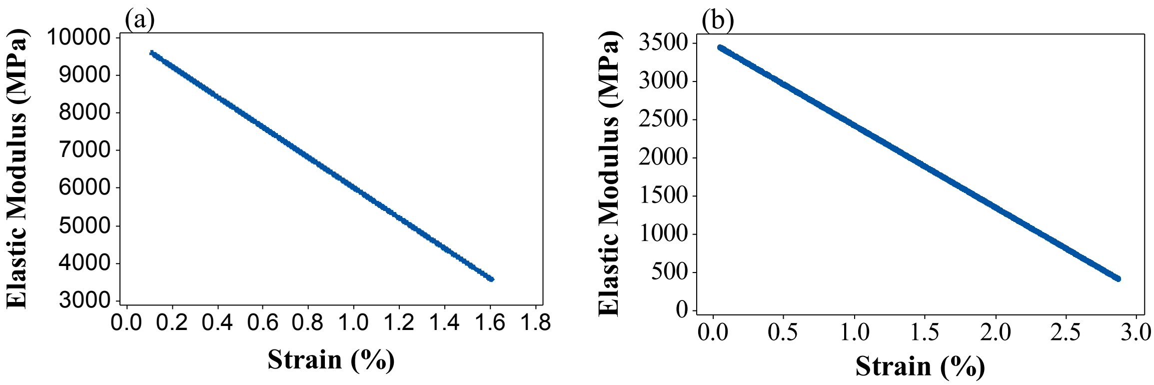

Fig 3: Variation of the elastic modulus. (a) PBT-GF30; (b) PMMA

5. Statistical analysis of model parameters

As well as exhibiting hyperelastic behavior, polymers are prone to manufacturing variability. This manufacturing variability must be considered in the process of selecting hyperelastic model for the analysis of polymers. In this study, inter-sample variation was observed on the experimental test of five samples of each material. It was apparent that the same level of variation was seen on the model parameters derived from the experimental data using MATLAB program. The Statistical analysis was performed on the obtained model parameters. The standard deviation which illustrates the variations of the parameters of the presented models is given in table 1. It can be seen that the variations in the model parameters increase with increasing order of parameters. There are relatively small amounts of variations in the parameters for Neo-Hookean, 2 and 3 parameters Mooney-Rivlin models. However, Mooney-Rivlin 5-parameter model and the three different orders of Ogden model exhibit large variations. Models with large standard deviation may not reliably model the material behaviour. The obtained mean value of the model parameters were used to construct average stress-strain behaviour and compared to the experimental stress.

Table 1: Models parameters statistics

Model	Parameters	PMMA		PBT-GF30	
		Mean	Standard deviation	Mean	Standard deviation
Neo-Hookean	C_{10}	8.848	0.287	25.04	0.923
Mooney Rivlin - 2 Par	C_{10}	7.886	0.557	24.29	0.214
	C_{01}	2.521	1.467	1.306	1.848
Mooney Rivlin - 3 Par	C_{10}	15.434	1.142	44.62	3.82
	C_{01}	-7.04	2.42	-19.8	2.75
	C_{11}	-0.6613	0.057	-2.75	0.566
Mooney Rivlin - 5 Par	C_{10}	13.94	10.01	72.1	55.6
	C_{01}	-5.67	9.42	-40.8	43.6
	C_{11}	-2.83	13.28	-121	139.8
	C_{20}	3.07	19.55	170	203.1
	C_{02}	-0.55	10.52	-70.2	92.8
Ogden - 1 Order	μ_{r1}	194	262	163.8	58
	α_{r1}	0.3085	0.187	0.453	0.163
Ogden - 2 Order	μ_{r1}	43.4	22.9	130	81.5
	α_{r1}	1.0237	0.152	1.121	0.508
	μ_{r2}	8.64	9.26	18.56	14.35
	α_{r2}	-4.038	1.081	-4.85	1.89
Ogden - 3 Order	μ_{r1}	40.8	24.9	55.02	17.97
	α_{r1}	0.826	0.351	1.296	0.352
	μ_{r2}	6.34	5.4	19.51	15.33
	α_{r2}	-4.563	0.61	-4.84	1.899
	μ_{r3}	7.59	6.53	123.8	111
	α_{r3}	1.684	0.522	0.897	0.711

6. Predictions based on average model parameters

The model stresses were constructed using the mean parameters from the statistical analysis and the results were compared to the experimental stress, see fig (4, 5). The 3 and 5 parameters Mooney-Rivlin model accurately represent the behaviours of PMMA and PBT-GF30 materials respectively. The generated stresses of both parameters of Mooney-Rivlin model match the experimental stress curve well. Although the 5-parameter model fitted reasonably well to the experimental data, care must be taken in using it, as the associated standard deviations are large which may result in larger confidence bands.

The Neo-Hooken and Mooney-Rivlin 2 parameter models show similar behaviour, both models are unable to match the experimental curve of both materials. They truly do not take notice of the increase in non-linearity in the behaviour of both materials, rather they show linear behaviour.

The Ogden model appears to be sensitive to the inter-sample variations. The stresses of 1st, 2nd and 3rd orders Ogden model generated using the mean value of the parameters are not in agreement with the experimental stress data. In the absence of inter-sample variations, the 1st and 2nd orders of the Ogden model can give a good fit to the experimental data. This shows that Ogden model is not ideal for modelling the behaviour of materials that are prone to manufacturing variability, such as polymers. However, for hyperelastic materials with properties that are not affected by the manufacturing process, the Ogden model can accurately replicate the behaviour.

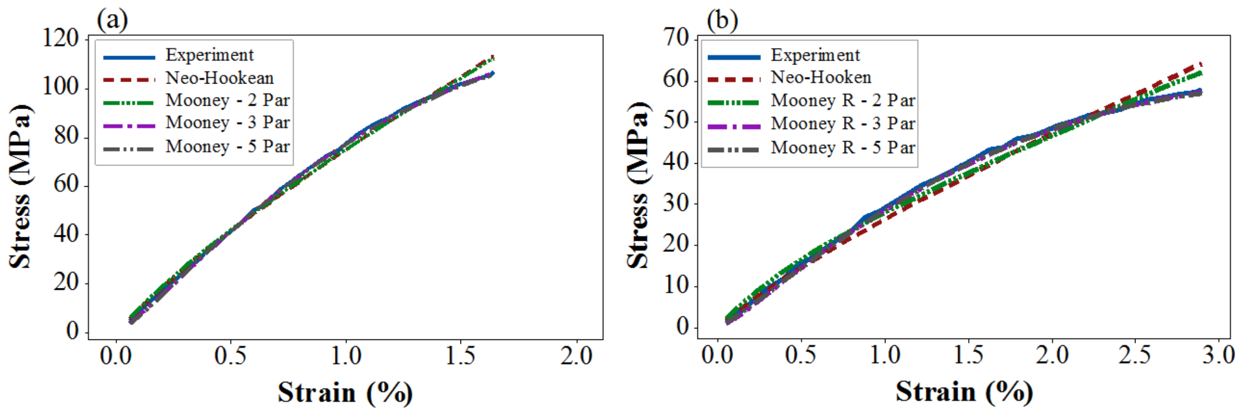

Fig 4: Stress-strain curves - experiment and models (Neo-Hookean / Mooney Rivlin). (a) PBT-GF30 (b) PMMA

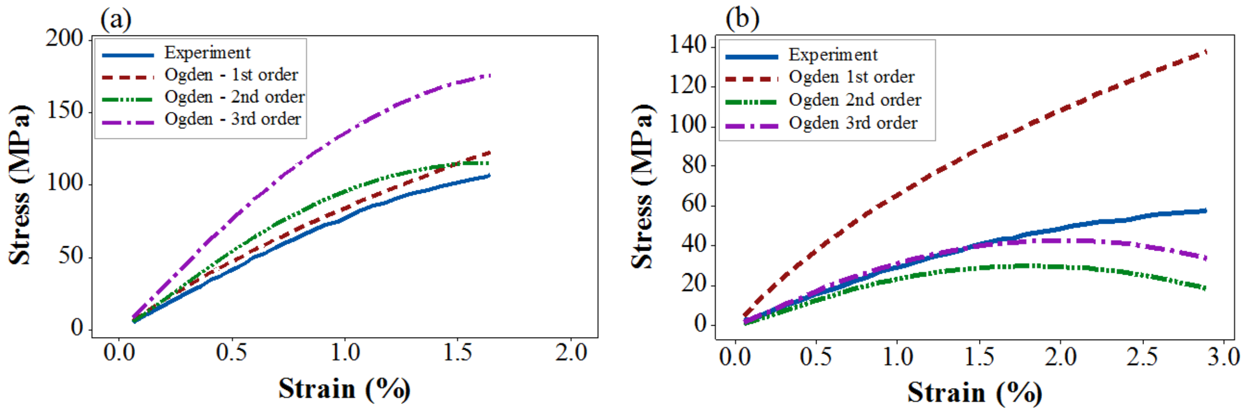

Fig 5: Stress-strain curves - experiment and models (Ogden). (a) PBT-GF30 (b) PMMA

7. Relationship between the model parameters

The relationship between the model parameters is studied. The influence of one parameter against the other is investigated using the Pearson correction coefficient (R) and P-value. The correlation coefficient gives a good understanding of the rate and direction of the relationship, while the P-value explains the significance of the relationship between the parameters. The values -1 and +1 represent perfect negative and positive correlations respectively. A P-value less than or equals to 0.05 is a good indicator of strong relationship between parameters.

Table (3) gives the values of the correlation coefficients and the P-values for all the model parameters for PMMA and PBT-GF30. For the 2-parameter Moony-Rivlin, C_{10} and C_{01} for PMMA are negatively related, but the influence of one to the other is not consistence. For the same model parameter, there is no relationship between C_{10} and C_{01} for PBT-GF30 material. These materials behaviors in 2 parameter model give a good understanding why Mooney-Rivlin 2 parameter stresses for both materials exhibit linear behavior. The model parameter C_{10} represents the linear elastic behavior, while the change in C_{01} means a shift from linear to non-linear behavior [6]. In the 3-parameter Mooney-Rivlin model, all the parameters for PMMA materials have a strong correlation with each other. As the C_{10} is increasing, C_{01} and C_{11} are decreasing at almost the same rate; hence they have strong negative relationship, while C_{11} and C_{01} are positively correlated and one has a strong influence to the other. For PBT-GF30 material again there is a good correction between the parameters, however the relationship between C_{01} and C_{11} is not strong. In the 5-parameter model, there is a linear relationship between C_{11} and C_{20}, only C_{01} and C_{20} show a weak relationship for PMMA material. For PBT-GF30 material there are linear relationships between C_{10} and C_{01} and between C_{11} and C_{20}. All the parameters have a strong relationship between each other for PBT-GF30 material.

Table 2: Model parameters correlation

		PMMA				PBT-GF30				
		C_{10}	C_{01}			C_{10}	C_{01}			
Mooney R - 3 Par	C_{10}	-0.995				-0.895				R
		0.000				0.040				P-value
	C_{11}	-0.982	0.990			-0.994	0.840			
		0.003	0.001			0.001	0.075			
		C_{10}	C_{01}	C_{11}	C_{20}	C_{10}	C_{01}	C_{11}	C_{20}	
Mooney R - 5 Par	C_{01}	-0.992				-1.000				
		0.001				0.000				
	C_{11}	-0.889	0.827			-0.934	0.929			
		0.044	0.084			0.020	0.023			
	C_{20}	0.891	-0.830	-1.000		0.935	-0.930	-1.000		
		0.043	0.082	0.000		0.020	0.022	0.000		
	C_{02}	-0.943	0.895	0.991	-0.991	-0.962	0.958	0.996	-0.997	
		0.016	0.040	0.001	0.001	0.009	0.010	0.000	0.000	
		μ_{r1}				μ_{r1}				
Ogden - 1st Order	α_{r1}	-0.859				-0.971				
		0.062				0.006				
		μ_{r1}	α_{r1}	μ_{r2}		μ_{r1}	α_{r1}	μ_{r2}		
Ogden - 2nd Order	α_{r1}	-0.732				-0.994				
		0.159				0.001				
	μ_{r2}	0.987	-0.612			0.984	-0.969			
		0.002	0.272			0.003	0.006			
	α_{r2}	0.810	-0.255	0.883		0.999	-0.997	0.984		
		0.097	0.679	0.047		0.000	0.000	0.002		

In the 1^{st} order Ogden model, the relationship between μ_{r1} and α_{r1} is less significance for PMMA, however, for PBT-GF30 both parameters have a strong relationship in opposite direction. In the 2^{nd} order of Ogden model, ar1 parameter doesn't have a correlation with other parameters for PMMA, μ_{r1} has a strong positive relationship with μ_{r2} and α_{r2} respectively, and μ_{r2} also shows a fairly strong relationship with α_{r1}. All the parameters for 2^{nd} Ogden model are strongly related for PBT-GF30 material.

8. Conclusions

The statistics of parameters of hyperelastic models specific to PBT-GF30 and PMMA materials used in automotive lamps were determined. The hyperelastic behaviour of both materials, a semi-crystalline and an amorphous, were characterised using appropriate hyperelastic models. The stress-strain curves of the materials were measured under uniaxial tension using a non-contact video gauge. Five samples each were tested to measure the effect of manufacturing variability. The model parameter statistics were determined, the mean value of the models' parameters were used to construct stress-strain curves and then compared to the experimental values. The variations in the model parameters increase with increasing order of parameters. There are relatively small amount of variations in the parameters for Neo-Hookean, 2 and 3 parameters Mooney-Rivlin models. The Mooney-Rivlin 5-parameter model and the three different orders of Ogden model showed large variations. The Neo-Hooken and Mooney-Rivlin 2 parameter models show similar behaviour, both models are unable to reproduce the experimental curves. The Ogden model appears to be sensitive to the inter-sample variations. The stresses of 1st, 2nd and 3rd orders Ogden model generated using the mean value of the parameters are not in agreement with the experimental stress data. The 3-parameter Mooney-Rivlin provided the most accurate prediction of the behaviour of both materials. The model showed excellent stability and is therefore the most appropriate model to represent variations due to manufacturing process. The detailed study of the correlation of the model parameters provided a good understanding of how the parameters are related to each other, enabling construction of complete probability distribution functions for further analysis.

The analysis of this study is based on smaller sample size of five. For more confidence, a larger size needs to be tested and analysed. Further to this there are other polymer components that makeup the LED lamp which also need characterisation.

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